Reflections on Practice

Project Maths
Tionscadal Mata
Development Team

Lesson Plan for [*Second Year, Proportional reasoning*]

For the lesson on [25/03/2015]
At [St. Mary's Academy, Carlow], [Mr. O'Leary's] class
Teacher: [Mr. O'Leary]
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1. **Title of the Lesson: Ratio and proportion - giving students an intuitive understanding through problem-solving**

2. **Brief description of the lesson:** We want students to understand problems which involve proportional reasoning. We want students to learn methods for solving such problems themselves. As part of this process we want students to recognize the concept of calculating a unit quantity and how this understanding can be applied to many different types of problems including percentages. The lesson is <u>one hour</u> in duration.

3. **Aims of the Lesson:**

Long-range/thematic goals:
I'd like my students to develop a positive disposition towards mathematics.
I'd like my students to learn to become effective problem solvers.
I'd like my students to be more independent learners.
I'd like my students to work effectively in groups and to be comfortable explaining their ideas to other students.
I'd like my students to recognize that they can do a lot of mathematics themselves.
I'd like my students to understand the importance of their thinking process as opposed to being really focused on an answer.
I'd like my students to enjoy their mathematics.
I'd like my students to develop an attitude of perseverance towards their mathematics.

Short-term goals
I'd like my students to apply proportional reasoning.
I'd like my students to recognise that there are different ways in which to tackle problems based on proportional reasoning.
I'd like my students to recognise the importance of unit quantity in solving proportional problems.
I'd like my students to understand how the operations of multiplication and division fit into proportional reasoning.
I'd like my students to recognise problems involving percentages greater than 100%.

4. Learning Outcomes

As a result of studying this topic students will be able to:

- Understand the concept of proportional reasoning
- Solve problems when given the cost of one quantity and asked for the cost of a different quantity.
- Relate the process of division to finding the cost of a unit.
- Relate the operation of multiplication to finding the cost of several units.
- Select suitable strategies for solving problems relating to proportional reasoning.
- Apply proportional reasoning to problems based around percentages.

4. **Background and Rationale**

Many students experience difficulty in understanding the concepts which underlie problems associated with proportional reasoning. Proportional reasoning is a core skill in mathematics and is useful in all branches of mathematics. At second level students encounter proportional reasoning all the time, in fractions, in percentages, in speed, in area and volume and many more situations.

Here is a simple example of a proportional-reasoning problem: *The price of a ticket is €88 which includes a 10% booking fee. What is the cost of the ticket alone?*
Students often solve these problems by following routine, without much understanding of why they're doing what they're doing. Accordingly when they encounter such problems in different situations they have limited ability to deal with them. We think that by presenting proportional-reasoning problems in a simple but specific way we can let students figure out how solve such problems and that by making some important connections we can show students that this type of reasoning is important in lots of different contexts. The hope is that by doing so students will understand and retain the skill for longer, they will see that they can do maths themselves and they will be exposed to the idea that a single approach can be used in many different situations.

5. Research

The Junior Cert. maths syllabus makes specific reference to problems based on ratio and proportionality:

3.3 Applied arithmetic	Solving problems involving, e.g., mobile phone tariffs, currency transactions, shopping, VAT and meter readings. Making value for money calculations and judgments. Using ratio and proportionality.	– solve problems that involve finding profit or loss, % profit or loss (on the cost price), discount, % discount, selling price, compound interest for not more than 3 years, income tax (standard rate only), net pay (including other deductions of specified amounts) **– solve problems that involve cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price) compound interest, income tax and net pay (including other deductions)**

Having examined a number of textbooks we feel that no single text deals with this syllabus learning outcome in a way that promotes deep understanding of the concepts.

6. About the Unit and the Lesson

The lesson will help students recognise problems based on proportional reasoning and solve such problems.
Students will be introduced to the lesson using a simple pricing problem and encouraged to estimate an answer to the problem. By the end of the lesson students should recognise that they can now do more than estimate the answer but that they understand how to calculate the exact value.
Students will start work on two simple matching activities. The first activity involves cards with certain masses and prices. Using proportional reasoning students should be able to find the missing price on the different cards. The cards are made to different sizes to give students a physical feel for the proportional nature of the relationships involved. The matching tasks in the first activity are designed to expose students to more difficult examples of proportional relationships so that they may gain a deeper understanding of the associated concepts and so that they are challenged. The aim of the first matching activity is to get students describing different ways to approach proportional problems and to then apply the number operations of division and multiplication to solve the problems.
The second matching activity is again based around pairs of cards. This time the relationships are expressed not in grams but in percentages. Under the guidance of the teacher students will see that these are just another example of a proportional problem. Under this guidance it is hoped that students will apply their newly-acquired knowledge to solve these problems.
Finally the class will revisit the introductory problem and students will be given the opportunity to recognise this as yet another proportional problem and will then get the chance to use their understanding to solve the problem.

7. Flow of the Unit:

Lesson		# of lesson periods
	Ratio and proportion	

1	• Distinguish between absolute comparison and relative comparison • See ratios as comparing part to part and fractions as comparing part to whole, where the quantities being compared have the same units. • See rates as the ratio of two quantities having different units. • Appreciate the importance of order when dealing with ratios • Find equivalent ratios • Divide a number into a given ratio • Recognise a proportion as a statement of equivalent ratios 5:2 = 10:4 or set up a proportion to find x as in 5:2 = 8: x	3 x 1 hour
2	• Distinguish between proportional and non-proportional situations recognising the multiplicative relationship that exists between the quantities in proportional situations as seen in tables, graphs and algebraic expressions	1 x 1 hour
3	• Use a variety of techniques including the unitary method, factor of scale and tables, to solve proportional tasks and to recognise that these techniques are all related • Solve problems involving proportional reasoning in different contexts • How to draw and interpret scaled diagrams	3 x 1 hour The first of which is the research lesson.

8. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction Students are presented with the following problem: You are given a ticket to a concert. The cost of the ticket includes a 10% booking fee. If the ticket costs €88, how much does the ticket cost? Students are asked to estimate an answer.	We want students to be engaged with the lesson. We want students to make an estimate of the ticket price. Do students understand that the ticket price must be less than €88? Are students' guesses reasonable? Do students have a rough idea of the value of 10% in this context?
2. Posing the Task Students are presented with a simple matching activity which consists of a number of cards of different sizes with associated masses (in grams) and prices. Students are asked to determine the	The teacher hands out the matching activity and the accompanying worksheet. Note: The activity has 10 parts. The teacher should choose which parts are suitable for their

cost of different masses.	students. The teacher circulates the room to check that students are on task. Are students able to use proportional reasoning to calculate the missing prices? Can students explain their approach to calculating the missing quantities? Do students recognize the importance of division for finding the value of a sub-multiple? Do students recognize the importance of multiplication for finding value of a multiple? Can students verbalise the application of division and multiplication to solving proportional-reasoning problems?
3. Anticipated Student Responses Students may not be able to use proportional reasoning. Students may be able to answer the questions but find it difficult to explain their thinking. Students may find the first number of problems (which involve division only) easy to deal with but find the latter problems more difficult (which involve division, followed my multiplication). Students may find the cost of different sub-multiples and use this to work out the new multiple. Students may find the cost of one unit and use this to work out the new multiple.	 It is important that students are encouraged to explain their approach. It is important that students don't simply talk about multiplication and division, rather they talk about finding the cost of a smaller quantity and using this to find the cost of a larger quantity. It is important that students make the connection between division and multiplication and the processes involved in proportional reasoning. The teacher asks students to explain why one problem is more difficult that another?
4. Comparing and Discussing The teacher represents problem A1 on the board with a diagram. The teacher asks students to explain their approach to solving the problem. The teacher represents problem E5 on the board with a diagram. The teacher asks students to explain their approach to solving this problem. The teacher represents problem I9 on the board with a diagram. The teacher asks students what's hard about this problem compared to the previous problems? The teacher asks students to communicate their ideas on how to approach this problem. The teacher summarises the approach of finding the cost of one unit and then finding the cost of X units, being careful to explain why this approach is needed for this example. The teacher identifies this method as the unitary method. The teacher explains to the students that what they are doing is known as proportional reasoning and explains what this means.	 We want students to focus on the reasoning behind solving the problem. We want students to explain their approach in their own terms (as opposed to focusing solely on the operations of multiplication and division). We want students to describe how division and multiplication fit into the process of dealing with proportional reasoning problems. We want students to recognize the relevance / importance of finding the cost of one unit. We want students to identify situations where it makes sense to find the cost of one unit. The teacher may ask students if the unitary method would have worked for the first two problems presented on the board. Do students understand what proportional reasoning means?

5. Posing the Task The teacher asks students if they think that this type of problem and approach to solving it only apply to scenarios involving grams and euro. The teacher presents students with their second matching activity. This one is based on percentages. The teacher asks students to look at the first pair of cards A1 and then asks them what they think of this problem. The teacher explains that this is simply another example of proportional reasoning. The teacher draws the first pair A1 on the board and asks students to make a statement about what we know. The teacher asks students to make a statement about what we are trying to find out. The teacher asks students to discuss what we could calculate which might be useful. The teacher writes details of students' approaches on the board. The teacher asks students to make a statement about how to solve the problem. The teacher asks students to attempt a selection of additional problems on the worksheet in their groups.	Can students identify other situations where proportional reasoning may apply? The teacher distributes copies of the second worksheet. Wait time is important here. Students need a minute to match the cards and think about the problem. Can students make statements of the form "We know that 150% is worth €300"? Can students make statements of the form "We want to know how much 100% is worth"? Can students identify useful sub-multiples of 150%? Can students suggest suitable strategies for tackling this problem? Can students make statements of the form "50% must be worth €100 so 100% must be worth €200"?
6. Anticipated student responses Students mightn't see the link between this activity and the previous one. Students might immediately turn off because of a dislike/fear of percentages. Students might have difficulty verbalising what they know about the problem and what they are being asked to find out. Students may give several different approaches. For example they might find what 50% is worth and then find what 100% is worth. They might find what 10% is worth and then find what 100% is worth. Students might find the pairs at the start of the worksheet straight-forward but may find the others quite difficult. Students may have difficulty communicating their approach to solving each problem. Students may try to divide and multiply in an ad hoc manner without trying to understand what they are doing.	It is important that students are given the chance to recognize the link between this activity and the previous one. This should be done by getting students to verbalise what they know, what they want to find out and how they might go about doing this. If different approaches are presented it is important to emphasise the merits of each approach and show that they are equivalent.
7. Comparing and discussing The teacher sketches pair C3 on the board, and asks different students to explain what they know, what they want to find out and how they might go about doing this.	Can students verbalise their ideas? Can students explain why Y102 is a more-difficult problem?

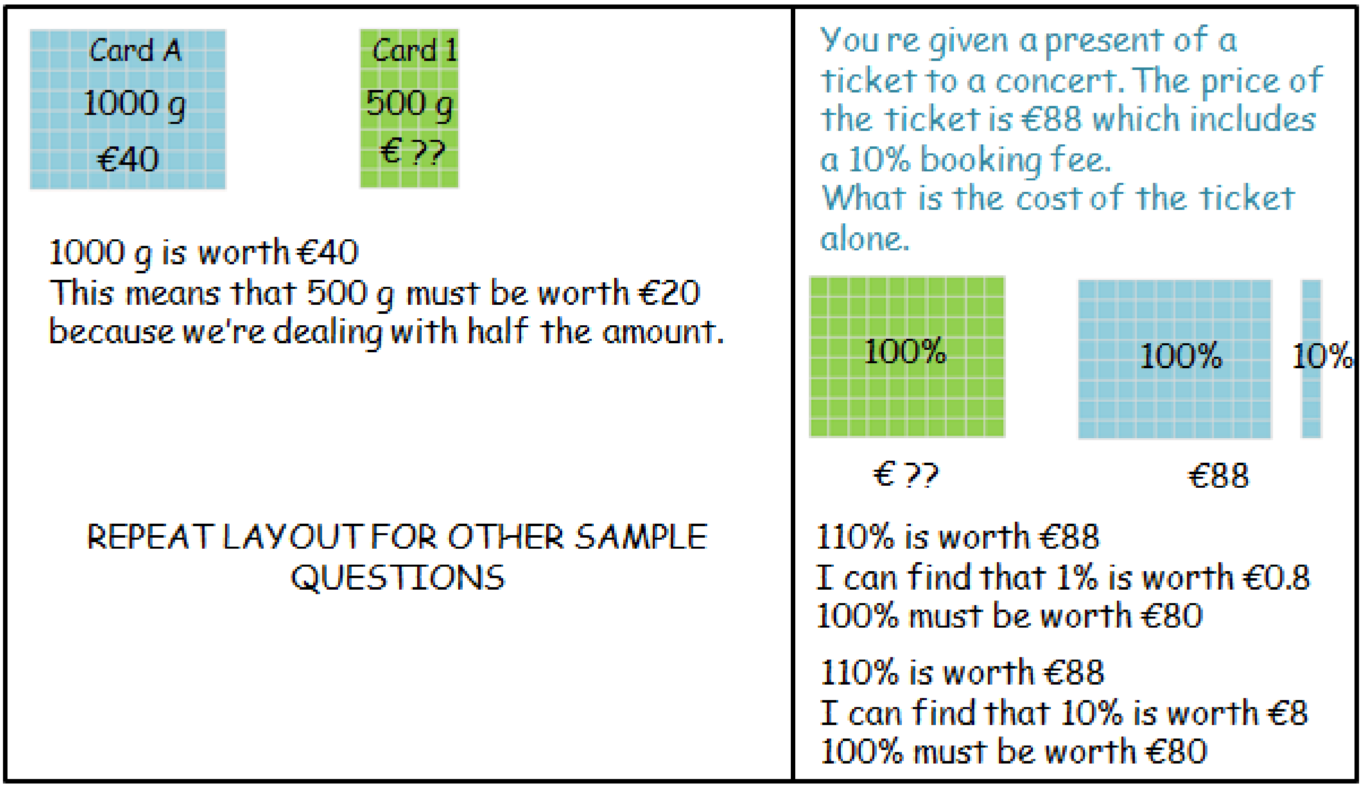
The teacher emphasises the fact that this is a proportional problem. The teacher sketches pair Y102 on the board and asks students why this is more difficult than the previous example. The teacher asks students to suggest how they could solve this problem. The teacher reinforces the idea of the unitary method and what it means and where it's useful.	Do students understand the unitary method and where it is useful?
8. Posing the task The teacher revisits the introductory task and asks students if they have any suggestions on how they might solve this problem. The teacher represents the cost of the ticket alone as a square on the board and identifies this square as representing 100%. The teacher identifies the price of this square as what we are looking for. The teacher draws the same square again and this time adds in an extra piece. The teacher asks students to identify what percentage each piece represents. The teacher writes in 100% on the square and 10% on the extra piece. The teacher asks how much this 100% costs and marks in €88 on the diagram. The teacher asks students to calculate the cost of the ticket alone.	It is important to draw relatively proportional shapes. The teacher may identify the 100% square as the price we are looking for.
9. Anticipated student responses Students may find it hard to understand that the final cost of the ticket is 100% of the actual cost. Some students may work out what 10% costs and then scale this up to 100%. Some students may require support to apply the unitary method to solving the problem.	Different approaches are to be commended as they show that students are thinking for themselves. The unitary method does not have to be used here. For some students it may make more sense to find the cost of 10% and use this to calculate the cost of 100%.
10. Comparing and discussing The teacher asks students to write their answers on their show-me boards and to hold them up. The teacher asks students to explain how they tackled the problem. The teacher asks students to compare their answer to their guess.	The teacher should try to get students to present different approaches. It is good for students to reflect on their answer and how it compares to their estimate from the start of the lesson.
5. Summing up The teacher asks students to explain what they have learned today? The teacher recaps on the concept of proportional reasoning. The teacher stresses that there are many ways to	

solve proportional problems but that the unitary method will work in any situation. Students are asked to make up two proportional problems for their homework.	

9. Evaluation

There will be three observers in the lesson as well as the teacher teaching the lesson.
Observer 1 will take pictures of student work.
Observer 2 will record student behaviour using an assessment wheel. This will include looking for evidence of students communicating with each other effectively, students understanding the main concepts of the lesson, students asking questions of each other and the teacher and the level of motivation.
Observer 3 will record details of the teacher-student interaction.

10. Board Plan



11. Post-lesson reflection

The lesson was very much a success with some areas where improvement could be made.

What worked well in the lesson:

- Students were motivated to estimate an answer to the introductory problem.
- Most student estimates were reasonable in that they were less than the total cost of the ticket.
- Most students worked well together in groups, cooperated with each other, helped each other and discussed their maths.
- Most students were motivated to try to solve problems.
- Most students were willing to offer suggestions on how to solve a problem or to describe how they solved the problem themselves.

- Some students suggested alternative approaches to solving a problem than the one suggested by other students.
- Students used their show-me boards when asked to display an answer to a problem.
- Students used an app on their ipads to help them divide and multiply quantities.
- Most students were interested in learning and were continuously engaging with the teacher to check their work and to ask for help.
- In the comparing and discussing parts of the lesson, the teacher made sure to involve all students in feedback and gave different students time to describe their approach to solving the problem. Where students focused on answers only, the teacher encouraged them to describe their approach.
- Some students demonstrated excellent ability in proportional reasoning. For example when tackling the problem of finding the cost of 100 g given that 150 g cost €30 one student explained “Because 150 g is worth €30, this means that 50 g must be worth €10 and 100 g must be worth €20”. With some encouragement this student explained his thinking to the rest of his group.
- The teacher encouraged students to describe the overall approach they were using from one question to the next. This helped students solidify what they were learning.
- Some students recognized the importance of division and multiplication to the solving of proportional problems.
- In one of the comparing and discussing sessions students were asked if they thought this approach to solving proportional problems would only work for mass and price. Students identified lots of other areas where this method would be useful, including some areas in other subjects they study.

What we would change in the lesson:

- We tried to do too much with the group of students this was tried with. We felt that spending more time on the activity relating mass and price may have helped more students develop a deeper understanding of proportional reasoning. The scope of the lesson very much depends on the students who sit in front of you. For some students we would recommend reducing the scope of the lesson while for better students we would try to cover all the content presented in the lesson plan.
- At the start of the lesson students really focused on their answers as opposed to their approach. It is important that students understand that a description of their approach is the most important part of the activity.
- Some students seemed intimidated by the extra adults in the room (including a film crew). This prevented them from engaging properly with the lesson. This shouldn’t be an issue with future lessons.
- We could have spent a little more time at the start explaining the first activity. Students didn’t realise that the cards were specifically created with relative sizes to help them understand the proportional nature of the problem. By taking the first pair in the first activity and asking students if they thought the missing price would be more or less than the one given, this idea could have been teased out.
- Students tended to focus on the number operations which provided them with solutions. For example when asked to describe what they did some students replied “I divided by… and multiplied by…”. We really want students to describe the process in everyday terms. For example statements of the form “I found the price of X grams and then I used this to find the price of 3X grams” would show a deeper level of understanding.
- Some students tended to look for an easy solution as opposed to thinking the problem through logically. For example, one student who obtained a correct answer to a problem explained his method as “knocking a zero off both of them”. This highlights the danger of students attempting to move straight to some form of number operation without understanding what they are doing.

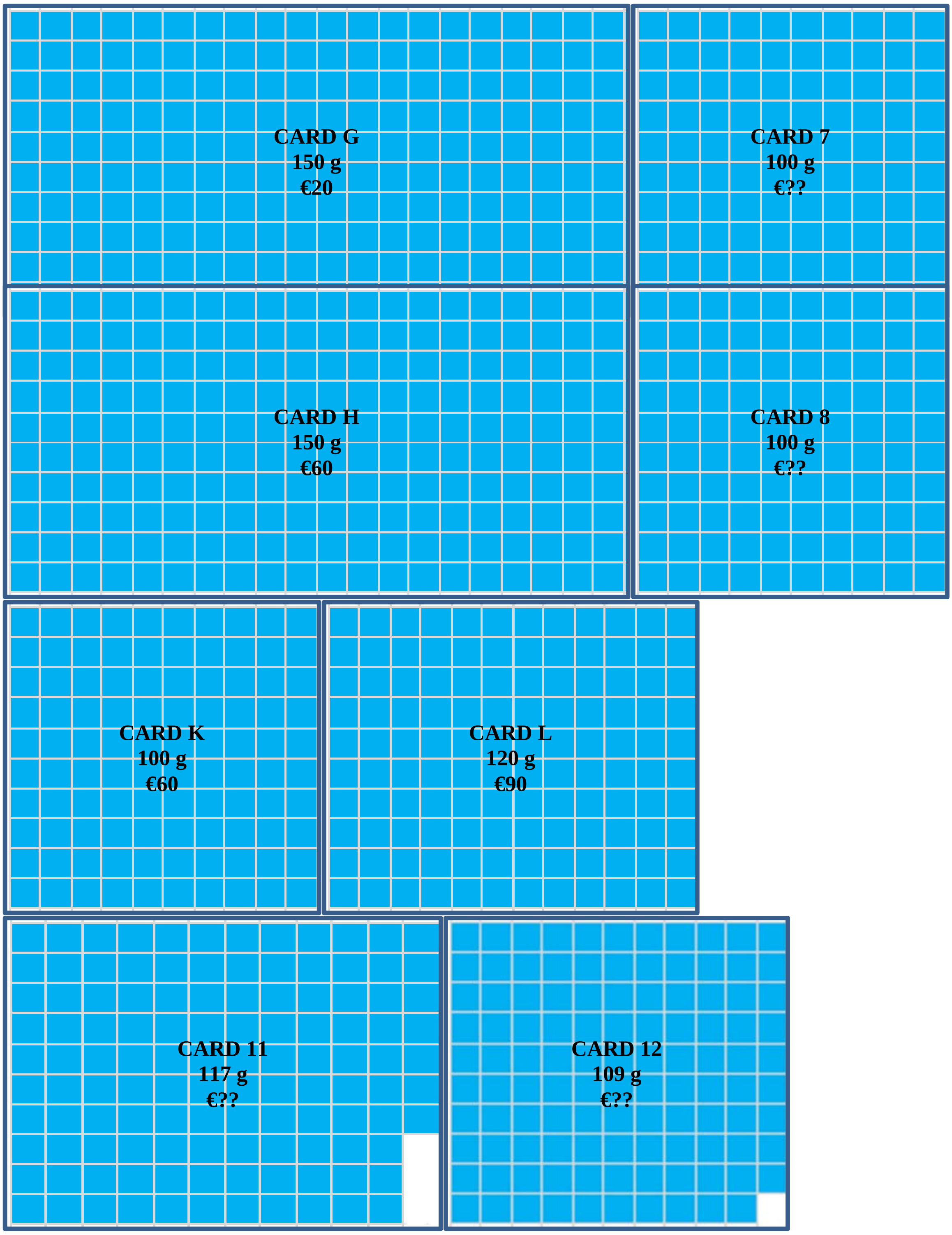
- Students took a long time sorting the pairs of cards. This reduced valuable thinking time. Perhaps there is a better way to present the activities. Alternatives include presenting the pairs of cards on a sheet of paper. On the flip side it should be said that students seemed to enjoy matching the cards and this seemed to get them more involved in the activity.
- Students didn't naturally write down an explanation of how they solved each problem. It is important to constantly remind them of the importance of doing so.
- Students may need extra support when moving from the mass-price activity to the percentages-price activity to let them recognize that the unitary method may also be used here. We found that as soon as students saw percentages, many of them considered the problems much too hard whereas in reality they were practically identical to the first activity.
- In the comparing and discussing session it is important that students are encouraged to explain why they used the approach they used. This explanation should not immediately focus on division and multiplication.

Summary:

We found the lesson to be successful but with areas where it may be improved. We designed a fairly general lesson in proportional reasoning but presented it to a group of students who often struggle with their maths. It is important when using a lesson plan to adapt it for the students who sit in front of you in a given lesson.

While the students in the class had difficulties, group work worked very well. This showed to us that group work isn't just something for the best students in a year group. It can work well for all students but only if the work is appropriate to those students' level.

It is important that students have some everyday understanding of what they are doing before applying numerical operations to solve a problem. If students focus on procedures and operations too soon they are likely to struggle with retaining what they have learned and when similar problems are presented in a different context they are less likely to be able to deal with them.


CARD G
150 g
€20
CARD 7
100 g
€??
CARD H
150 g
€60
CARD 8
100 g
€??
CARD K
100 g
€60
CARD L
120 g
€90
CARD 11
117 g
€??
CARD 12
109 g
€??

CARD A
1000 g
€40

CARD B
1000 g
€20

CARD C
1000 g
€60

CARD 1
500 g
€??

CARD D
1000 g
€5

CARD E
1000 g
€80

CARD F
1000 g
€25

CARD 2
500 g
€??

CARD 3
250 g
€??

CARD 4
250 g
€??

CARD 5
300 g
€??

CARD 6
300 g
€??

CARD I
250 g
€80

CARD 9
100 g
€??

CARD J
250 g
€25

CARD 10
100 g
€??

CARD 7
100 %
€??

CARD 8
100 %
€??

CARD 9
100 %
€??

CARD G
167 %
€500

CARD H
113 %
€800

CARD I
115 %
€800

CARD J
130 %
€250

CARD 10
100 %
€??

CARD K
200 %
€45

CARD 11
100 %
€??

CARD L
200 %
€70

CARD 12
100 %
€??

CARD A
150 %
€300

CARD 1
100 %
€??

CARD 5
100 %
€??

CARD B
150 %
€600

CARD 2
100 %
€??

CARD 6
100 %
€??

CARD C
250 %
€800

CARD 3
100 %
€??

CARD D
250 %
€250

CARD 4
100 %
€??

CARD E
120 %
€600

CARD F
120 %
€900

Percentages Worksheet:

Find the cost of the quantity given on the smaller card

Card Letter	Card Number
A	1

What is the question asking?

Answer:

Explanation:

Card Letter	Card Number
B	2

What is the question asking?

Answer:

Explanation:

Card Letter	Card Number
C	3

What is the question asking?

Answer:

Explanation:

Card Letter	Card Number
D	4

What is the question asking?

Answer:

Explanation:

Card Letter	Card Number
E	5

What is the question asking?

Answer:

Explanation:

Card Letter	Card Number
F	6

What is the question asking?

Answer:

Explanation:

Card Letter	Card Number
G	7

What is the question asking?

Answer:

Explanation:

Card Letter	Card Number
H	8

What is the question asking?

Answer:

Explanation:

Card Letter	Card Number
I	9

What is the question asking?

Answer:

Explanation:

Card Letter	Card Number
J	10

What is the question asking?

Answer:

Explanation:

Card Letter	Card Number
K	11

What is the question asking?

Answer:

Explanation:

Card Letter	Card Number
L	12

What is the question asking?

Answer:

Explanation:

Grams Worksheet:

Find the cost of the quantity given on the smaller card

Card Letter	Card Number
A	1

What is the question asking?
Answer:
Explanation:

Card Letter	Card Number
B	2

What is the question asking?
Answer:
Explanation:

Card Letter	Card Number
C	3

What is the question asking?
Answer:
Explanation:

Card Letter	Card Number
D	4

What is the question asking?
Answer:
Explanation:

Card Letter	Card Number
E	5

What is the question asking?
Answer:

Explanation:

Card Letter	Card Number
F	6

What is the question asking?
Answer:
Explanation:

Card Letter	Card Number
G	7

What is the question asking?
Answer:
Explanation:

Card Letter	Card Number
H	8

What is the question asking?
Answer:
Explanation:

Card Letter	Card Number
I	9

What is the question asking?
Answer:
Explanation:

Card Letter	Card Number
J	10

What is the question asking?
Answer:
Explanation: