

Athmhachnamh ar Chleachtas

COLÁISTE
IOGNÁID



Ní neart go cur le chéile

Athmhachnamh ar Chleachtas – Réamhfhocal

Cé go ngníomhaíonn an oide mar theagascóir agus mar stiúrthóir, ní mór dó/dí a bheith mar thaighdeoir chun cur go leanúnach lena bhforbairt ghairmiúil. Comhoibriú agus comhroinnt eolais atá i gceist sa phróiseas seo.

- Is mian linn léachtóireacht sa seomra ranga a sheachaint.
- Is mian linn deiseanna a chur ar fáil d'fhoghlaimoirí óga, deiseanna a chuidíonn leo smaointe bunúsacha a shealbhú.
- Is mian linn tacú leo ionas gur féidir leo a bheith ina bhfoghlaimoirí neamhspleácha.

Mar sin, ní mór dúinn fadhbanna a cheapadh, fadhbanna a ghríosáíonn foghlaimoirí chun a gcuid machnaimh a mhíniú. Tabharfar tús áite do na céimeanna seo a leanas:

- Dífreálú sa cheacht chun freastal a dhéanamh ar léibhéil éagsúla intleachtúla i measc an ghrúpa ranga
- Fiosracht a chothú trí ceachtanna gníomhacha taitneamhacha gan coinne agus ceachtanna comhoibríoch a thionól
- Straitéisí ceistiúcháin a fhorbairt chun cur lena scileanna fadhb-réiteacha – ní leor freagra a lorg, ní mór dóibh slite éagsúla a mholadh chun fadhb a réiteach
- Measúnú ar fhoghlaim – cuidiú le foghlaimoirí a gcuid straitéisí féin a fhorbairt ar mhaithe le measúnú a dhéanamh ar a gcuid oibre féin

Bronnann na céimeanna seo uile muinín ar an bhfoghlaimoir.

Foireann – Plean

Rinneamar ransú smaointe in éindí mar ghrúpa taighde agus glacadh leis go raibh deacrachtaí foghlama le sárú san Ailgéabar. Síleadh nár chuidigh an sean-nós, is é sin algartam a úsáid ar dtús ar mhaithe le coincheapa a shealbhú. Bhí an baol ann go raibh céimeanna gan ciall á meabhrú. Glacadh leis go forleathan go raibh deacrachtaí ar leith ag baint le hoibrithe chodán. Chreideamar gur chóir freastal a dhéanamh ar choincheap an chodáin, ar a gcur in ord agus ar chóibhéis sula gcuirfí tús le hoibrithe chodán.

D'fhéachamar inár gcuid taighde ar léirithe éagsúla – léirithe pictiúrtha, láimhsitheacha, briathartha, samhlatacha, agus saolta – a chuideódh le greim daingean a fháil ar choincheapa. D'fhéachamar freisin ar ábhar a bhain le teagasc na gcodán, ábhar a bhí cruthaithe cheana féin ag oidí Mata, ina measc, forbhreathnú, tástáil diagnóiseach, deighilt, cóibhéis, agus oibrithe. Roghnaíomar díriú ar chodán a chur in ord agus foghlaimoirí a chumasú chuige sin trína straitéisí féin a fhorbairt. Ba mhian linn na huirlisí cuí agus áiseanna foghlama a úsáid mar scafall agus ní mar mhaidí croise. Roghnaíomar stiallacha codán, balla codán, agus uimhir líne mar shamhailteacha maithe meabhracha.

Oide – Plean

Ó tharla nach raibh aon rang Mata don Teastas Sóisearach idir lámha ag an oide, bheartaigh sí tástáil diagnóiseach a dhéanamh ar dtús ar ghrúpa-ranga sa chéad bhliain. Theastuigh uaithi tréimhse ranga eile a chaitheamh leo ar mhaithe le caidreamh a chothú agus deacracht dá laghad a aimsiú sula dtabharfaí aghaidh ar an gceacht taighde.

Ceacht Taighde – Aidhm

Straitéisí chun codáin a chur in ord a fhorbairt, ag baint leas as a dtuiscintí ar choincheapa codán in ionad comhainmneoir a úsáid.

Torthaí foghlama fadtéarmacha: greim daingean ar choincheapa codán a chuideoidh le buanmháistreacht ar Ailgéabar a chothú.

Introduction

Phase 2 of Professional Development – “Reflections on Practice”

Buíochas le Sheelagh Clowry, our Regional Development Officer, for her commitment and enthusiasm. We commend the Project Maths Development Team (PMDT) for this development initiative to assist teachers in leading their own Continuous Professional Development (CPD) into the future. We have now arrived at the core element: observation and discussion of the research lesson. This collaborative effort should assist in developing effective teaching strategies that enable our students to become independent learners.

Teaching through problem-solving is a familiar element of our craft. However, rather than make a bee-line for the solution, we now try to provide students with an opportunity to explore the many ways to arrive at a solution. The skilfully designed active collaborative lesson is the key to our success. The practice of documenting student thinking and rating progress via formative assessment is, perhaps, not quite so familiar.

We propose that teachers who wish to be progressive and improve education methodologies should be allocated formally recognized school hours to enable their collaborative work.

We are a team numbering three teachers who collaborate across a geographical span of 35-70 kilometres. Two of us function on a daily basis though Irish. Consequently, team communications are bilingual (Irish & English). From the very beginning, our research and planning activities have taken into consideration the recommendations of the inspectorate.

Mathematics Lesson Plan – Research Lesson

Lesson plan developed by: Monica Turley, Máire Uí Chonghaile, Maireád Uí Dhonnchadha

Location: Coláiste Chroí Mhuire, An Spidéal

Date: 4 Feb 2015

Duration: 14.40 – 15.20

Class: First Year Mixed Ability Group

Class teacher: Máire Uí Chonghaile

Language: Gaeilge agus Béarla

Title of the lesson: *Strategies for ordering fractions (besides using common denominators or decimals)*

Brief description of the lesson: To help students use conceptual understanding of fractions to compare and order fractions. Students will engage in an activity that will lead them to discover the ordering strategies for themselves.

Aims of the lesson:

- Students will develop the concept of fraction size
- Students will visualise the fractions and order them according to size
- Students will develop ordering strategies *not based on common denominator*
- Students will discover these ordering strategies for themselves

Learning outcomes: At the end of this lesson, students, when given two fractions, will be able to:

- judge the relative size of each fraction
- show the relative positions of fractions on the number line
- demonstrate flexibility in their choice of strategy to compare fractions

Background and Rationale: The syllabus reference states: ‘students need to learn strategies for computation that can be applied to any numbers, in this case the set of rational numbers; implicit in such computational methods are generalisations about numerical relationships involving the operations being used’ (see Appendix 1, 3.1 Number systems).

In the wider scheme of things, research indicates that trouble spots in algebra come from an incomplete understanding of fraction concepts. The use of algorithm in a purely procedural way without thinking about each step has probably created more problems than it has solved. Conceptual understanding is crucial to procedural competence with fraction operations.

In partnership with the PMDT, we have come together as a group to focus on aspects of teaching and learning that will enable students to develop strategies for working in number systems (in particular working with fractions), which they can successfully apply in Algebra later.

Our team will address the development of the concept of fraction size through using ordering strategies not based on common denominators with a view to avoiding the pitfall of the activity becoming a purely mechanical procedure. The emphasis will be placed on visualising the fractions and thinking about the meaning of both parts of a fraction.

Research: Our first point of reference was the Junior Certificate Mathematics Syllabus for examination from 2016 (see Appendix 1, Strand 3, Topic 3.1 Number Systems, p.22). We reviewed material that showed the difficulties students have with fractions. Research shows that children need to conceptualise fractions as quantities before they are introduced to conventional symbolic algorithms for operations on fractions. A lack of understanding means that algorithms develop bugs e.g. inappropriately cross-multiplying fractions. It is believed that part of the reason students encounter difficulties is that an insufficient amount of time is spent on the concept of a fraction and on ordering and equivalence of fractions before operations on fractions are introduced.

We then looked at the website resource *An overview of teaching and learning fractions*, which was produced by the PMDT in 2009. Having studied this material, which was created by teachers locally, we could not find material on ordering fractions.

We sourced the Teaching and Learning Plan *The Multiplication of Fractions – PMDT 2010* and gave consideration to the resources recommended including fraction strips, fraction circles, fraction stacks, number lines, and fraction wall (p.3).

About the Unit and the Lesson:

Unit: According to the First year handbook 2014 (see Appendix 2), there are four sub-sections in syllabus section 3.1 named Number systems N, Z, Q, and Ratio and Proportion. We will address Number system Q. It is interesting that Number system Q preceded Number system Z in the original handbook 2011. The research goal of the lesson is to enable students to develop ordering strategies not based on finding common denominators.

In order to proceed satisfactorily, we must not assume that students have an understanding of fraction concepts. Consequently, we have decided to assess students' current understanding. Prior to the research lesson, students will complete the *Diagnostic Test* on fractions, which is available on the Project Maths website (see Appendix 3).

We have also decided to devote another lesson to partitioning – *Partitioning the whole/unit into equal parts* (see Appendix 4) – in order to explore the meaning of the numerator and denominator in a fraction. We want conceptual understanding to take root before we branch forward into the research lesson Ordering Fractions.

Lesson: In the research lesson, each student will be provided with a tool-kit containing a laminated coloured fraction wall, coloured fraction strips, two laminated white A4 sheets (one with grid, the other blank) and two white board markers (one red, one black). It is hoped that the blank sheet will serve as a blank canvas to work on and also as a useful tool to communicate ideas to the student/class group.

During the lesson an *activity sheet* of questions (see Appendix 5) will be presented to each student in order to engage and excite them to respond to the task. Students may refer to the fraction wall or fraction strips or they may draw a number line to help them to develop their reasoning strategies. As students engage in the learning task, the teacher will assess how students are approaching the task. In order to assess student understanding, a review will take place after every three questions whereby students will present their reasoning for their findings. The teacher will provide the necessary scaffolding through appropriate questioning in order to raise the level of their understanding. When they reach the final question, they may discover the need for an alternative strategy to those which they have just used so skilfully. This paves the way for the next lesson: the introduction of equivalence and common denominators.

In summing up, students will be asked to summarise all the ways that they found to compare fractions through reasoning. Follow-up tasks will be assigned upon completion of the lesson. These will be differentiated to allow for learner diversity.

Flow of the Unit (Appendix 2)

Assessment for learning: Best practice employs formative assessment for learning as distinct from summative assessment of learning. Formative assessment is forward-looking and learner-driven whereas summative assessment is backward-looking and content-driven. The constant feedback of formative assessment greatly assists learning but, more importantly, students are encouraged to take responsibility for their own learning. In general, the teacher would use a number of strategies to support learning, including:

- share learning outcomes at the start of the lesson
- encourage and develop good communication skills, seek questions rather than answers
- encourage discovery learning, co-operative problem solving through challenging activities

The teacher also uses learning logs both for student and teacher reflection (see Appendices 7, 8, 9). We believe that more formative assessment strategies need to be embedded in and integrated with practice.

In the early stages of this reflective work, the RDO furnished us with much material including ideas for Assessment for Learning in Mathematics shared by Dr. Máire Ní Ríordáin (NUIG). We chose the following five for our research lesson: Numbers 9, 17, 21, 31 and 27 (see Appendix 6). Upon reflection, and in the interest of time efficiency, we decided that students (working in pairs) could display their responses on the laminated blank A4 sheet and review instead of to-ing and fro-ing to the board with post-its/sheets. We contend that, while innovative approaches to teaching and learning are desirable, engaging and enjoyable, they must be time efficient. Therein lies the challenge for the teacher.

Flow of the lesson		
Teaching activity	Points of consideration	Anticipated Student Responses
<p>Introduction</p> <p>Review learning from previous lesson - initial fraction concepts</p> <p>Check homework</p> <p>Present a pair of fractions which when rounded to a whole number cannot now be ordered!</p> <p>Launch</p> <p>Tool kit</p>	<p>Natural numbers allow us to count. Lets measure Sean's height. Not a whole number. The need for fractions.</p> <p>Teacher questions "what did we learn in the previous lesson?"</p> <p>Role of fraction parts <i>O/H 1 Appendix 10</i></p> <p>Comparing pairs of fractions using estimation only.</p> <p>When we look at fractions such as $\frac{7}{8}$ and $\frac{3}{4}$ how we decide which is bigger?</p> <p>Today we will try to come up with some other ways of deciding which of two fractions is the biggest by using what we already know about fractions</p> <p><i>P.P.1 Codáin Appendix 11</i> (Note I've kept the title simple with a view to asking students at the close of the lesson to suggest a better title.)</p> <p>In the previous class you worked in groups, today I would like you to work in pairs on the following tasks. You must make a decision about which fraction is biggest and you must explain your decision.</p> <p>When deciding, think about what the denominator of a fraction means and what the numerator of a fraction means.</p> <p>After each group of three questions, we will ask you to report your findings and reasons.</p> <p>You may use your tool kit to help you make your decision and to help you give a reason for your decision.</p>	<p>A metre and a bit.</p> <p>The numerator, the denominator The whole, the part</p> <p>Common denominator</p>

Flow of the lesson		
Teaching activity	Points of consideration	Anticipated Student Responses
Fraction strips	O/H 2 <i>Appendix 13</i> Teacher explains how to use fraction strips.	
<p>Posing the task (Ordering fractions) Activity sheet <i>appendix 6</i></p> <p>Scaffolding</p> <p>Teacher continues to circulate and observe strategies.</p> <p>Teacher asks a couple of groups to report back and justify their answers.</p>	<p>Q 1 (Unit fractions - using 0 as a benchmark) $1/9$, $1/17$ Mindful of the role of denominator, students should reason readily. If not, teacher will refer them back to share size - fraction wall. <i>Appendix 12</i></p> <p>Q 2. (Same denominator) $3/7$, $5/7$ Guidance may be needed - teacher questions appropriately "What is the same and what is different about these two fractions?" Use the tool kit. Can you think of a similar question?</p> <p>Q 3. (Same numerator) $3/5$, $3/7$ What is the same and what is different about these two fractions?" Can you think of a similar question?</p> <p>By now, students are engaging with the meaning of both parts of the fraction i.e. The numerator and the denominator</p>	<p>R 1. Pieces/parts are smaller R 2. Denominator is greater so fraction is smaller. R 3. Looking at fraction wall one is smaller than the other.</p> <p>R 1. There are more shares in $5/7$ R 2. There are more sevenths in $5/7$ than in $3/7$ R 3. The denominators are the same and $5 > 3$ R 4. The denominators are the same but the numerator is greater. R 5. The numerator is greater so it is closer to the whole. R 6. Same size shares, 5 of them $>$ 3 of them.</p> <p>R 1. The shares are larger in $3/5$ than $3/7$ R 2. $3/5$ is greater because the denominator is smaller. R 3. The denominator is different, the numerator is the same, so look at the denominator. R 4. Fifths are greater than sevenths, so three of the bigger shares are greater than three of the smaller shares.</p>

Flow of the lesson		
<i>Teaching activity</i>	<i>Points of consideration</i>	<i>Anticipated Student Responses</i>
<p>Teacher listens to students talking through the activity and is alert for any misconceptions</p> <p>Teacher checks are there many different strategies among the students</p> <p>Teacher asks different groups to present their strategies so they all get to see the different viewpoints which are valid.</p>	<p>Q 4. (Using 1 as benchmark) $2/3, 3/4$ They might be asked "Are the fractions closer to 0, $1/2$ or 1?" Realising that they are both approaching 1 "How close?" "How can you use Q 1. to help you?"</p> <p>Q 5. $9/8, 4/3$ They might be asked "Are the fractions closer to 0, $1/2$ or 1?" "Are the fractions smaller than or greater than 1?" "How much more?" "How can you use Q 1. to help you?"</p> <p>Q 6. $5/3, 5/7$ They might be asked "Are the fractions closer to 0, $1/2$ or 1?" Are the fractions smaller than or greater than 1? "How can you use Q 1. to help you?"</p> <p>O/H 3 Teacher explains the use of the double number line for Q 5. <i>Appendix 13</i></p>	<p>R 1. $3/4$ is closer to 1 on the number line R 2. $3/4$ is closer to 1 on the fraction wall R 3. $2/3$ is within a third of 1 but $3/4$ is within a fourth and so is closer, so it is greater.</p> <p>R 1. $4/3$ goes further past 1 R 2. $1/3$ is greater than $1/8$ so $4/3$ is closer to 2. R 3. $1/3 > 1/8$ so $1 + 1/3$ is greater than $1 + 1/8$</p> <p>R 1. $5/3$ is greater than 1, $5/7$ is not even close to it R 2. $5/3$ is greater than 1, $5/7$ is less than 1 R 3. $5/3$ is greater than the whole R 4. $5/7$ is on the fraction wall but 1 and $2/3$ is not because it is greater. R 5. Numerators are the same but thirds are greater than sevenths.</p>
<p>Teacher continues to circulate and listen to students discussion.</p>	<p>Q 7. (Using $1/2$ as a benchmark) $3/8, 4/10$ They might be asked "Are the fractions closer to 0, $1/2$ or 1?" How close? How can you use Q 1. to help you?</p>	<p>R 1. Using fraction wall $4/10$ is closer to $1/2$ R 2. Using fraction $4/10$ closer to the whole. R 3. $3/8$ is $1/8$ less than $1/2$ but $4/10$ is $1/10$ less than $1/2$, so is bigger. R 4. $3/8$ is closer to 0</p>

Flow of the lesson		
<i>Teaching activity</i>	<i>Points of consideration</i>	<i>Anticipated Student Responses</i>
<p>Reliance on fraction wall so far is now challenged.</p> <p>Teacher asks different groups to present their strategies so they all get to see the usefulness of some strategies over others.</p>	<p>Q 8. $4/6, 7/12$ They might be asked "Are the fractions closer to 0, $1/2$ or 1?" How close? How can you use Q 1. to help you?</p> <p>Q 9. $6/14, 9/16$ They might be asked "Are the fractions closer to 0, $1/2$ or 1?" How close? How can you use Q 1. to help you?</p> <p>O/H 3 <i>Appendix 13</i> Teacher may refer to number line illustrations in the interest of time efficiency.</p>	<p>R 1. $4/6$ is gone further past a $1/2$ R 2. $4/6$ is closer to 1 R 3. $4/6$ is a $1/6$ more than a $1/2$ and $7/12$ is only a $1/12$ more than a $1/2$</p> <p>R 1. There are no $1/14$'s on a fraction wall! R 2. $9/16$ is greater than a $1/2$. R 3. $6/14$ is a $1/14$ less than a $1/2$ whereas $9/16$ is $1/16$ greater than a $1/2$.</p>
<p>By now the teacher ensures that a sense of fraction size is applied at ease.</p> <p>By now, some independent learners in an effort to maintain a sense of fraction size drive the lesson in search of equivalence in order to compare fractions.</p>	<p>Q 10. $7/24, 31/32$ They might be asked "Are the fractions closer to 0, $1/2$ or 1?" How close? How can you use Q 1. to help you?</p> <p>Q 11. $3/5, 5/9$ "Think about the different strategies you have used. Are they helpful?"</p> <p>Discovery learning will have taken place and the link to the next lesson is revealed.</p>	<p>R 1. $31/32$ is so close to 1 and $7/24$ is closer to 0 R 2. $31/32$ is much closer to 1. R 3. $31/32$ is much closer to the whole. R 4. $7/24$ is less than a $1/2$, $31/32$ is greater than $1/2$</p> <p>R 1. $3/5$ is bigger on the fraction wall. R 2. $3/5$ is half of a fifth more than a $1/2$ but $5/9$ is only half of a ninth more than $1/2$ so $3/5$ is greater. R 3. By doubling each fraction then $6/5 > 10/9$. because 1 and $1/5$ is greater than 1 and $1/9$. R 4. Try to make the numerators the same. R 5. Try to make the denominators the same.</p>

Flow of the lesson		
<i>Teaching activity</i>	<i>Points of consideration</i>	<i>Anticipated Student Responses</i>
Teacher asks students to summarise all the ways that they found to compare fractions through reasoning.	"Write down at least three strategies that you used today."	R. 1 Same denominator, compare numerator R 2. Same numerator, compare denominator R 3. How close is the fraction to 0 R 4. How close is the fraction to a 1/2 R 5. How close is the fraction to 1
Closure Teacher reminds students of what they discovered themselves.	PP2. <i>Appendix 14</i> "Straitéisí" Note the word "tagarmharc" is finally introduced	
Teacher asks students to suggest a better title for the lesson.	PP1. <i>Appendix 11</i> "Codáin"	
Teacher assigns follow up task	<i>Appendix 15, 16</i> This should serve to increase students flexibility in their choice of strategy to compare fractions Students are asked in addition to draw up a sheet of similar questions	

Evaluation: As members of our team are geographically dispersed between Galway and the Connemara Gaeltacht, we decided that the research lesson would take place in An Spidéal. The observers were be internal and external, the latter being Monica from Galway. We have drawn up comprehensive *Student Observation Record Sheets* (see Appendix 17). Observation will include:

- Is there evidence that the objectives in the lesson plan are clear and that they are met?
- Is there evidence that meaningful tasks are provided, tasks that capture students' attention and help students achieve the learning objectives?
- Is there evidence that the students are fully engaged in problem-solving tasks and thus encouraged to generate their own questions?
- Is there evidence that the previous lesson was of value to their conceptual understanding of a fraction?
- Do students receive appropriate support for any misconceptions and misunderstandings?
- Is there evidence of oral literacy (correct mathematical language) and collaboration of ideas?
- Is there evidence of students' enthusiasm and enjoyment of the learning experience?
- Is assessment for learning an integral part of the lesson thus extending and securing students' learning?
- Are students becoming independent learners and taking responsibility for their own learning?

Finally, summative assessment, though content-driven, could take place in the form of collection of students' work.

Board Plan: The classroom in which the research lesson will take place has two white boards across one wall. Another wheelie board will be introduced to the right of their field of view. Any one of these boards may command their attention as the need arises.

Board No. 1 to the left of their field of view will serve to accommodate the overhead transparencies (three in all)

Board No. 2 in the centre will accommodate the PowerPoint presentation – launch: Codáin; closure; Straitéisí. It will also serve as a blank canvas for spontaneous, creative sketches and scribbles.

Board No. 3 a colourful board to the right of their field of view will serve as a reminder about necessary vocabulary – Eochairfhocal (see Appendix 18).

In addition, vantage points around the room will serve to display the fraction wall: Balla Codán.

Post-Lesson Reflection:

We believe that this research lesson was an effective one where students' learning was enhanced and the goal was accomplished, namely to develop a number of strategies to enable students to order fractions.

Careful planning provided the students with a learning experience that supported their learning, thus enabling them to become independent learners. Mathematical proficiency was evident, namely conceptual understanding, procedural fluency and adaptive reasoning. Working in pairs prompted students to generate their own questions. Frequent reporting of their reasoning advanced their learning. They also supported each other when in difficulty. Throughout the lesson, the students were very engaged. Discussion with peers provided opportunity to explain their thinking. Rich learning took place. The teacher served as a source of support and inspiration.

The greatest challenge for the teacher was time efficiency. Sometimes, at the start of a lesson, when time permits, a video link is used to capture students' attention. On this occasion, however, hastening the start of the lesson in order to jump-start the students' involvement, the teacher simply used a meter stick to measure a student's height. Having checked the homework, the teacher used an overhead transparency in order to review homework, all in an effort to save time. Fortunately, there were no problems with the homework. The teacher was anxious to allow sufficient time to conceptualise the problems posed by the forthcoming main activity of the lesson.

The teacher believes that the key to the success of this research lesson was the partitioning that the students had learned in the previous lesson. The need to visualise fraction size prompted students to make the connection between both lessons.

Provision of tasks and learning aids:

Tool -kit: Firstly, it served to exclude any other distractions on the desk. Secondly, the large marker writing rendered clarity for effective collaboration and communication of ideas. The inclusion of the grid was unnecessary and may have been a hindrance to some. They did not use it.

Visual aids: The display of keywords was essential to cater for those *ar bheagán Gaeilge*. The official language of instruction in CCM, An Spidéal, is Gaeilge. The teacher and the majority of the class group were native Gaeilge speakers. However, the lesson was sometimes conducted bilingually in order to accommodate students/observers *ar bheagán Gaeilge*.

Student activity sheet: We are satisfied that the series of questions was skilfully designed to guide students along a path of discovery, the use of different strategies enabling them to order fractions.

Q1. (**using 0 as a benchmark**) They were at ease with the unit fraction and the concept of share size.

Q2. Same denominator, Q3 Same numerator – the role of fraction parts enabled them to reason here. Same denominator, compare numerator. Same numerator, compare denominator. Following the first three questions, they were able to explain their reasoning with confidence.

Q4, Q5, Q6 (**using 1 as a benchmark**) In this trio, Q4: $\frac{2}{3}$, $\frac{3}{4}$ presented most difficulty. A third less than 1 compared to a quarter less than 1 was confusing for some. It was easier to grasp an eighth greater than 1 compared to a third greater than 1 in Q5: $\frac{9}{8}$, $\frac{4}{3}$. In fact, the teacher was not prompted to use the double number line illustration for Q5 on the overhead transparency. We propose that, in future, this trio should be questioned in reversed sequence starting with $\frac{5}{3}$, $\frac{5}{7}$ where one fraction is less than 1 and the other is greater. This secures 1 as a benchmark and prompts them to look out for it again.

Q7, Q8, Q9 (**using $\frac{1}{2}$ as a benchmark**) In a similar fashion, we recommend reversing the sequence again, thus securing a $\frac{1}{2}$ as benchmark first and then prompting a search for it thereafter. In addition, Q9: $\frac{6}{14}$, $\frac{9}{16}$ would appear sooner, so the realisation for some students that fourteenths were not on the fraction wall forced them to seek an alternative to leaning on the crutch!

Q10: $\frac{7}{24}$, $\frac{31}{32}$ (**using $\frac{1}{2}$ as a benchmark**) $\frac{7}{24}$ and $\frac{31}{32}$ were so far apart – $\frac{7}{24}$ approaching 0 and $\frac{31}{32}$ approaching the whole – that they disregarded the $\frac{1}{2}$ as a benchmark.

Q11: $\frac{3}{5}$, $\frac{5}{9}$. They were about to discover that the previous strategies were no longer helpful. One student suggested aloud that the numerators be made the same (perhaps she meant the denominator). The teacher quickly realised that the two 5's might confuse some students and suggested they work with the original fraction pair $\frac{7}{8}$, $\frac{3}{4}$, which she wrote on the board. This was an impulsive decision to modify the plan in order to maximise students' learning. By now, time was pressing and some students excitedly thought aloud in search of equivalence, the link to the next lesson.

The teacher quickly reviewed the five strategies that were successfully used in class, finally introducing the word “tagarmharc”. In other words, though they had never known the word, the students were benchmarking.

Unfortunately, time did not allow for student reflection on a better title for the lesson, which, at the time, was named ‘Codáin’. The homework sheets were hurriedly distributed. It was a most enjoyable lesson.

Unusual responses:

Language difficulty: “níos lú píosáí” / “píosáí níos lú”

Misconception: “numerator closer to denominator”

Manipulative: shading a circle as at primary level

Finally, we would like to emphasise that it was the drawing-up and the implementation of the student observation record that ensured that the whole experience was worthwhile both for student and teacher. It was pivotal to the reflection on teaching and learning.

Focal Scoir – Afterword

During the analysis and revision, we examined a list of questions most of which are answered by now in this document. Nonetheless, we will now revisit one of those questions: “***Teaching through problem solving: Is this a change in culture for us?***”

No. Engaging students in a task for which the solution is not immediately obvious has always permeated mathematics. However, *arriving at* the correct solution is not as satisfying or as enriching an experience as *exploring* the many ways to find the correct solution, so the latter learning experience may potentially represent a change in culture. It could be summarized with the axiom: it’s less about the destination and more about the journey.

Our research lesson happened to involve a small class group of students in the presence of the facilitator/teacher and two observers. We surmise that, in a large class of students, a teacher could have difficulty monitoring exchanges within smaller groups.

Syllabus

Strand /Topic Title

Learning outcomes

Students should be able to

Strand 3: 3.1: Number systems

Students explore the operations of addition, subtraction, multiplication and division and the relationships between these operations – in the first instance with whole numbers. They will explore some of the laws that govern these operations and use them and models to reinforce the algorithms they commonly use. Later they review these operations in the contexts of rational numbers and refine and revise their ideas.

Students will devise strategies for computation that can be applied to any number. Implicit in such computational methods are generalisations about numerical relationships involving the operations being used. Students will articulate the generalisation that underlies their strategy, firstly in common language and then in symbolic language.

- investigate models such as decomposition, skip counting, arranging items in arrays and accumulating groups of equal size to make sense of the operations of addition, subtraction, multiplication, and division. They use these methods including the inverse operations
- investigate the properties of arithmetic – commutative, associative and distributive laws and the relationships between them
- appreciate the order of operations – use of brackets
- investigate models, such as the number line, to illustrate operations such as subtraction, multiplication and division in **Z**
- use the number line to order numbers in **N**, **Z**, **Q** (and **R** for HL)
- generalise an particularisation of arithmetic operations
- investigate models to help think about the operations of addition, subtraction, multiplication and division of rational numbers
- consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value
- analyse solution strategies to problems
- begin to look at the idea of mathematical proof
- calculate percentages
- use the equivalence of fractions, decimals and percentages to compare proportions
- consolidate their understanding and their learning of factors, multiples and prime numbers in **N**
- consolidate their understanding of the relationship between ratio and proportion
- check a result by considering whether it is of the right order of magnitude

The Common Introductory Course will lay the foundation for conceptual understanding which learners can build on subsequently.

Appendix 2

Suggested Sequence of topics-

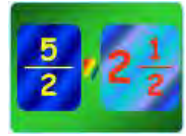
Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
Section 1	3.5	1.1	Sets	9	
	3.1	1.2	Number system N	9	
	3.1	1.3	Number system Z	10	
	3.1	1.4	Number system Q	11	
	3.1	1.5	Ratio and Proportion	12	

1. Flow of the Unit:

Handbooks would be useful here

Shows how the research lesson fits into a larger unit. Briefly describes what students learn before and after the research lesson.

Lesson		# of lesson periods
	Lessons on number system Q	
1	<ul style="list-style-type: none"> Diagnostic test on fractions 	1 x 40min.
2	<ul style="list-style-type: none"> Lesson on partitioning (See T&L on partitioning) 	1 x 40min.
3	<ul style="list-style-type: none"> Ordering of fractions 	2 x 40 min. (#1 = research lesson)
4	<ul style="list-style-type: none"> Addition and subtraction of fractions using estimation of the answers 	2 x 40 min.
5	<ul style="list-style-type: none"> Addition and subtraction of fractions using common denominators (preferably least common multiple) 	2 x 40 min.
	<ul style="list-style-type: none"> Multiplication of fractions and addition and subtraction of fractions 	2x 40 mins
	<ul style="list-style-type: none"> Division of fractions and multiplication and addition and subtraction of fractions 	2x 40 mins



Student Activity 1 (Lesson 3)

Ordering fractions using basic fraction concepts, without the use of common denominators

1. Shade in $\frac{1}{3}$ of each of the following figures.



Does $\frac{1}{3}$ of two items always represent the same amount? _____

What does the size of an area representing $\frac{1}{3}$ depend on?

2. Using a rectangle as a model, **draw** a picture to show what $\frac{3}{7}$ means.

What is the denominator of the fraction $\frac{3}{7}$?

What does the denominator of a fraction tell you?

What is the numerator of the fraction $\frac{3}{7}$?

What does the numerator of a fraction tell you?

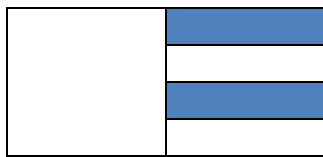
Complete the following using a fraction : $\frac{3}{7}$ is three "lots of " _____.

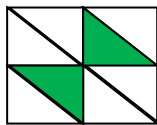
- 3.



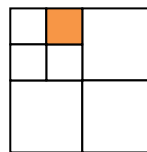
What fraction of this circle is shaded? _____

4. What fraction of each figure below do you think is shaded?









5. Arrange these fractions from smallest to largest: $\frac{1}{5}, \frac{1}{25}, \frac{1}{3}, \frac{1}{8}, \frac{1}{987}, \frac{1}{2}$

Explain your answer. _____

6. Draw 2 rectangles, one underneath the other, **which have the same length and width**. Partition each rectangle into 5 equal parts. Shade in $\frac{2}{5}$ of one rectangle and $\frac{3}{5}$ of the other rectangle.

Which fraction is bigger: $\frac{2}{5}$ or $\frac{3}{5}$? _____

Explain your answer: _____

7. Draw 2 rectangles, **which have the same length and width**, one underneath the other. Partition one rectangle into 5 equal parts and partition the other rectangle into 4 equal parts.

Shade in $\frac{3}{5}$ of one rectangle and $\frac{3}{4}$ of the other one.

Which is the bigger fraction $\frac{3}{5}$ or $\frac{3}{4}$? _____

Explain your answer. _____

8. Show using diagrams which of these two fractions is biggest, $\frac{3}{8}$ or $\frac{5}{6}$?

Explain your answer without using common denominators. (Hint: Compare to $\frac{1}{2}$)

9. Show using diagrams which of these two fractions is biggest, $\frac{4}{9}$ or $\frac{7}{12}$?

Explain your answer without using common denominators. Hint: Compare to $\frac{1}{2}$

10. (i) Which of these two fractions is **closest to one**:

$\frac{5}{6}$ or $\frac{2}{3}$? _____

(ii) Draw two rectangles with the same length and width. Shade in the area representing $\frac{5}{6}$ on one rectangle and the area representing $\frac{2}{3}$ on the other rectangle?

Explain your answer to part (i) using what you know about the unit fractions $\frac{1}{6}$ and $\frac{1}{3}$

Which is bigger $\frac{5}{6}$ or $\frac{2}{3}$? _____

Explain _____

11. Which is bigger $\frac{9}{8}$ or $\frac{4}{3}$? _____

Explain your answer. _____

Some drill bit sizes

Gauge inches

BS1 1/8"

BS2 3/16"

BS3 1/4"

BS4 5/16"

BS5 7/16"

BS5A 1/2"

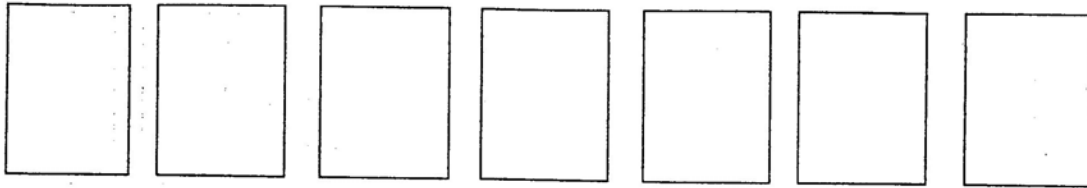
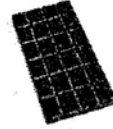
BS6 5/8"

BS7 3/4"

Appendix 4

Roinn 7 barra go cothrom idir ceathrar.

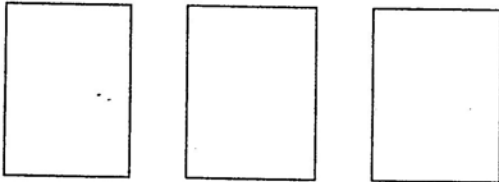
Student Activity 1: Divide 7 bars equally /fairly among 4 people



Describe how you did this. Déan cur síos ar an gcaoi a ndearna tú é seo.

Roinn 3 barra go cothrom idir ochtar.

Student Activity 2a – Divide 3 bars equally among 8 people

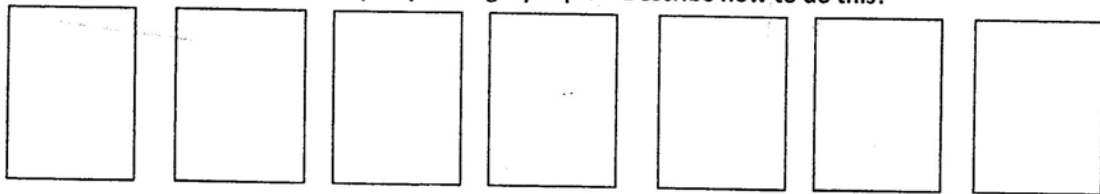


Describe how you did this:

Déan cur síos ar an gcaoi a ndearna tú é seo.

Roinn 7 barra go cothrom idir ochtar.

Student Activity 2b – Divide 7 bars equally among 8 people – Describe how to do this?



Déan cur síos ar an gcaoi a ndearna tú é seo.

Appendix 5

In the following questions, choose which fraction is the bigger fraction of the two fractions given.

Explain your choice, without using common denominators. Use the fraction wall or make your own drawings.

1. Which is bigger: $\frac{1}{9}$ or $\frac{1}{17}$?

_ is the bigger fraction. $\frac{\square}{\square} > \frac{\square}{\square}$

Reason:

2. Which is bigger: $\frac{3}{7}$ or $\frac{5}{7}$?

_ is the bigger fraction. $\frac{\square}{\square} > \frac{\square}{\square}$

Reason:

3. Which is bigger: $\frac{3}{5}$ or $\frac{3}{7}$?

_ is the bigger fraction. $\frac{\square}{\square} > \frac{\square}{\square}$

Reason:

4. Which is bigger: $\frac{2}{3}$ or $\frac{3}{4}$?

_ is the bigger fraction. $\frac{\square}{\square} > \frac{\square}{\square}$

Reason:

5. Which is bigger: $\frac{9}{8}$ or $\frac{4}{3}$?

___ is the bigger fraction. $\frac{\square}{\square} > \frac{\square}{\square}$

Reason:

6. Which is bigger: $\frac{5}{3}$ or $\frac{5}{7}$?

_ is the bigger fraction. $\frac{\square}{\square} > \frac{\square}{\square}$

Two reasons:

7. Which is bigger: $\frac{3}{8}$ or $\frac{4}{10}$?

Reason:

_ is the bigger fraction.

$$\frac{\square}{\square} > \frac{\square}{\square}$$

8. Which is bigger: $\frac{4}{6}$ or $\frac{7}{12}$?

Reason:

_ is the bigger fraction.

$$\frac{\square}{\square} > \frac{\square}{\square}$$

9. Which is bigger: $\frac{6}{14}$ or $\frac{9}{16}$?

Reason:

_ is the bigger fraction.

$$\frac{\square}{\square} > \frac{\square}{\square}$$

10. Which is bigger: $\frac{7}{24}$ or $\frac{31}{32}$?

Reason:

_ is the bigger fraction.

$$\frac{\square}{\square} > \frac{\square}{\square}$$

11. Which is bigger: $\frac{3}{5}$ or $\frac{5}{9}$?

Reason:

_ is the bigger fraction.

$$\frac{\square}{\square} > \frac{\square}{\square}$$

Appendix 6

Measúnacht le haghaidh na foghlama

- Dul i mbun cumarsáide – ag obair i bpéirí, féadfar freagra na ceiste a chur ar bhileog ‘post it’ agus í a ghreamú ar an gclár. An n-aontaíonn gach duine?
- Athrú ról – deis a thabhairt don fhoghlaimoir ceist a chur ar an ngrúpa ranga.
- Straitéisí a chur i gcomparáid – aiseolas ó bhéal agus díospóireacht a cheistíonn fiúntas na straitéisí sin.
- Is féidir straitéisí eile a úsáid go sciliúil de réir mar is cuí m.sh. foghlaimoirí a fhorbraíonn a gceisteanna tóraíochta féin chun tuiscint a bpáirtí a thástáil.
- Stiúradh seiftiúil ceachta i dtreo an chéad ceacht eile – ceacht a lorgáinn ceisteanna rud a chruthaíonn nasc leis an gcéad ceacht eile.

Tuigtear gur mó foghlama a dhéanann foghlaimoirí agus iad i mbun cíoradh agus réasúnú ná ó theagasc dhíreach. De bhrí go dtéann an fhoghlaimoir i ngleic leis an ábhar go gníomhach, bronann na straitéisí/seifteanna seo muinín ar an fhoghlaimoir.

40 Ideas for Assessment For learning in Mathematics

1. List three things you have learned today
2. List three things your neighbour has learned today
3. 60 second challenge – sum up
4. Write the new words you have learned in this lesson and what they mean in mathematics
5. Write three/five top tips for...
6. The answer is ... what is the question?
7. Take one minute to compose two sentences in your head to explain what we have learnt and how we have learnt it, using the key words from the lesson
8. Where can you use/apply this skill in other areas of mathematics or in other subjects?
9. In pairs, answer this question on a post-it/sheet, stick it on the board and review. Does everyone agree?
10. Show your work to your neighbour, work in pairs to set targets
11. Self assessment – record what you've learnt, any difficulties you have had and set your personal targets
12. Prediction - what do you think will happen next?
13. True/false game- ask students questions and in pairs, they decide whether it is true or false and state why.
14. Jigsaw feedback – groups work on different parts of task then reform to share findings
15. Group 'show and comment' on what was learnt
16. Feedback to whole class by one or two groups only (use rota or roll of dice).
17. Change of role – student as teacher. What questions would you ask the class and why?
18. Groups of three, numbered 1-3. Put statements on the board which individuals must explain to group

19. Quick fire oral 'quiz' to review/revisit learning
20. Giving wrong answer(s). Why is this wrong?
21. Comparing strategies. Which is the most efficient and why?
22. Matching games or cards to consolidate ideas
23. Make a 'mini book' summarising or revising key ideas and vocabulary
24. Revisit objective of the lesson self assessment – students indicate how they feel with respect to achieving the objective
25. Students write their own questions based on the objective of the lesson (these questions can then be selected randomly and used with the rest of the class)
26. Give an equation or number sentence and ask 'what else do you know?' based on the main teaching (eg web diagram)
27. Link the lesson to new learning – next lesson/next year
28. Visualisation based on the learning and vocabulary
29. Setting up a homework task eg: find examples at home or research on the internet and present findings to the class.
30. Teacher led probing questions to test understanding
31. Students develop their own probing questions to test their partner's understanding
32. Aide memoirs – students devise their own ideas/mnemonics eg picture/visual clues to the meaning of key words (eg parallel) linked to objective
33. Give students a known fact and an unknown fact, in pairs/groups find the route from one to the other.
34. Sentences with missing words and a selection of given words to fit in
eg: _____ lines are lines that intersect to form a 90 degree angle.
35. Questions with alternative answers posted around the walls and students go to the answer they think is correct and explain why (could give 10 seconds to reconsider)
36. Give the solution to a problem that students have to improve (eg graph, data collection, collecting like terms)

37. 'Same or different?' – give group of shapes/expressions/graphs and students identify what is the same and what is different about them
38. Give students groups of shapes/expressions/graphs etc and students sort into 2 groups/3 groups according to their own classifications (you could give the number items in each group)
39. Selecting the information required to solve a problem (words, pictorially or numbers)
40. Web diagram with false information – students have to select the correct information

Appendix 7

Log foghlama: Tuairisc an fhoghlaimeora C.C.M.

Ainm:

Ábhar:

Topaic:

Dáta:

Cad a d'fhoghlaim mé san topaic seo?

An cuid ba suimiúla dom?

Rudaí nach dtuigim fós?

Teastaíonn cúnamh breise uaim le...

Bhí an taithí fhoghlama seo:

Suimiúil Leadránach Oideachasúil Taithneamhach
Gan tairbhe Fiúntach Dúshlánach Cabhrach

Síniú: _____

Appendix 9

Log foghlama: Tuairisc an Mhúinteora C.C.M.

Oide:

Ábhar:

Rang:

Dáta:

Topaic:

Torthaí foghlama:

Cad ar éirí go maith leis?

Cad nár éirí leis?

Cén fáth?

Cén chaoi ar féidir liom é seo a fheabhsú? Aon ghníomhaíocht san áireamh.

Aon tuairim eile?

Síniú: _____

Cleachtaigh meastachán a thabhairt ar chodáin
 Cén uimhir is gaire don codán? 0, $\frac{1}{2}$ nó 1

Let's Practice Estimating Fractions

Which number is the fraction closest to? 0, $\frac{1}{2}$, or 1:

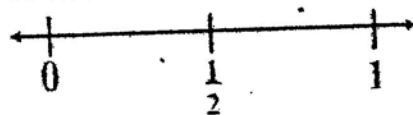
Fraction	0	$\frac{1}{2}$	1
$\frac{5}{12}$			
$\frac{9}{10}$			
$\frac{2}{11}$			
$\frac{17}{19}$			
$\frac{7}{13}$			
$\frac{3}{20}$			
$\frac{6}{14}$			
$\frac{1}{16}$			
$\frac{15}{17}$			

Úsáid garbh luachana m.sh. 0, $\frac{1}{2}$, 1, 2, 3, 4, chun na codáin seo a leanas a chur i gcomparáid.

Let's Try It!

Use estimation to compare the following fractions.

Round each fraction to 0, $\frac{1}{2}$, or 1



$$\frac{3}{4} \square \frac{4}{8}$$

$$\frac{1}{10} \square \frac{2}{3}$$

$$\frac{7}{9} \square \frac{2}{5}$$

$$\frac{6}{12} \square \frac{9}{10}$$

$$3\frac{5}{6} \square 3\frac{1}{3}$$

$$2\frac{7}{8} \square 3\frac{1}{4}$$

Codáin

- * Aidhm: Aithne a chur ar **mhéid** chodáin
Machnamh ar bhrí chodáin
- * Tascanna: Codáin a **chur in ord**
- * Foghlaim: Stráitéisí a úsáid chun codáin a
chur in ord

Q3.

$\frac{3}{5}$



$\frac{3}{7}$



Q4.

$\frac{2}{3}$



$\frac{3}{4}$

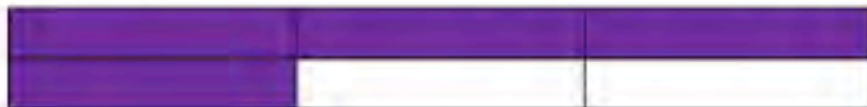


Q5.

$\frac{9}{8}$



$\frac{4}{3}$



Straitéisí

Aonad chodáin:

Roinnteoir mór – scar beag
Roinnteoir beag – scar mór

Ainmneoir céanna:

Uimhreoir mór – go leor scaranna
Uimhreoir beag – beagán scaranna

Uimhreoir céanna:

Ainmneoir mór – scar beag
Ainmneoir beag – scar mór

Tagarmharcanna:

0 _____ $\frac{1}{2}$ _____ 1

Úsáid na **stiallacha codán** chun cabhrú leat na codáin seo a leanas a chur i gcomparáid.

Comhlánaigh an bosca ag úsáid $<$, $>$ nó $=$.

Let's Try It!

Use $>$, $<$, or $=$ to compare the following fractions
Use the fraction strips to help you.

$$\frac{1}{2} \quad \square \quad \frac{3}{5}$$

$$\frac{4}{12} \quad \square \quad \frac{3}{8}$$

$$\frac{3}{4} \quad \square \quad \frac{9}{12}$$

$$\frac{7}{10} \quad \square \quad \frac{6}{8}$$

$$\frac{2}{6} \quad \square \quad \frac{1}{3}$$

$$\frac{3}{4} \quad \square \quad \frac{4}{5}$$

Codán Phéiri

Cén codán is mó? Breac síos an stráitéis a d'úsáid tú.

Fraction Pairs	Which Fraction is larger? Record your strategy.
a. $\frac{3}{8}$ $\frac{7}{8}$	
b. $\frac{1}{2}$ $\frac{5}{8}$	
c. $\frac{3}{4}$ $\frac{4}{5}$	
d. $\frac{2}{4}$ $\frac{4}{8}$	
e. $\frac{2}{4}$ $\frac{4}{2}$	
f. $\frac{3}{8}$ $\frac{5}{6}$	
g. $\frac{5}{6}$ $\frac{7}{8}$	
h. $\frac{3}{4}$ $\frac{7}{9}$	

Lean ort ag úsáid codán phéiri eile le leaschodáin m.sh. $\frac{5}{3}$, $\frac{9}{6}$ Léirigh gach péire acu ar uimh.líne ag úsáid tagarmharcáil cuí.

Repeat the above activity with other improper fractions e.g. $\frac{5}{3}$, $\frac{9}{6}$ Order the pairs by placing them on a benchmarks number line.

Student Observation Record

BEGINNING OF LESSON:

Observe level of difficulty with homework/previous class - Partitioning the whole/unit into equal parts.

If no difficulty tick the box for each student. If student has difficulty please identify issues:

	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6
(i) Identification of the whole						
(ii) Use of partitioned drawings/fair shares						
(iii) Use of shading/fraction - no. of shares						
(iv) Use of words						
(v) Use of arithmetic sentences						
Questions asked by students:						

Student Observation Record

DURING LESSON

Observe student understanding and interaction. If no difficulty tick the box for each student.

If student has difficulty please identify issues.

	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6
(i) Roll of denominator						
(ii) Roll of numerator						
(iii) Bigger fraction						
(iv) Reasoning/Strategies						
(v) Use of correct mathematical vocabulary						

Student Observation Record

DURING LESSON:

Observe student interaction. If no difficulty tick the box for each student.

If student has difficulty please identify issues.

	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6
(i) Questions asked to teacher						
(ii) Questions asked to partner/other group members						
(iii) Identify when student understood each strategy						
(iv) Identify when student understood earlier strategies may not be helpful						
(v) Other observations						

Student Observation Record

LESSON CONCLUSION: Observe student interaction. Check student level of confidence with use of different strategies.						
	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6
(i) Rate student ability to judge the relative size of fractions using fraction wall.						
(ii) Rate student ability to scale the no. line in order to show relative positions of fractions.						
(iii) Rate student flexibility in their choice of strategy to compare fractions - above or benchmarking.						
(iv) Rate students effective use of relational operators - $<$, $>$, $=$						

Student Observation Record

LESSON CONCLUSION: Check student level of confidence with ordering fractions.						
	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6
(v) Rate students grasp of fraction concept						
(vi) Other observations						
(vii) Issues that need to be addressed in the next class						
(viii) Recommended changes to lesson plan						

Eochairfhocail

Codán Páirt den iomlán nuair a roinntear an t-iomlán i bpáirteanna cothroma.
m.sh. ocht milseán nó píotsa amháin idir ceathrar.

Aonad chodáin An scar cothrom.

Uimhreoir Líon na scaranna cothroma.
(An t-uimhreoir)

Ainmneoir Líon na bpáirteanna cothroma ina roinntear an t-iomlán.
(An t-ainmneoir)

Ceartchodán Codán atá < 1

Leaschodán Codán atá > 1

Uimhir mheasctha Leaschodán

Balla codán

Stiall chodáin

Uimhirlíne

Ciorcal codáin