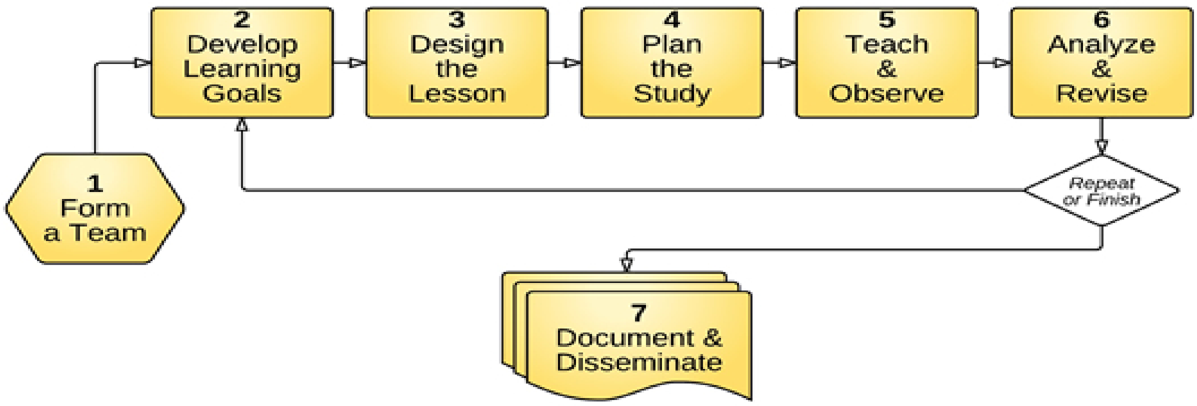
Reflections on Practice



Project Maths Tionscadal Mata Development Team

Lesson Plan for [Junior Cert, Strand 4 Algebra section 4.2 and 4.5]

Lesson : 10th March 2015
St. Patrick's, Lacken Cross, Second Year
Teacher: Michael Walsh
Lesson plan developed by: Michael Walsh, Sinead O'Malley, Pauline Kilcullen

1. **Title of the Lesson:** Walking the talk: Generating and interpreting graphs using a motion sensor.

2. **Brief description of the lesson:** Students get involved in creating graphs (and tables) which show the relationship between displacement (distance from a point in a particular direction) and time, for a moving object, using a Go!motion detector to detect the position of the object. They discuss the patterns and relationships revealed by these graphs. This allows them to make sense of other linear graphs.

3. **Aims of the Lesson:**
I'd like to build my students' enthusiasm for mathematics by engaging them with stimulating activities
I'd like my students to see the mathematics they are studying as being practical

   I'd like my students to associate mathematics with sense-making
I'd like my students to discuss the mathematics they are studying and to learn from each other's viewpoints.
I'd like my students to connect and review the concepts that they have studied already and to see the relationship of these concepts to new topics and to other subjects.

4. **Learning Outcomes:**

   As a result of studying this topic, students will be able to:
   - Understand that a displacement time graph representing a stationary object is represented by a horizontal line, whose y-intercept is the distance from a point (in this case the sensor)
   - Understand from practical experience that moving at a constant speed is represented on a displacement-time graph by a line with a constant non-zero slope
   - Understand that all graphs representing constant speed generated with the detector we use, will be linear graphs **not** passing through the origin.
   - Understand that it is not possible to generate a linear graph passing through the origin with this equipment, as distance from the sensor would have to be zero for this to happen
   - Identify the variables and constants in the situation and understand the difference between them
   - Understand the difference between independent and dependent variables

- Explore graphs of motion
- Make sense of quantitative graphs and draw conclusions from them
- Make connections between the shape of a graph and the story of a phenomenon
- Describe both quantity and change of quantity on a graph
- Use graphs to represent phenomena quantitatively
- Understand that we are limiting ourselves here to a situation with just two variables but that in many contexts there are more than two variables

5. Background and Rationale

- Students are required to interpret linear, quadratic and exponential functions in graphical form. (Our focus here will be on linear graphs.)
- Students are required to use tables, diagrams and graphs as tools for representing and analyzing linear, quadratic and exponential patterns and relations.

- Students in the past have had difficulties with graphs; recognizing the connection between graphs and their algebraic representation and in understanding the difference between different types of graphs.
  When students actually generate graphs themselves they gain an understanding of the variables and constants involved in a practical way. They can control the constant starting value and the speed and note the change in the graph when these starting value and speed are changed. They can identify which of the variables is dependent on the other one.
- Students also have had problems realising the relevance and use of graphs, typically in science, to understand relationships. I want them to see that having generated the graph they can read information from it such as distance from the sensor at the start, speed of walking, direction of walking, whether or not speed is constant etc.
- Students need to understand that linear graphs not passing through the origin do not represent proportional relationships
- We propose to facilitate this learning by using a Go!motion detector to create graphs. We propose to use the data collected and the graph produced from the data to investigate the relationship between the variables position (in metres) and time (in seconds).
- It is envisaged that students would be able to predict the graphical representation given the story associated with a situation of co-variation and form algebraic expressions for the relationship between the variables. This is also an important skill for science class.

6. About the Unit and the Lesson

Unit

Representing situations with tables, diagrams and graphs (4.2 Junior Certificate Syllabus)
See also 4.5 below from the Junior Certificate Syllabus:

4.5 Relations without formulae	Using graphs to represent phenomena quantitatively.	– explore graphs of motion – make sense of quantitative graphs and draw conclusions from them – make connections between the shape of a graph and the story of a phenomenon – describe both quantity and change of quantity on a graph

The unit will consist of introducing students to the graphs generated by data they created themselves from their own motion in front of the motion detector. The students will understand in a practical way how linear graphs are generated. Later they will study the relationship between consecutive unit numeric outputs created by the device and formulate the conditions for a linear relationship.

Lesson

Creating linear graphs and understanding:

- The connection between the shape of the graph and the story of a phenomenon
- The relationship between the slope of the graph and the rate of change of position with respect to time i.e. speed in a given direction

Use the Go!motion detector to collect data on the distance a student is from the detector at any time. The aim here is to get students to try to create a linear graph. Students will predict the appearance of a graph from a verbal statement of how position varies with time. Students will then draw a graph based on this verbal statement. Students then compare their predicted graph with the graph produced by the motion detector using the conditions given in the verbal statement. These conditions can be varied. Finally, how well students have understood the task is assessed by asking them to walk in front of the motion detector in such a way that their motion produces a graph which matches a pre-drawn linear graph. They compare the two graphs (and tables given time) and reason about the differences and similarities.

7. Flow of the Unit:

Shows how the research lesson fits into a larger unit. Briefly describes what students learn before and after the research lesson.

Lesson		# of lesson periods
	Title Applied measure	
1	• Revise first year coordinate geometry	1 x 30min.
2	• Midpoint of a line segment	1 x 30min.
3	• Slope of a line	1 x 30 min.
4	• Using graphs to represent phenomena quantitatively	1 x 30 min. **(#1 = research lesson)**
5	• Discuss rates of change and the y-intercept	1 x 30 min.
	• Generalise and explain patterns and relationships in words and numbers.	1 x 30 min.

8. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction **Introduction:** Students are shown the motion detector, connected to the laptop. It is explained to them that the motion detector measures the distance of an object from it at any g time (using reflected waves). A blank graph of how far the object is from the motion sensor versus time is displayed on the screen. What variable is on the x-axis? What variable is on the y-axis?	Students read time on the x-axis and position on the y-axis. Students may need to be reminded to read the units also.
2. Posing the Task Today we are going to try to show our understanding of speed, distance and time by graphing our own motion in front of the motion detector. We will look at graphs of position versus time when **(i) An object is stationary in front of the detector** **(ii) An object is moving at constant speed away from the detector** **(iii) An object is moving at a constant**	Students will be given a sheet with space to graph a prediction and the actual outcome of each of the motions we study. Students then compare their predictions to the actual outcome. Students carry out each task i.e. they first predict the graph given the conditions, then produce the actual graph using the detector with the subject walking in front of the detector. This is followed by a discussion, before moving onto the next task. (The subject should hold an object such as a hard backed

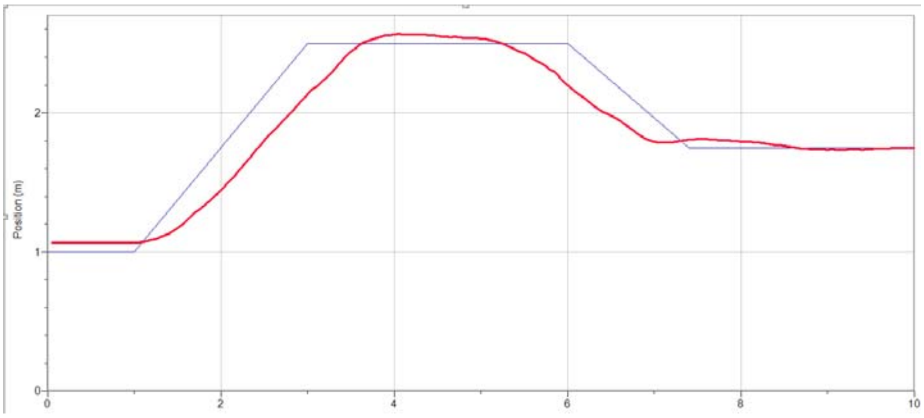
speed (different than in (ii)), away from the detector **(iv) An object is moving at a constant speed towards the detector** **(v) An object is moving at a constant speed (different from that in (iv)) towards the detector**	A4 copy in front of them to act as a good reflector for the waves.)
3. Anticipated Student Responses Some students may not know how to start when predicting the position time graph for a stationary object. A student may draw a linear graph with a non-zero slope to represent a student at rest with respect to the Go!motion device. Students may have difficulty moving at a **steady speed** away from or towards the detector. They might accelerate or decelerate. Students may have difficulty predicting the position time graph for a student moving towards the detector from a faraway position. When asked to calculate speed from the graphs, students may not relate initially to their prior knowledge of what speed is. They may say distance over time when prompted but may not relate this initially to the slope of the graph.	If students have a problem predicting the graph when the object is stationary in front of the detector, ask them how far away it is after t s, after t+1 seconds, after t+2 seconds etc. Students could also be asked to predict the position-time graph if the fixed distance from the detector was increased or decreased. The student might need to try walking at a steady speed before data is collected. The class needs to articulate what it means to walk at a steady speed. If they accelerate initially they can compare the graphs for steady speed and the graph showing acceleration. If students are showing a lack of understanding, ask "Where is the student at time t and how do you represent this" and then repeat the question for time t+1 etc… and also refer back to what a steady speed means. Ask students to define speed. Ask them for the units of speed. "If speed is 3metres per second, how many metres have you travelled in 1 second, 2 seconds etc." Use this to find your speed from the graph. "Does the distance from the detector increase or decrease each second?" "Given that you use the slope of the graph to find the speed, how would slope be different for a speed of 3 metres per second away from the detector and a speed of 3 metres per second towards the detector?"
4. Comparing and Discussing Students initially predict the graphs for each situation on their own. Having made an attempt on their own they can then share their prediction with a fellow student. They should be able to defend their prediction.	We are trying to let the students explore rates of change that are constant and non-constant but the focus in this lesson is on constant rates of change. Students will have a prediction to compare with the actual graph. The student walking in front of the Go!motion detector may not carry out the task at hand as directed but this will allow students to discuss what should have happened and what actually happened.
Final assessing of the learning: Students demonstrate how well they have understood the above tasks (i) to (v) by walking in front of the detector at an appropriate speed to **match a pre- drawn**	The pre-drawn graph is shown on the board. Groups discuss how they will walk to match it. One group is chosen to do the walk. One member of the group is nominated by the group to "walk the walk" and needs to be clear on what they have to do before the "Collect

graph of position versus time.	data" button is pressed. It is the group's responsibility to ensure that this person is briefed properly beforehand. The class discusses how well the task was carried out and if necessary it is repeated with the same graph. Other groups repeat the task with different graphs.
5. Summing up	To summarise, the students are given example questions e.g. from Junior Cert exam papers on graphs of motion and other sources. The students will be required to match the rate of change to each situation and discuss their reasoning.

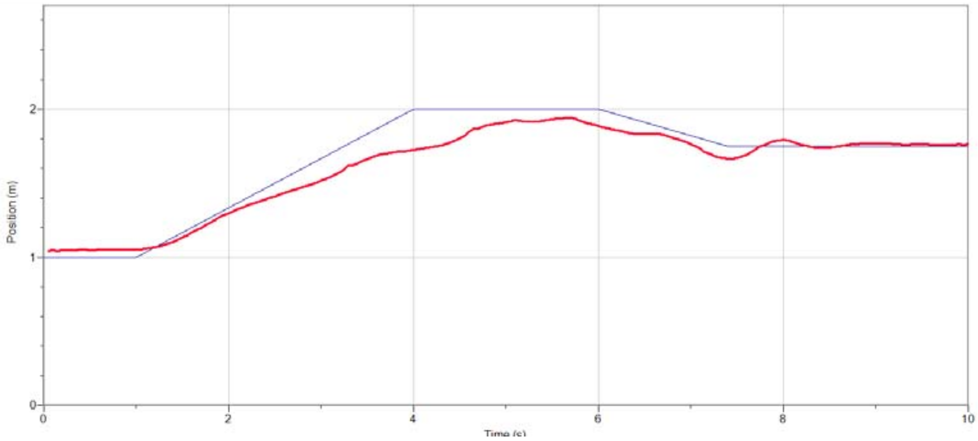
9. Evaluation

- To observe students we plan to have some teaching staff, preferably maths teachers present. They will make notes about students' responses and interactions with the class. These teachers will also have been involved in planning the lesson.
- The observers will have detailed sheets where they can place their observations. They will be noting the students' initial responses to the material at the beginning of the class. The students will then hopefully show an evolving understanding of the topic and this will be noted by the observers. Difficulties students have and misconceptions will also be noted.
- At the beginning of the class I will ask the students about their understanding of a previous topic on the relationship between speed, distance and time. The students will then generate graphs representing situations of differing speeds. My questioning will be to check understanding of the linear relationship between position and time when speed is constant by using these graphs. Following this, students will be asked to interpret other graphs of motion. They will be questioned about what sort of movement in front of the Go!motion detector would be necessary to generate these graphs.
- The observers will focus on students thinking in relation to speed i.e. rate of change of position with time. They are to focus on difficulties students have and any misconceptions that arise.
- I plan to collect the student worksheets after the class to check the students' ability to explain their graphs as well as their ability to predict a graph given a " story "on which the graph is based.

10. Board Plan



Screen capture 1,
This student shows an understanding of the distance he must start at to match the y-intercept. He also shows an understanding of moving at constant speed away from the detector (positive slope) and moving at constant speed towards the detector (negative slope).Finally he has an understanding that the horizontal line segment represents remaining at rest.



Screen capture 2
This student had much difficulty at his first few attempts but eventually he did quite well. The group he was part of were not giving him clear instructions.

11. Post-lesson reflection

- What are the major patterns and tendencies in the evidence? Discuss.

  Students had a good understanding of what the graphs should look like. Students drew predictions that were correct when I verbalised the situation. Students demonstrated prior knowledge of co-ordinate geometry. Clear use of the language was lacking. "He started slower" when the student had a slow reaction time.

- What are the key observations or representative examples of student learning and thinking?

  All the students got involved in the lesson. Students enjoyed the task given to them at the end of the lesson. A student asked for a metre stick to measure how far he would need to stand from the detector to match the y-intercept on the pre-drawn graph. Then he realised that he could just use the detector to find this exact distance. Students got quite involved in directing the chosen student to carry out the task at hand from their respective groups i.e. walking to match the graph

- What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?

  Students had difficulty expressing the co-variation of the variables time and distance. Early on in the lesson, students did not express that the time was changing and only identified position as a possible variable. Students couldn't verbalise the fact that while the distance was remaining constant the time was varying when they stood stationary in front of the detector while it was collecting data.
  Later in the lesson students were trying to match a graph given to them. One group understood the task and completed it quite well. The other group had difficulty completing the task. It is maybe worth pointing out that the student chosen to match the graph didn't grasp the fact that moving away from the motion detector at a constant rate generated a line with positive slope. He seemed to fail to understand the concept and was confused by his group mates and their instructions. This group needed a bit more time discussing the concepts of positive and negative slope.

- In what ways did students achieve or not achieve the learning goals?

  Students achieved a deeper understanding of graphs of motion. They displayed this by correctly predicting each of the graphs they were set to graph. They gave correct predictions and in most cases gave an explanation why there prediction was correct or if not why it was different to the actual graph.

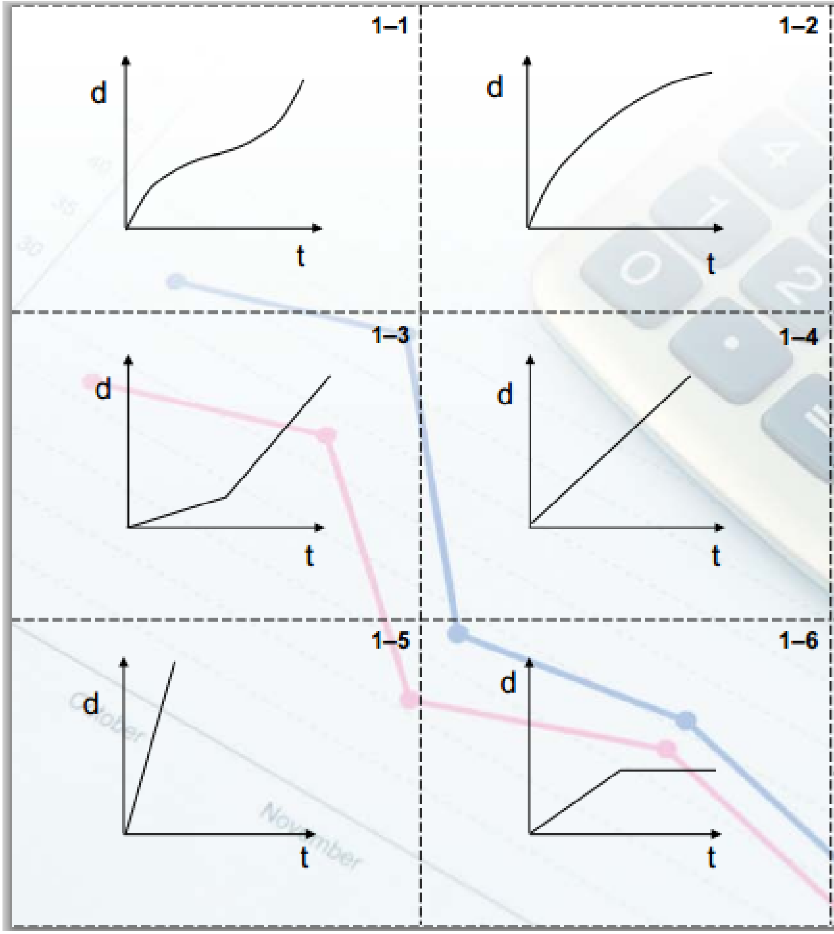
- Based on your analysis, how would you change or revise the lesson?
  I would focus more on keywords and ensuring that students give clear explanations to each of the graphs. Perhaps we tried to do too much in one lesson.

- What are the implications for teaching in your field?
  I repeated this lesson for an older group. I found that this lesson can help students with understanding graphs. I also believe that I will repeat this lesson for my second years when they are in third year to consolidate the concepts.

Student worksheet for predicting graph shapes

Prediction	Actual	Explanation and Keywords
		
		

Additional matching exercise : http://www.projectmaths.ie/workshops/workshop4/UnderstandingGraphs.pdf)



1–A : Moving at a steady constant pace.

1–B : Moving at a fast pace moving gradually to a slower pace.

1–C : Moving at a fast steady pace.

1–D : Moving fast, then slowing slightly, then going faster again.

1–E : Moving at a steady pace, then stops for a period of time.

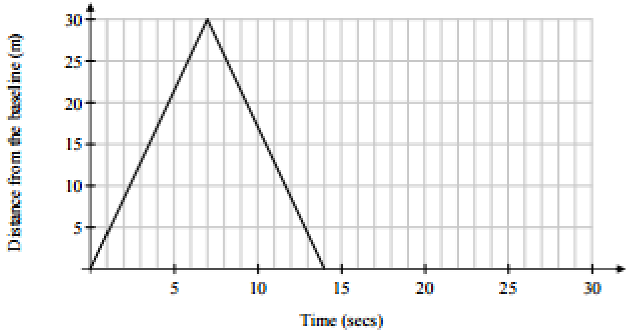
1–F : Moving at a slow pace and then rapidly increases.

Question 8 **(Suggested maximum time: 10 minutes)**

The students in a PE class are doing a fitness test.

Each student runs from the *baseline* of the gym to the *halfway line* of the gym, and back again. This is called a *lap*. They run a number of laps in the fitness test.

The graph below shows Miriam's distance from the baseline for her first lap.



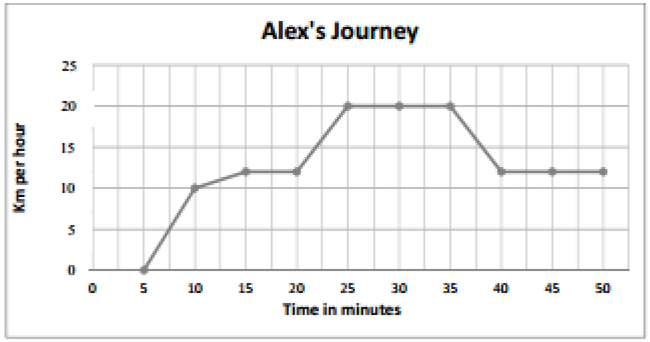
(i) From the graph, how long did it take Miriam to complete her first lap?

(ii) From the graph, how far is the *baseline* of the gym from the *halfway line* of the gym?

(iii) For her second lap, Miriam increases her average speed to 5 metres per second. On the diagram, continue the graph to show her distance from the baseline over the course of this lap.

Question 15 **(Suggested maximum time: 5 minutes)**

The graph below shows some details about a journey Alex made by bicycle.



Alex waited for his friend before he set off on his journey.

(a) How long did he wait before setting out?

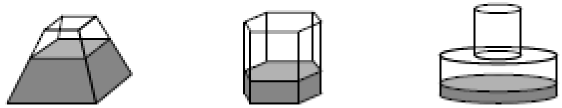
(b) What was Alex's highest speed during the journey?

(c) For what length of time was Alex travelling at the highest speed?

(d) How far did Alex travel at the highest speed?

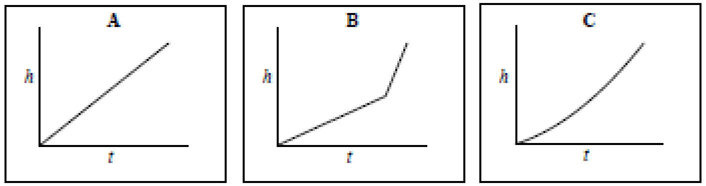
Question 6 **(Suggested maximum time: 10 minutes)**

Below are three containers, labelled **1**, **2**, and **3**.
Water is poured into each container at a constant rate, until it is full.



1 **2** **3**

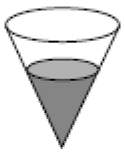
The three graphs, **A**, **B**, and **C**, show the height of the water, *h*, in the containers after time *t*.

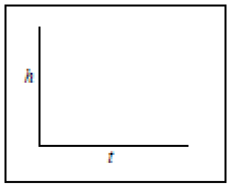


(a) Write **A**, **B**, and **C** in the table below to match each container to its corresponding graph.

Container	**1**	**2**	**3**
Graph			

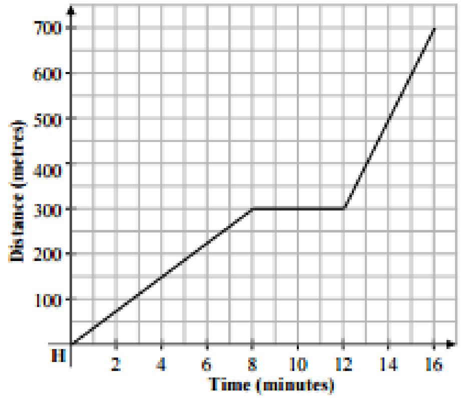
(b) Another container is shown below. Water is also poured into this container at a constant rate until it is full. Sketch the graph you would expect to get when plotting height (*h*) against time (*t*) for this container.





Question 7 **(Suggested maximum time: 10 minutes)**

Angela leaves home (H) at 5 pm to go to football practice, which is 700 m away. The graph shows her journey, on foot, to football practice.



(a) One of the stories below matches Angela's journey.
Place a tick in the box beside the correct matching story.
(**Note:** Only **one** story matches Angela's journey).

Story	**Tick one story (√)**
Angela walks at a constant pace and stops at 5.08 for four minutes. She then walks at a slower pace and arrives at practice at 5.16.	
Angela walks at a constant pace and stops at 5.12 for four minutes. She then walks at a faster pace and arrives at practice at 5.16.	
Angela walks at a constant pace and stops at 5.08 for five minutes. She then walks at a faster pace and arrives at practice at 5.16.	
Angela walks at a constant pace and stops at 5.08 for four minutes. She then walks at a faster pace and arrives at practice at 5.16.	
Angela walks at a constant pace and stops at 5.08 for four minutes. She then walks at the same pace and arrives at practice at 5.16.	

(b) Mary also lives 700 m from football practice, but cycles to practice. She leaves home five minutes after Angela. She cycles at a constant pace and arrives at practice two minutes before Angela.
Represent Mary's journey on the graph above.