Reflections on Practice

Lesson Plan for Third Year ordinary level, Multiplying linear algebraic terms

For the lesson on 13/3/2015
At Ardscoil na Mara, Enda Donnelly's third year class
Teacher: Enda Donnelly
Lesson plan developed by: John Hartery, Irene Murray, Enda Donnelly

1. Title of the Lesson: How to multiply expressions of the form a(bx+c)

2. Brief description of the lesson: Student led investigation of the distributive law using algebra tiles and a variety of related representations.

3. Aims of the Lesson:

Long term:

I'd like my students to have confidence when performing skills in algebra.
I'd like my students to connect and review the concepts that we have studied already.
I'd like my students to become more creative when devising approaches and methods to solve problems.
I'd like my students to retain material longer and better.
I want to help my students be better prepared to factorise in algebra.

Short term:

I'd like my students to be able to multiply expressions of the form a(bx+c)

4. Learning Outcomes:

As a result of studying this topic, students will be able to:

- multiply terms of the form a(bx+c)
- verbalise the distributive property for expressions of the form a(bx+c)
- use Algebra tiles and drawings on gridded paper to represent expressions of the form a(bx+c)
- expand expressions of the form a(bx+c)
- collect like terms
- If time allows; simplify such expressions as a(bx+c)-d(ex+f).
- If time runs out this will be tackled in the next class.

5. Background and Rationale

This third year group have covered this topic previously (more than once) and have used the array model before. As a pre check for this exercise they were asked (with no warning or revision) to attempt nine expansions; ranging from 2(x+1) up to (2x+1)(x+3). Only one student in the class got all nine correct and many struggled with 2(x+1) and x(x+1).

Expanding brackets and grouping like terms in algebra correctly is a regular problem as it has an effect on some students ability to do questions in many topics e.g. co-ordinate geometry and also their understanding of later topics e.g. factorisation.
We decided to look at other ways to teach and reinforce this topic to help students retain this material.

In the future we would like to explore the links between this topic and the topic of area and volume.

6. Research

Algebra tiles - online, on the board and student tiles
Precheck nine expansions given to class
www.projectmaths.ie Workshop 5 booklet
Lesson study by Vocational College, Wexford Town on www.projectmaths.ie
http://a4a.learnport.org/page/algebra-tiles was used at the meetings to show how versatile algebra tiles can be for many algebraic skills and concepts.
We also looked at some of the growing patterns used in the Project Maths Seminar. We chose to use algebra tiles as some members of the school are using them effectively in class while others have never used them in class. Other models for introducing this topic can be looked at in later years.

7. About the Unit and the Lesson

Algebra tiles will be introduced using physical tiles and a website projected on the whiteboard. We anticipate some problems with the algebra tiles.
After practicing using algebra tiles and drawing array models for some different expressions we will revisit the pre-test questions and look at some of the student errors. The student errors should be interesting for the class and will hopefully lead to some good discussions on what went wrong and opportunities for students themselves to verbalise how to fix the errors and avoid these types of errors in the future.
We hope by using the tiles most students will see that the multiplication is distributed.
Some students have changed maths levels in school and were not in this class when the pre-test was done. All students have covered this topic before.
We will hopefully have made progress during the class and at least temporarily clear up some of the pre-existing misconceptions.

8. Flow of the Unit:

From page 26 of the first year handbook (Lesson idea 1.24)
This material is section 4.6 of the Junior Certificate syllabus (page 30)

Lesson		**# of lesson periods**
	Title: Algebraic Expressions	
1	The use of letters to represent quantities that are variable	1 x 40min.
2	Indices in algebra	2 x 40min.
3	Terms coefficients and expressions	2 x 40 min.
4	How to add terms	2 x 40 min.
5	How to generate algebraic expressions from simple contexts	1 x 40 min.
	How to evaluate expressions	1 x 40 min.
	How to multiply terms and expressions including use of brackets and the distributive law using a model such as the array model	3 x 40 min. **(#1=research lesson)**

9. Flow of the Lesson

Resources needed:
"Can We Fix It" PowerPoint
Student algebra tiles packs
http://a4a.learnport.org/page/algebra-tiles
Student mini-whiteboards and markers (black and red)

Teaching Activity	Points of Consideration
1. Introduction With the pre-test questions projected on the whiteboard it will be explained that we are going to revisit this topic in what we hope will be an interesting and helpful way. We must learn how to use algebra tiles to model expressions. Ask students to represent 2x+3 using tiles and by drawing on the whiteboard. Also 3x-2 and (x+3). Make up one themselves for their partner.	Anticipated problems *"An x is 5 ones"* Would tiles work if an x was the same length as 3.27 singles?
2. Key Task 1 The key question is "What would the following look like if made using algebra tiles? 2(x + 3)"	Additional questions to aid and further the learning: How do you double (x+3)? Use the Algebra tiles to show what (x+3) looks like. Use the Algebra tiles to show what double (x+3) looks like. Describe how you doubled (x+3) (i.e. verbalising the process.) Draw (x+3) on your gridded paper (or mini whiteboard) and draw 2(x+3). Describe what the brackets do. How is 2(x+3) different to 2x+3?
3. Comparing and Discussing Groups share how they were able to do the task. Students may form 2(x+3) in different arrangements.	Which is correct? Which is neatest? Which is easiest to see what you have?
4. Two Consolidation Tasks Students will represent 3(2x-4) and 3x+4+2x-2 using algebra tiles and by drawing array models.	Do students grasp the idea of a "zero pair"?
5. Using Student Errors to Promote Discussion	Anticipated student responses: R1: They forgot to double the 1.

Students discuss some of the errors below in their groups and share their thoughts with the rest of the class. They can build or draw the representations to help explain their thinking. Explain the mistakes that were made in the following: 2(x+1)=2(x)+2(1)=2x+1=3x 2(x+1)=2x,1x 2(x+1)=2x+1 2(x+1)=2x 2(x+1)=2x+3 3(x-2)=3x+6 3(x-2)=3x-2 3(x-2)=-2x×3=-6x 3(x-2)=3x-2 4(2x+3)=4x,0x 4(2x+3)=8x+3 4(2x+3)=6x+12 x(x+1)=2x+1x $x(x+1)=x^2+1$	R2: They only multiplied the x by 2. R3: They should have doubled both parts of the bracket. R4: Each term in the bracket should have been multiplied by the number outside the bracket. R5: You can only add like terms. R6: Plus times minus is minus. R7: Positive by negative is negative. R8: They mixed up adding and multiplying. R9: x+x is 2x and x by x is x^2. R10: x by 1 is x.
6. Summing up For homework the students will draw the algebra tile representations as well as using the traditional written work. 1. 2(x + 4) 2. 3(2x + 1) 3. 2(3x – 2) 4. 4(2x + 3 - 1)	Homework will have differentiated levels of questions.

10. Evaluation

Prior to the lesson three students will be videoed answering the following questions about algebra:
(i) How confident are you about doing maths?
(ii) What do you think when you hear the word algebra?
(iii) Have you ever used colours or shapes to learn algebra?
(iv) Why do we use brackets in maths?
During the lesson two observers will observe two pairs of students each. A third observer will take photos of students work and of the work on the classroom whiteboard and whatever is

displayed by the data projector. This third observer will also observe two pairs when not taking photos.
We have quite a detailed set of "Pair/Group Observation Record" sheets for the observers to fill in. These sheets focus on the main concepts in the lesson and there is also three questions to ask students about their understanding and confidence with the tiles/array method and the distributive property.
Immediately after the lesson the same three students as before will be videoed answering the following questions about algebra:
(i) How confident are you about doing maths?
(ii) What do you think when you hear the word algebra?
(iii) Now that you have used colours and shapes to learn algebra; is there any difference?
(iv) Why do we use brackets in maths?

11. Board Plan

The "Can We Fix It" PowerPoint contains
(i) all the questions from the pre-test (see Appendix),
(ii) images of the various algebra tiles (front and back i.e. positive and negative),
(iii) questions and solutions to the following tasks: 2x+3, 3x-2, (x+3), 2(x+3), 3(2x-4), 3x+4+2x-2,
(iv) images of various student errors from the pre-test which are in the Flow of the Lesson above and
(v) homework questions

12. Post-lesson reflection

- ➔ Students were more positive towards algebra and more confident in their abilities to deal with problems. The use of colours and shapes, peer corrections and group discussion was helpful.
- ➔ Having alternatives to representing algebraic expressions as symbols by using the tiles or a sketch might have made this group of students a bit more independent. Some groups self-corrected their work.
- ➔ Students could recognise errors in their work, the tiles and their modelling of problems helped them to visualise what was abstract for some students up to today. In one of the observed groups they used the tiles incorrectly at first for a few questions but then they went on to
  (i) self-correct (the question was (x+3) and they had x^2+3), or
  (ii) correct their work after listening to a student verbalising the process "double everything" (the question was 2(x+3) and they had 2x+3) or
  (iii) correct their work after the teacher asked them a question about their work (the question was 3(2x-4) and they had 6x+12).

- ➔ The idea of multiplying everything inside the bracket was lost on some students before the lesson. They had a vague of idea of the method but had not seen the purpose/reason before. Brackets and the distributive law was of concern to them, a variety of answers were possible. Using the algebra tiles students had various approaches but the modelling of problems resulted in the correct answer.
- ➔ Students very quickly understood what the different tiles represented. They could identify and model problem expressions e g. 3x+4 very quickly. Doubling using brackets caused some errors which were sorted with peer or teacher intervention. Their ability to model expressions developed very quickly. The students I observed said that on a scale of 1 to 10 their understanding of expressions now would be rated 8, ability to work with brackets rated 8 and their confidence of arriving at the correct answer rated at 9 out of 10.
- ➔ The final question students did before looking at student errors was 3x+4+2x-2. Many students represented this correctly but didn't go on to simplify it as 5x+2. Some time was given to discuss this but more time would be needed for students to get comfortable with it.
- ➔ The verbalisation of some students' errors was clear. 2(x+1)=2x "forgot 2 by 1", 2(x+1)=2x+1 "just took away brackets", 2(x+1)=2x+3 "did 2x put plussed the 2 and the 1".
- ➔ The lesson needed more time. Very quickly the teacher moved to the pace of understanding of the class.
- ➔ It would have been better if the slides had contained images of the various algebra tiles (front and back i.e. positive and negative) or if we had a poster with these permanently on the wall.
- ➔ The benefit of visualising algebra through shape and colour cannot be underestimated. Necessary to introduce this idea of tiles 1st year and 2nd year and then move to array model with tiles until students are confident to see the colours and shapes in their mind.
- ➔ Similar lessons had been done with other class groups using just the tiles. The combination of the tiles and the whiteboards brought a lot of extra value.
  - Potentially embarrassing errors can be fixed on the boards
  - Different pictorial representations can be used by students
  - Ideas can be shared and boards can be held up: peer teaching
  - Ultimately the students need to be able to show written work for homework and in exams.
- ➔ One pair worked on the problem 2(x+1) and had 2x+2 in the tiles and 2x in their symbolic representation. Hopefully a little more work with the tile, sketch and symbol representations will mean this pair can be proficient in all of the representations.

13. Appendix

Appendix A Pre-check questions

1. $2(x+1)$

2. $3(x – 2)$

3. $4(2x + 3)$

4. $x(x+1)$

5. $x(x – 2)$

6. $x(2x + 3)$

7. $(x+1)(x+2)$

8. $(x-2)(x+3)$

9. $(2x+1)(x+3)$

Pair/Group Observation Record

BEGINNING OF LESSON:

Observe level of difficulty. If no difficulty tick the box for each Pair/Group. If Pair/Group has difficulty please identify issues.

	Pair/Group 1	Pair/Group 2	Pair/Group 3
(i) Terms in algebra and their tiles			
(ii) Representing x and representing units			
(iii) Representing expression using pre drawn model			
(iv) Multiplying or combining terms using pre drawn model			
(v) Multiplying with brackets using pre drawn model			
Questions asked by Pair/Groups:			

Pair/Group Observation Record

DURING LESSON:

Observe Pair/Group interaction. If no difficulty tick the box for each Pair/Group. If Pair/Group has difficulty please identify issues.

	Pair/Group 1	Pair/Group 2	Pair/Group 3	
(i) Questions asked to teacher				
(ii) Questions asked to other group members				
(iii) Identify when students understand/ verbalise distributive property for class				
(iv) Students set and test each other to reinforce learning. Observe methods used				
Other observations				

Pair/Group Observation Record

LESSON CONCLUSION:
Observe Pair/Group interaction. Check Pair/Group level of understanding and confidence with tiles/array method and the distributive property.

	Pair/Group 1	Pair/Group 2	Pair/Group 3	
Students rate understanding of tiles /array method, **Scale 1-10 where :** **1= poor**				
Students rate understanding of Distributive Property, **Scale 1-10 where :** **1= poor**				
Rate their confidence with this method of arriving at answer. **Scale 1-10 where :** **1= poor**				
Other observations				