

**Topic: Alternate Angles**

**Year group: First Year**

**Lesson Plan taught:** 19<sup>th</sup> January 2016

**At** Scoil Chonglais, Baltinglass, Co. Wicklow, First Year class

**Teacher:** Ms. Laura Hogan

**Lesson plan developed by:** Laura Hogan (Scoil Chonglais, Baltinglass, Co. Wicklow),

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### Title of the Lesson

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## ***“An Alternative approach to Alternate Angles”***

### Brief description of the lesson

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Using prior knowledge of angles to determine that all alternate are equal in measure. And the converse, if a transversal makes equal alternate angles on two lines then the two lines are parallel.

### Aims of the lesson

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- a) **Short term aims:** We would like our students to
  - i. Gain an insight into the language of Geometry (e.g. transversal, alternate, corresponding, etc.).
  - ii. Recognise the relationship between equal angles, parallel lines and a transversal.
  - iii. Appreciate the connection between prior knowledge and new material.
  - iv. Identify that all alternate angles are equal in measure.
- b) **Long term aims:** We would like our students:
  - i. to appreciate that there are multiple approaches to solve mathematical problems and each students is equally valid.
  - ii. to develop skills to become independent learners.
  - iii. to know that sometimes in mathematics it is not necessary to always use numbers.
- c) We would like to support our students in developing their literacy and numeracy skills through discussing ideas.

### Learning Outcomes

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As a result of studying this topic students will be able to:

- a) Recognize alternate angles and that they are equal in measure.
- b) Identify that a transversal is formed when a line cuts two parallel lines at two distinct points.
- c) Use, compare and contrast multiple approaches when identifying angles of equal measure.

## Background and Rationale

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In a typical lesson to introduce angles of equal measure using a transversal and parallel lines, the lesson often starts with the teacher drawing a diagram and highlighting the equal angles. However, this limits the thought process of the student in terms of the possibility of discovering these angles by themselves. Thus, it is not clear if the students understand the concepts of alternate angles or corresponding or if they are just parroting the words and actions of their teacher without any real understanding. Students are not always encouraged to develop their ability to investigate the angles themselves, consequently curtailing their understanding.

If the equal angles are introduced at the beginning of the class, students are denied the opportunity to develop their own abilities and to develop mechanisms for approaching visual geometry questions.

Using a variety of diagrams and multiple resources for encouraging investigation of angles students should be able to measure angles and locate matching pairs of angles. This will encourage them to draw conclusions about possible emerging rules for finding angles of equal measure about the transversal.

## Research

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In preparation of this lesson plan the following materials have been used:

- a) Junior Certificate Mathematics Guidelines for Teachers (DES 2002)
- b) Junior Cert Syllabus Strand 2 Section 2.1 – Synthetic Geometry p. 18 of syllabus
- c) Concepts for Junior Cert – Section 6.5 p. 51-52 of syllabus
- d) First Year Handbook: Lesson Idea 1.13 (CIC) and Lesson Idea 1.14 (CIC) p. 18 and p.19 (from the Maths Development Team’s website [www.projectmaths.ie](http://www.projectmaths.ie))

## About the Unit and the Lesson

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According to the Junior Certificate Mathematics Syllabus for examination from 2016 (Strand 2, Section 2.1 Synthetic geometry, pages 17-19) students at both Higher and Ordinary Level be able to apply the results of Theorem 3: “If a transversal makes equal alternate angles on two lines then the lines are parallel”. And also the converse of this Theorem.

(see Syllabus: Parallel Lines section 6.5, pp. 51-52)

**Theorem 3** (Alternate Angles). *Suppose that A and D are on opposite sides of the line BC.*

*(1) If  $|\angle ABC| = |\angle BCD|$ , then  $AB \parallel CD$ . In other words, if a transversal makes equal alternate angles on two lines, then the lines are parallel.*

*(2) Conversely, if  $AB \parallel CD$ , then  $|\angle ABC| = |\angle BCD|$ . In other words, if two lines are parallel, then any transversal will make equal alternate angles with them.*

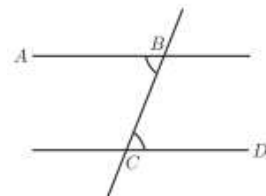


Figure 4.

Furthermore, “it is intended that all of the geometrical results on the course would first be encountered by students through investigation” (Geometry Course, section 8.2).

From Junior Certificate Mathematics Guidelines for Teachers (2002, p.20): *“Synthetic geometry is traditionally intended to promote students’ ability to recognise and present logical arguments. More able students address one of the greatest of mathematical concepts, that of proof, and hopefully come to appreciate the abstractions and generalisations involved. Other students may not consider formal proof, but should be able to draw appropriate conclusions from given geometrical data. Explaining and defending their findings, in either case, should help students to further their powers of communication.”*

This lesson is designed to get students to draw on their prior knowledge of parallel lines and angles to attempt to discover a new and interesting fact about alternate angles. Student have participated in multiple synthetic geometry lessons to date and this lesson hopes to guide them to gain an insight into discovering connections between angles in an unfamiliar situation. From the hand-out and their previous knowledge of parallel lines, measuring angles and triangles they will become more creative in finding multiple approaches to solve any one problem.

### Flow of the Unit


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Synthetic Geometry 2, First Year Teacher Handbook based on 2016 syllabus

Lesson	Details	# of lesson periods
1	<p>Introduction to geometry and the language of geometry through discussion and verbalising concepts while also referencing geometry in the world around us. Introducing the plane, point’s, and lines.</p> <p>Revision of angles, types of angles, estimating and measuring angles from primary school. The correct use of a protractor mopping up misconceptions and misuse of the protractor.</p> <p>Recap of parallel and perpendicular lines from primary school.</p>	3 x 40 min Classes
2	<p>Introduction to the concept of a theorem and vertically opposite angles. Vertically opposite angles are equal in measure (Theorem 1)</p> <p>Recap on types of triangles from primary school.</p>	2 x 40 min Classes
3	<p>Introduction to alternate angles and transversals through investigation. Comparing multiple methods of recognising equal angles to show that alternate angles are always equal in measure.</p>	1 x 40 min Class <b>(research lesson)</b>
4	<p>Follow up lesson, corresponding angles are equal in measure through a similar approach to research lesson, building on the concepts and skills students acquired during the research lesson</p>	1 x 40 min Class

5	<p>Triangles continued:</p> <ul style="list-style-type: none"> <li>- The angles in any triangle add up to <math>180^\circ</math>. (Theorem 4)</li> <li>- Each exterior angle is equal to the sum of the interior opposite angles. (Theorem 6)</li> <li>- Congruent triangles.</li> </ul>	2 x 40 min Classes
6	<p>The use of a compass and introduction to constructions.</p> <ul style="list-style-type: none"> <li>- <b>Construction 5:</b> Line parallel to a given line, through a given point.</li> <li>- <b>Construction 6:</b> Division of a line segment into 2 or 3 equal segments, without measuring it.</li> <li>- <b>Construction 8:</b> line segment of a given length on a given ray.</li> <li>- <b>Construction 9:</b> Angle of a given number of degrees with a given ray as one arm.</li> </ul>	2 x 40 min.

### Flow of the Lesson

Teaching Activities and Students' Anticipated Responses	Points of Consideration
<p><b>1. Introduction (5 minutes)</b></p> <p><b>Prior Knowledge:</b> Prior knowledge will be tested and revised with the use of a number of diagrams.</p> <ol style="list-style-type: none"> <li>1) <i>[From Diagram]</i> What are the properties of the following pair of lines?</li> <li>2) <i>[From Diagram]</i> How do you know that these two lines are parallel?</li> <li>3) <i>[From Diagram]</i> Can you identify each of the following types of angles?</li> <li>4) How do we identify each of these types of angles?</li> <li>5) How could we check to see if an angle is acute (<math>&lt;90^\circ</math>)</li> <li>6) <i>[From Diagram]</i> Can you tell me anything special about the angles in the diagram?</li> <li>7) <i>[From Diagram]</i> Can you identify each of the types of angles labeled in the diagram?</li> <li>8) <i>[From Diagram]</i> What can you tell me about the angles in the diagram?</li> </ol>	<p>Teacher displays relevant images on the board if needed to extract the required information from students.</p> 

## 2. Posing the Task (5 minutes)

Problem for today: **Which of the following angles are equal? Use as many methods as you can to answer this question**

On the worksheet, you are given four identical diagrams each with four labeled angles. (See Appendix 1, 2 & 3). Your task is to identify which angles are equals. You must do this on your own.

Diagram A:

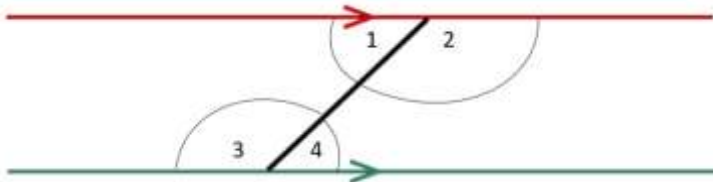


Diagram B:

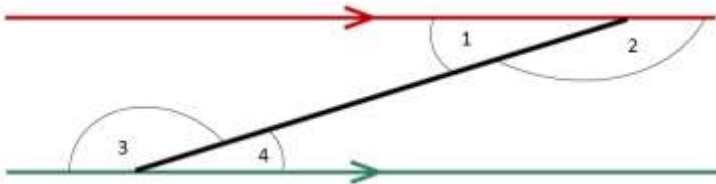
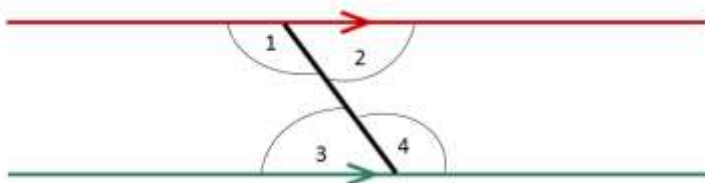


Diagram C:



You are encouraged to do this in as many ways as you can. To assist you, there are several resources available, including protractors, scissors, rulers and Geo Strips.

The teacher will read out the given task in order to make sure that all students are aware of what is being required of them and that we are looking for multiple approaches.

Materials required: blank sheets/trace paper, scissors, protractor, ruler, pencil and Geo Strips.

The teacher will circulate the room and observe students work. At this point the teacher will decide on students to bring to the board and in what order.

## 3. Students individual work (5 minutes)

Students will be given 5 minutes to work on the problem on their own. There will be three different worksheets (all with slightly different diagrams). This will mean students next to each other will have slightly different problems and will be less likely to try and work together.

Teacher circulates the room assessing students' work to plan how to orchestrate the presentation of students' work on the board and class discussion.

#### 4. Anticipated Student Responses:

First let's look at Diagram A.

##### Student 1

Measuring each of the angles labeled 1 – 4, we can see that :

$$\sphericalangle 1 = 45^\circ$$

$$\sphericalangle 2 = 135^\circ$$

$$\sphericalangle 3 = 135^\circ$$

$$\sphericalangle 4 = 45^\circ$$

Therefore  $\sphericalangle 1 = \sphericalangle 4$  and  $\sphericalangle 2 = \sphericalangle 3$

Now let's look at Diagram B.

##### Student 2

Measuring each of the angles labeled 1 – 4, we can see that :

$$\sphericalangle 1 = 20^\circ$$

$$\sphericalangle 2 = 160^\circ$$

$$\sphericalangle 3 = 160^\circ$$

$$\sphericalangle 4 = 20^\circ$$

Therefore  $\sphericalangle 1 = \sphericalangle 4$  and  $\sphericalangle 2 = \sphericalangle 3$

Now let's look at Diagram C

##### Student 3

Measuring each of the angles labeled 1 – 4, we can see that :

$$\sphericalangle 1 = 125^\circ$$

$$\sphericalangle 2 = 55^\circ$$

$$\sphericalangle 3 = 55^\circ$$

$$\sphericalangle 4 = 125^\circ$$

Therefore  $\sphericalangle 1 = \sphericalangle 4$  and  $\sphericalangle 2 = \sphericalangle 3$

##### Student 4

Using Tracing paper, draw one of the angles. Compare this angle to the other angles by placing the tracing paper over the other angles. From this it is clear that  $\sphericalangle 1 = \sphericalangle 4$  and  $\sphericalangle 2 = \sphericalangle 3$ .

##### Student 5

Using Geo strips, one of the angles is replicated and then compared to the angles in the diagram. This is then repeated for the other angles. From this comparison it is clear that  $\sphericalangle 1 = \sphericalangle 4$  and  $\sphericalangle 2 = \sphericalangle 3$ .

##### Student 6

Using a scissors, the student will cut out each of the angles 1 – 4. These are then compared to the angles in the diagram. From this comparison it is clear that  $\sphericalangle 1 = \sphericalangle 4$  and  $\sphericalangle 2 = \sphericalangle 3$ .

Students' names will be written beside their work to give them ownership of their work. When referring back to the work, the students' names will be used.

### 5. Comparing and Discussing: Céardaíocht (10 minutes)

Allow students 1, 2 and 3 to come up in quick succession, without any discussion. At the end of the third student's presentation engage the whole class in a discussion about the differences (size of the angles) and the similarities (parallel lines and transversal). The word transversal' should be introduced at this point.

Next allow students 4, 5 and 6 to come up to present their ideas (if there is a student who used this method. If there are any other methods that students used, they can discuss at this point.

Finally, the students will be posed the question:

**"Can you make up a rule, based on your work, to say when angles would be equal".**

From this, the students will hopefully be able to come up with the idea of

- between the parallel lines
- opposite sides of the 'transversal'
- 'Z' Shape

If the students refer to the line that goes from one parallel line to the other, the word 'transversal' should be reiterated.

At this point, the students will be given the word "Alternate" to describe angles.

The teacher will lead a discussion in describing student findings through correct terminology i.e. angles on the opposite side of the transversal, internal opposite angles, etc.

Q. Which methods have similarities?

Q. How are methods different?

Q. How would you describe the position/location of the angles that are equal in measure?

### 6. Individual work on Follow-up Task (5 mins)

Each student will then be given the following two diagrams and asked to use one of the previously discussed methods to identify any equal angles within the diagram.

Diagram D:

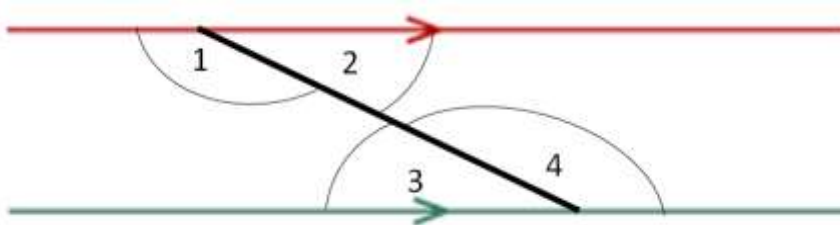
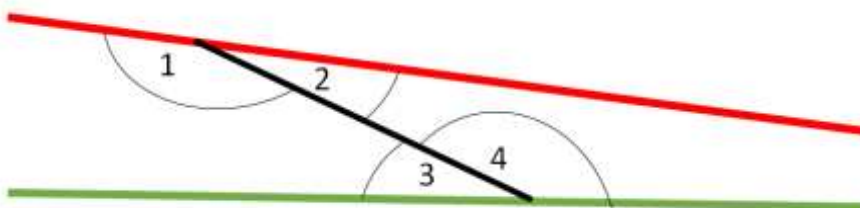


Diagram E:



<p><b>7. Céardaíocht for Follow-up Task (5 minutes)</b></p> <p><b><u>Student 7</u></b> Using any of the previous methods, the students will have discovered that <math>\sphericalangle 1 = \sphericalangle 4</math> and <math>\sphericalangle 2 = \sphericalangle 3</math>.</p> <p><b><u>Student 8</u></b> Using any of the previous method, the student will have come to the conclusion that in this diagram, none of the angles are equal.</p> <p><b>Question is posed to the class:</b> In one case, there are equal angles and in the other no angles are equal. We have seen four cases where angles are equal and one case where it was not. Was there something about this diagram that might have caused this?</p> <p><b>Follow on question posed to the class:</b> What could we say about this?</p> <p>The idea of “converse” should be introduced here,</p>	
<p><b>8. Summing up (5 minutes)</b></p> <p>Recap of all the information from this class. When we are given two parallel lines and a transversal, the angles inside the parallel lines and on opposite sides of the transversal are called “Alternate Angles” and are equal in measure.</p> <p>Conversely, if we have equal angles in such a set up, we know that the lines must be parallel lines.</p> <p>See Appendix 5 for Homework. Homework is a follow on question that will lead into Corresponding angles in the next lesson.</p>	<p>Reinforce the findings in today’s lesson through use of correct terminology associated with the lesson and also that there were multiple avenues to the same result.</p>



## Evaluation: Plan for observing students

- A seating plan provided by the teacher
- Two observers, 8 students per observer, both observers using pen and paper with observation sheet (Appendix 6).
- Types of student thinking and behavior observers will focus on:

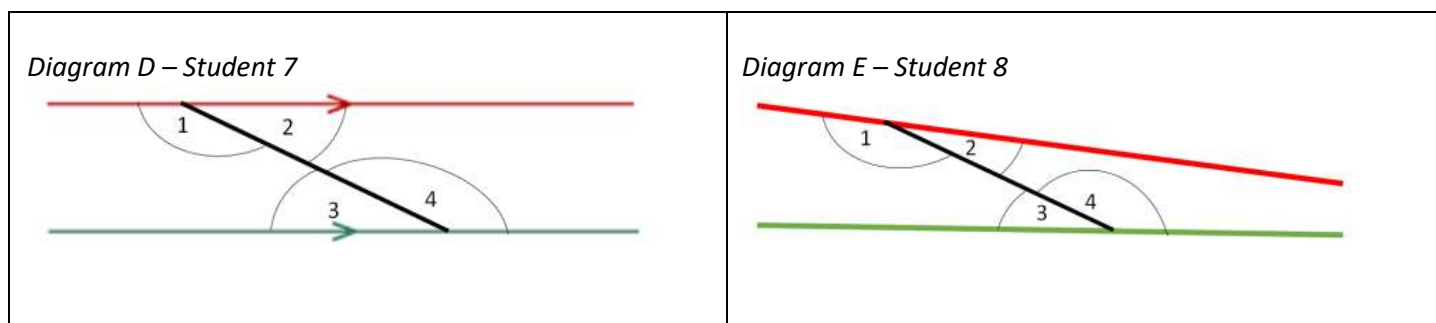
Introduction, posing the task	Can students recall the answer to all questions asked during the prior knowledge phase? Was wording of the task clear? Questions asked by students
Individual work	What methods are used? Are prompts required? How long do students spend on the task? What kind of questions do students ask? Do they persist with the task?
Discussion	Are students attentive to what is happening on the board? Are clarifications needed to presenters' board work? Did the discussion promote student learning?

- Additional kinds of evidence collected (exemplars of students' work, photographs, end of lesson reflection)

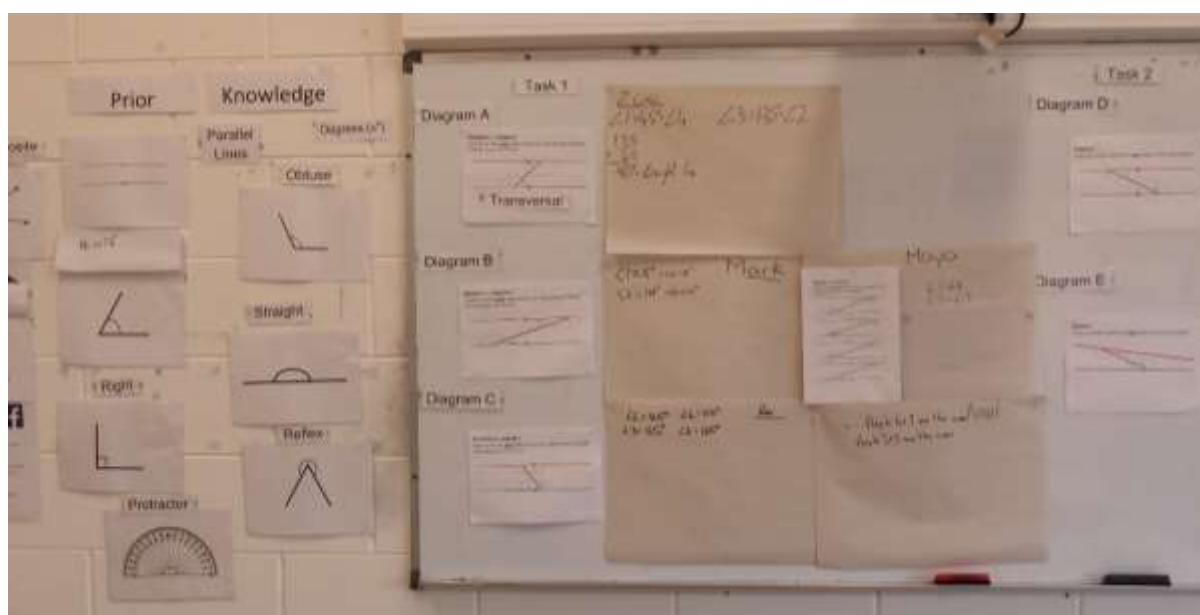
## Board Plan

Prior Knowledge	Method 1 (Student 1, 2 & 3)	Method 2 (Student 4)	Method 3 (Student 5)	Method 4 (Student 6)
Diagrams.	<i>Diagram A - Student 1</i> 	<i>Diagram A - Student 4</i> 	<i>Diagram A - Student 5</i> 	<i>Diagram A - Student 6</i> 
Keywords.		<b>OR</b>	<b>OR</b>	
Students answers to questions asked.	<i>Diagram B - Student 2</i> 	<i>Diagram B - Student 4</i> 	<i>Diagram B - Student 5</i> 	<i>Diagram B - Student 6</i> 
	<i>Diagram C - Student 3</i> 	<i>Diagram C - Student 4</i> 	<i>Diagram C - Student 5</i> 	<i>Diagram A - Student 6</i> 

### Follow up:



### Picture of Board Work:



### Post-lesson reflection

#### What are the major patterns and tendencies in the evidence?

We found that students were very quick to ask for clarity, this could be related to the fact that the lesson was of an unfamiliar style. Students tended to work with resources that were familiar to them rather than try new methods/unfamiliar resources, we felt some students wanted to use new methods but were reluctant to do so as most did not know what geostrips were.

#### What are the key observations or representative examples of student learning and thinking?

Most students tried to use their prior knowledge to help them to get a solution. Most students tried to use the terminology  $\angle 1 + \angle 2 = 180^\circ$  and  $\angle 3 + \angle 4 = 180^\circ$  to make connections.

One student came to the conclusion on worksheet 2 that the angles in worksheet 2 could not be equal because the lines were not parallel.

Students used the logic that they had to use the tracing paper to try and explain that angles were equal in measure.

**What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc?**

Students misused the phrase vertically opposite angles in trying to describe the position/location of the angles in the diagrams.

Student notation was good but was not relevant here ( $\sphericalangle 1 + \sphericalangle 2 = 180^\circ$  and  $\sphericalangle 3 + \sphericalangle 4 = 180^\circ$ )

**In what ways did students achieve or not achieve the learning goals?**

All short term learning goals listed below were met by all students in the class.

**Short term learning goals:** We would like our students to

- i. Gain an insight into the language of Geometry (e.g. transversal, alternate, corresponding, etc.).
- ii. Recognise the relationship between equal angles, parallel lines and a transversal.
- iii. Appreciate the connection between prior knowledge and new material.
- iv. Identify that all alternate angles are equal in measure.

Aim (i) was achieved with the use of transversal was evident in follow up lessons. Students also achieved aim (ii) by identifying that if lines were not parallel then angles could not be equal. Aims (iii) and (iv) were also successfully achieved as all students used prior knowledge to identify angles of equal measure with some students using knowledge beyond this. Students were also able to describe the location of alternate angles and agreed that they would always be equal in measure.

We feel that all of the long term learning goals below have not been met fully by students but that they are on the path to achieving them with more exposure to this type of approach to solving problems. These goals are to be achieved over multiple lessons. One suggestion might be to limit how they can carry out a follow up problem.

**Long term goals:** We would like our students:

- i. To appreciate that there are multiple approaches to solve mathematical problems and each students is equally valid.
- ii. To develop skills to become independent learners.
- iii. To know that sometimes in mathematics it is not necessary to always use numbers.

We also feel that students were supported in developing their literacy and numeracy skills through discussing ideas and the correct use of math's language in the prior knowledge section and follow up lessons.

**Based on your analysis, how would you change or revise the lesson?**

1. Issue/Element that you would change is the timing and the flow of the lesson. We felt that timing allowed on the lesson proposal was unrealistic but the lesson did allow for this as follow up task was given for homework and the homework activity was used as a starter for the following lesson which worked well as students reported feeling of safety in doing the task at home and not being watched.

Recommendation: to be taught in a one hour lesson or over two classes or give worksheet 2 as homework and homework as a follow up lesson starter.

2. Issue/Element that you encountered was that students were reluctant to use resources on the table in front of them. Students were unwilling to ask/talk as they were not use to more than one body in the room. Geostrips also caused distraction when some students were at the board.

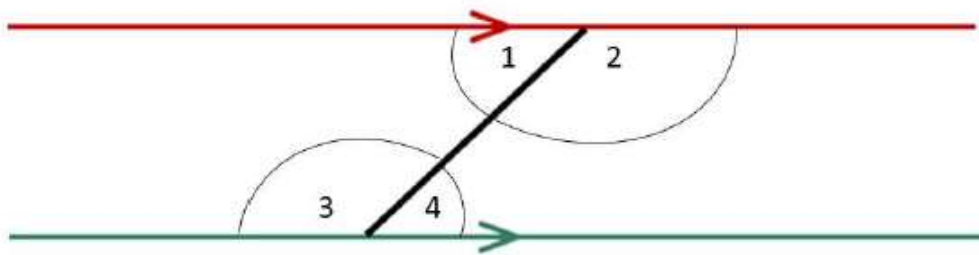
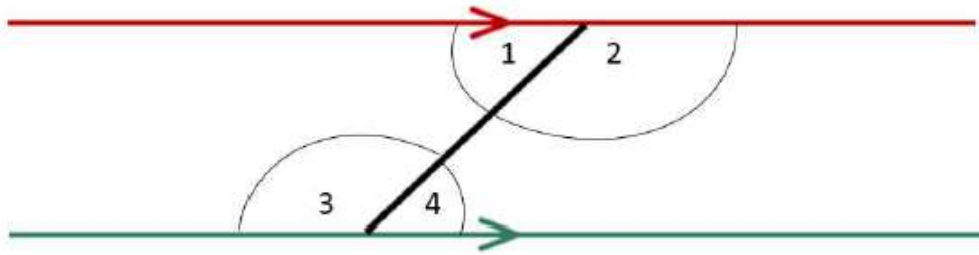
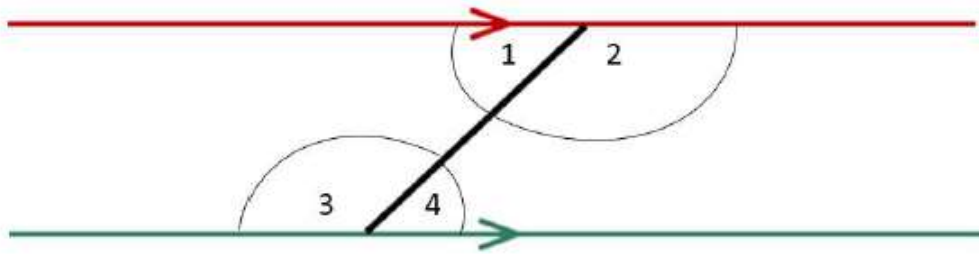
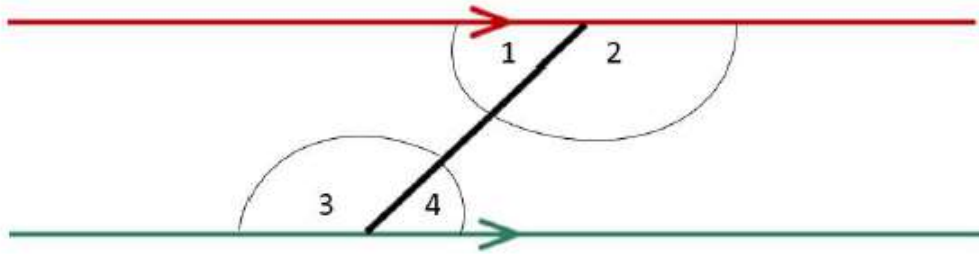
Recommendation: introduce student to resources before they start the task or else allow them to get up and get the resources themselves as they wish.

**What are the implications for teaching in your field?**

We feel that alternate and corresponding angles are usually poorly answered questions as it doesn't seem to stick with students. We feel that if students are introduced to the topic in a more investigative way will help the content to stick.

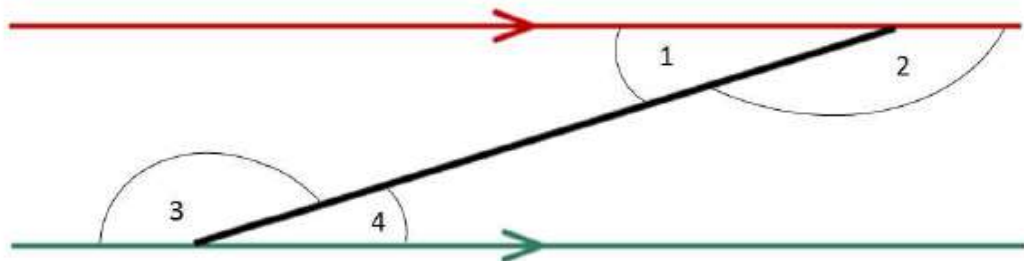
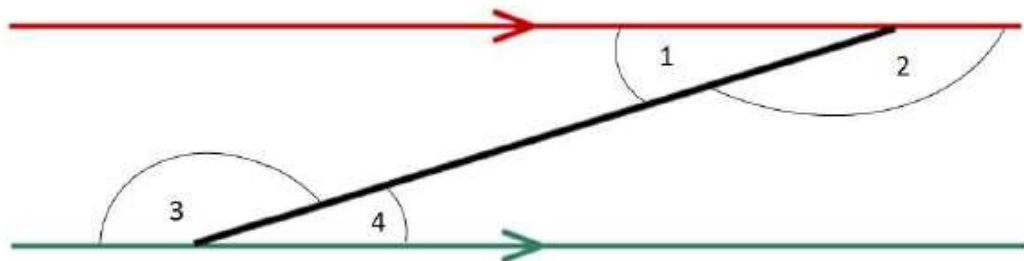
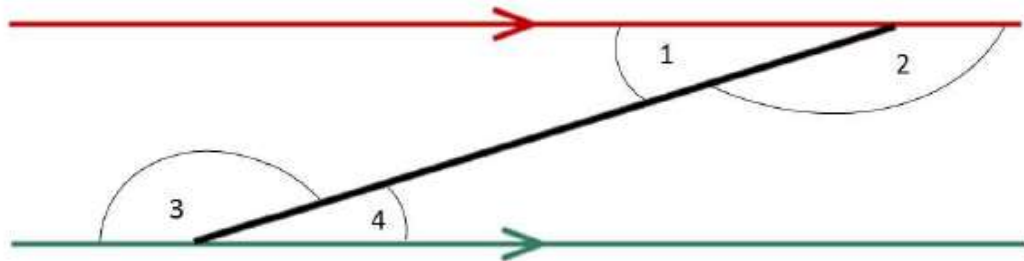
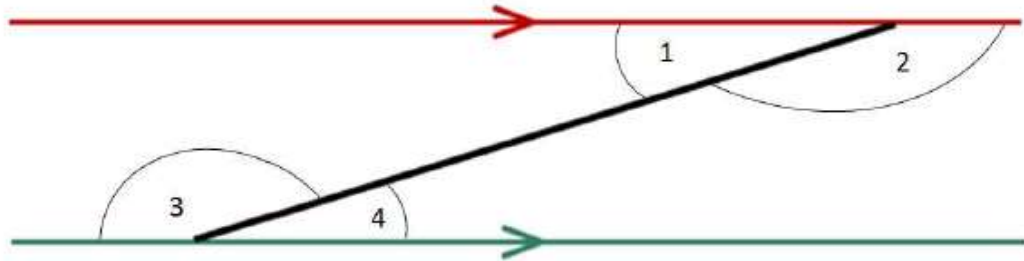
Worksheet 1 - Diagram A

Identify as many equal angles as you can, using as many different methods as you can think of.



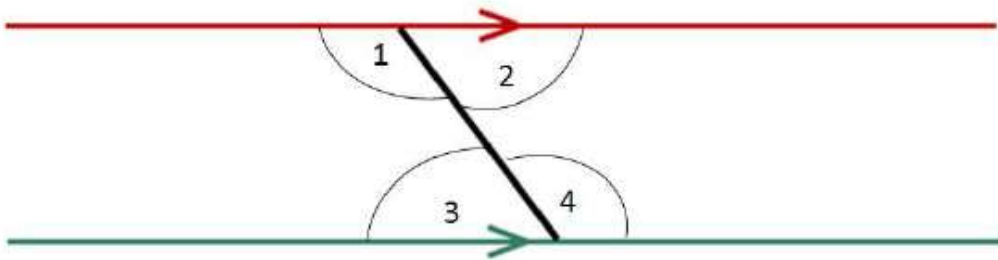
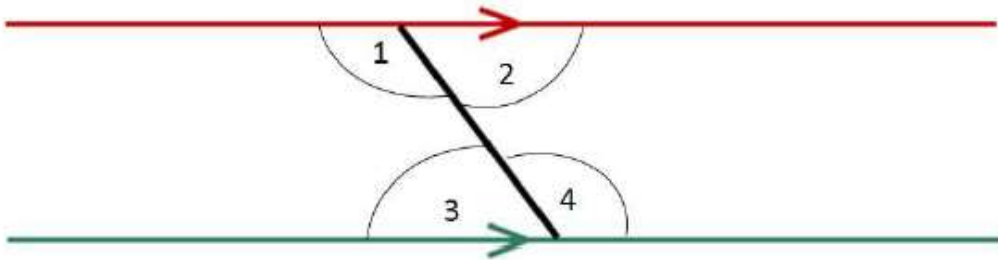
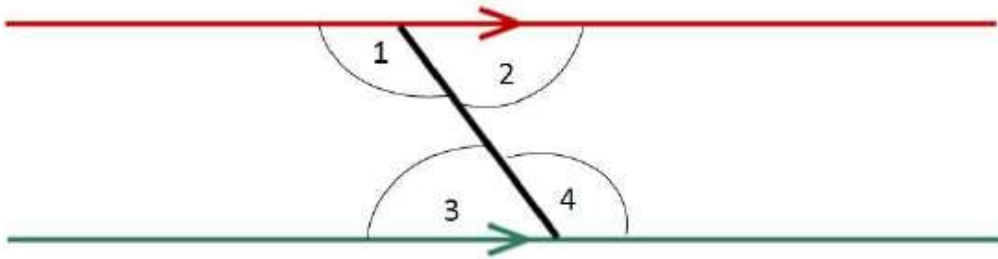
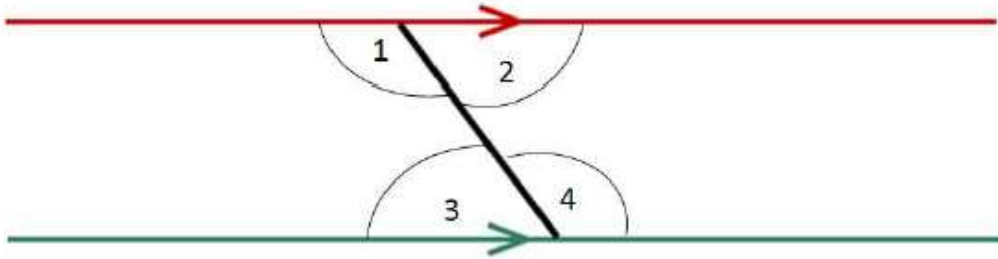
### Worksheet 1 - Diagram B

Identify as many equal angles as you can, using as many different methods as you can think of.



### Worksheet 1 - Diagram C

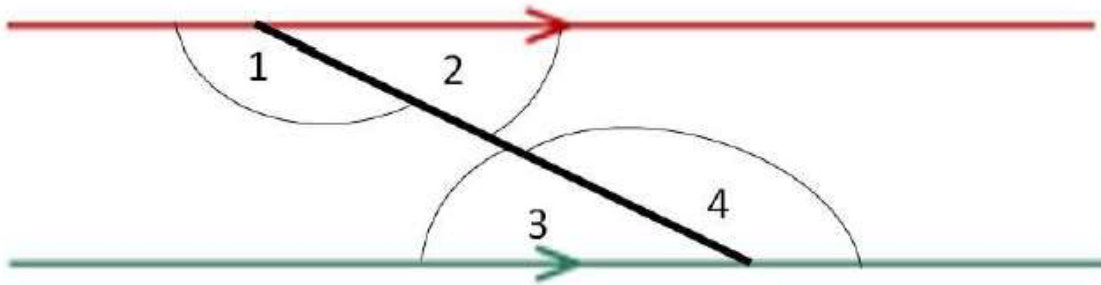
Identify as many equal angles as you can, using as many different methods as you can think of.



## Worksheet 2

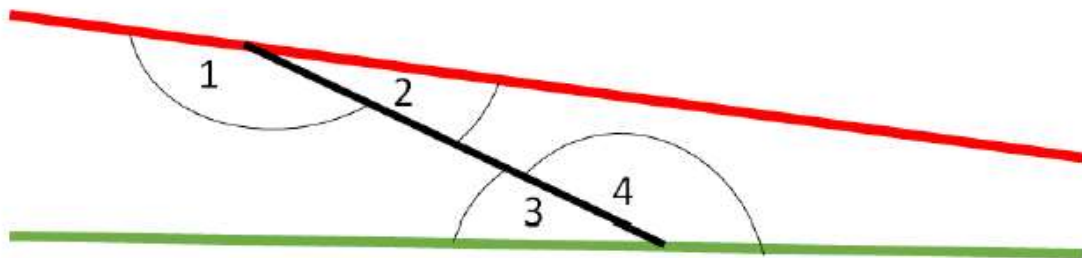
### Diagram D

Using any method, identify all equal angles in the below diagram.



### Diagram E

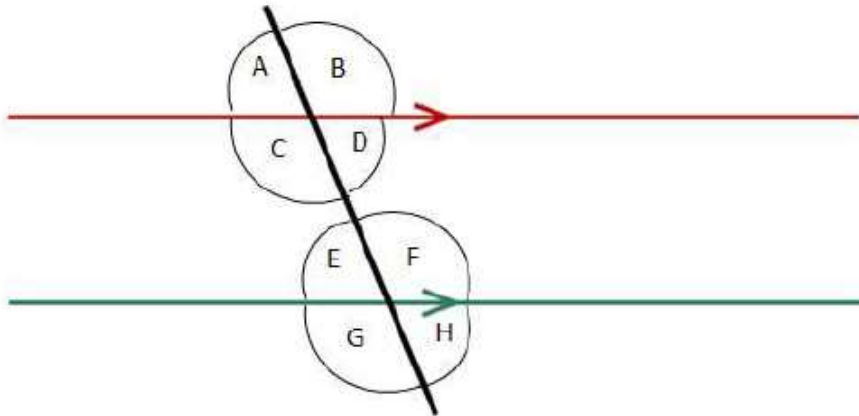
Using any method, identify all equal angles in the below diagram.





### Worksheet 3 - Homework

**Ex 1:** Identify as many equal angles as you can, using as many different methods as you can think of.



**Ex 2:** For any of these pairs of equal angles, can you explain why they are equal? (*Hint: Alternate Angles, Vertically Opposite Angles*)

**Ex 3:** For the pairs of angles you cannot explain, do you think they would always be equal when a pair of lines with a transversal is drawn? (*Hint: Does the pair lines need to be drawn in a special way?*)