

## *Lesson Title: Deepening the understanding of the properties of a parallelogram*

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**Group:** Second Year Mixed Ability

**Topic:** Geometry

**School :** St. Tiernan's Community School

**Lesson Teacher :** Deirdre Newell

**Lesson Plan Developed By:** Sandra Gilmore, Thérèse Ruane, Deirdre Newell and Lynn Anderson.

### *Lesson Plan for Second Year*

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#### AIMS OF THE LESSON:

This lesson should create opportunities where:

- Students can express their thoughts in logical steps using words, numbers, diagrams, tables, graphs and activities using concrete models.
- Students can deepen their understanding of shape and properties, students can develop the skill of listening to each other's thoughts by identifying the differences and similarities in their methods.
- Students can learn that there are a variety of ways to solve problems and they are all equally valid.
- Students can develop an appreciation that mathematics can be used to communicate thinking effectively through the effective use of mathematical language and terminology
- Students can realise that algebra is a tool for making sense of certain situations
- Students can become independent learners
- Students are given the opportunity to become more creative when devising approaches and methods to solve problems
- Students' enthusiasm for the subject can further develop by engaging them with stimulating activities

- Students can identify, connect and review the concepts that we have studied already

#### LESSON SPECIFIC GOAL:

Students should identify what makes a parallelogram special and be able to identify the similarities and differences when comparing parallelograms to other quadrilaterals.

#### LEARNING OUTCOMES

As a result of studying this topic students will be able to:

- Make connections between diagonals and parallelograms (key words: bisection, midpoint)
- Identify the unique attributes of parallelograms through analysis and problem solving, leading to the comparison with all other quadrilaterals (reinforcement: Homework question and lesson 2)
- Identifying quadrilateral types based on the criteria and through developing the ability to spot differences and similarities through comparison
- Using and connecting synthetic geometry in a coordinate geometry problem

#### BACKGROUND AND RATIONALE

According to the curriculum students who study the geometry strand will

*“construct a variety of geometric shapes and establish their specific properties or characteristics solve geometrical problems and in some case present logical proofs, interpret information presented in graphical and pictorial form, analyse and process information presented in unfamiliar contexts, select appropriate formulae and techniques to solve problems” (NCCA,2013)*

The formal proofs Theorem 9 and theorem 10 are dependent upon a student’s understanding about its basic characteristics and those of parallel lines. Students are often informed of these connections and have to accept them as true with little or no reasoning/ investigation. Through this lesson and building upon students own investigative and logic work, it is intended that they will learn from each other to develop a thorough understanding of the value of properties of shapes in particular a parallelogram.

## RESEARCH

*“Investigating links with primary school curriculum, familiarising ourselves with their use of language and understanding of shapes”*

First and Second year Teacher Handbooks, based on the 2016 syllabus

## UNIT OF STUDY: COORDINATE GEOMETRY OF THE LINE

### FLOW OF THE UNIT:

Lesson		# of lesson periods
CIC	Students have learned how to plot points, find the midpoint by formula and to observe translations	7
1	Revision of reading and Plotting points, the quadrants	1
2	Midpoint Formula revision, focusing on coordinate and integer work	1
<b>3</b>	<b>Reflection on Practice Lesson: Problem Solving</b>	<b>1</b>
4	Reinforcement: comparison of other quadrilaterals, using their properties to solve problems	1
5	Slope: An introduction	1

### FLOW OF THE LESSON

Teaching Activity	Points of Consideration
<b>1. Introduction</b> <b>Recap of prior knowledge to include reading a point from a Cartesian plane, Formulae we remember from last year and key properties of a parallelogram(if known)</b> <b>5min</b>	Students will have been asked to bring in set square, ruler, compass and scissors for class. They have all completed the CIC. They also have an additional knowledge of calculating the midpoint, distance between two points, and plotting points.  The title of the lesson will not be given to the students as it will provide too much of a lead and make take away from the advantages of investigative learning.

<p><b>2. Posing the Task</b>  <i>E is equidistant from all four vertices. Find the coordinates of E in as many ways as possible</i>  <b>3min</b></p>	<p>Give students an appropriate time to read through the question after the teacher has read it through once before.</p> <p>Clarify any issues with the language, e.g., what does equidistant mean? What are vertices? What is a vertex?</p> <p>The students will be given at least three occasions to express if they are having difficulty or require clarification of the problem.</p>
<p><b>3. Anticipated Student Responses</b>  <b>See Appendix 1</b></p> <p><b>10 min students work</b></p>	<p>R1: Cut out the parallelogram and use the diagonals to find the centre point.</p> <p>R2: Use measurement to find the midpoints of sides and midpoints of these</p> <p>R3. Find the midpoint of the diagonal using measurement.</p> <p>R4: Bisected a diagonal to get the midpoint.</p> <p>R5: Formula used to find the midpoint of the diagonal.</p> <p>R6: Use the knowledge that the diagonals bisected each other to get the equidistant point</p>
<p><b>4. Comparing and Discussing 15 min</b></p>	<p>Students will develop their language skills through explaining how they did and what they did to find the solution. They will provide the reasons behind each action.</p> <p>This creates an opportunity where they can develop appreciation for the variety of different approaches each of equal value.</p> <p>The progression will be from the most basic solution to the most advanced based on the lesson plan aim, using the parallelogram properties to solve problems.</p>
<p><b>5. Summing up</b>  <b>5 min Reflection</b></p>	<p>Students will reflect on what They learned today?</p> <p>Did they learn anything at the beginning from the prior knowledge?</p> <p>What did they learn from solving the problem?</p> <p>What did they learn from listening to other students explain their solutions to the problem?</p>

## EVALUATION

- Teachers observing will take responsibility for particular students.
- Students' work will be recorded by photographing, any questions will be recorded and the way the work of each student developed will be recorded including any evidence of self-correction.
- Teachers involved developed a student observation record sheet to assist in the recording of all work.
- In preparation for this work, all teachers observing are given a copy of the lesson plan and are expected to be familiar with the aims and goals of the lesson.
- Lesson Note will not be used but we will aim to record data for students in a chronological order, in order to record each student's development during the task.
- Teachers involved will assess the knowledge of parallelograms at the beginning of the class, the students' demonstration of knowledge of different approaches to solving the problem and record any evidence of Self correction.
- The evidence we intend to gather will be: Student work, photographic evidence of board, photographic evidence of student work, interesting questions arising from the task, student misconceptions and any evidence of deep learning.



### POST-LESSON REFLECTION

- Students were highly engaged and enjoyed the challenge set.
- All students found the correct solution and were all able to validate their work using at least one alternative method.
- Students were given an opportunity to self –correct (incorrect use of brackets in distance formula)
- Students were given an opportunity to provide solutions and verbalise reasons behind answers given.
- Students found the whole class discussion interesting, they enjoyed listening to other students' explanations and expressed any difficulties they had in understanding alternative methods.
- Students initially were slow to discuss the properties of parallelograms, however once they gained confidence most properties were identified.
- Students had a very clear understanding of the task
- Most students solved the problem on the first go by the construction of the two diagonals, the challenge then became to find another valid solution.
- No misconceptions arose during this problem.
- The value of selecting an appropriate problem and creating a forum for discussion is immense. Students were intrigued by their classmates' solutions.
- The students did come with two alternative solutions that we had not anticipated. One student constructed the diagonal and used the distance formula to find E. Another student used the division of a line segment to find the midpoint of each side and connected these to find E.

### RECOMMENDATIONS

- Time

The length of time of the lesson proved challenging at the end. To create a greater opportunity for discussion the lesson should be approximately 5-10 min longer. This should allow for students to verbalise what they learned, to compare and analyse the different types of solutions.

The worksheet could include a reflection for the end of the lesson, or this could be done in the students exercise copy.

- Questions that we asked at the end of the class?
  - What did you learn from doing this task?
  - List four things you know about parallelograms?
  
- Boardplan

Although the idea of a boardplan is a great one it proved difficult to implement due to the limitations of Irish classrooms.(board space/room space)

- Observation Sheets

Observation record sheets proved too detailed and difficult to use.

Substantial photographic evidence and main events were noted providing a good record of the lesson. (LessonNote could ease recording)

If we were conducting a research lesson again we would design less complicated observation sheets and focus more on gathering additional photographic evidence.

## Appendix 1: Anticipated Students responses:

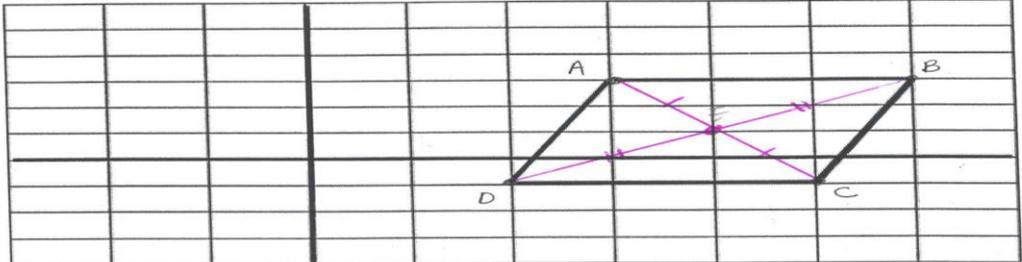
(using original worksheet), the worksheet which will be used in the lesson is in the positive quadrant. This was adjusted so that students who have difficulty with integers would not be penalised for this.

*Construction*

Geometry Investigation:

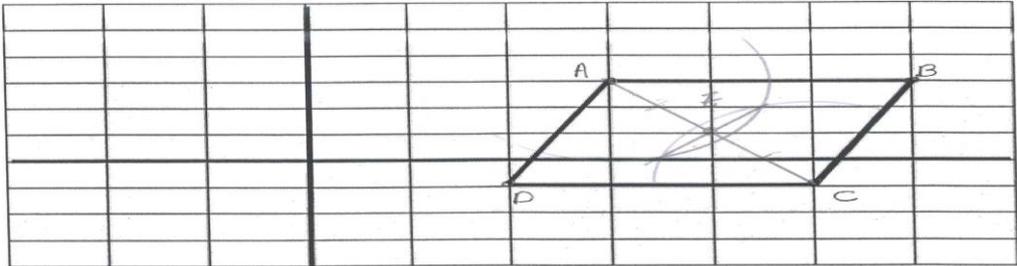
In as many ways as possible find the coordinate E, given that E must be equidistant from all the vertices of the parallelogram ABCD. Please explain each method.

*Constructed diagonals*



Explanation:  
 I knew diagonals bisect each other in parallelogram. so I drew  $[AC]$ ,  $[BD]$ . E was the point I was looking for.

*Bisected*

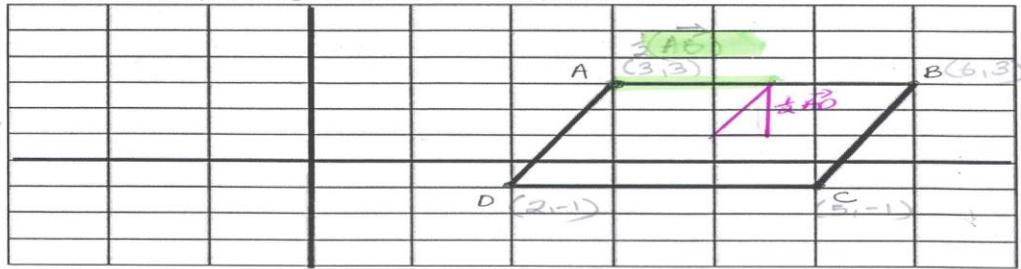


Explanation:  
 I bisected the diagonal  $[AC]$  to get the midpoint.  $[AC]$  is equidistant from B and D. The point I was looking for is their point of intersection.

Geometry Investigation:

In as many ways as possible find the coordinate E, given that E must be equidistant from all the vertices of the parallelogram ABCD. Please explain each method.

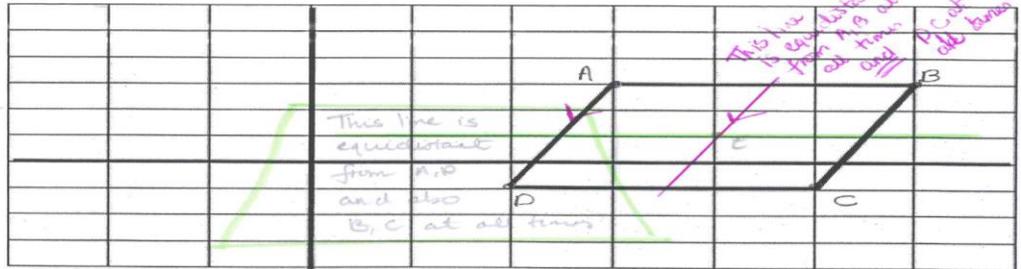
Translation.



Explanation:

Using the fact that opposite sides do exactly the same thing, this means that equidistant from half of the movement AB and half of movement AD

Properties of parallel lines.



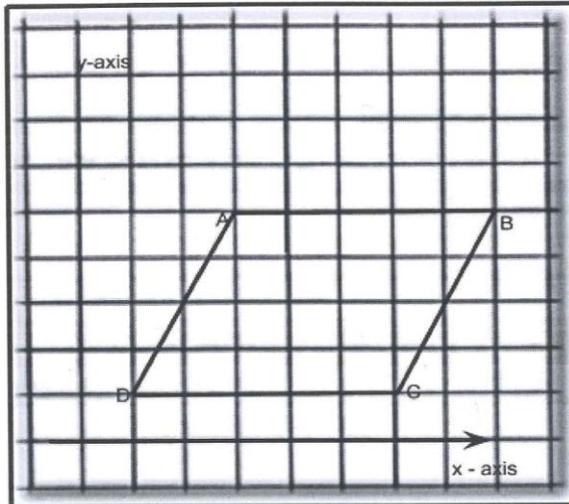
Explanation:

4 units distance between AD and BC  
 The green line is equidistant from AD and BC  
 3 units distance between AB and DC  
 ⇒ Pink line = parallel line to AB midway.  
 E is point on both lines so it is equidistant from AB and DC and AD and BC

## Appendix 2: Worksheet

### **Geometry Investigation:**

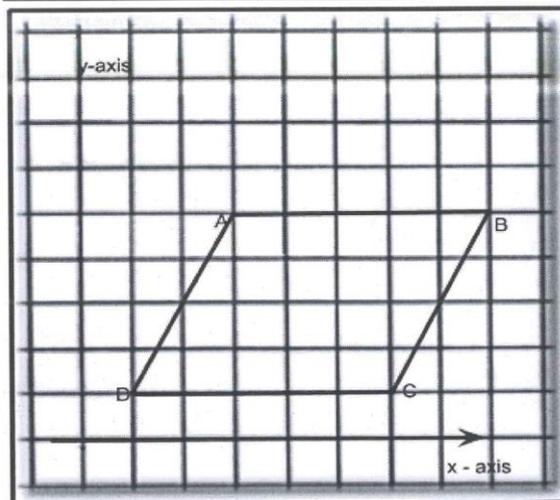
In as many ways as possible find the coordinate E, given that E must be equidistant from all the vertices of the parallelogram ABCD. Please explain each method.



Explanation:

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Explanation:

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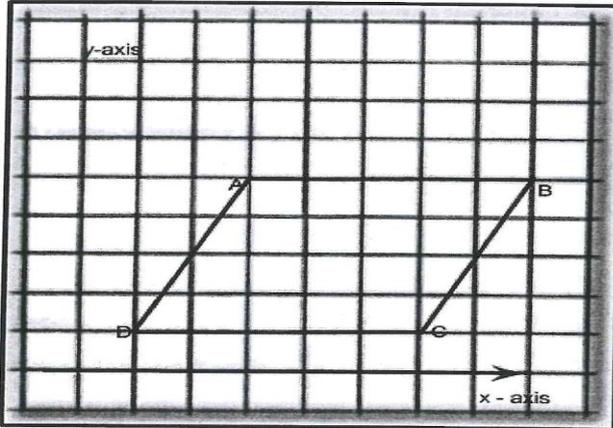


## Appendix 3: Planned Board Plan Solutions

①

### Geometry Investigation:

In as many ways as possible find the coordinate E, given that E must be equidistant from all the vertices of the parallelogram ABCD. Please explain each method.

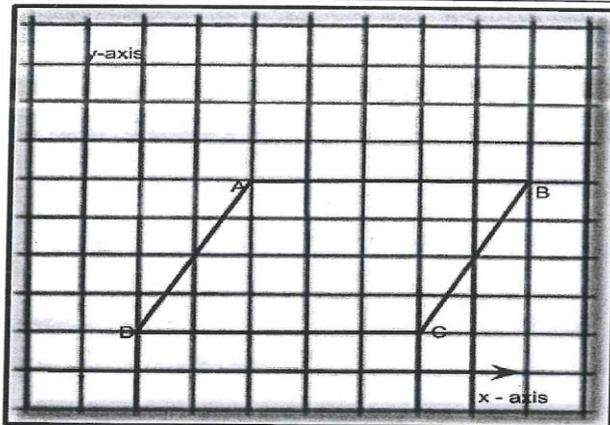


Constructed diagonals

Explanation:

I knew diagonals bisect each other in a parallelogram. so I drew  $[AC]$ ,  $[BD]$  E was the point I was looking for.

②



Bisected.

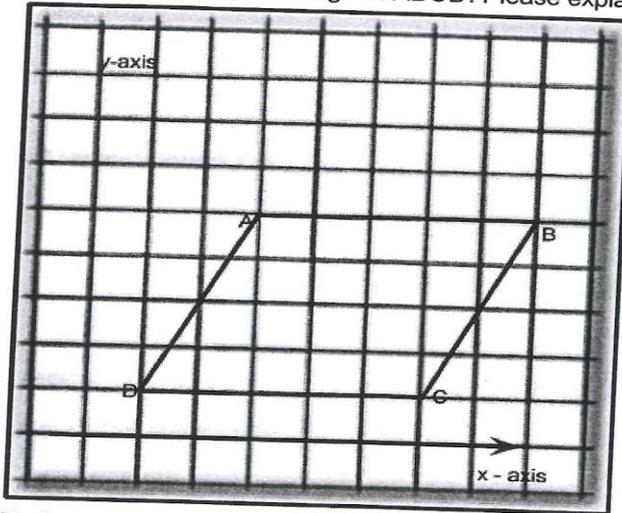
Explanation:

I bisected the diagonal  $(AC)$  or  $[BD]$  to get the mid point.  $(AC)$  is equidistant from  $B$  and  $D$  or  $(BD)$  is equidistant from  $A$  and  $C$ . The point I was looking for is their point of intersection.

**Geometry Investigation:**

In as many ways as possible find the coordinate E, given that E must be equidistant from all the vertices of the parallelogram ABCD. Please explain each method.

③

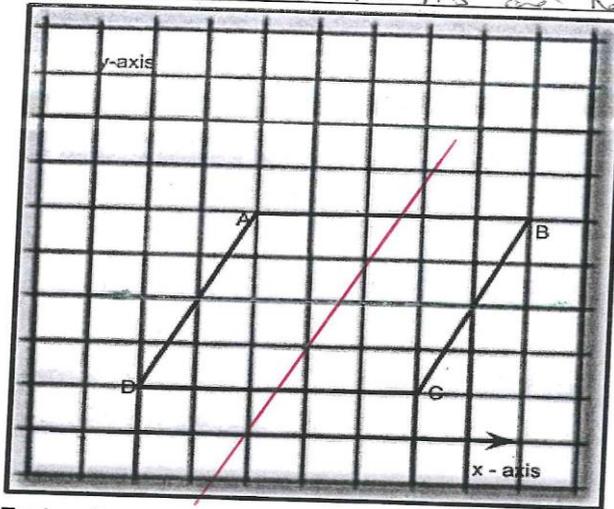


Translation.

Explanation:

Using the fact that opposite sides do exactly the same thing. This means that equidistant would be half of the movement AB and half AD.

④



Properties of a parallel lines

Explanation:

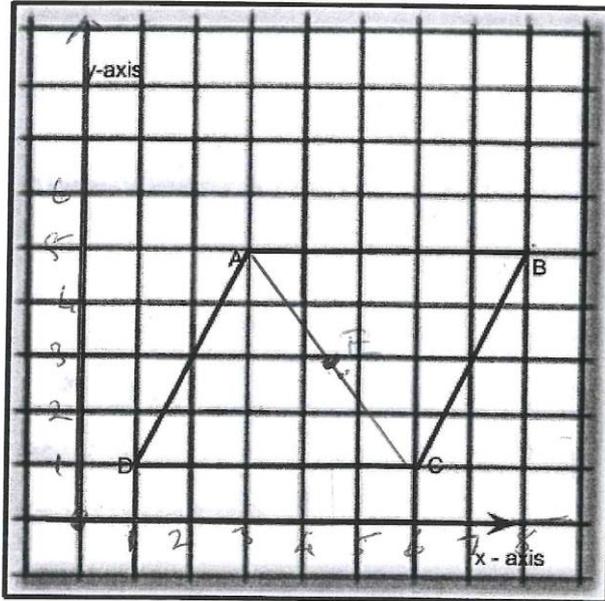
4 units between [AB] and [DC]. The green line is equidistant from [AB] and [DC], 5 units between [AD] and [BC]. Red line - parallel line to [AD] midway. E is the point on both lines so it is equidistant from [AB] and [DC] and [AD] and [BC].

## Formulare Dgeed.

### Geometry Investigation:

In as many ways as possible find the coordinate E, given that E must be equidistant from all the vertices of the parallelogram ABCD. Please explain each method.

6

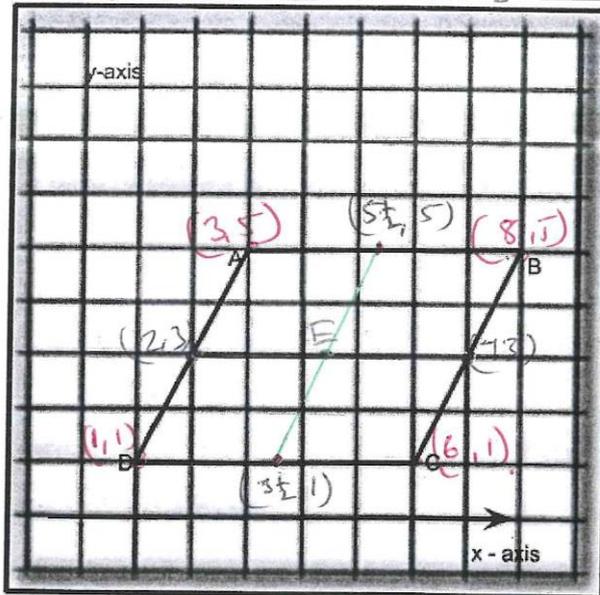


A	B
$(3, 5)$	$(8, 5)$
$C(6, 1)$	$D(1, 1)$
$\frac{3+6}{2}, \frac{5+1}{2}$	$\left( \frac{8+1}{2}, \frac{5+1}{2} \right)$
$\frac{9}{2}, \frac{6}{2}$	$\left( \frac{9}{2}, \frac{6}{2} \right)$
$(4\frac{1}{2}, 3)$	$(4\frac{1}{2}, 3)$

Explanation:

I know parallelogram diagonals bisect each other. The point I want to find is the point E is the midpoint of the diagonal [AC] or [BD]

5



Explanation:

Using the midpoint formula find the midpoint of [AD] and [BC]. Connect points midpoint for [AD] and [BC] connect. The point of intersection of the two line segments is E.