

Reflections on Practice

*Topic Of Lesson : A Quick Way To Add
Numbers - Strand 3*

Year Group : 5th Year - Ordinary Level

For the lesson on 8/2/2016

At St. Flannan's College, Martina Scully' class
Teacher: Martina Scully

Lesson plan developed by: Clare Reflections on Practice Group: Desmond Fry, Anna Quilter, Olive Dillon, Pearse Ryan, Peggy Lynch and Martina Scully.

Lesson Plan

1. **Title of the Lesson: A quick way to add numbers**
2. **Brief description of the lesson:** We want students to come up with as many ways as possible to sum numbers from one to ten. We want students to recognise the limitations of number. We aim to generalise the sum from one to any number. The following resources will be available to students linking cubes, desk mats, “shut the box” games, coloured disks, calculators, pen and paper.
3. **Aims of the Lesson:** The main aim for students is to realise the value of algebra and that there can be limitations to using number only. Students should experience a learning experience which illustrates that there are many different ways to solve a problem. We want to students to feel comfortable in their uncomfotableness – to recognise that they are very able and utilise their mathematical abilities to solve unfamiliar mathematical problems.

Examples of long-range/thematic goals:

- I’d like my students to realise the limitations of using numbers only and to appreciate the benefit of algebra.
- I’d like my students to derive formula in general and why we use formulae.
- I’d like students to have ownership of their own work.

Examples of short-term goals(content goals specific to the lesson)

- I’d like my students to sum from one to ten in as many ways.
- I’d like my students to derive a formula for the sum of arithmetic terms.
- I’d like my students to realise the limitations of using numbers only and to appreciate the benefit of algebra.
- I’d like my students to understand why we use the formula for the sum of an arithmetic sequence.

4. **Learning Outcomes:**

As a result of studying this topic students will be able to:

- Sum 1 to n and n to m
- See the need for algebra
- Approach problem solving using a variety of different methods.

5. **Background and Rationale**

The focus of the lesson is introducing students to how to sum a number of numbers. The rationale includes the need for students to realise the value of algebra while encountering restrictions that can arise with number. This lesson can be taught in conjunction with patterns, sequences and series.

6. **Research**

The planning team spent a considerable amount of team brainstorming solutions to the problem. Resources used for research included text books, the internet, the syllabus and discussion among fellow maths teachers. Hands-on resources were used when attempting to solve the problem to encourage a wide range of solutions.

7. About the Unit and the Lesson

This lesson can be taught as part of Strand 3: Number and is aimed at an Ordinary Level class group. An excerpt from the syllabus reads as follows:

Strand 3: Number

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to
3.1 Number systems	<ul style="list-style-type: none"> - appreciate that processes can generate sequences of numbers or objects - investigate patterns among these sequences - use patterns to continue the sequence - generate rules/formulae from those patterns 	<ul style="list-style-type: none"> - generalise and explain patterns and relationships in algebraic form - recognise whether a sequence is arithmetic, geometric or neither - find the sum to n terms of an arithmetic series

(Syllabus for Leaving Certificate Maths p.31). Also the end of each strand refers to enhancing problem-solving skills.

Students learn about	Students should be able to
4.5 Synthesis and problem-solving skills	<ul style="list-style-type: none"> - explore patterns and formulate conjectures - explain findings - justify conclusions - communicate mathematics verbally and in written form - apply their knowledge and skills to solve problems in familiar and unfamiliar contexts - analyse information presented verbally and translate it into mathematical form - devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

8. Flow of the Unit:

Lesson		# of lesson periods
	We decided to teach this lesson as a stand alone lesson. The students have recently finished Statistics and have experience of approaching mathematics problems in a variety of ways.	80 minutes research lesson
	The lesson gives way to an introduction to the value of algebra and is a useful start point for looking at series in patterns. The following lesson will look at summing.	

9. Flow of The Lesson	
Teaching Activity	Points of Consideration
<i>How to add numbers quickly</i>	
1. Introduction <i>Sum = adding</i>	
2. Posing the Task Task one: Sum up the numbers from one to ten in as many ways as you can	<i>Encourage students to ask questions at this stage before the task begins. Engage some questions of students to ensure they understand the task.</i>
3. Anticipated Student Responses R 1: $1+2+3+4+\dots+10=55$ using mental arithmetic and/or a calculator. (1 to 10 and 10 to 1). R:2 Rectangle with 11 rows and 10 columns or vice versa based on the triangular pattern of the linking cubes. R.3: Pairing R.4: Factors R.5: 5(11)	<i>All students should be able to access this task due to R1. Students who arrive at the solution will then be encourage to continue solving the task in a variety of different ways. The hands on resources will be available to the students. Students who arrive at the formula at this stage will not be informed that this is the end goal (as the variety of tasks engaged in is also critical). When solutions are selected for boardwriting those who arrived at the formula will not be selected for presentation of this until task 2.</i>
4. Comparing and Discussing <i>The order of the anticipated responses above (R1 TO R5) is the suggested order for the boardwriting.</i>	<i>Student's engagement is critical at this stage as they are given the opportunity to display and present their work and engage in a dialogue with their peers and teacher.</i>
2. Posing the Task Task Two: Sum up the numbers from one to one hundred in as many ways as you can.	
3. Anticipated Student Responses R.1: 2ND difference = constant R:2: median(n) R:3: $n(n+1)/2$ R:4: $n/2(n+1)$	
4. Comparing and Discussing	<i>The discussion here will focus on how the students work generates a formula.</i>
2. Posing the Task Task three: Sum 51 to 100.	<i>This will be given as a homework task if necessary.</i>
3. Anticipated Student Responses R1: $51+52+53+\dots+100$ using calculator R2: Sum 1 to 100 and sum 1 to 50 using a formula. Calculate their difference.	
4. Comparing and Discussing	<i>Discussion of these solutions can be looked at in detail in the next class.</i>
Summing Up Nest day activities will look at summing numbers in different patterns i.e even numbers, odd numbers....	

10. Evaluation

All team members are involved in observing the lesson. Photos of the students' work will be taken to document what is happening. Notes of the students' work will be made as relates to the anticipated responses. The observing team members will not interfere in the class and will not interact with the students once the class has begun.

11. Board Plan

The board plan will run over two boards from left to right and will build from the introduction, through task 1, task 2 and task 3 using the actual responses as anticipated in the above order.

12. Post-Lesson Reflection

The teacher teaching this lesson did an excellent job of holding everything together, monitoring and observing the students' work, selecting students to present their work and encouraging students to put their ideas together. With encouragement the students arrived at the concept of $5(11)=55$.

This lesson had several downfalls. The first of which was the reduction of the lesson time from the anticipated 80 minutes to 55 minutes due to an unanticipated change in timetabling due to pre examinations. As a result the learning in task 2 was a little rushed and task 3 was not reached.

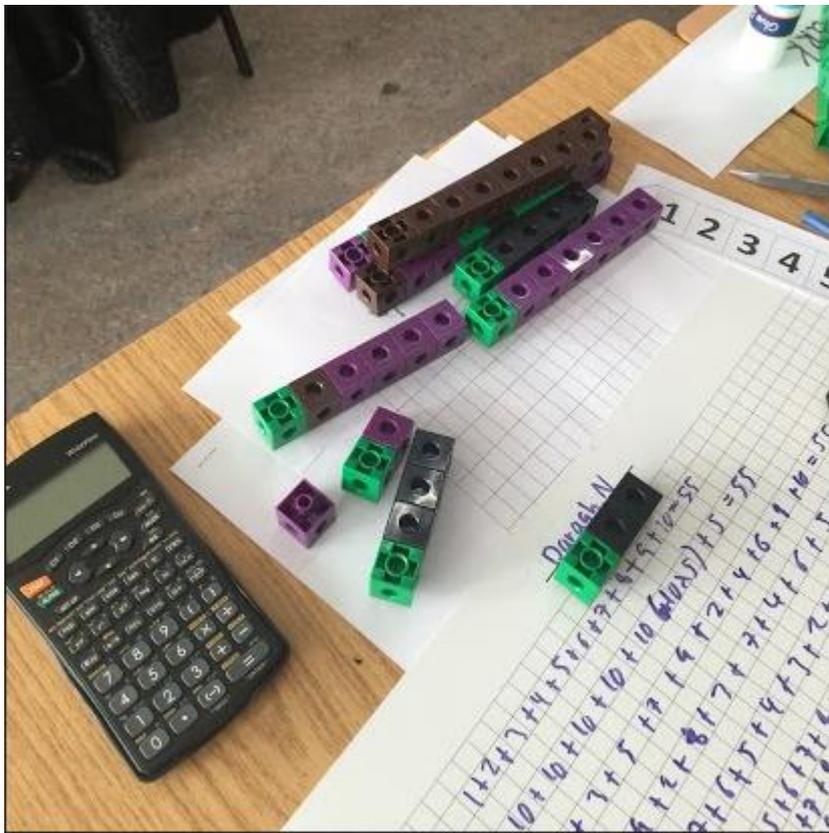
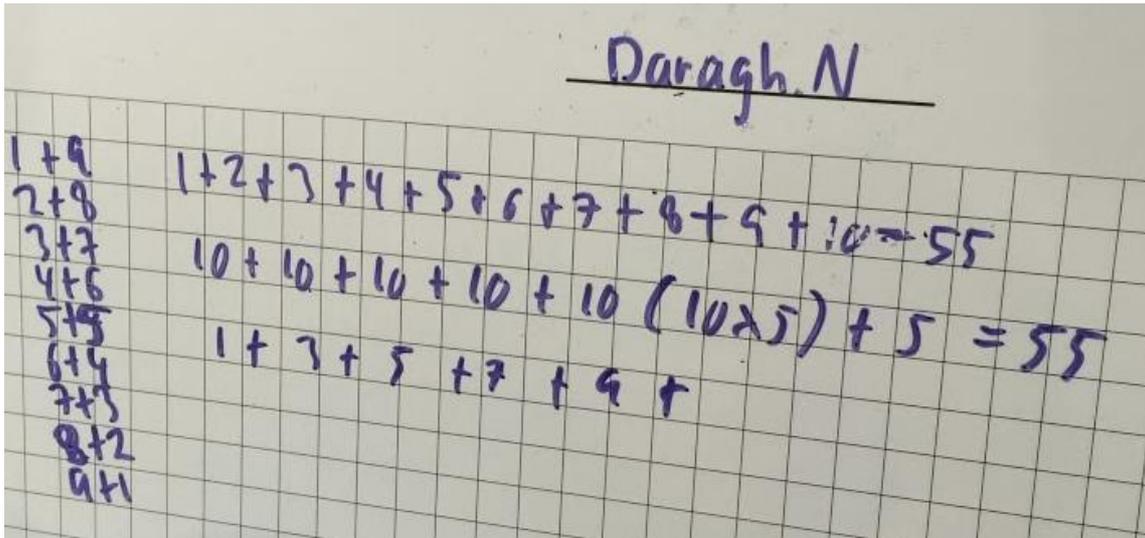
The students appeared unnerved by the number of observers which was indeed a little disproportionate to the number of students (18 students and 6 observers). The students appeared very self conscious and did not act in the confident fashion they normally demonstrate with their teacher in class. Students reported afterward to their teacher that they found the photographing of their work extremely intimidating. The group latterly felt that perhaps because these were senior cycle students they felt more self conscious when observed?

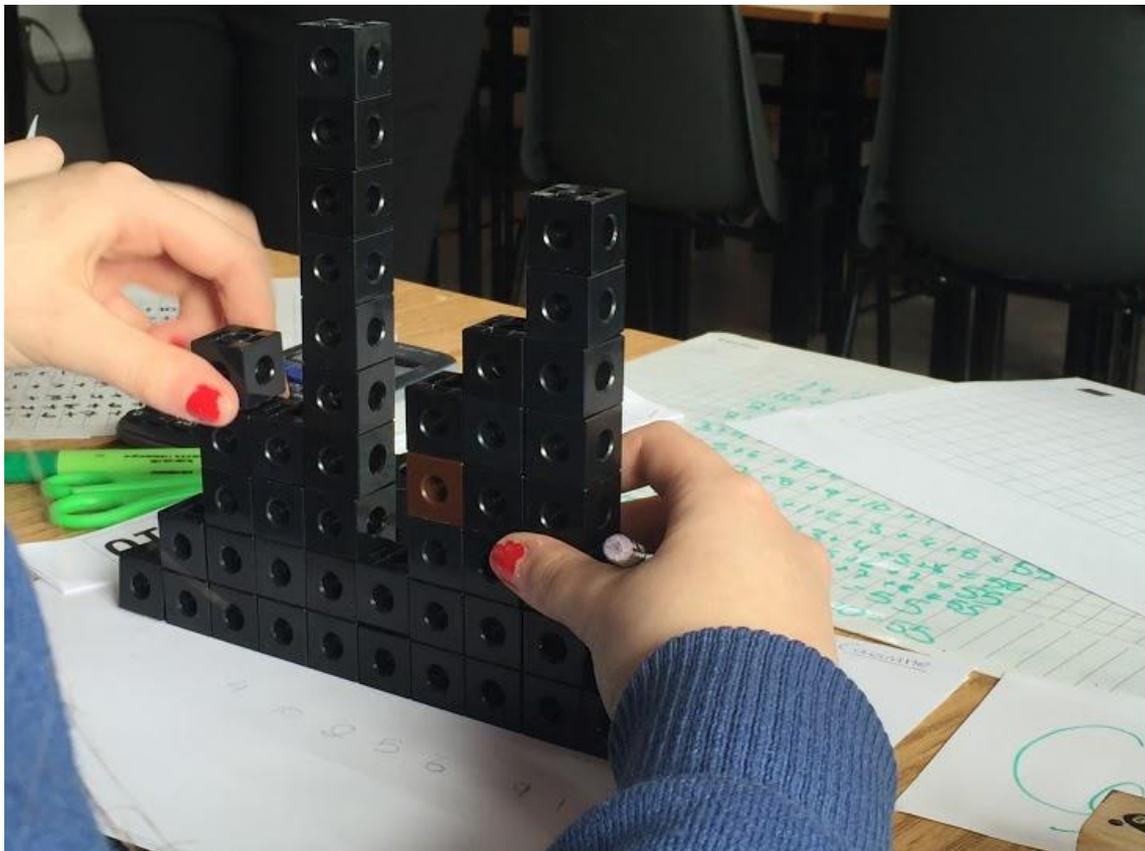
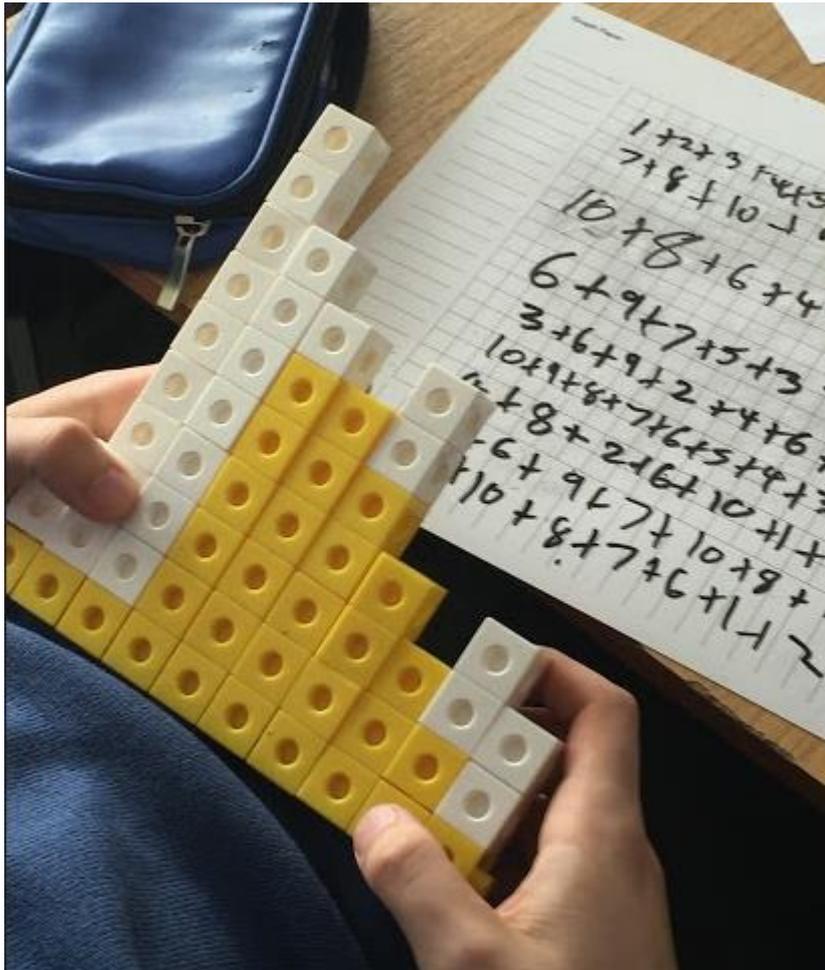
With some careful scaffolding by the teacher during the board-writing stage of task 1, task 2 was much more successful and students were able to utilise their knowledge of pairing etc to solve this in a variety of ways. For homework the students were asked to derive a formula for summing sequential numbers from 1 to 10.

During the post lesson discussion it was decided that students had difficulty with the concept of sum which the planning group had not anticipated in advance. As a result students focused on adding numbers repeatedly and in a variety of combinations (often adding random pairs such as $4+10$ etc) with little focus on what these numbers added up to. The group also realised that while the students were asked to sum the numbers from 1 to 10 in a variety of ways, the word 'ways' led to confusion. While the planning group meant methods the students focused on the arrangement of numbers.

A group member tried this lesson with a different group of students, mixed ability transition years, with huge success and many students arriving at the formulae. Key differences were a 70 minute lesson (which was still tight on time and task 3 was not arrived at in class), time spent discussing the misconceptions mentioned above relating to 'sum', 'ways' etc and no intimation for senior cycle students by observers. This lesson plan was very successful when this amendments were made.

13. Pictures of Student Work







100 pairs
100 pairs

100 pairs

Fact
 1, 3, 6, 10

Quadratic:
 $T_n = an^2 + bn + c$

Homework:
 Derive a formula for the sum of the numbers 1 - n

Each pair = 101 100 numbers
 $\frac{100 \text{ pairs}}{2} \times 101$
 $\frac{100 \text{ pairs}}{2} \times \text{Sum of the pairs}$
 50 pairs *