Finding the minimum combined area possible by sizing two rectangles

Ardscoil Na Mara, Tramore
Third Year, Higher Level

For the lesson on 10/3/2016
At Ardscoil Na Mara, Emily Campbell's third year higher level class
Teacher: John Hartery
Lesson plan developed by: Emily Campbell, John Hartery, Gobnait Ni Scanaill, Erica Owens and Ciara Power

1. **Title of the Lesson:** Finding the minimum combined area possible by sizing two rectangles.

2. **Brief description of the lesson:** A problem solving lesson involving rectangular area, patterns and functions.

3. **Aims of the Lesson:**

Long term:

I'd like my students to appreciate that mathematics can be used to solve real world problems
I'd like my students to appreciate that algebra is a tool for making sense of certain situations
I'd like to foster my students to become independent problem solvers
I'd like my students to become more creative when devising approaches and methods to solve problems
I'd like to emphasise to students that a problem can have several equally valid methods of solution
I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities
I'd like my students to connect and review number patterns in a different context

Short term:

I'd like my students to understand that noticing patterns and using algebra are both relevant techniques that can be applied to problems that initially may seem unconnected.
I'd like my students to be able to represent the area of a rectangle (or other 2D shape) using variables.

4. **Learning Outcomes:**

As a result of studying this topic students will be able to:
Write an expression in variables for the area of a rectangle and find the value of the variable that minimizes (or maximizes) that area.

5. **Background and Rationale**

We recognize that students are challenged by spatial reasoning and particularly by geometry problems in an unusual context.

From the geometry and trigonometry section (page 20, strand 2) of the junior cert syllabus:

Students learn about	Students should be able to
2.5 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

6. Research

We researched Irish students' performance in state exam questions and also in recent PISA studies and found that Irish students do not perform as well in "space and shape" problems.
We decided to look for a problem involving geometry that is suitable for junior cert maths students. One group member suggested a version of a question from the 2015 junior cert sample paper 2 and this is what we ended up working on.

7. About the Unit and the Lesson

This lesson spans a few units; namely patterns, functions and geometry.
This third year class group worked on number patterns in first and second year and should be familiar with creating tables and graphs. They have also covered the concept of a function.

8. Flow of the Unit:

From page 26 of 3rd year teacher's handbook

Section 4: Algebra

Lesson Idea 3.21

Title
Relations approach to algebra- revision and extension from second year material

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to develop generalising strategies and ideas (considering those of others), present and interpret solutions, explaining and justifying methods, inferences and reasoning in the following:
    - The use of tables to represent a repeating pattern situation
    - Generalise and explain patterns and relationships in words and numbers
    - The use of tables, diagrams , graphs words and formulae as tools for representing and analysing linear patterns and relations
        - Discuss rate of change and the y - intercept. Consider how these relate to the context from which the relationship is derived and identify how they can appear in a table, in a graph and in a formula.
        - Write arithmetic expressions for particular terms in a sequence
        - Find the underlying formula written in words from which the data is derived
        - **Find the underlying formula algebraically from which the data is derived (linear, quadratic)**

From page 32 of 3rd year teacher's handbook

Section 5: Functions

Lesson Idea 3.28

Title
Functions: interpreting and representing linear, quadratic and exponential functions in graphical form

- **How to find maximum and minimum values of quadratic functions from a graph**

Lesson		# of lesson periods
1	Minimise area of 2 rectangles	(#1 = research lesson)
2	Further investigation of area/quadratic questions	2 x 40 min
3	Related questions with circles, volume etc.	1 x 40 min.

9. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction Is 3 + 4 = 4 + 3? Is there a general form of this? To emphasise we like to generalise mathematical results so they can be applied in multiple cases. *Very quick check to make sure all students know how to get the area of a rectangle*	Yes! a + b = b + a (written by teacher if no student gives it) Ensure students work with area, not perimeter.
2. Posing the Task Power point slides on garden problem. Allowing 12 minutes for individual student work on the problem.	Additional student questions for clarification? Teacher check round each student's initial work to ensure they have started and understand task.
3. Anticipated Student Responses R1: <u>Trial and improvement</u>: student tries a few different positions of the lamp, works out the area of grass each time and picks out the minimum. They may draw some positions out on paper provided. R2: <u>Visual inclination</u>: student who reckons maximum patio area is when the lamp is halfway along to make two equal area sections of patio. They then position the lamp at 5m and work out area of grass. R3: <u>Ordered list/table</u>: student moves position of the lamp along in one metre increments, works out the area of grass each time and picks out the minimum.	Students were encouraged to take risks with their problem solving? MATH -Mistakes Allow Thinking to Happen.. no single right answer but many approaches were encouraged.. go beyond and see how many ways you can solve this problem Observe student work and offer individual suggestions Can you see a pattern? How can this pattern help you find an answer? How did you find that out? Why do you think that? Who has the same answer as this? Who has a different solution? Are everybody's results the same?

R4: Graph from table: R5: Algebraic expression: Creates an algebraic expression of the form: Area = $x^2 + (10 – x)^2$. Substitutes values in for x and picks out the minimum. R6: Graph from algebra: Creates an algebraic expression of the form: Area = $x^2 + (10 – x)^2$. Substitutes values in for x, graphs area versus x and picks out the minimum. R7: from a list of numbers spots that area is a quadratic function and uses numbers to work out a, b and c (for $ax^2 + bx + c$).	
4. Comparing and Discussing We would be prepared to go through responses R1 to R6. Working towards presenting R6 as the "most sophisticated" and transferable approach.	Results were recorded by student and students were invited to the board in the order from R1 Student answers were pinned to board for class to see and teacher initiated questions Students were invited to comment on the work, the answers from student at the board and compare to their own methods
5. Summing up Homework?	Attempt at structured problem solving Recap steps in stating problem, outline the results achieved and the results expected. Who had a different solution? Are everybody's results the same? Why/why not? What would happen if we changed side to 100m? Benefit of a rule/formula. Trigonometry was a solution from students which was not in our original result. Short exploration of this. Quick demonstration of how students could extend the lesson themselves using alternative result methods. Some generalisation of method from example to rule and reference to mathematical method of solution

10. Evaluation

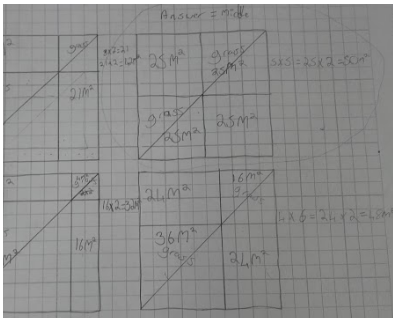
- There will be 6 other teachers in the room. The lesson will be done in room FF10 (more space, massive whiteboard)
- 5 observers will remain stationary and monitor 6 students each using the observation sheet as in appendix A. One observer will walk around the room with a camera to take pictures of student's work.
- Students will hand up all their work at the end of the lesson for closer analysis.

11. Board Plan

R1

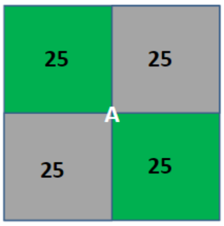
R1: Trial and Improvement

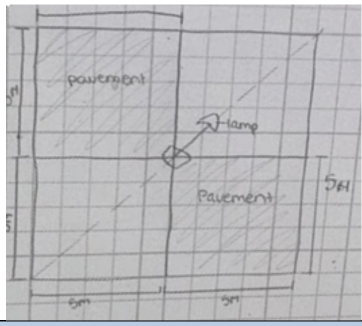
- X 10 + 0 X 0
- ‹ 9 + 1 X 1
- 8 + 2 X 2
- 7 + 3 X 3
- 6 + 4 X4
- 5 + 5 X 5
- 4 X 4 + 6 X 6
- 3 X 3 + 7 X 7
- 2 X 2 + 8 X 8
- 1 X 1 + 9 X 9
- 0 X 0 + 10 X 10



R2

R2: Visual inclination



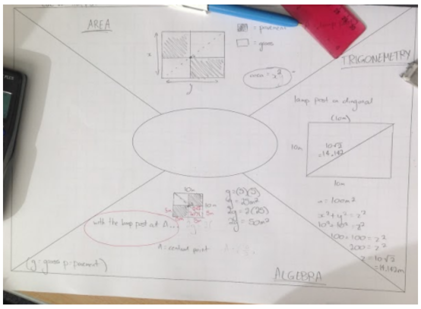


R3

R3: Ordered list/table:

Metre	Area 1	Area 2	Total Area
0	0	100	100
1	1	81	82
2	4	64	68
3	9	49	58
4	16	36	52
5	25	25	50
6	36	16	52
7	49	9	58
8	64	4	68
9	81	1	82
10	100	0	100

R4



R5

R5: Algebraic expression

- Area = $10^2 + 0^2$
- Area = $9^2 + 1^2$
- Area = $8^2 + 2^2$
- Area = $7^2 + 3^2$
- .... To
- Area = $(10- x)^2 + x^2$

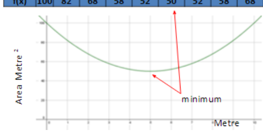
- Area = $(10- x)^2 + x^2$
- Sub in values
- Find 5 as minimum

R6

R6: Graph from Algebra

- Area = $(10- x)^2 + x^2$
- Table

x	0	1	2	3	4	5	6	7	8	9	10
x^2	0	1	4	9	16	25	36	49	64	81	100
$(10-x)^2$	100	81	64	49	36	25	16	9	4	1	0
f(x)	100	82	68	58	52	50	52	58	68	82	100



R7

List of numbers, quadratic function, find a,b,c & $T_n = an^2 + bn + c$

100	82	68	58	52	50	52	58	68	82	100

ff -18 -14 -10 -8

Diff 4 4 4

$T_n = an^2 + bn + c$ from above 2a = 4, a =2

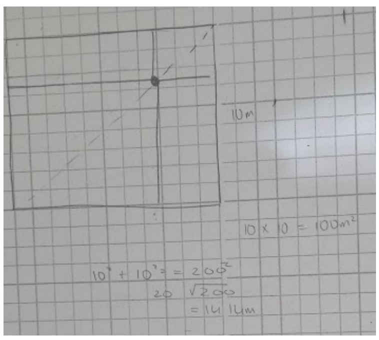
$T_n = 2n^2 + bn + c$

$T_0 = 2(0)^2 + b(0) + c = 122$, c =122

$T_1 = 2(1)^2 + b(1) + 122 = 100$, b = -24

$T_n = 2n^2 - 24n + 122$

Trigonometry approach that we had not foreseen.



12. Post-lesson reflection

Due to unforeseen circumstances, the lesson was taught to a class by a teacher who is not the class's normal maths teacher.

- Possibly due to their being in unfamiliar surroundings, understanding the problem was an issue. To avoid this being an issue again, more time could be spent explaining that the lamp can be placed anywhere along the buried cable.

- The vast majority of the work done by the students was on drawing different sized triangles. This suggests a strength in this area, but possibly a need for work on other techniques. We were especially surprised we didn't see more ordered lists or tables.

- Students recognised the quadratic pattern formed when dealing with area and were happy that this happens naturally in real life contexts.

- Time constraints on the day meant the learning goals were not fully achieved in the 40 minutes. The problem was revisited afterwards and students worked on writing an expression in variables for the area of the rectangles.

- There was good interest and positive feedback from the students afterwards. One student said "we learned more by doing it this way"

Appendix A: Observation sheet used by observing teachers

DURING INDIVIDUAL STUDENT PROBLEM SOLVING TIME: **Observe student responses.**						
	Student 1	**Student 2**	**Student 3**	**Student 4**	**Student 5**	**Student 6**
(i) Understood problem						
(ii) Drew relevant sketches						
(iii) Made a table						
(iv) Drew a graph						
(v) Created an algebraic expression						
(vi) Graphed an algebraic expression/function						
(vii) General term of quadratic						
Other observations						

DURING BOARD WORK/CEARDAIOCHT: **Observe student engagement with what is presented on the board.**						
	Student 1	**Student 2**	**Student 3**	**Student 4**	**Student 5**	**Student 6**
Rate student engagement with board work. Scale 1-3 where : 1= poor						

2 = some engagement 3 = completely engaged						
Other observations						
Issues that need to be addressed in the next class						
Recommended changes to lesson plan						