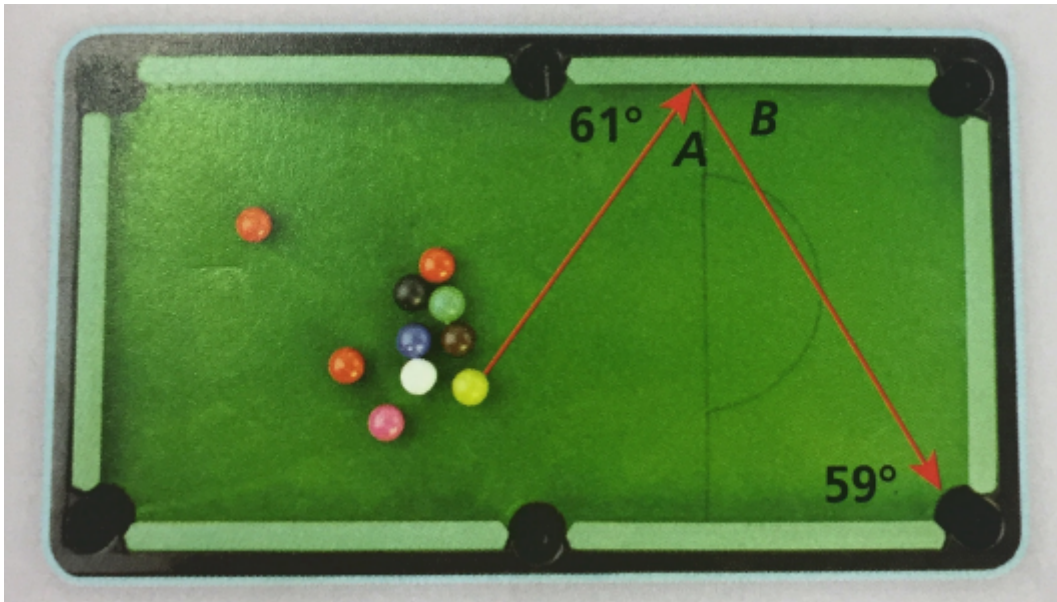


First year group – Are You Snookered?



For the lesson on
January 11th at
11.15a.m.

At Wilson's Hospital School, Multyfarnham,
To a First Year Class
Teacher: Olive Butler

Lesson plan developed by: Carmel Coleman (Wilson's Hospital School)
Olive Butler (Wilson's Hospital School)
Joanna Byrne (Mullingar Community School)

1. Title of the Lesson: Are You Snookered? (Angles)

2. Brief description of the lesson

The lesson builds on students' prior learning of synthetic geometry, angles and lines in particular. Using a contextualized problem students are asked to discover in how many ways they can evaluate the missing angles. Students will be challenged to apply their prior knowledge in a previously unseen context.

3. Aims of the Lesson:

Short term aim;

- We would like our students to be able to apply their knowledge of the various types of angles to enable them to solve unseen problems in Geometry in a variety of ways.

Long term aims;

- I'd like to foster my students to become independent learners (key skill managing myself - being able to reflect on my own learning).
- I'd like to emphasise to students that a problem can have several equally valid solutions
- I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities (key skill staying well- being positive about learning and managing information and thinking- being curious).
- I'd like my students to connect and review the concepts that we have studied already. These concepts are vertically opposite angles, alternate angles, corresponding angles, straight angle, full rotation and three angles in a triangle sum to 180 degrees (key skill; managing information and thinking- critical and creative thinking)
- We would like students to develop their literacy and numeracy skills through discussing prior knowledge and varied problem solutions¹.

4. Learning Outcomes:

As a result of studying this topic students will be able to:

- Relate their understanding of angles to a conceptual problem.
- Use the relationship between parallel lines and angles to find the missing angles.
- Present and discuss varied solutions with their peers (key skill; communication –using language, using number, listen and expressing myself, discussing and debating, being creative – exploring options and alternatives, being numerate – expressing ideas mathematically)

5. Background and Rationale

(a) Relationship to the syllabus

The Junior Certificate Mathematics syllabus encompasses five strands and Synthetic Geometry falls under the strand Geometry and Trigonometry. Each strand is sub-divided into topics where a description of the topic is given (what the students learn about) and learning outcomes are detailed (what the student should be able to do).

^{1 1} This Lesson Proposal illustrates a number of strategies to support the implementation of Literacy and Numeracy for Learning and Life: the National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020 (Department of Education & Skills 2011).

Strand 2, subsection 2.1 (page 18) Synthetic Geometry notes that student should be able to show that:

- Vertically opposite angles are equal in measure.
- A transversal makes equal alternate angles on two lines then the lines are parallel (and converse).
- Two lines are parallel if and only if, for any transversal, the corresponding angles are equal.
- Each exterior angles of a triangle is equal to the sum of the interior opposite angles.

The syllabus notes that teaching and learning methods need to be differentiated in order to meet the needs of all learners (page 11). It is noted that every opportunity to make connections across the strands should be taken (page 11), and that problem solving is ‘integral to mathematics learning’ (page 10). The syllabus states:

“...encouraging learners to share, explain and justify their solution strategies...learners [can] develop robust and deep mathematical understanding as well as confidence in their mathematical ability.” Junior Certificate Mathematics Syllabus Page 11

Page 20 of the Junior Certificate Syllabus also outlines student learning outcomes as they relate to geometry.

(b) Difficulties students have had in the past with the subject matter.

- Not being able to calculate unknown angles.
- Not applying their previous knowledge to complete more complicated questions
- Not constructing lines to solve a geometrical problem

(c) The thematic focus of this lesson study (larger goals the team will try to address and why).

- Problem solving
- More than one way to complete a question.
- Not waiting for teacher instructions: Foster independent learning.

6. Research

- Junior Certificate Guidelines for Teachers (DES 2002, Government Publications Sales Office).
- First Year Handbook (PMDT).

- Junior Certificate textbook.
- Junior Certificate Mathematics Syllabus (DES 2016, Government Publications Sales Office).
- The Maths Development Team website, www.projectmaths.ie
- Website; www.nrich.maths.org
- Literacy and Numeracy for Learning and Life (DES 2011).

7. About the Unit and the Lesson

This lesson is designed to get students to draw on prior knowledge to attempt more challenging questions. Students will have participated in multiple lessons thus far in Synthetic Geometry (Strand 2) and so we hope to guide them in understanding how to calculate unknown angles from information presented in a given problem and recognising steps that can be taken. From the diagram and their previous knowledge of triangles and parallel lines they will visually become more creative in finding an unknown angle.

It follows the Junior Certificate Mathematics Syllabus 2016, strand 2, section 2.1 Synthetic geometry, page 17-19.

8. Flow of the Unit:

Lesson	Title: Geometry 1 – Triangles and Quadrilaterals	# of lesson periods
1	<ul style="list-style-type: none"> • Lines, angles and parallel lines. Resource: 1 st Year Handbook Section 3 & 8	2 x 40 min.
2	<ul style="list-style-type: none"> • Angles of a triangle including interior and exterior angles. 	2 x 40 min.
3	<ul style="list-style-type: none"> • Quadrilaterals 	1 x 40 min.
4	<ul style="list-style-type: none"> • Parallel lines and triangles 	3 x 40 min.

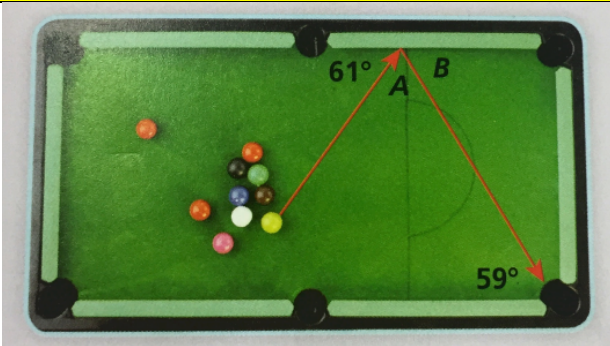
9. Flow of the Lesson

Teaching Activity	Points of Consideration
<p>1. Introduction (6 minutes)</p> <ul style="list-style-type: none"> • Welcome students. • Prior Knowledge. <p>Using the information, we have learned to date what can you tell me about each of the following?</p> <ul style="list-style-type: none"> ✓ Lines ✓ Angles ✓ Triangles ✓ Parallel Lines ✓ Parallel lines, triangles and their angles. ✓ Exterior angles 	<p>Teacher draws relevant images on the board (PowerPoint presentation may be used) to help extract required information.</p>
<p>2. Posing the Task (4 minutes)</p> <p>Today's task will involve understanding and</p>	<p>Having received the problem, ensure that students are aware that each problem is the same</p>

using all the information we have been taught to date to solve this task.

You are all given A3 sheet with a problem that requires solving. The sheet is divided into 6 sections all of which have the same problem.

Problem Posed: Find angles A and B in as many ways as possible.



and that there is more than one solution to this problem.

Read out the given task and ensure that students are aware what is being required of them.

Required:

Students will need the following; set squares, ruler, and pencil.

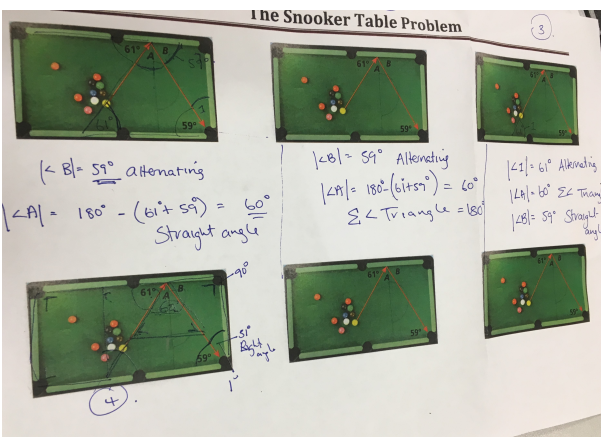
3. Individual Student Work (10 minutes)

Instruct students that they have 10 minutes to solve the given problem as many ways as they can think of.

During this ten minutes circulate room to prepare and plan for board work and Class Discussion.

This in between desk assessment is crucial to the success of the class discussion to follow. The teacher will circulate and choose the students that will present their solution on the board.

4. Comparing and Discussing



We will focus on getting students to use the correct mathematical language when discussing and presenting the various solutions.

We will ensure students use the correct mathematical equipment if they draw any constructions lines when presenting their solution.

	<p>We will know students are benefiting from the discussion by asking for questions when a possible solution has been presented. The teacher will also be watching for this when she circulates the room during individual student work time.</p>
<p>5. Summing up</p> <p>The teacher will ask the class if there are any similar solutions on the board and get a student to say which solutions should be grouped together and why.</p> <p>The class suggested that</p> <ol style="list-style-type: none"> 1. There are multiple and different ways of finding solutions 2. They need to show their work clearly 3. They need to reference theorems when using them. 	<p>Teacher asked class to add another solution to their work from the variety of solutions presented to the class for homework</p>

10. Evaluation

- What is your plan for observing students?

A seating plan is provided by the teachers giving the lesson. There are three observing teachers and each teacher in the observing group has selected an area of the room to circulate and observe student activity.

- Lesson data

Data will be recorded with pen and paper and an I Pad will be used to take photos of student workings/ misconceptions/ lack of understanding to understanding etc.

- Observational strategies

Walk around the section of the room assigned to the teacher. Just observe and try not aid in student thinking by hinting at solutions (even if students look for help we will try and avoid

this as we are trying to foster independent thinkers and learners). If taking a photo request permission from the student beforehand.

- Types of student thinking and behaviour the observers will focus on

- Stage 1 Prior Knowledge

- When recalling prior knowledge watch for students who seem confused or lack understanding at the early stage of the lesson.
- Are students clear of the task they have to complete
- What questions do students ask in prior knowledge or are any misconceptions raised?

- Individual work by students

- Are students able to start the problem
- How many solutions are students finding?
- What equipment are students using to find solutions i.e. rulers/ set squares etc.
- Are students constructing any lines/ triangles to aid solutions
- Are students “stuck” and how do they react to this feeling/ event – do students just give up or ?
- What questions are students posing to teachers observing / each other?
- What comments are students making to themselves as they work individually?
- How long do students spend working on the task?
- Are they able to apply the prior knowledge discussed to the problem on the page?
- Are there any “different” solutions that the teachers did not think of?

Discussion

- Are students attentive? What are they doing when solutions being presented i.e. taking notes / chatting to others around them about the solution/ having an “ah ha” moment etc.
- What questions do students pose to the presenter?
- How is the board work presented by the students – any clarification needed or any correction
- Use of mathematical language by the student presenting and by students posing questions.
- Did the discussion aid understanding and give students other strategies when dealing with a mathematical problem.

- Additional kinds of evidence will be collected

- Samples of work
- Photos of misconceptions/ errors/ unique solutions


- Photos of board work
- Photos when presenting

11. Board Plan.

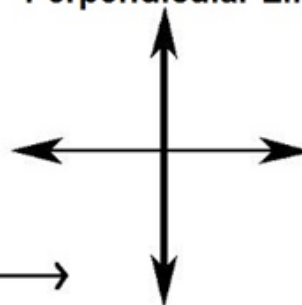
Prior Knowledge using a PowerPoint Presentation

LINES

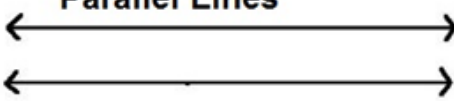
Intersecting Lines




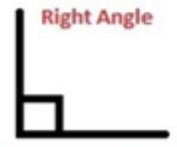




Perpendicular Lines



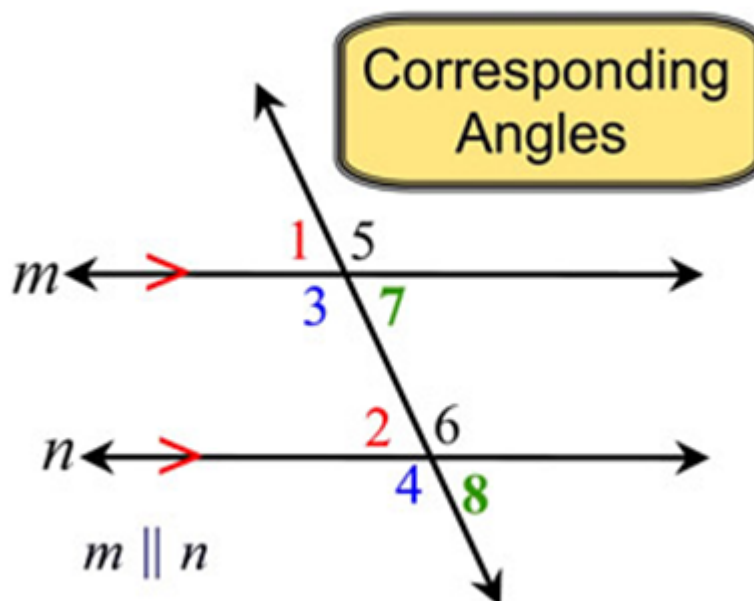
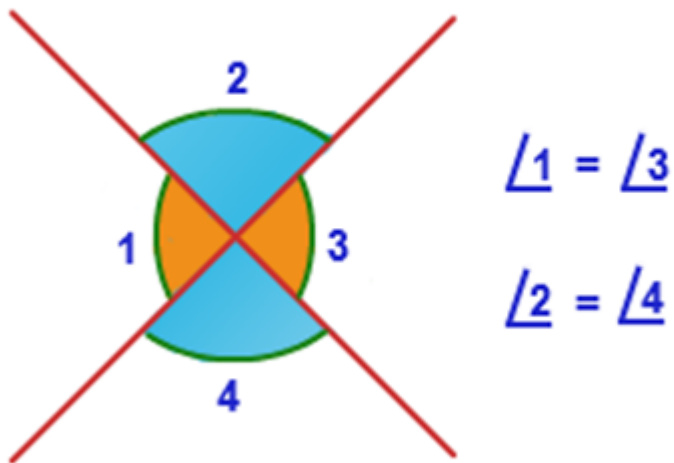
Parallel Lines



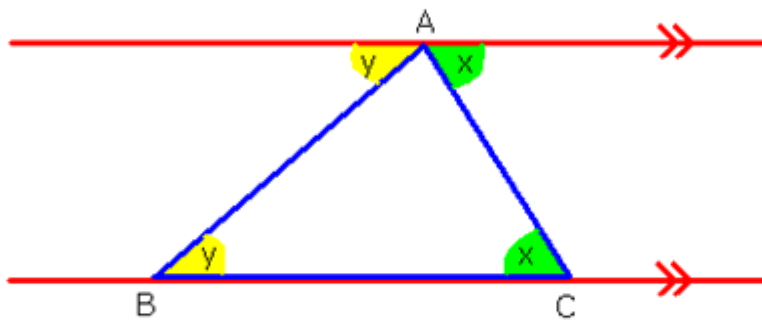
ANGLES

<p>Acute Angle</p>  <p>Less Than 90 Degree</p>	<p>Right Angle</p>  <p>Exact 90 Degree</p>	<p>Obtuse Angle</p>  <p>Greater than 90 degree and less than 180 degree.</p>
<p>Straight Angle</p>  <p>Exact 180 degree.</p>	<p>Reflex Angle</p>  <p>Greater than 180 degree.</p>	<p>Full Rotation</p>  <p>Exact 360 degree.</p>

VERTICALLY OPPOSITE ANGLES






ALTERNATE ANGLES






TYPES OF TRIANGLES

Triangles Based on Sides

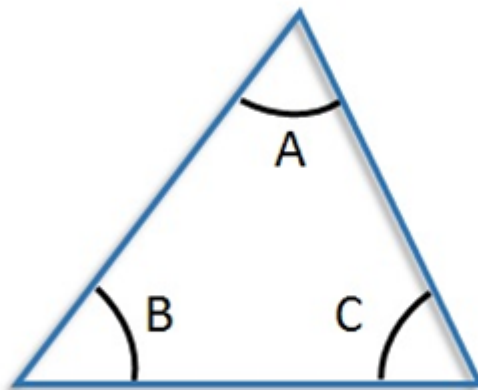
<p>Scalene</p> 	<p>Isosceles</p> 	<p>Equilateral</p> 
<p>Length of all sides are different</p>	<p>Length of two sides are equal</p>	<p>Length of all sides are equal</p>

Triangles Based on Angles

<p>Acute</p> 	<p>Right</p> 	<p>Obtuse</p> 
<p>Each angle is $< 90^\circ$</p>	<p>One angle is $= 90^\circ$</p>	<p>One angle is $> 90^\circ$</p>



ANGLES IN A TRIANGLE





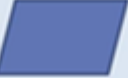



$$A + B + C = 180^\circ$$



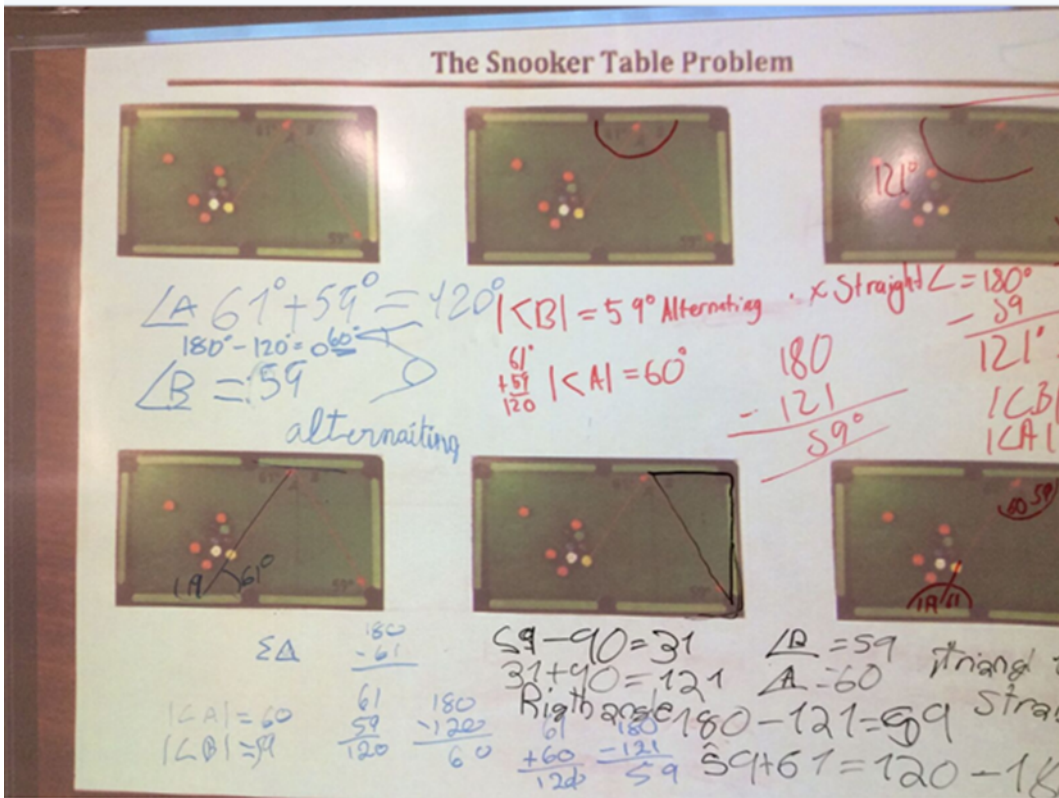
QUADRILATERALS

Types of Quadrilateral

<p>square</p> <p>4 right angles</p> <p>4 equal sides</p> <p>Opposite sides are parallel</p> <p>All sides the same length</p> 	<p>rhombus</p> <p>0 right angles</p> <p>4 equal sides</p> <p>Opposite sides are parallel</p> <p>All sides the same length</p> 	<p>kite</p> <p>0 right angles</p> <p>2 sets of equal sides</p> <p>No sides are parallel</p> <p>2 pairs of sides the same length</p> 
<p>rectangle</p> <p>4 right angles</p> <p>4 equal sides</p> <p>Opposite sides are parallel</p> <p>Opposite sides the same length</p> 	<p>parallelogram</p> <p>0 right angles</p> <p>2 sets of equal sides</p> <p>Opposite sides are parallel</p> <p>Opposite sides the same length</p> 	<p>trapezium</p> <p>0 right angles</p> <p>2 sets of equal sides</p> <p>1 set of sides are parallel</p> <p>sides can be any length</p> 



Student solutions



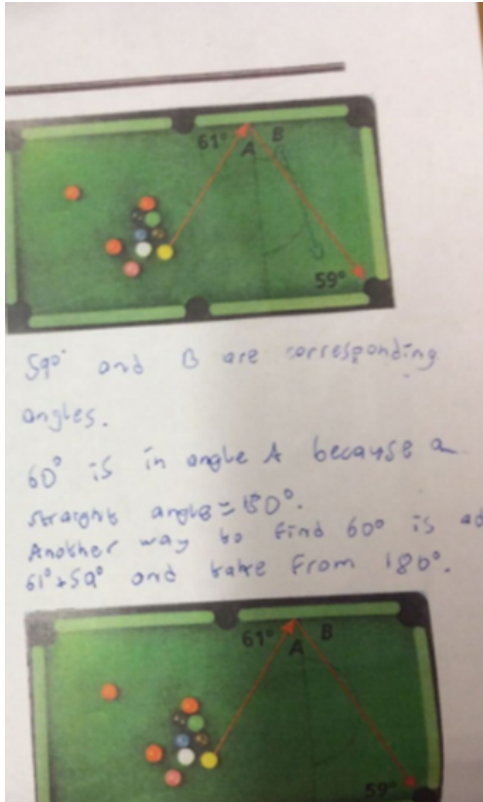
12. Post-lesson reflection

- Major patterns and tendencies in the evidence

Teacher agreed that some students have a language barrier which was noted when students had problems verbalising presentation or misspelling words such as triangle was spelled trianl (missing e at end). A percentage of the class are non-native English speakers.

The language used by the presenters did however improve as more students presented which was interesting.

Students confused the type of angles and the names i.e. called alternate angles corresponding etc.



Key observations or representative examples of student learning and thinking?

Students generally were excellent verbally in using mathematical language when presenting their solutions. The explanations were clear and prior knowledge was applied. However they did not put this down in writing on the board with their solutions i.e. students would give the theorem ‘alternate angles’ however not write this beside the mathematics. The theorem had been referenced in their solution on the page but they failed to do the same when putting solutions on the board.

The task was engaging and the majority of students seemed to enjoy the challenge.

- What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?

-Some students did not see that the Snooker table edges are a pair of parallel lines.

-Others did not spot that by extending the transversal, they could create a triangle.

-One student wrote that angle B was 61° as it was vertically opposite to the angle of 61°

-Two students were ‘stuck’ one said she needed more thinking time and the other tried drawing different triangles between the transversals, however did not spot any connections with triangles and angles.

-some students spotted an application of the alternate angle theorem that the teacher had not thought of.

- some students mixed up the names of corresponding and alternating angles with each other

when referencing them in the solution they presented. One student claimed ‘I know it but what’s the word?’

- In what ways did students achieve or not achieve the learning goals?

The majority of students found one method in finding the solution with many starting other methods. The observation group agreed with more time these students may have had more success. One student had a total of four different methods of solving the problem.

Two students did not achieve the learning goal however they did write down what they spotted. One had spotted a straight angle and the other had tried drawing triangles in.

Teachers felt that if students could not visualise the parallel lines or the triangle the students had difficulty starting the task.

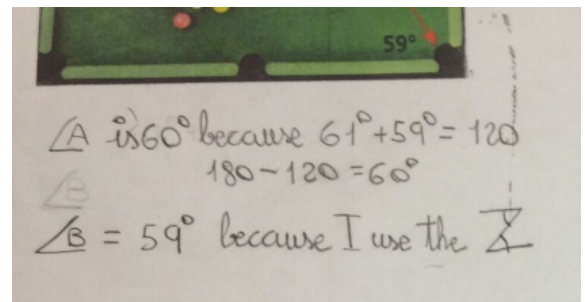
- Based on your analysis, how would you change or revise the lesson?

More thinking time to allow for development of methods started by students towards a solution.

Encourage students more to explain in words and not only verbally what they are presenting on the board.

- What are the implications for teaching in your field?

- Mathematical written literacy was a weakness with some students. All presenters were verbally quite strong however this was not apparent in board work with misspellings and theorems not written down beside mathematics.



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The teachers agreed to look at a ‘Mathematical Language Check List’. The school has already a ‘Literacy Language Check List’ as part of the Literacy strategy however this is too broad and some parts are irrelevant to the mathematics classroom. The teachers suggested points like – correct symbols, clear presentation of work, include all steps, and include theorems when used.

Application or existence of parallel lines in ‘real life’ contexts has to be discussed as this appeared to be an issue for some students.