

# Reflections on Practice

# Lesson Plan for [*First Years, Coordinating the plane*]

For the lesson on [12/03/2015]

At [Good Counsel College, New Ross, Wexford], [Shirley Dempsey's] class

Teacher: [Shirley Dempsey]

Lesson plan developed by: [Shirley Dempsey, Sinead Furlong, Brigid Kennedy, Sean Mac Cormaic]

- 1. Title of the Lesson:** Coordinate geometry – thinking of coordinates as distances
- 2. Brief description of the lesson:** To help students understand the need for a coordinate system and what a useful coordinate system looks like. To help students come up with a coordinate system for themselves using their own language.

### **3. Aims of the Lesson:**

#### **Overarching aims**

I'd like my students to recognise that very often maths comes from the need to deal with a real-world problem,

I'd like my students to understand that discussing ideas is an important part of mathematics.

I'd like my students to value working as part of a group to solve a problem.

I'd like my students to recognise that there are different ways to tackle a given problem.

I'd like my students to see the connections between maths and other subjects they study.

I'd like my students to recognise that they can bring a lot to maths just by thinking about a problem.

I'd like my students to find their maths lessons engaging, stimulating and challenging.

I'd like my students to be comfortable presenting their ideas to their class mates and to me.

I'd like my students to learn to work more independently.

#### **Lesson-specific aims**

I'd like my students to understand the need for a coordinate system.

I'd like my students to recognise the presence of coordinate systems in everyday life.

I'd like my students to understand the importance of a starting or reference point for defining a coordinate system.

I'd like my students to understand that distances from a starting point are a really effective way for describing different locations.

I'd like my students to become fluent in the language of describing positions on the plane.

I'd like my students to become fluent in using mathematical notation when describing positions on the plane.

I'd like my students to recognise the convention of stating a position as a horizontal distance followed by a vertical distance.

I'd like my students to understand the form of the coordinates of points on the horizontal axis and on the vertical axis.

### **4. Learning Outcomes:**

As a result of studying this topic students will be able to:

- plot points on the plane
- describe points on the plane
- understand coordinates as distances
- calculate distance between two horizontal points or two vertical points

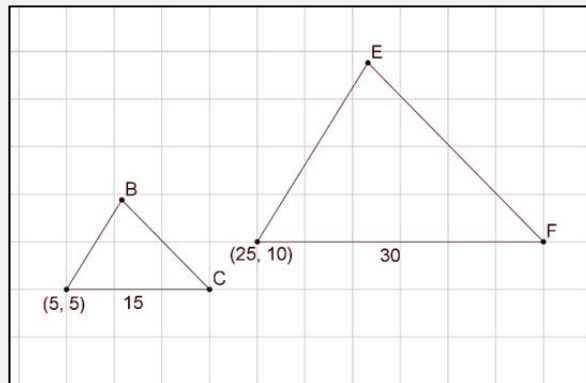
### **5. Background and Rationale**

The idea for this lesson came from observing the difficulties students have with understanding coordinates as distances from a reference point. These difficulties hinder student understanding in a number of areas including the application of Pythagoras's Theorem for determining distance,

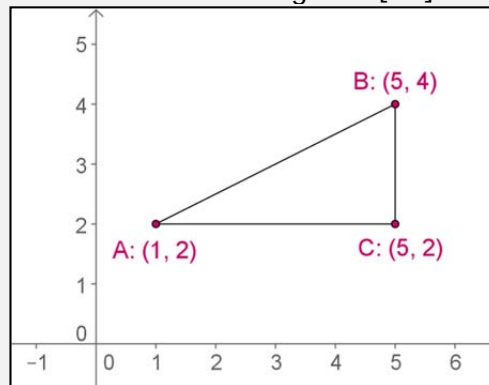
application of the length of a line segment formula and understanding the equation of a circle centred on the origin and not centred on the origin.

As examples of this, it has been observed that many students have difficulty with the following questions:

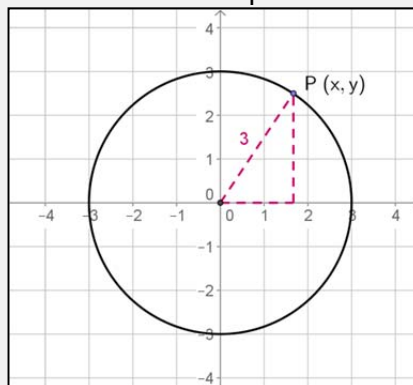
Write down the coordinates of C and F.



Calculate the length of [AB]



Write down a relationship between x and y.



We hope that by approaching the introduction to coordinate geometry in a particular way we will help to prepare students for such questions.

The syllabus states that students should be able to:

- coordinate the plane
- locate points on the plane using coordinates

by the end of first year. These are the specific learning outcomes we are focusing on but we are also trying to lay solid foundations for the related learning outcomes further along the sequence of learning.

Historically students find coordinating the plane relatively straight-forward, if a little boring. We hope that by presenting the lesson in a particular way we will obtain higher levels of engagement and understanding.

There are a few problems which students have on a recurring basis with coordinating the plane. The first problem is the order of the coordinates. We hope that by letting students understand coordinates for themselves this will not be such an issue.

The second problem that students have is with locating points on the axes. We hope that by students understanding the importance of a starting point when defining a coordinate system and that by understanding coordinates as distances from the starting point that they will naturally come up with coordinates of points on the axes themselves. This will be a good test of the approach we are using.

During the lesson emphasis will be placed on the use of suitable language for describing locations. For example we are going to encourage students to describe points as "across 3" or "back 2" or "up 5" or "down 1". We hope that this will give students a deeper understanding of what a coordinate is and will allow them to problem-solve around coordinate geometry more easily. While the lesson has a number of specific aims we also want it to fit into our overall approach to the teaching and learning of mathematics. We want students to learn to work independently of the teacher, to work together effectively, to try out different approaches, to be comfortable explaining their thinking and to recognize mathematics as useful and doable. We want students to enjoy their maths lesson and feel challenged by it.

## 6. Research

We are not certain as to the best language to use when describing the location of points. We had lengthy discussions as to whether we should draw from the language used in everyday coordinates systems (such as maps) and speak about moving north, south, east or west. We settled on the language of "across", "back", "up" and "down" but are still unsure if this is the best way to communicate the idea of a coordinate system to students. We will have to see how it works during the lesson.

We are going to use animated PowerPoint to support teaching and learning in the classroom. Throughout the lesson, students will work on short activities aimed at helping them understand the basic concepts of coordinates.

The common introductory course for first years states that students should learn to coordinate the plane and to locate points on the plane using coordinates. We think that this can be done in a way which makes students think about the underlying maths, how coordinates make sense and the usefulness of mathematics.

## 7. About the Unit and the Lesson

To start the lesson students will be presented with a simple task of describing the location of four points. Out of this task will come the ideal of using a map to locate points and what the important features of a map are: namely a grid and a starting point.

Students will then be given the opportunity to describe the location of a number of points using a numbered grid by describing how each point is located relative to the starting point. During this task it is hoped that students will develop their own language for describing the location of a point.

Students are then presented with the idea of coordinate notation as a shorthand version of their own language description. Students will be challenged to understand how this coordinate notation relates to their understanding of locating points and will be expected to become comfortable moving between their own description of a location, the mathematical notation used for same and how this location looks on a coordinated plane.

## 8. Flow of the Unit:

Lesson		# of lesson periods
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	Introduction to coordinate geometry	
1	<ul style="list-style-type: none"> <li>How to coordinate the plane</li> </ul>	1 x 40min.
2	<ul style="list-style-type: none"> <li>How to locate points on the plane using coordinates</li> </ul>	1 x 40 min.

## 9. Flow of the Lesson

Teaching Activity	Points of Consideration
<p><b>1. Introduction</b></p> <p>Four points, A, B, C and D are shown on the board.</p> <p>Students are asked to describe the position of these points.</p> <p>Students are asked to describe the position of A in relation to B.</p>	<p>Pupils may explain the position of the points of the board such as 'top left' or 'bottom right'.</p> <p>Can students use everyday language to describe the positions of the four points.</p>
<p><b>2. Posing the Task</b></p> <p>Students are asked what would make this task easier to complete.</p> <p>Students are asked if they know of any ways used to describe locations in the real world.</p> <p>Students are asked to discuss the key features of any map.</p> <p>Images of maps are shown on the board.</p> <p>The existence of a grid is highlighted.</p> <p>Pupils are asked what else is needed to make a map easy to use.</p> <p>Students are asked to describe the location of Wexford on a map of Ireland.</p> <p>Students are asked what these numbers on a map means?</p> <p>Pupils are asked to draw a horizontal line and a vertical line on the centre of their laminated graph paper using a marker.</p> <p>The teacher asks students to identify where the starting point on the map is.</p> <p>Students are asked to insert numbers onto the grid.</p> <p>The teacher describes each line as a number line.</p> <p>The number lines are inserted on the coordinated plane displayed on the board.</p> <p>Students are now asked to insert points A, B, C and D onto their grid.</p>	<p>Can students identify maps as a real-world example of locating points?</p> <p>Can students describe the key features of a map?</p> <p>Do students identify a grid as a useful tool when describing locations?</p> <p>Do students recognize that numbers or letters are also needed to make good sense of a map?</p> <p>Can students explain that the numbers on a map of the world refer to how far north or south you are from the equator and how far east or west you are from London?</p> <p>Do students understand the concept of a starting or reference point?</p> <p>Some pupils may not draw the axes in the centre of the page and some may not draw the axes on the heavier lines in the graph page. Make sure to check each page.</p> <p>Students may not have equal distances between each number nor include negative numbers.</p> <p>Can students look at the coordinated plane on</p>

<p>Students are encouraged to describe how they located each point on the plane with specific reference to the starting point.</p>	<p>the board and figure out where to plot the points on their own plane?  Can students describe how they knew where to locate each point?  Do students start to use their own description of each points location e.g. "out 3 and up 4" etc.?</p>
<p><b>3. Anticipated Student Responses</b>  Students should find the activity engaging as it is based on real-life and involves them solving some basic problems.  Most students should find the task fairly straightforward.  Some students might find it difficult to correctly place numbers on their planes.  Some students might find it difficult to transfer the four points on the board to their own coordinated plane.  Some students might find it difficult to explain their reasoning for drawing the four points where they did.  Some students will come up with their own language for describing how they located each point.  Some students might choose to talk about locating the points as a vertical distance followed by a horizontal distance.</p>	<p>Show-me board are of great benefit in this activity as they allow the teacher to monitor the progress of every student in the class. The teacher can target particular students for additional help or to share their thinking accordingly.</p> <p>While students have to understand the convention of <math>(x, y)</math> as opposed to <math>(y, x)</math> there is no need to have this discussion yet.</p>
<p><b>4. Comparing and Discussing</b>  The teacher asks different groups how they located each point.  The teacher encourages students to explain their thinking.  The teacher uses the animated PowerPoint to reinforce the idea of movement horizontally followed by movement vertically.</p>	<p>If different groups use different approaches to locating each point it is important to recognize these differences.  If some students choose to talk about the vertical distance before the horizontal distance the teacher should highlight this difference in approach and ask students if this is okay.</p>
<p><b>5. Posing the Task</b>  Students are presented with the short-hand notation for describing the location of point A.  Students are asked to explain what this short-hand notation means in their own terms?  Students are asked to name the three remaining points using the same short-hand notation.</p>	<p>Can students verbalise the meaning of the coordinates?  Do students use language such as "left 4 units", "back 4 units", "4 units in the negative direction" etc.?  It is important the teacher encourages students to read the short-hand notation as a description of the location of the points.</p>
<p><b>6. Anticipated Student Responses</b>  Students may not link the short-hand notation to the work they have previously done.</p>	<p>The opportunity may arise here to discuss the agreed order of coordinates. Students who</p>

<p>Students may not see the efficiency of the short-hand notation for locating points.</p> <p>Students may write the coordinates in the incorrect order.</p> <p>Students may miss out on including the appropriate signs on their coordinates.</p>	<p>treat the vertical distance first can be identified through their show-me boards. The teacher could show two conflicting answers to locating the same point. This could form the basis for a discussion on needing to agree an order for the short-hand notation.</p>
<p><b>7. Comparing and Discussing</b></p> <p>The teacher asks different groups to explain their choice of short-hand notation.</p> <p>The teacher highlights the efficiency of the short-hand notation in locating a point.</p> <p>The teacher highlights the problem with the short-hand notation in that the order of the coordinates must be agreed by everyone.</p>	<p>The teacher may use some simple examples to help students remember the order of coordinates. Some examples include “h comes before v in the alphabet so we always write the horizontal coordinate followed by the vertical coordinate” and “A plane must travel along the runway before it can rise into the air”.</p>
<p><b>8. Posing the task</b></p> <p>Students are presented with a table with a description of the location of a number of points using everyday language.</p> <p>Students are instructed to write the short-hand notation for the location of each point.</p> <p>Students are encouraged to explain their thinking.</p> <p>Students are encouraged to mark each point in on their coordinated plane.</p>	<p>This is a “think, pair, share” activity.</p> <p>Can students rewrite the locations using coordinate short-hand?</p> <p>Do students get the order of the coordinates correct?</p> <p>Do students relate the negative sign to “back” and “down” and the positive sign to “forward” and “up”?</p> <p>Can students explain why they wrote the coordinates in the way they did?</p>
<p><b>9. Anticipated student responses</b></p> <p>Most students should have little difficulty with this task.</p> <p>Some students will write the short-hand notation in the incorrect order.</p> <p>Some students may have difficulty with signs.</p>	<p>Students who list the coordinates in the incorrect order can be readily identified using their show-me boards. It is important that the teacher takes time to discuss with those students why this is now incorrect.</p>
<p><b>10. Comparing and Discussing</b></p> <p>The teacher asks all students to show their answers on their show-me boards.</p> <p>The teacher draws in an additional point on the coordinated plane and intentionally writes its coordinates in the incorrect order. The teacher asks students to identify the error and to explain why it is an error.</p> <p>The teacher draws in an additional point on the coordinated plane and intentionally writes its coordinates in the correct order but with incorrect signs. The teacher asks students to identify the error and to explain why it is an error.</p> <p>The teacher asks students if there is anything interesting about the final point in the table.</p> <p>The teacher identifies that this point lies on the vertical number line.</p>	<p>Did students successfully complete the task?</p> <p>Do students recognize the importance of the agreed order when writing locations using coordinate short-hand?</p> <p>Do students recognize the important role that signs play in a coordinate system?</p> <p>Do students understand that some points can lie on the number lines?</p> <p>Do students understand what a coordinate of the form <math>(0, y)</math> means?</p> <p>Do students understand what a coordinate of</p>

	<p>the form <math>(x, 0)</math> means?</p> <p>The teacher could take this opportunity to get the class to do some algebraic reasoning. Students could be asked what the coordinates of any point on the horizontal number line would look like and why this must be so.</p>
<p><b>11. Posing the task</b></p> <p>Students are presented with a table of coordinates and are asked to write out the long description of what these coordinates mean. Students are encouraged to plot each point on their coordinated plane.</p>	<p>This is a “think, pair, share” activity.</p> <p>Can students interpret coordinates in terms of everyday language?</p> <p>Can students use the every-day language as a support to graph each point on their coordinated plane?</p> <p>Can students interpret the coordinates of the point <math>(0, -4)</math> using everyday language and graph this point correctly?</p>
<p><b>12. Anticipated student responses</b></p> <p>Students may find it easy to describe the coordinates in everyday terms.</p> <p>Students may struggle with describing the meaning of negative coordinates.</p> <p>Students may find it difficult to interpret the coordinates of points on the number lines.</p> <p>Some students may still struggle with signs and order.</p>	<p>It is important that students are encouraged to discuss the meaning of the coordinates using everyday language.</p> <p>If students struggle interpreting the coordinates of the point on the number line, it is important to support these students in their thinking using appropriate questioning.</p>
<p><b>13. Comparing and discussing</b></p> <p>The teacher asks different groups to describe each pair of coordinates in their own terms.</p> <p>The teacher takes time to remind students of the importance of order and signs.</p>	
<p><b>14. Posing the task</b></p> <p>Students are presented with a list of coordinates and are asked to graph them on their coordinated plane.</p>	<p>This is a “think, pair, share” activity.</p> <p>It is important to encourage students to verbalise the meaning of each pair of coordinates.</p> <p>Some students may find it beneficial to draw lines out from the vertical axis and up from the horizontal axis to locate each point.</p>
<p><b>15. Anticipated student responses</b></p> <p>Most students should have little difficulty with this.</p> <p>Some students will have problems with order and signs.</p> <p>Some students may struggle with points located on either axis or the point located at the origin.</p>	<p>The teacher can use this activity to assess how the learning outcomes were met by everyone. It is important to support students in their thinking around points located on either axis or at the origin, with appropriate questioning.</p> <p>If students get different answers it is important to encourage them to understand the discrepancy.</p>
<p><b>16. Comparing and discussing</b></p> <p>The teacher asks different groups for their answers to each part of the task.</p> <p>The teacher commends the class for their excellent work.</p>	



## 17. Summing up

The teacher asks students what they have learned during the lesson.  
The teacher encourages students to summarise the most important concepts discussed in the lesson.

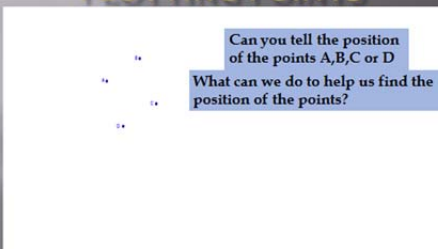
## 10. Evaluation

There will be three observers in the lesson along with the teacher. Observers will record instances of the following:

- student engagement with the lesson
- student understanding of the lesson content
- student difficulties with the lesson content
- if students work collaboratively as a group and stay on task while participating in group work
- if students ask questions of each other and of the teacher and the type of questions they ask
- if students are willing to explain their reasoning to the teacher and to the class

## 11. Board Plan


### PLOTING POINTS



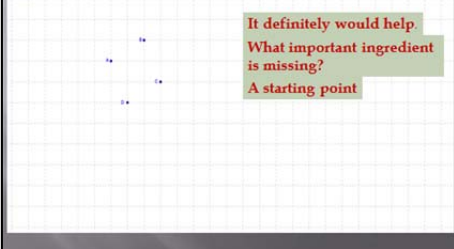
Can you tell the position of the points A,B,C or D

What can we do to help us find the position of the points?

Look at the following



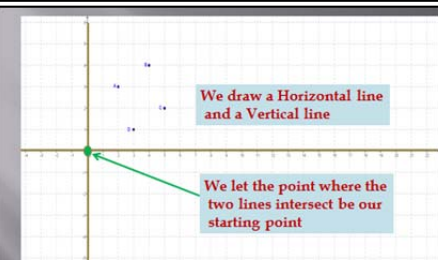
Would a grid help us?



It definitely would help.

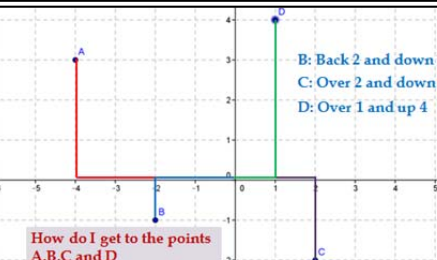
What important ingredient is missing?

A starting point



We draw a Horizontal line and a Vertical line

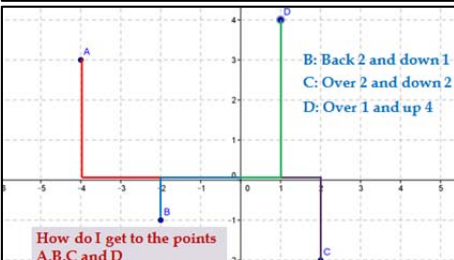
We let the point where the two lines intersect be our starting point



B: Back 2 and down 1  
C: Over 2 and down 2  
D: Over 1 and up 4

How do I get to the points A,B,C and D

A: Back 4 and up 3



B: Back 2 and down 1  
C: Over 2 and down 2  
D: Over 1 and up 4

How do I get to the points A,B,C and D

A: Back 4 and up 3

Complete the following table

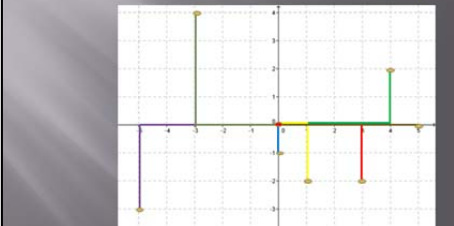
Description	Shorthand
Over 2 and down 3	(2, -3)
Back 3 and Up 5	(-3, 5)
Over 7 and Up 6	(7, 6)
Back 8 and Down 12	(-8, -12)
Back 5 and Down 5	(-5, -5)
Over 8 and Down 11	(8, -11)
Over 0 and Up 7	(0, 7)

Complete the following table

Description	Shorthand
Over 5 and Down 3	(5, -3)
Back 4 and Down 7	(-4, -7)
Over 3 and Up 9	(3, 9)
Over 5 and Down 3	(5, -3)
Stay and Down 4	(0, -4)
Over 8 and stay	(8, 0)
Back 4 and Down 6	(-4, -6)

Using the same Axes or Grid, plot the following points

(3, -2), (4, 2), (-5, -3), (0, -1), (1, -2), (-3, 4), (5, 0), (0, 0)



## 12. Post-lesson reflection

The lesson proved a great success with students engaged in the lesson at all times. Using show-me boards worked really well as it helped to keep all students on task and allowed the

teacher to observe which students were having difficulties with each task.

In spite of our best efforts and perhaps partly because of the approach we used, many students still had difficulties remembering the order in which coordinates should be stated and also the need for this convention. In hindsight we think it might be good to get students to use their show me boards to write the coordinates of a particular point and let students see that different students write the coordinates in different ways i.e.  $(x,y)$  and  $(y,x)$ . The point could then be made that this represents a problem and that we need to agree on the order of coordinates. A simple memory aid like "An airplane must travel horizontally before going up" might be useful for helping students remember the agreed order.

It was surprising to us that students used the term "axis" when discussing the task presented to them. The teacher specifically did not mention this word and yet many students were aware of it already. Where did this knowledge come from? Perhaps from primary school or from science class.

Students were presented with pre-labelled axes for most of the tasks they worked on. Towards the end of class, students were presented with un-labelled axes for one task and many of them struggled with filling in values on the axes. This was surprising as we almost took it for granted that students would be able to do this. Many were not.

Students liked the use of the terms "over", "back", "up" and "down" and could readily use this language to describe the location of points.