Year Group: 3rd Year
Level: Higher Level
Topic: Geometry & Trigonometry (Strand 2)

Date of Research Lesson: 18/1/17
At Coláiste Chill Mhantáin
Teacher: Mariosa O'Callaghan
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Title of the Lesson: 3D Cubes.

Brief description of the lesson
By engaging with this problem, students will become more confident dealing with three-dimensional shapes and how Trigonometry can be used to help interpret these shapes.
The task requires the students to work out the lengths of the sides in right – angled triangles, but the interesting part is that they need to be able to visualize the triangles to be able to solve the problem.

Aims of the Lesson
Short-term
We would like our students to:
- Apply their knowledge of The Theorem of Pythagoras in everyday problem solving. (Key Skill: Being Numerate).
- Use their knowledge of similar triangles to find the unknown lengths in the question. (Key Skill: Being Numerate).
- Become more familiar with three – dimensional shapes and be able to interpret the shapes using their knowledge of Pythagoras's Theorem. (Key Skill: Being Creative).

Long-term
We would like our students to:
- Appreciate that mathematics can be used to solve real world problems. (Key Skill: Being Creative).
- Develop a positive disposition towards problem solving. (Key Skill: Being Numerate).
- Connect and review the concepts that they have previously engaged with as part of their Mathematics course. (Key Skill: Managing Information and Thinking).
- Appreciate that it might take time to solve a problem and that they may need to use their imagination to creatively find a solution to the problem. (Key Skill: Managing Myself).
- Develop their literacy and numeracy skills through discussing different solutions to this problem. (Key Skills: Being Literate and Being Numerate).

Learning Outcomes

As a result of studying this topic students will be able to:
- Apply their understanding of Similar Triangles and Pythagoras's Theorem to solve unknown lengths.
- Apply a variety of methods to solve more complex problems.
- Use their imagination to interpret three dimensional shapes.

- Discuss different methods and listen to other student's as they explain their way of problem solving.

Background and Rationale

In previous lessons, students have learned about Pythagoras's Theorem, Similar Triangles and solving algebraic fractions. Students are familiar with three dimensional shapes, but not in this more complicated context.

One of the major challenges of this lesson is for the student to interpret a three dimensional shape by the use of a two dimensional shape.

In effect they need to take out the right – angled triangles to begin to discover more information about the three – dimensional shape. In doing this, students need to use their imagination and develop their creative thinking.

The concept of similar triangles plays a significant role in both Junior and Senior Cycle as does the students interpretation of three dimensional shapes. These concepts are dealt with in Strand 2: Geometry and Trigonometry of the Mathematics syllabus for Junior Certificate and Leaving Certificate.

Space and Shape is one of the PISA (the Programme for International Student Assessment) mathematical content areas. It involves understanding perspective and interpreting views of three – dimensional shapes from various perspectives. The content area draws on geometry, spatial visualisation, measurement and algebra.

In developing this lesson, the group wanted to encourage students to engage in more complex, higher – level tasks and to explore solving problems in novel ways.

Research

The following materials were used in the development of this lesson proposal:

- Junior Certificate Mathematics Guidelines for Teachers.
- The Teacher Handbooks from The Maths Development Team's website. These were particularly useful in planning the entire unit.
- Models to facilitate the learning – the rods with connectors.
- Recent findings from the Programme for International Student Assessment.
- Mathematics Syllabus for Junior Certificate.
- The State Examination Papers for Junior Cert Higher Level Mathematics.

About the Unit and the Lesson

This lesson proposal aims to encourage students to recall their prior knowledge which they have gained from the unit to attempt a more challenging question.

Students begin the unit with Trigonometry and begin their investigation and understanding of Pythagoras's Theorem, as mentioned in the Third Year Teacher Handbook Section 9.

Students will be able to apply the theorem of Pythagoras to solve right – angled triangle problems of a simple nature involving heights and distances and also be confident solving problems involving surds, as stated in the Junior Certificate Mathematics Syllabus, Section 2.3.

Students will investigate Theorem 12 and Theorem 13 involving similar triangles.

As part of the unit, students will have applied the results of these theorems to solve problems. In the study of Synthetic Geometry, the geometrical results "should first be encountered by learners through investigation and discovery."(Strand 2: Geometry and Trigonometry, Junior Certificate Mathematics Syllabus).

Students will have had experience with three dimensional objects in their study of 3D solids, including the nets of solids as stated in section 3.4 Applied Measure from the Junior Certificate Mathematics Syllabus. It will be important that students revisit this in a previous lesson to the research lesson. Students will be handed a model of a 3D solid and they will be able to fold it out and pick out all of the right – angled triangles which they see.
In the research lesson, the challenge for the students is to draw on all of this prior knowledge to creatively solve a problem including all of these concepts.

Flow of the Unit:

Title: Geometry & Trigonometry	**# of lesson periods**
These lessons will involve the students in investigating and understanding: • Pythagoras's Theorem. • The properties of right – angled triangles generated by Pythagoras's Theorem. • Apply the theorem of Pythagoras to solve right – angled triangles of a simple nature involving heights and distances.	5
These lessons will involve the students in investigating and understanding: • Angles and Parallel Lines.	2
These lessons will involve the students in investigating and understanding: • Theorem 12. • The meaning of similar triangles and the difference between similar and congruent triangles. • Theorem 13.	4
This lesson will involve the students in investigating and understanding: • 3D Shapes – students will be required to visualise all of the right – angled triangles when they examine the net of cuboids and rectangular cuboids.	1
<u>Research Lesson.</u> This lesson will involve the students in investigating and understanding: • The use of Pythagoras's Theorem in solving 3D shapes. • Problem Solving using similar triangles.	1

These lessons will involve the students in investigating and understanding: • Theorem 14.	2

Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction **(Prior Knowledge - 5 minutes)** • How many right angles are there in this diagram? • What is the connection between the lengths of the sides in a right- angled triangle? • Is there anything you can tell me about these triangles and why?	The teacher uses the construction set to draw two parallel lines on the board. The teacher then draws a perpendicular line (a transversal) through the two parallel lines. The teacher then draws another transversal which forms two triangles. Ensure that the students are aware where the right – angled triangles are. Indicate this on the diagram. Encourage students to use the words transversal, corresponding angles, right – angled, similar triangles etc. Models of the question may be help to some students. 
2. Posing the Task **(15mins)**	The task is read out clearly to the students.

<table>
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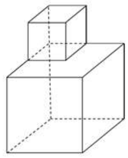
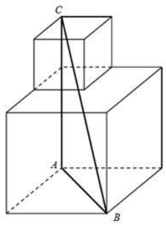
<td>All students are supplied with the worksheet below.

A shape is made by placing a small cube on top of a larger one as shown. The cubes have edges of length 1 unit and 2 units.

The right-angled triangle ABC is constructed inside this shape as shown.
The length of |BC| is √17cm.
The line segment [AB] is a diagonal of the base of the shape.
(i) Find |AB|. Give your answer in surd form.


(ii) Find the length of part of the line BC that is inside the larger cube.</td>
<td>All students are required to work through the problem using the model as an aid if they need it.

The teacher circulates the room assessing students’ work to plan how to orchestrate the presentation of students’ work on the board and the class room discussion.

The teacher may need to help students who are finding the task very challenging.</td>
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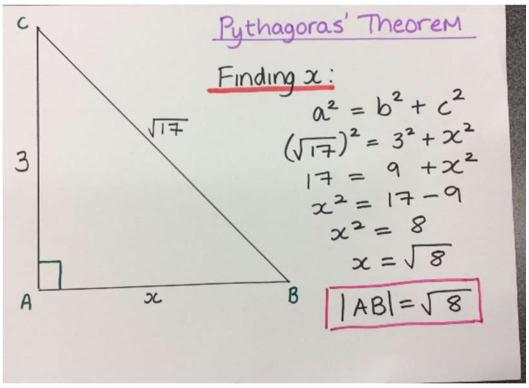
<td>3. Anticipated Student Responses:
We anticipate the following responses from students:
(i) Pythagoras’s Theorem


I see that you spotted the larger right – angled triangle. Can you show me where that is?

(ii)</td>
<td>Give the students a chance to verbally summarise their solution to the problem.

Ask questions. This will help the other students in the class.

Why did you split $\sqrt{17}$ into 3 parts?</td>
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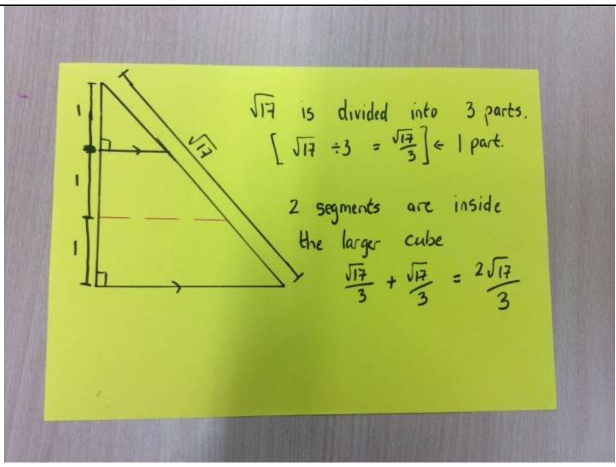
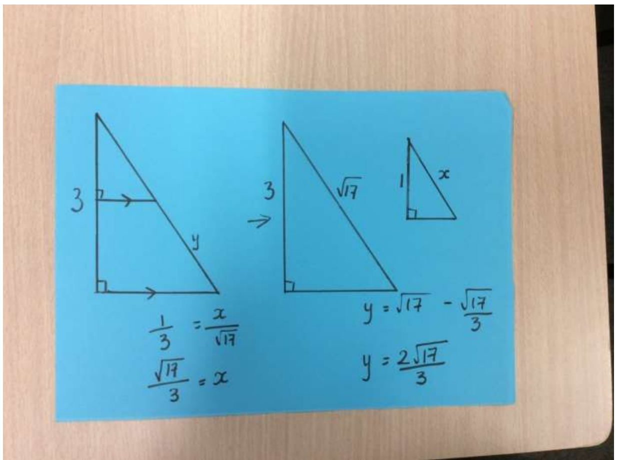
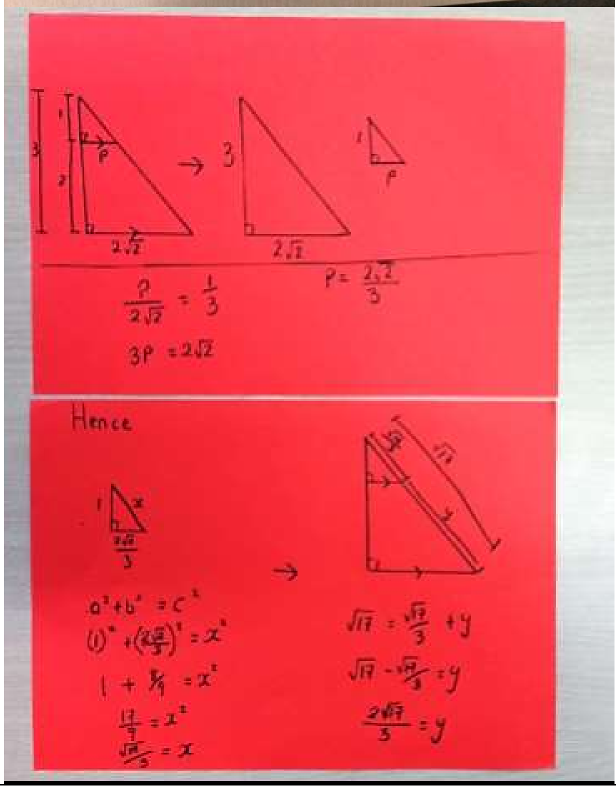
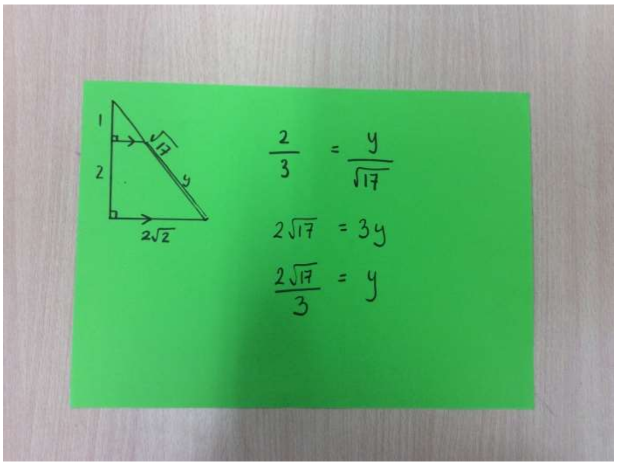


Where did you get the 3 from in your diagram? I see that you have split it into two triangles. Very good. What are these two triangles called?





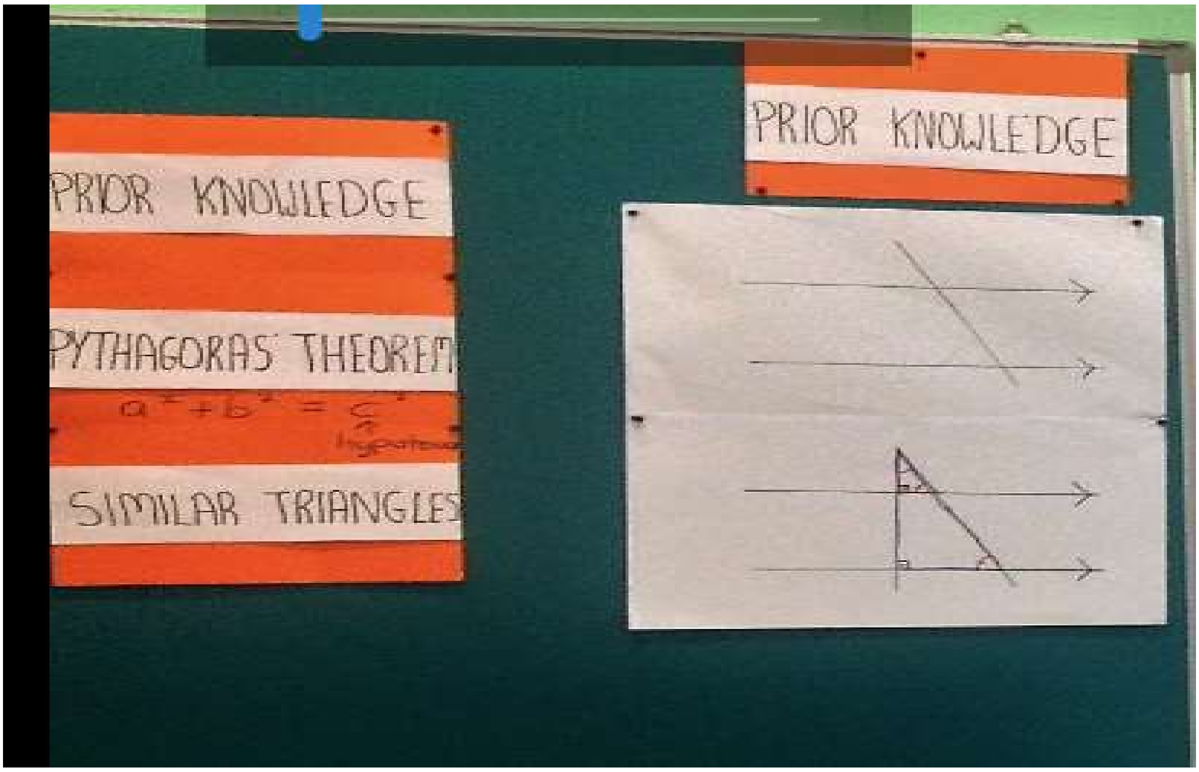
	
4. Comparing and Discussing **(15 Mins)** As a result of walking around the room and taking note of the students work, we will now ask students to come up to the board and show their different methods to the class.	There needs to plenty of space on the board for the board work. The teacher will have printed out the task, enlarged it and placed it on the board. The teacher can then number the sections and the responses so that the board work is very clear for the pupils to read. The teacher needs to make sure that pupils are happy to showcase their work on the board. See above images for possible student responses. Possible questions and explanations from teacher are also noted. Summarise each solution once the individual students have presented their work. Guide students through the solutions that they are unfamiliar to them.
5. Summing Up /Reflection **(5 Mins)** What did you learn today? What part of the task was the most challenging? **<u>Homework:</u>** The teacher asks the students to try to find another method to answer part (ii) of the question and that the worksheets will be collected the next day.	Ensure to reinforce the important aspects of today's class.

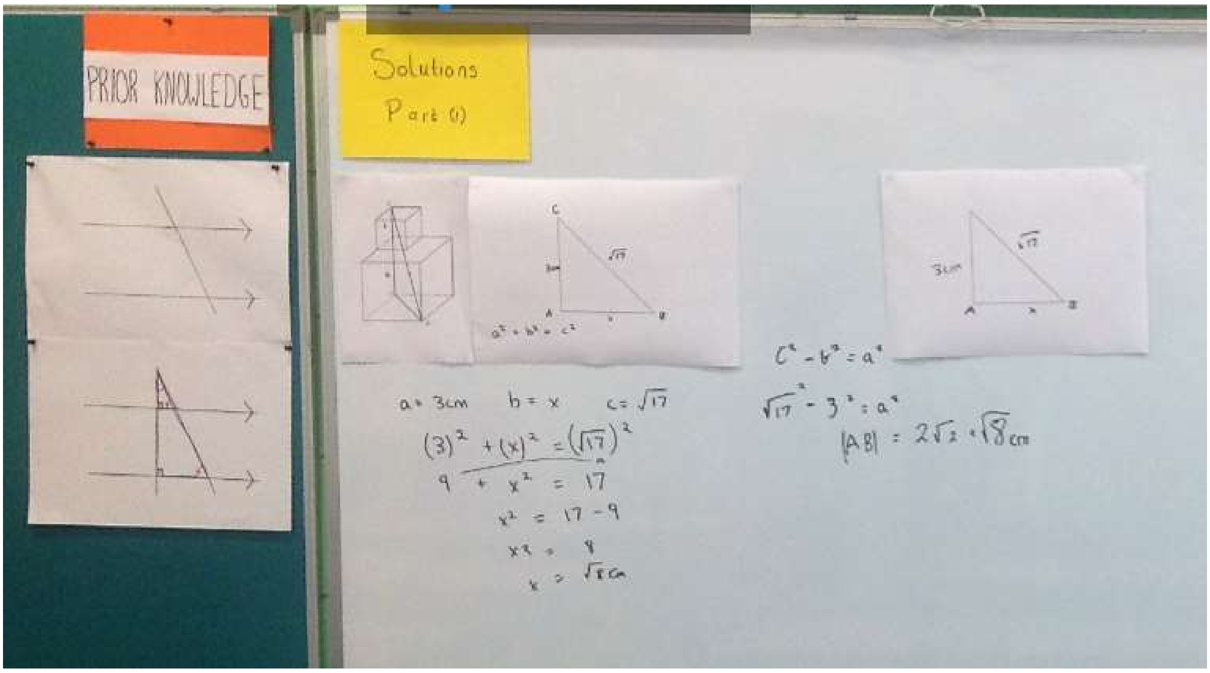
Evaluation: Plan for observing students

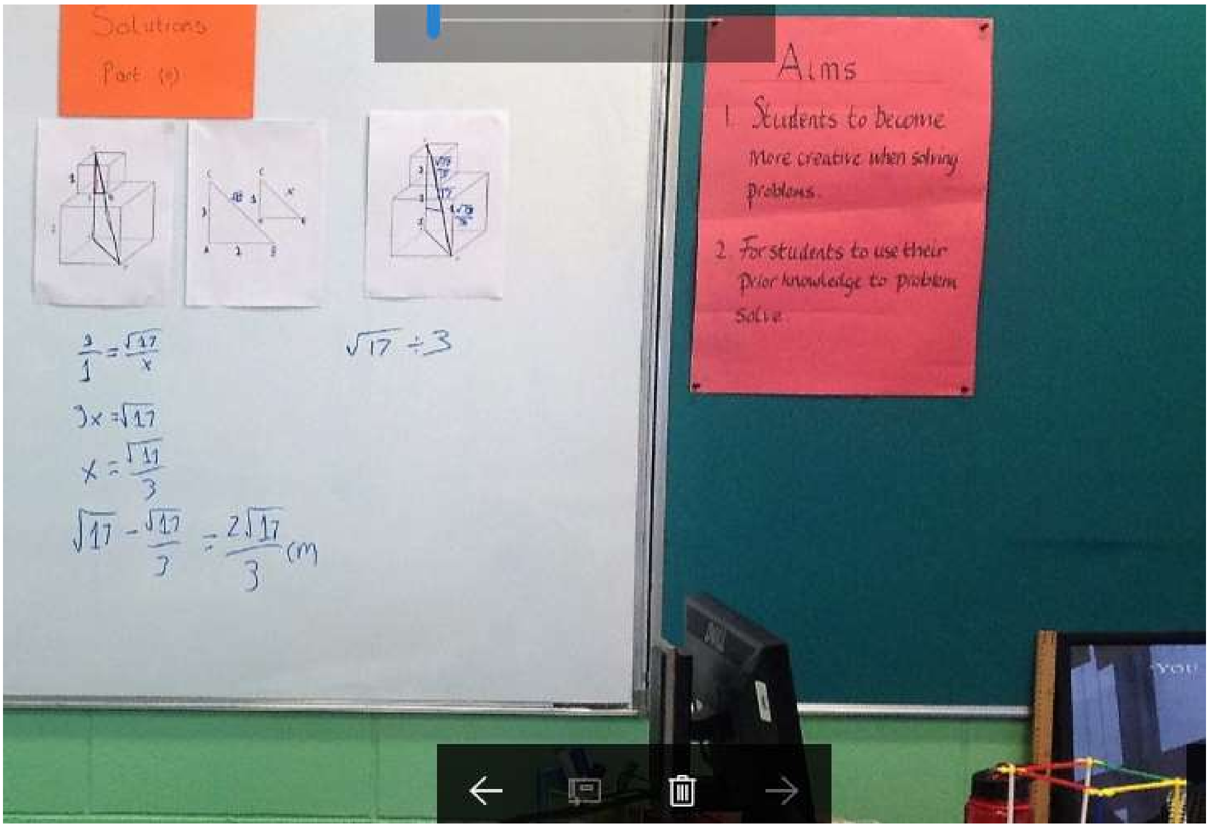
- A seating plan provided by the teacher.
- Three observers, 7 students per observer, all observers use the lesson observation sheet and iPads to take images.
- Types of students thinking and behavior observers will focus on:

Introduction, posing the task	Can students recall their prior knowledge on Pythagoras's Theorem & Similar Triangles? Are the students capable of understanding the wording of the task? Questions asked by the students.
Individual Work	Are the students able to recognize a 2D shape within a 3D shape? Are the students capable of completing both parts of the task at hand? Are prompts required? What kind of questions were asked by the students? How long do the students spend on the task? Do the students persist with the task at hand?
Discussion	Are all students observant to what is happening and being displayed on the board? Are any clarifications needed to presenters' board work? Did the discussion promote student learning?

Board Plan


PRIOR KNOWLEDGE
PYTHAGORAS THEOREM
$a^2+b^2=c^2$
SIMILAR TRIANGLES
PRIOR KNOWLEDGE


PRIOR KNOWLEDGE
Solutions
Part (i)
$a = 3cm$ $b = x$ $c = \sqrt{17}$
$(3)^2 + (x)^2 = (\sqrt{17})^2$
$9 + x^2 = 17$
$x^2 = 17 - 9$
$x^2 = 8$
$x = \sqrt{8}cm$
$c^2 - b^2 = a^2$
$\sqrt{17}^2 - 3^2 = a^2$
$|AB| = 2\sqrt{2} = \sqrt{8}cm$



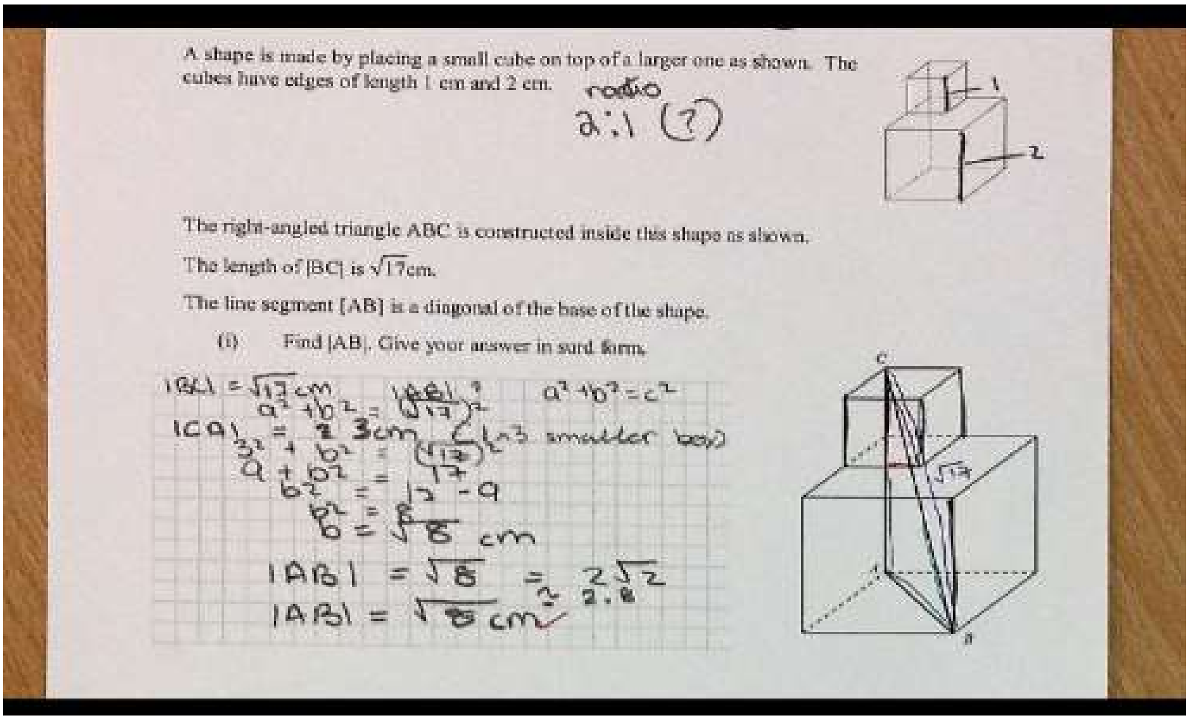
Post-lesson reflection

What are the major patterns and tendencies in the evidence?

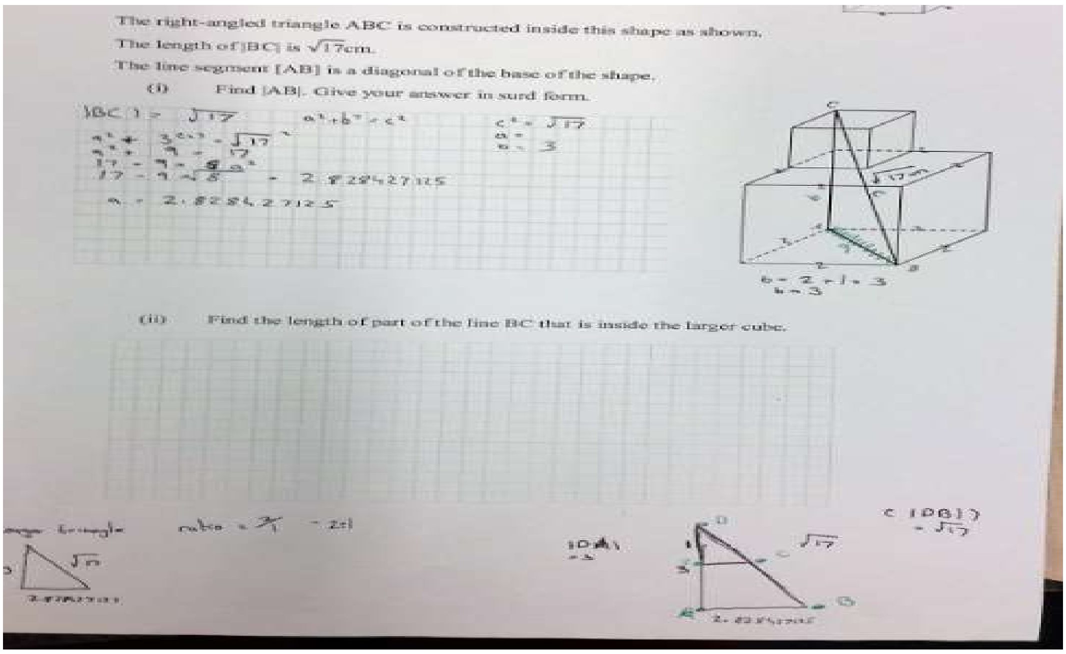
Students must have a clear understanding of what they are asked to do.
We found that when the students were presented with part (i) of the task they all tended to use Pythagoras's Theorem. We were quite concerned to see that some students applied this Theorem incorrectly and were unsure of the identities of the different lengths.
The students who completed Pythagoras, despite some errors in part (i), were confident to continue with part (ii) of the task.
It was noted that many students showed an indication of their ability to visualise in 3D. This became apparent when reviewing the worksheets post research lesson.
It was also observed that many students started their work by labelling the various lengths on the diagram (measurements had not been given in the diagram).

What are the key observations or representative examples of student learning and thinking?

From the beginning students were engaged in the task – all students identified the formula for Pythagoras's Theorem. They were very clear from the outset what was required of them.
Many of the students were capable of noticing the right-angled triangle within the 3D shape. They were then able to use this triangle and apply Pythagoras's Theorem to find the unknown length.



In part (ii) of the question, it was noted that more of the students had difficulty with the problem. Students in general had difficulty with the idea of ratio. It was obvious that they knew what they needed to find, but many, but not all students has some difficulty with part (ii) of the task.

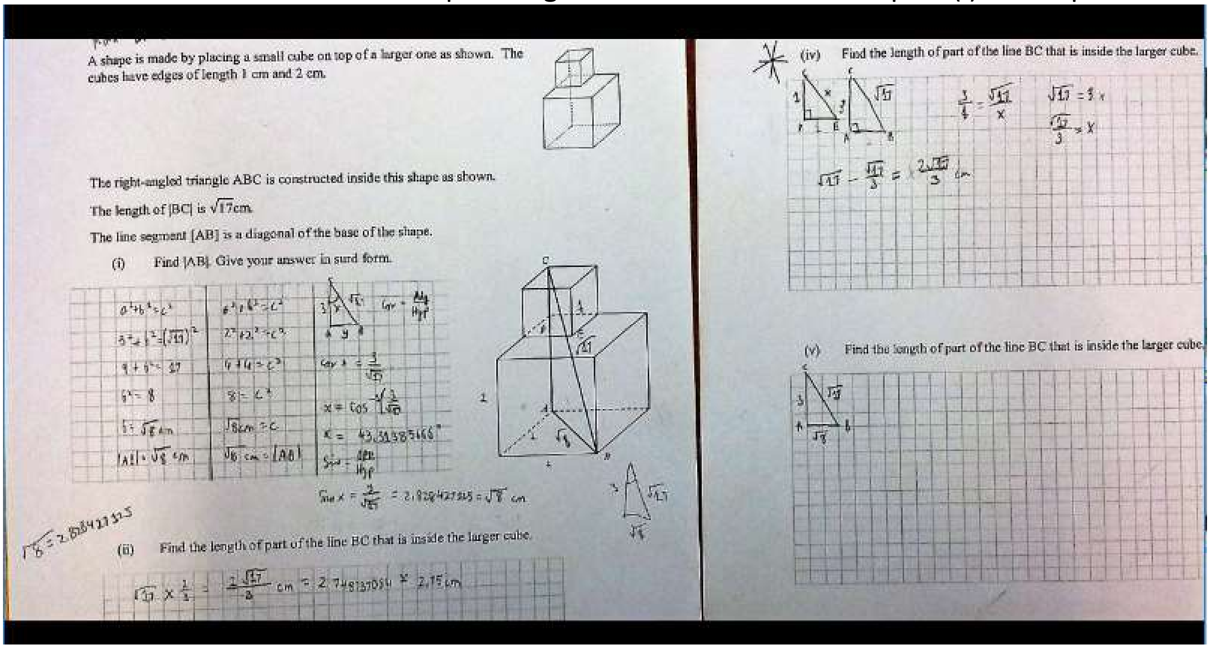


What does he evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?

All students were capable of substituting into Pythagoras's Theorem. Some students, however, substituted incorrectly. All students also placed measurements onto their worksheets which showed their understanding of the 3D shape.
The task of identifying the part of the line BC that is inside the larger cube caused some students some difficulty. Very few students re-drew the triangle outside the cubes. Students were more confident labelling the triangle within the cubes. Even though students did not re-draw the triangle, they displayed an understanding of their knowledge of right-angled triangles.

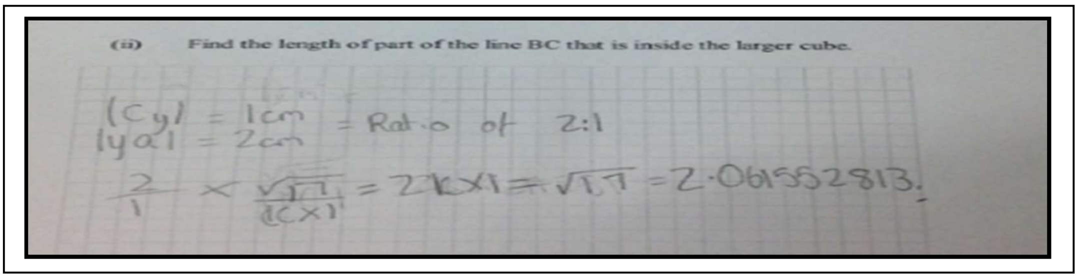
Four students in the class were successful in completing the task and obtaining the correct answer. Other students found the task challenging at times, especially working out problems using surd form and visualising the correct ratio for solving the similar triangles.

One student introduced the concept of trigonometric ratios to solve part (i) of the problem.



This was particularly impressive as the student needed to use the trigonometric ratios twice to solve part (i). This student had three methods to solve part (i). This showed that this was a task which suited the different ability levels in the classroom.

Interestingly, all students showed some knowledge of ratio. Most students however, used the incorrect ratio of 2:1 as opposed to 3:1. This caused problems in obtaining the correct solution.



In what ways did students achieve or not achieve the learning goals?

All students applied their knowledge of Pythagoras's Theorem, hence showing their ability to use the theorem in a problem solving exercise. Although all students recognised that the problem involved Similar Triangles, more than half of the class struggled with ratio. All students showed an awareness of three-dimension. This became evident from the measurements that the students placed on both cubes. This also reinforces their ability to understand the task at hand.

The students who had difficulty with part (ii) of the question would be more confident dealing with a similar example at another stage as when the students came up to the board to explain how they got the answer to the problem it resulted in reinforcing the concept of ratio to the other in the class.

Based on your analysis, how would you change or revise the lesson?

It might be beneficial if students had recognised a common Pythagorean Triple during the prior knowledge. This would have helped students in substituting correctly into Pythagoras' Theorem. This could have been introduced by inviting a student up to the board and showing the Pythagorean triple on the board by use of a right – angled triangle.

When the students were presented with the problem, it may have been useful to some students to use the model to further enhance their own thinking. This may have helped them with the idea of ratio and how it connected with the similar triangles in the problem. Not all students were able to identify the similar triangles, and ratio caused problems.

For part (i) of the problem, students were very confident in identifying the fact that they were dealing with Pythagoras's Theorem. It could be worth telling students that they could try and complete part (i) of the question in a few different ways, or maybe even mention the Trigonometric Ratios in the Prior Knowledge section.

In noticing that some pupils had difficulty with the ratios, it might have been beneficial to explore that in the Prior Knowledge section.

Use of the models, even when the pupils were working might have helped them to pick out the similar triangles.

What are the implications for teaching in your field?

Taking part in Lesson Study has certainly resulted in more conversation about teaching and teaching methods adopted by different teachers of Mathematics. Having taken part in the process of planning a lesson for Structured Problem Solving, has certainly made all of us think more about our methods of teaching. It has also helped us to develop ideas for our teaching and encouraged us to think about how the student might view the problem. It is very interesting to sit down and think of some of the responses the pupils might come up with for a problem. The board work gives students who may have had another method of solving the problem, a chance to show his/her peers in the class. We all like the way that it encourages pupils to verbalise their solution.

All of us agreed that giving the pupils too much information to solve a problem can take from the active learning part of the lesson which they really need to engage in to get gain some of the benefits from the lesson.

It can be surprising what the pupils can learn from each-other, especially during the board-work when the pupils get an opportunity to verbalise their own solution to their peers as they show their work on the board. All of us would also agree that when students get working on a problem, they are getting engaged in the task at hand and that once they get the opportunity to do that, and to see solutions from their peers, they are becoming active learners who are developing a positive disposition towards investigating, reasoning and problem – solving.