

“An Average Problem”

Lesson plan developed by:

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and

Jill Condell (Temple Carrig)

Title: An Average Problem

Topic: Measures of Centre – Averages of Statistical Data

Class: *2nd Year Higher Level*

Lesson: 1 hour

Date: *26th January 2017*

School: *Woodbrook College*

Teacher: *Angela Dwane*

A Brief Description of the Lesson

This lesson is a problem solving investigation into the appropriate selection of measures of central tendency (mean, mode and median). Students will be provided with a set of data and a choice of three numerical measures of central tendency. The problem requires students to calculate the mean, median and mode. They may use statistical diagrams or otherwise to explain and justify why in this instance one numerical measure of central tendency is preferred over another.

Aims of the lesson

Short Term:

By the end of this lesson, students should be able to :

- Explain why the mean is only one type of ‘average’
- Recognise why it is important for data to be arranged in order when calculating the median
- Compare the mean, median and mode
- Explain why a particular numerical average is the best representation and most suitable numerical average depending on the data provided
- Identify and explain the effect of an outlier in the context of central tendency

Long Term Goals for Students:

- To encourage students to develop their literacy and numeracy skills through interpreting dataⁱ.
 - To support the principles of the Junior Cycle Frameworkⁱⁱ
 - To broaden students understanding of measure of central tendency.
 - To allow appreciation of the relevance of Mathematics in everyday life.
 - That students will experience meaningful Mathematics.
 - That students will experience independent learning.
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- To build enthusiasm for the subject.
- To develop cross-curricular links with Business Studiesⁱⁱⁱ.

Learning Outcomes

As a result of studying this topic students will be able to evaluate the purposes and appropriateness of the use of mean, median and mode.

Background and Rationale

When teaching Measures of Centre Tendency students often fail to understand the importance of the median and the mode as measures of central tendency and focus solely on the mean. Students do not consider the type of data when applying the techniques for calculating the mean mode and median. In addition students often fail to ignore the significance of an outlier when interpreting their results. Lastly students often have a lot of difficulty with interpreting and analysing data sets and verbalising their findings. We felt that if we presented the problem using salaries this might generate interest and enthusiasm for the problem. We hope that by allowing the students to investigate the problem for themselves they will experience the three averages of statistical data in a more meaningful way and this will help to make students more aware of why one measure is preferred over another based on the given data set.

The following elements from the Junior Cycle Syllabus were considered in the development of the lesson:

- Use of a variety of summary statistics to describe data; central tendency (mean, median, mode).
- Higher Level Extension: recognize existence of outliers.
- Analyse, interpret and draw conclusions from data.
- Devise, select and use appropriate mathematical models.

The focus of this Lesson Study is to develop and broaden students' understanding of central tendency and how it is used to analyse data.

Research

- Junior Certificate Mathematics Syllabus. (See Appendix 1)^{iv}.
- 2nd Year Teachers Handbook^v.
- Junior Certificate Mathematics – Guidelines for Teachers^{vi}
- Maths Development Team's Website www.projectmaths.ie^{vii}
- Junior Cert Textbooks^{viii}

About the Unit and the Lesson

- We have chosen a problem that simulates a real-life situation in order to generate students' interest.
- The problem facilitates an investigative mode of learning where students are encouraged to recognise various measures of central tendency.
- The use of real world data in terms of money encourages discussion, allowing students to draw conclusions about the data.
- The data set is designed to facilitate recognising outliers.
- The data is presented in the form of a list and the statements are presented in the form of text based sentences.

Flow of the Unit

Syllabus: Strand 1 Statistics & probability Pages 15 and 16 (Appendix 1)

Lesson		Number of lesson periods
1.	<p><u>Classification of data</u></p> <p>Revisit the data cycle</p> <p>Classifying data (Numerical; Discrete/Continuous) (Categorical; Nominal/ordinal)</p> <p>Resource: Data Handling Cycle (Page 2, 3)^{ix}</p>	1 x 40 min.
2.	<p><u>Collecting Data</u></p> <p>Revisit statistical investigation from 1st year and the steps involved.</p> <p>Conduct new investigation.</p> <p>Resource: Data Handling Cycle (Page 1) Handbook page 25 Lesson Idea 2.23</p> <p>Census at school data tool^x</p> <p>Students' excel file generated last year with arm-span and height.</p>	2 x 40 min.
3.	<p>Graphing</p> <p>Recall Graphs from first year</p>	3 x 40 min

	<p>Introduce Histograms and Pie Charts</p> <p>Resources: Handbook Lesson 2.26 Page 27</p> <p>Census at school data</p> <p>Excel investigation</p>	
4.	<p>Frequency Tables</p> <p>Tally Charts for recording large quantities of data and converting to frequency tables</p> <p>Introduce grouped frequency distributions</p> <p>Resources: Handbook Lesson 2.25 page 27</p>	3 x 40 min.
5.	<p>Measures of Central Tendency</p> <p>Investigation through problem solving</p> <p>Formalisation of techniques to find mean mode and median</p> <p>Discuss when it is appropriate to use each measure of center</p> <p>Resources: Research lesson, Handbook page 27</p>	3 x 40 min.
6.	<p>Measures of Central Tendency</p> <p>Investigating the appropriate use of each of the measures of center</p>	1 Research Lesson 60 mins
7.	<p>Measures of Spread</p> <p>Range and Interquartile Ranges</p> <p>Back to Back Stem and Leaf</p>	3 x 40 min

	Resources: Handbook page 27, PM Website	
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Flow of the Lesson

Activity	Points of Consideration																									
<p>Introduction (15 mins)</p> <p>Recap of prior learning:</p> <ul style="list-style-type: none"> • Percentages • Collecting Data (Tally count, frequency tables, ordered list) • Statistical Diagrams (Bar Chart, Histogram, Pie Chart, Stem and Leaf, Dot Plot etc...) • Measures of Centre (mean, mode, median) <div style="border: 1px solid black; padding: 10px; margin-top: 20px;"> <p>Prior Knowledge</p> <p>Below is the number of goals scored by a soccer team in 21 matches:</p> <table style="width: 100%; text-align: center; border-collapse: collapse;"> <tr> <td style="padding: 5px;">4</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">0</td> </tr> <tr> <td style="padding: 5px;">0</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">3</td> <td style="padding: 5px;">3</td> </tr> <tr> <td style="padding: 5px;">1</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">1</td> <td style="padding: 5px;">3</td> <td style="padding: 5px;">4</td> </tr> <tr> <td style="padding: 5px;">3</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">2</td> <td style="padding: 5px;">4</td> <td style="padding: 5px;">1</td> </tr> <tr> <td style="padding: 5px;">2</td> <td></td> <td></td> <td></td> <td></td> </tr> </table> <p>(a) How would you represent the data using an appropriate statistical diagram?</p> <p>(b) How would you calculate?</p> <ul style="list-style-type: none"> • the mean • the median • the mode <p>(c) What percentage of students scored below the mean amount of goals?</p> </div>	4	1	2	1	0	0	1	2	3	3	1	2	1	3	4	3	2	2	4	1	2					<p>Recap of prior knowledge, student led.</p> <p>Recap on measures of center through a short revision problem. Oral response.</p>
4	1	2	1	0																						
0	1	2	3	3																						
1	2	1	3	4																						
3	2	2	4	1																						
2																										

Introduction (15 mins)

Recap of prior learning:

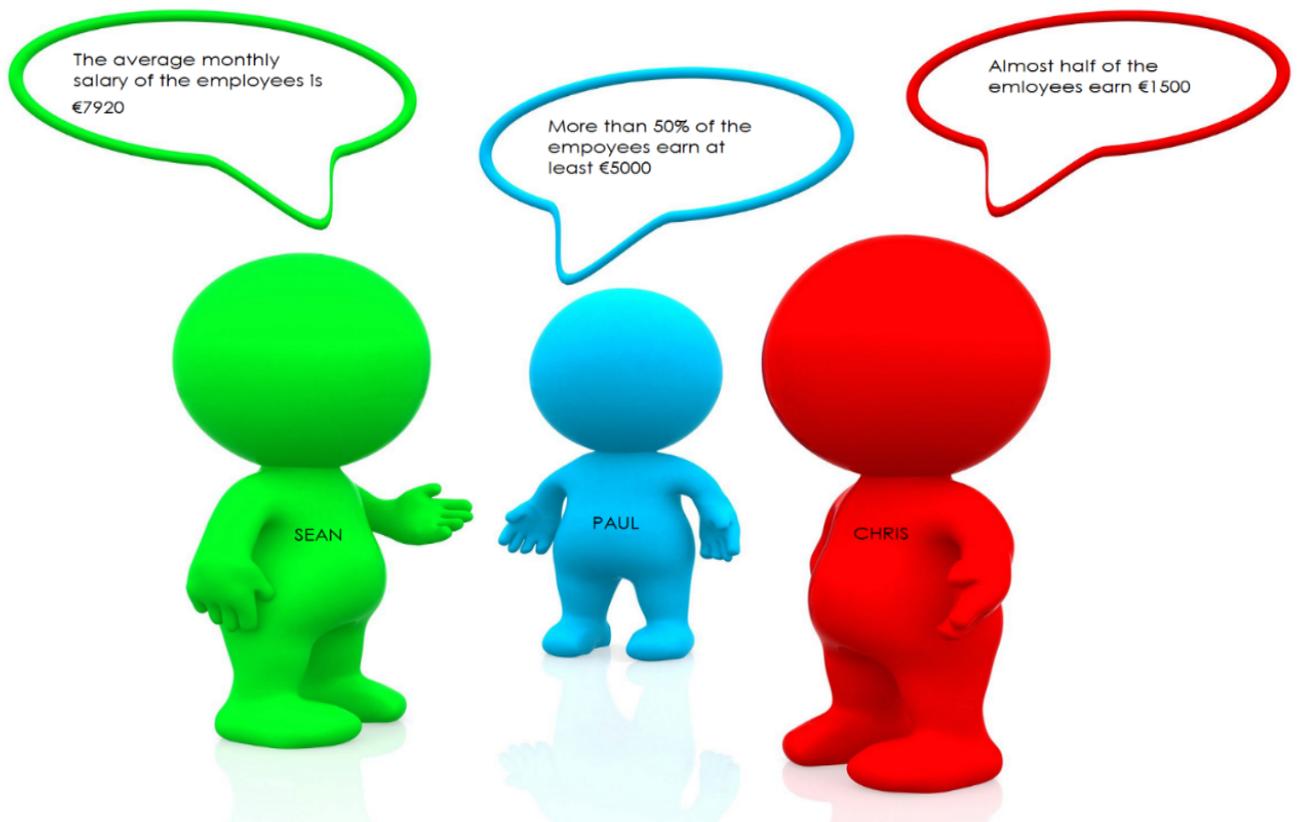
- Percentages
- Collecting Data (Tally count, frequency tables, ordered list)
- Statistical Diagrams (Bar Chart, Histogram, Pie Chart, Stem and Leaf, Dot Plot etc...)
- Measures of Centre (mean, mode, median)

Recap of prior knowledge, student led.

Recap on measures of center through a short revision problem. Oral response.

Prior Knowledge

Below is the number of goals scored by a soccer team in 21 matches:



2. Posing the problem (10 mins)

The Problem:

The monthly salaries (in €) of 25 employees in a company are recorded:

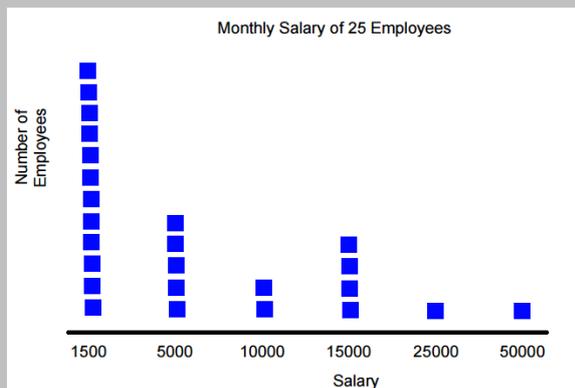
2. Posing the problem (10 mins)

The Problem:

The monthly salaries (in €) of 25 employees in a company are recorded:

1500 10000 1500 5000 15000 50000 1500 25000 15000 1500 1500
5000 1500

1500 5000 1500 1500 15000 5000 1500 10000 15000 1500 1500 5000



...the statements that contradict each other. Who gives the best picture of how much money the employees earn in the company per month?

Explain your answer.

Explain to students that when they pick one of the names as giving the best



E3: Students order the data from biggest to smallest

Expected Responses:

R1: Séan gives the best picture of what the employees earn per month.

$$\text{Total Salary: } 12(15000) + 5(5000) + 2(10000) + 4(15000) + 1(25000) + 1(50000) = 198000$$

Total People: 25

$$\text{Mean} = 198000 / 25 = \text{€}7920$$

Reason: The mean is the average of what employees earn per month.

R1 Counter: I didn't pick Séan because € 50,000 euro is an outlier and exaggerates the average monthly earnings.

R2: Chris gives the best picture of what the employees earn per month.

Student may refer to frequency table and/or bar chart

Reason:
12 is the highest frequency so the modal

Let each student calculate the mean, median and mode and give their reason(s) for choosing which of the three statements captures the best picture of how much the employees in the company earn per month in their copies.

Use a seating chart to note each student's calculation of mean, median & mode. More importantly and in line with the aims of the lesson their statistical diagrams and reason(s) for whether they choose mean, median or mode as their preferred numerical value of central tendency.

Prepare for organizing the whole-class discussion.

salary is €1500

Percentage of employees who earn
€ 1,500 = $12/25 \times 100\%$
=48%

R2 Counter:

I didn't pick Chris even though his answer is correct because € 1500 is the minimum value

€1500 does not give any information about the amount the other employees in the company earn.

R3: Paul's gives the best picture of what the employees earn per month.

Students make an ordered list of the data:

1500 1500 1500 1500 1500 1500 1500 1500
1500 1500 1500 1500 5000 5000 5000 5000
5000 10000 10000 15000 15000 15000
15000 25000 50000

Students may use a dot plot (see above)

Median = € 5,000

Reason : The number of employees who earn at least € 5,000 = 13

Percentage of employees who earn at least € 5, 000= $13/25 \times 100\%$ = 52 % which is greater than 50%

The statement that more than 50% of the employees earn at least € 5,000 is correct. It also gives the best picture of how much money the employees earn in the company since the statement allows us to infer the amount that the rest of the employees earn.

Potential Pitfall:

Students may struggle to recognise that this figure is not giving a good reflection of the data set as a whole.

Potential Pitfalls:

Students may struggle with interpreting the median in the context of the whole company.

3. Anticipated Student Responses.

General expected methods of approach:

E1: Students attempt to organize the data into a frequency table

Frequency Table

Salary (€)	1500	5000	10000	15000	25000	50000
Number of Employees	12	5	2	4	1	1

E2: Students attempt to visualise the data using a graph.

E3: Students order the data from biggest to smallest

Expected Responses:

Let each student organize the data in their own way. An ordered list, frequency table and statistical diagram allow students to approach the problem in a logical and ordered way.

Each of the expected approaches will be pre-prepared to aid with the classroom discussion.

4. Comparing and Discussing (25 mins)

For each answer above:

1. Ask one of the students who came up with one of the choices to show their answer to the class but not their reason(s)
2. Let other students explain how the student reasoned their choice
3. Let the student who came up with one of the choices to justify if the other student's reason(s) are correct.

Repeat the above for the three choices so that students can draw the following conclusions:

Anticipated Conclusions:

1. The mean is usually preferred over the median and mode because all of the values in a set of data are used to

Discuss will be structured to lead the student volunteers to the conclusion that the median is the preferred measure of center in this situation.

Students will be given opportunities to change their opinion throughout the discussion.

By providing an opportunity for students to explain to other student's reasons for why they picked Séan, Chris & Paul in this order helps students see why Séan & Chris are not the best statements.

This helps students to understand why the median is

Evaluation

Observation Plan

- Each observer will take a position with a group of 8 students.
- Observers will photograph samples of students.
- Most common methods will be recorded.
- Which method students tried first will be noted.

Logistics

- Aoife and Jill will observe.
- Data will be photographed on the iPad.
- A seating plan will be given to each observer. (Appendix 2)

Observational Strategies

- Group dialogue will be recorded.
- Any questions students ask each other will be recorded.
- The first method approached will be recorded.
- The most commonly attempted approach will be noted.
- Any misconceptions will be recorded.
- Do the students remain focused on the task for the whole session?

Evidence

- Photographs of the method.
 - Written description of approaches taken.
 - List of time spent on each method.
-

Board Plan

<p>The average monthly salary of the employees is €7920</p>	<p>Frequency table</p> <table border="1"> <thead> <tr> <th>Salary (€)</th> <th>1500</th> <th>3000</th> <th>5000</th> <th>10000</th> <th>15000</th> <th>20000</th> <th>50000</th> </tr> </thead> <tbody> <tr> <td>Number of Employees</td> <td>12</td> <td>5</td> <td>2</td> <td>4</td> <td>1</td> <td>1</td> <td>1</td> </tr> </tbody> </table> <p>Monthly Salary of 25 Employees</p>	Salary (€)	1500	3000	5000	10000	15000	20000	50000	Number of Employees	12	5	2	4	1	1	1	<p>Calculations:</p> $12(15000) + 5(5000) + 2(10000) + 4(15000) + 1(25000) + 1(50000)$ <p>Total Salary: = 198000 Total People: 25</p> $\text{Mean} = \frac{198000}{25} = \text{€}7920$ <p>I chose Sean because the mean is the average of what employees earn per month.</p>	<p>I didn't pick <u>Seán</u> because €50,000 euro is an outlier and exaggerates the average monthly earnings.</p>
Salary (€)	1500	3000	5000	10000	15000	20000	50000												
Number of Employees	12	5	2	4	1	1	1												
<p>Almost half of the employees earn €1500</p>	<table border="1"> <thead> <tr> <th>Salary (€)</th> <th>1500</th> <th>3000</th> <th>5000</th> <th>10000</th> <th>15000</th> <th>20000</th> <th>50000</th> </tr> </thead> <tbody> <tr> <td>Number of Employees</td> <td>12</td> <td>5</td> <td>2</td> <td>4</td> <td>1</td> <td>1</td> <td>1</td> </tr> </tbody> </table> <p>Monthly Salary of 25 Employees</p>	Salary (€)	1500	3000	5000	10000	15000	20000	50000	Number of Employees	12	5	2	4	1	1	1	<p>Calculations:</p> <p>12 is the highest frequency</p> <p>Mode = €1500</p> <p>Percentage of employees who earn €1,500 = $12/25 \times 100\%$ = 48%</p> <p>I chose Chris because 48% is almost half of the employees.</p>	<p>I didn't choose Chris because although 48% is almost half of the employees, it is the lowest possible value.</p> <p>There are another 52% of the employees earning more than this that are not represented by this figure.</p>
Salary (€)	1500	3000	5000	10000	15000	20000	50000												
Number of Employees	12	5	2	4	1	1	1												
<p>More than 50% of the employees earn at least €5000</p>	<p>Monthly Salary of 25 Employees</p>	<p>Calculations:</p> <p>1500 1500 1500 1500 1500 1500 1500 1500 1500 1500 1500 1500 5000 5000 5000 5000 5000 10000 10000 15000 15000 15000 15000 25000 50000</p> <p>Median = € 5,000</p> <p>I chose Paul because the number of employees who earn at least €5,000 is 13</p> <p>$13/25 \times 100\% = 52\%$</p>	<p>Paul's is the best statement since the statement allows us to infer the amount of money the rest of the employees earn.</p>																

6. Post-lesson reflection

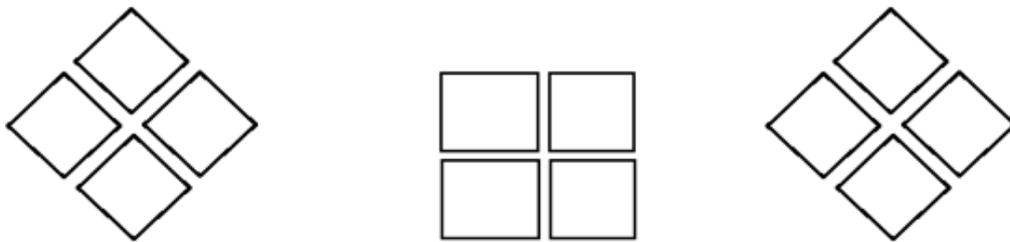
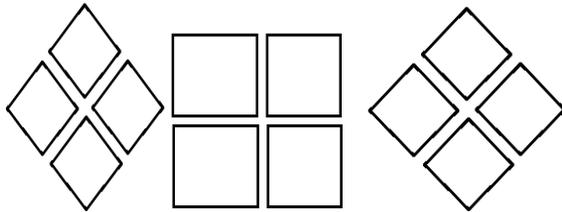
To be filled out later.

- What are the major patterns and tendencies in the evidence? Discuss
- What are the key observations or representative examples of student learning and thinking?
- What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?
- In what ways did students achieve or not achieve the learning goals?
- Based on your analysis, how would you change or revise the lesson?
- What are the implications for teaching in your field?

Appendix 1: Excerpts from Syllabus

<p>1.4 Statistical reasoning with an aim to becoming a statistically aware consumer</p>	<p>Situations where statistics are misused and learn to evaluate the reliability and quality of data and data sources. Different types of data.</p>	<ul style="list-style-type: none"> – engage in discussions about the purpose of statistics and recognise misconceptions and misuses of statistics – work with different types of data: <ul style="list-style-type: none"> categorical: nominal or ordinal numerical: discrete or continuous in order to clarify the problem at hand – evaluate reliability of data and data sources
<p>1.5 Finding, collecting and organising data</p>	<p>The use of statistics to gather information from a selection of the population with the intention of making generalisations about the whole population. Formulating a statistics question based on data that vary, allowing for distinction</p>	<ul style="list-style-type: none"> – clarify the problem at hand – formulate questions that can be answered with data – explore different ways of collecting data – generate data, or source data from other sources including the internet – select a sample from a population (Simple Random Sample) – recognise the importance of representativeness so as to avoid biased samples – design a plan and collect data on the basis of above knowledge
<p>1.6 Representing data graphically and numerically</p>	<p>Methods of representing data. Students develop a sense that data can convey information and that organising data in different ways can help clarify what the data have to tell us. They see a data set as a whole and so are able to use proportions and measures of centre to describe the data. Mean of a grouped frequency distribution.</p>	<p>Graphical</p> <ul style="list-style-type: none"> – select appropriate methods to represent and describe the sample (univariate data only) – evaluate the effectiveness of different displays in representing the findings of a statistical investigation conducted by others – use pie charts, bar charts, line plots, histograms (equal intervals), stem and leaf plots and back-to-back stem and leaf plots to display data – use appropriate graphical displays to compare data sets <p>Numerical</p> <ul style="list-style-type: none"> – use a variety of summary statistics to describe the data: central tendency – mean, median, mode variability – range, quartiles and inter-quartile range – recognise the existence of outliers
<p>1.7 Analysing, interpreting and drawing conclusions from data</p>	<p>Drawing conclusions from data; limitations of conclusions.</p>	<ul style="list-style-type: none"> – interpret graphical summaries of data – relate the interpretation to the original question – recognise how sampling variability influences the use of sample information to make statements about the population – draw conclusions from graphical and numerical summaries of data, recognising assumptions and limitations
<p>Students learn about Students should be able to</p>		
<p>1.8 Synthesis and problem-solving skills</p>	<ul style="list-style-type: none"> – explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions. 	

Appendix 2: Seating Plan



ⁱThe lesson aims to support the National Strategy to Improve Literacy and Numeracy among Children and Young People 2011- 2020 by presenting data for students to analyse and interpret.

ⁱⁱ This lesson adopts some of the following points from the 8 principles of the Junior Cycle Framework. “High expectations of learners” “be creative and innovative” “encourages participation, generates engagement and enthusiasm, and connects with life outside the school.” “Inclusive of all students” “enables students to build on their learning to date” “support students in developing greater independence in learning”. The lesson also supports the following statements of learning. “The student communicates effectively... devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills”

ⁱⁱⁱBusiness Studies Syllabus Strand 1 Personal Finance “1.2 Identify and classify sources of income” Strand 2 “In this strand, students learn about being enterprising, the functions of an organisation and the business environment.”

^{iv} “Junior Cert Maths Syllabus Foundation, Ordinary and Higher for Examination from 2016” pages 15 and 16

^v Lesson Idea 2.23 page 25 Lesson Idea 2.26 page 27

^{vi} Guidelines pages 23 also statistics lesson ideas pages 52 - 57

^{vii} <http://www.projectmaths.ie/for-teachers/junior-certificate/#> Information and Resources in Statistics and Probability Section in particular the modular courses.
<http://www.projectmaths.ie/documents/PDF/RoadMapForAStatisticalInvestigation.pdf?strand>

^{viii} A variety of textbooks were considered and examples studied, however no direct problems or information came from a particular text.

^{ix} <http://www.projectmaths.ie/documents/PDF/TheDataHandlingCycle.pdf?strand>

^x <http://www.censusatschool.ie/>