5th Year Higher Level Class: Developing Frieda's Field

For the lesson on January 17th, 2017.

At St Plunket's College, Swords Road, Whitehall, Dublin 9.

Teacher: Susan Cribbin

Lesson plan developed by: Eimer Nic an Rí, Susan Cribbin, Niamh Fagan and Lucy de Faoite.

Title: Developing Freda's Field

The students will expand on and adapt their understanding of the ½ base × perpendicular height formula using trigonometric ratios to find the area of a non-right-angled triangle.

Aims

The students should

- Appreciate that we can use mathematics in real-life contexts
- Be able to communicate their ideas effectively to their peers using appropriate language
- Be prepared to think of various approaches to solving problems and to think creatively
- Be able to manipulate formulae
- Be able to connect various areas of the syllabus
- To understand there is more than one way to find the area of a triangle
- To apply the use trigonometric ratios to find the area of a triangle
- To explain how the two formulae ½ base x perpendicular height and ½ ab sin C relate to each other

Learning Outcomes

At the end of the lesson the students should be able to:

- Recognise the best way to find the area of a non-right-angled triangle, depending on the context.
- To use the sine ratio to find the perpendicular height of a triangle when not given
- To appreciate that the trigonometric ratios can be used to solve problems in non-Pythagorean triangles.
- To develop the formula ½ ab sin C

Background and Rationale

As there are three possible methods of finding the area of a triangle on the syllabus, it is of benefit to explore the connection between the formulae. The students are not used to situations where the perpendicular height is not given. They will be given the opportunity to consolidate their understanding of the trigonometric ratios and understand how the ½ base x perpendicular height formula can be adapted.

Research

Leaving certificate maths textbooks – while considering what topic we would like to focus on

Exam papers – to help us create a suitable problem

Project maths handbook – to assist in planning a logical sequence of lessons

Teaching and learning plans – trigonometry

About the Unit and Lesson

The unit will start by checking the students' prior knowledge about triangles and basic geometrical constructions and by promoting an appreciation of how a mathematical understanding of triangles is useful in daily life. The unit will continue by focusing on right-angled triangles and Pythagoras' Theorem. The students also know how to manipulate formulae. The students will learn to use the trigonometric ratios to solve a range of problems, with an emphasis on real-life contexts. The lesson itself will present a new context to the students, a non-right-angled triangle, and will require them to use their prior knowledge to discover the formula ½abSinC. The students will then progress to the sine rule and cosine rule, to solving problems relating to right-angled and non-right-angled triangles alike.

Flow of the Unit

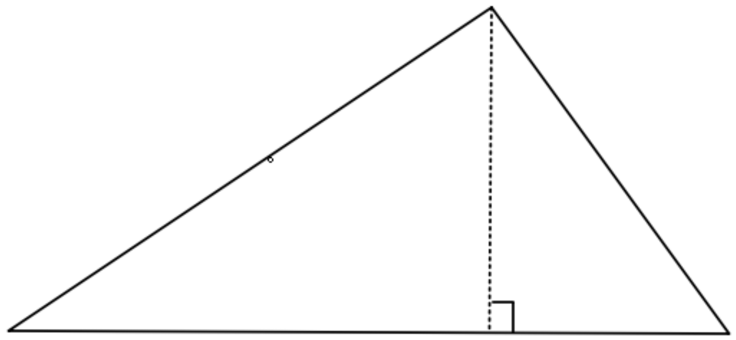
Lesson	Description	No. of Periods
1	Triangles, constructions, labelling Pythagoras' Theorem	1
2	Trigonometric Ratios	2
3	Area of a triangle: ½abSinC	1
4	Application of the formula	1
5	Sine rule, cosine rule	1

Flow of the Lesson

<table>
<tr><th>Activity and Expected Responses</th><th>Points of consideration</th></tr>
<tr><td>1. Introduction
The students will be asked about what they know so far about triangles (types and labelling) and trigonometry (the sine, cosine, tangent ratios and Pythagoras' theorem) and will also be reminded of the area of a triangle formula (1/2 base x perpendicular height).</td><td>This will be done swiftly through pre-prepared posters (see Appendix 1)</td></tr>
<tr><td>2. Posing of Problem: Developing Freida's Field
Freida has a triangular field that she wants to sell to developers. She needs to find its area.

The students will be asked to draw a plan of Freida's field and asked how they might use their prior knowledge to calculate the field's area.</td><td>(See Appendix 2) A short discussion will follow about not being able to directly being able to find the perpendicular height directly by physical measurement.</td></tr>
<tr><td>3. Students working individually for 10 minutes</td><td>Teacher circulates the room assessing the students' work, deciding which order in which to show their work on the board.</td></tr>
<tr><td>4. Class discussion
Let us look at how you think Freida could or should go about this.

<u>Expected response 1:</u>
Draws a perpendicular, possible measurement of the perpendicular



<u>Expected response 2:</u>
Literal measurements – lengths and angles
Likely to be in conjunction with expected response 1</td><td>The teacher goes to the student who draws the perpendicular. The student explains what they have done and why.

The teacher then chooses work where the student literally measures side and angles. The student explains what they did, and why.</td></tr>
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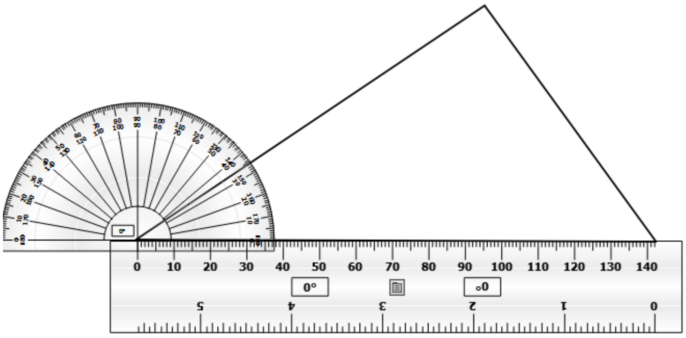

</td>
<td>The student who may complete a rectangle is then chosen. The student explains what they did, and why.</td>
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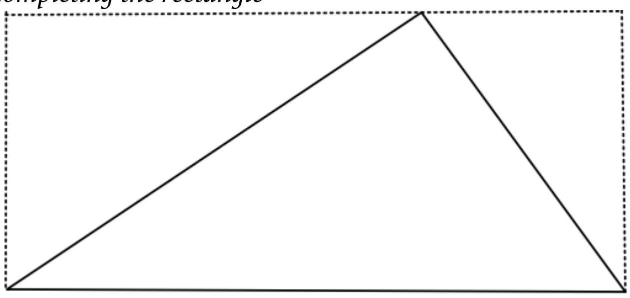
<td>
<u>Expected response 3:</u>
Completing the rectangle

</td>
<td>The student who may label the triangle, this may be in conjunction with literal measurements and use of trig ratios. The student explains what they did, and why.</td>
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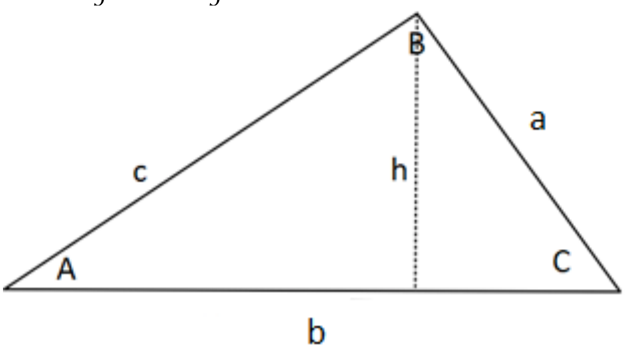
<td>
<u>Expected response 4:</u>
Labelling the triangle


</td>
<td>The students who uses some or all of the trig ratios is then chosen. The student explains what they did, and why.</td>
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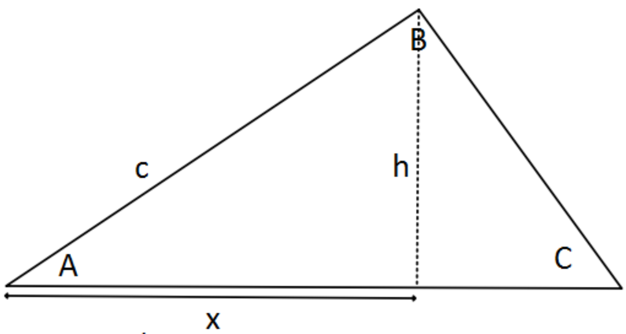
<td>
<u>Expected response 5:</u>
Using some or all of the trig ratios


cos A = x/c
sin A = h/c
tan A = h/x
</td>
<td>The students who manipulates formulae to find h. The student explains what they did, and why.</td>
</tr>
<tr>
<td>
<u>Expected response 6:</u>
Manipulation of the formulae, possible rejection of the use of cosine/tangent/both ratio(s)
</td>
<td>The students who manages to</td>
</tr>
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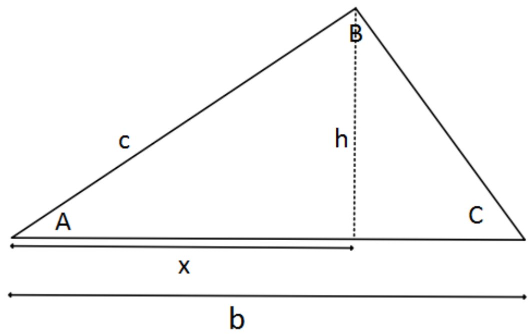
ccosA = x *x.tanA = h* *csinA = h* *<u>Expected response 7:</u>* *Adapt ½ (base)(perpendicular height) formula to find ½ abSin C*  *h = cSinA* *Area = ½ bcSinA*	develop the formula ½abSinC. The student explains what they did, and why.
5. **Summing up** In anticipation that the students will have developed the formula, they will be asked to reflect on what they have learned. They will be given another triangle with lengths of sides given, and asked to apply the formula to find the area. In the event that students do not achieve the lesson goal, they will be asked to use the most advanced method on the board and to continue working on it for a further 10 minutes to further develop the formula and reflect on what they have learned in class.	(See Appendix 3)

10. **Evaluation**

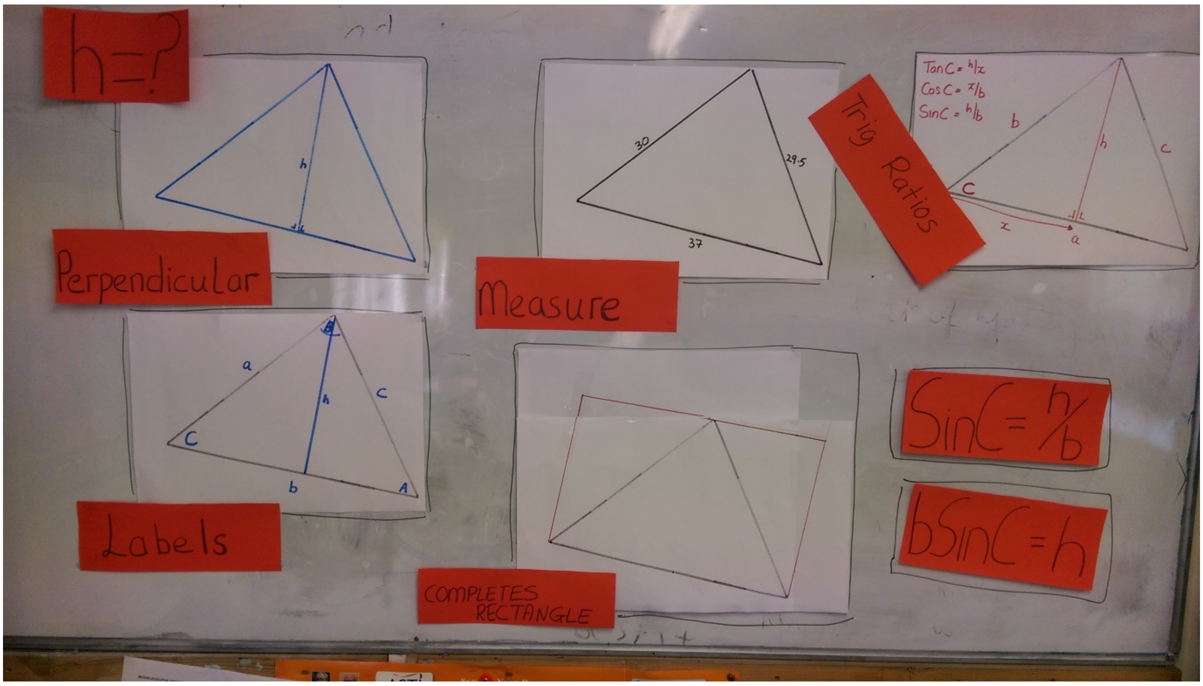
What is your plan for observing students?

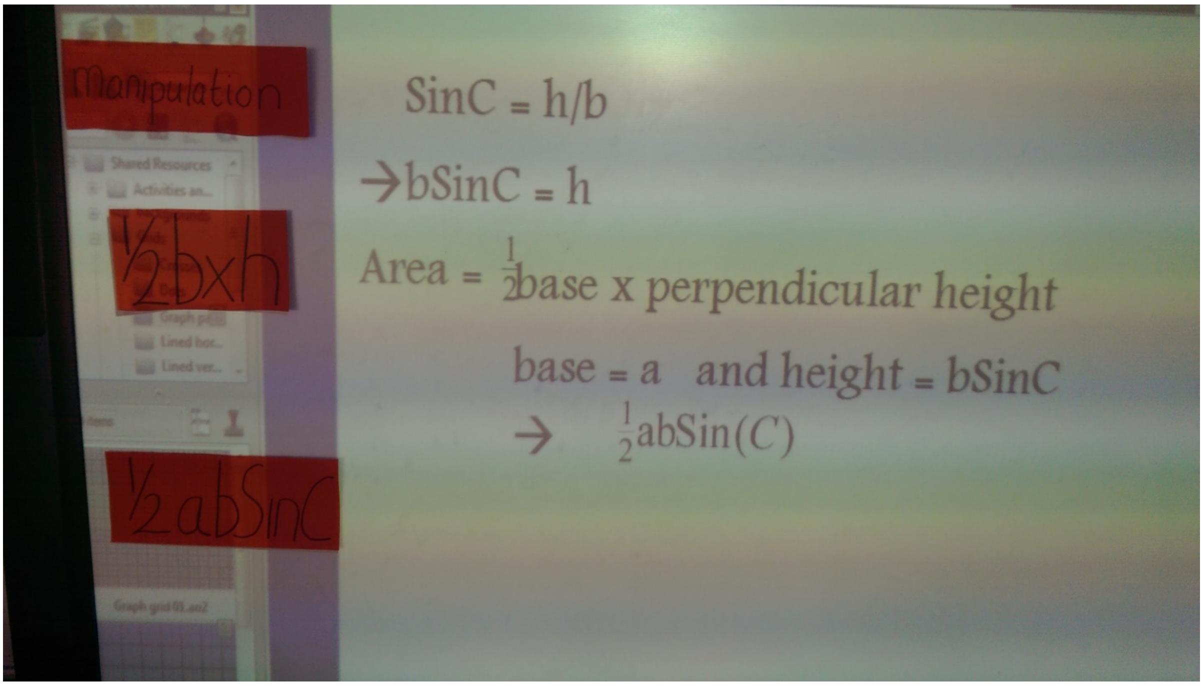
- As there will be three observers, we will divide the class into three groups of four students (there is a total of 12 students in the class)
- We will observe and record each student's construction, labelling and use of prior knowledge to solve the problem. All approaches will be taken into account. Photographs may be taken of students' work.
- All observers will have a copy of the class seating plan and will focus on taking notes on the page and back up with photographs when possible.
- Observers will focus on length of time spent thinking, on actual written work, on questions asked and possible interaction with other students.
- Observers will watch for any misconceptions surrounding the problem and note any assumptions they make without proof. Specific observations during *Hatsumom/Kikanshido* include:
    - Can the students label the sides and angles of the triangle?
    - Can the students recall and use the trigonometric ratios?
    - Can the students recall and use the area of triangle formula ½ base x perpendicular height?

- Do the students understand the wording of the task?
    - Must the teacher say more than anticipated?
    - What different approaches are being used and are there any we had not thought of?
    - Are the students persisting with the task?
    - How long does the task take the students?
    - Do the students have any questions/are there any prompts required?
- Specific observations during *Ceardaíocht* include:
    - Can the students clarify their individual approaches?
    - Are the students engaging with other people's methods?
    - Must the teacher clarify a student's method?
- The teacher will lead the discussion (to e*xpected responses: drawing perpendicular, labelling triangle, actual measurements, completing the rectangle)* by asking questions like:
    - How does this help us?
    - What was your aim?
    - Do other students agree?
    - Do we encounter any problems with this approach?
- The teacher will lead the discussion (to e*xpected responses: using trigonometric ratios)* by asking questions like:
    - Are any of the ratios useful to us?
    - Do we encounter any problems with using tan/cosine?
- *(if student/students develop(s) formula)*
    - How can we use the sine ratio?
    - How does the formula change if we use a different base?
- Did the discussion promote learning/allow students to achieve the lesson objectives?
- Observation team will collect students' work at the end of the class and photograph it where appropriate. (See Appendix 4)

II. Board Plan

Board Plan of Anticipated Student Responses





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12 Post-lesson reflection

What are the major patterns and tendencies in the evidence? Discuss

The teacher, (Susan) noticed that the students were engaged in the task and were interested in solving the problem. However, the teacher (Susan) found one major pattern was that the majority of students found it difficult to make the leap from the image to the page, which flummoxed many of them. One other thing observed by the group was that accuracy and precision in drawing the triangle was limited, which was also a factor considered by the group during the *Meitheal Machnaimh* process of Lesson Study. All students managed to draw a perpendicular height, but some focussed on finding the area of two separate areas. There were some questions asked about measuring and one sneaky student looked in the maths formula booklet, found the formula, but was unable to use it. This student later went on to, all but, solve the problem at the end of *Ceardaíocht*.

Another thing that we anticipated was that students would be reluctant to show their work on the board. We were correct in our assumption, and, as a result the lesson may not have gone as well as we would have hoped.

Two things that were not anticipated in *Meitheal Machnaimh* were that students would label the diagram incorrectly, and that students would divide the triangle and try to find two separate areas. (One student actually explained his measurements in 'steps', when brought to the board. This was not what he had written down on his page, but somehow when he was put on the spot, he added to what he had already written.)

At the end of the discussion, we came to the conclusion that another five minutes would have seen most students achieving the lesson goal. Perhaps had we provided a diagram of the triangle and allowed them to focus on labelling and working to the task, they may have achieved the lesson goal.

What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?

Evidence suggests that no students were confused by the task, but they did have difficulty developing a method to solve the problem. Most students were hoping to be given actual measurement and when numbers were not give, they found it difficult to make the leap to using variables to represent the angles and sides, even though they could correctly label a triangle during prior knowledge. When the perpendicular height was drawn in, they found the labelling problematic.

In what ways did students achieve or not achieve the learning goals?
Unfortunately, the goal of the lesson was not fully achieved. The group agreed that more focus on board work. Students recognised that the use of the trigonometric ratios would be appropriate but failed to substitute into the ½ base x height formula. Therefore two out of four of the learning goals were achieved.

Based on your analysis, how would you change or revise the lesson?

The cohort agreed that we would change the focus of prior knowledge, perhaps not putting so much emphasis on different types of triangles and Pythagoras' Theorem. Although they fit in well in the overall flow of the unit, perhaps more emphasis should be placed on the trigonometric ratios themselves and how they can be used in right angled triangles to solve real life problems. Focus should also be placed on the manipulation of formula and substitution.

The use of the interactive whiteboard as a means of showing board work was not entirely productive in the lesson, as labelling was problem and students were not exposed to the full flow of the lesson. This will be looked at for the next cycle as we know that board work is an essential part of lesson study.

More emphasis should be put on construction of angles and lines using the proper equipment. Both teachers and students should become more comfortable using the white board construction sets, hence, allowing for more accurate constructions.

Emphasis should be placed on students' correct use of mathematical language during C*eardaíocht*, and we should endeavour to help students' confidence in showing their work and speaking to their peers.

The group that took part in the lesson study are a 5^{th} year (mixed ability) honours group. The learning outcomes for this lesson, however, would be more suited to a higher level honours group.

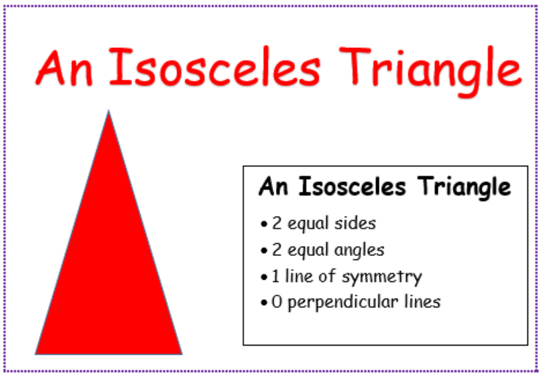
What are the implications for teaching in your field?

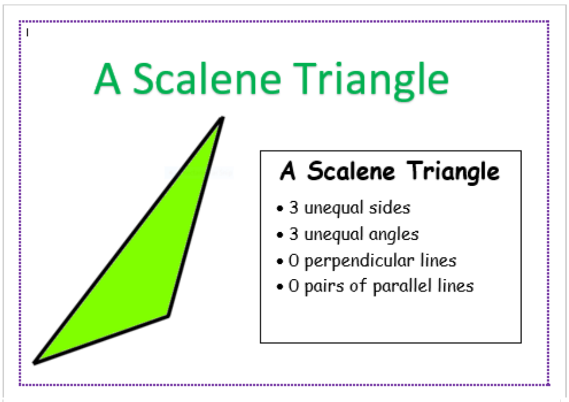
As teachers, we have had the opportunity to discuss our ideas around this topic and have also had the opportunity to learn from our students. Sometimes we can over estimate students' abilities and understanding of this topic. While they understand the concept, their application of certain skills may need more practice once the knowledge is acquired.

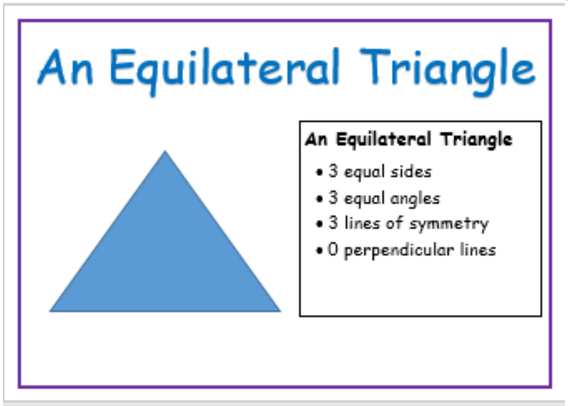
It also allowed us to look at the order in which topic fits in to the overall unit and the manner in which we present prior knowledge, as sometimes too much focus can be placed on one ideas, leading students to focus on that concept as they endeavour to solve the problem at hand. (In this case, the final concept in prior knowledge was Pythagoras' Theorem, leading many students to focus on that while trying to solve the problem.)

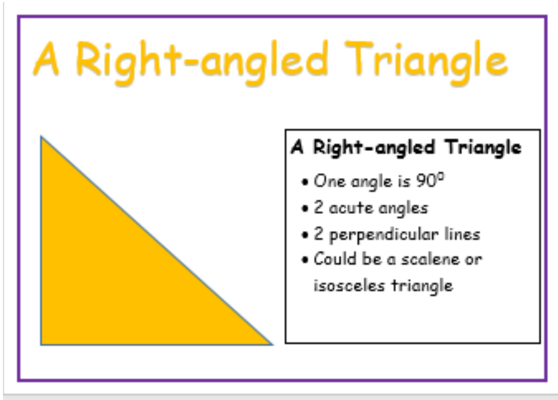
Having been exposed to this type of pedagogy, teachers are beginning to enjoy exploring topics in this manner. Lesson Study provides a solid foundation for teachers to make connections between their own previous knowledge and that of other teachers. It also provides opportunity to encounter possible new content which they learn from their peers through *Meitheal Machnaimh* and their students during *Ceardaíocht*.

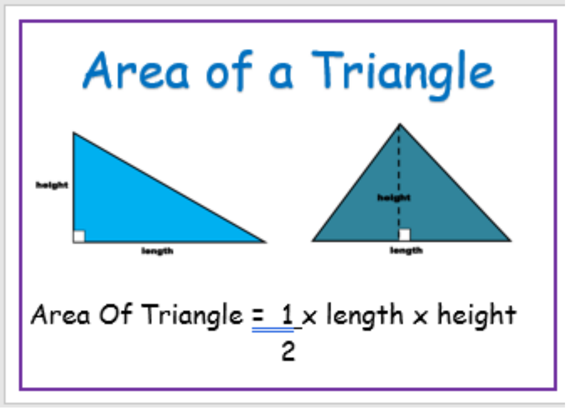
Appendix 1

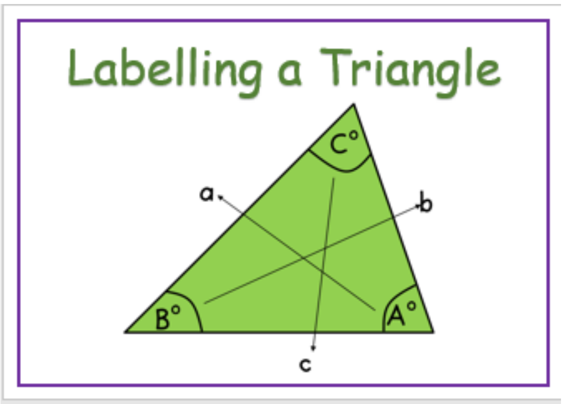

An Isosceles Triangle
An Isosceles Triangle
• 2 equal sides
• 2 equal angles
• 1 line of symmetry
• 0 perpendicular lines

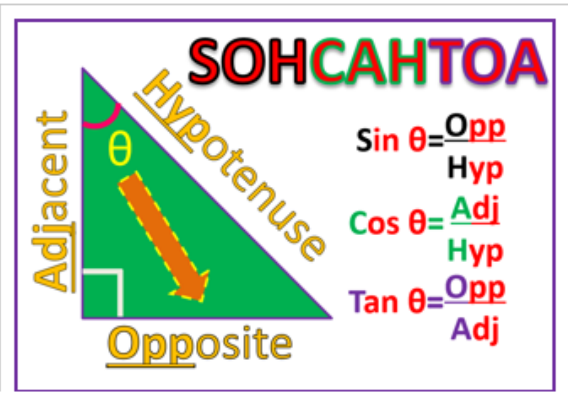

A Scalene Triangle
A Scalene Triangle
• 3 unequal sides
• 3 unequal angles
• 0 perpendicular lines
• 0 pairs of parallel lines

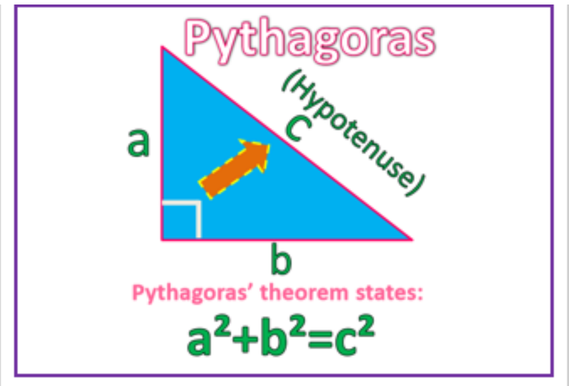

An Equilateral Triangle
An Equilateral Triangle
• 3 equal sides
• 3 equal angles
• 3 lines of symmetry
• 0 perpendicular lines


A Right-angled Triangle
A Right-angled Triangle
• One angle is 90°
• 2 acute angles
• 2 perpendicular lines
• Could be a scalene or isosceles triangle


Area of a Triangle
height
length
height
length
Area Of Triangle = 1/2 x length x height


Labelling a Triangle
C°
a
b
B°
A°
c


SOHCAHTOA
Hypotenuse
Adjacent
θ
Opposite
Sin θ= Opp/Hyp
Cos θ= Adj/Hyp
Tan θ= Opp/Adj


Pythagoras
(Hypotenuse)
C
a
b
Pythagoras' theorem states:
a²+b²=c²

Display of posters for prior knowledge

Teacher using posters for prior knowledge investigation

Appendix 2

Frieda inherited land from her uncle. She decided to sell the land to developers and needed to know the area of the field. Frieda walked the perimeter of the land and found that it was triangular in shape.

The problem was that Frieda could not accurately measure the perpendicular height of the field.

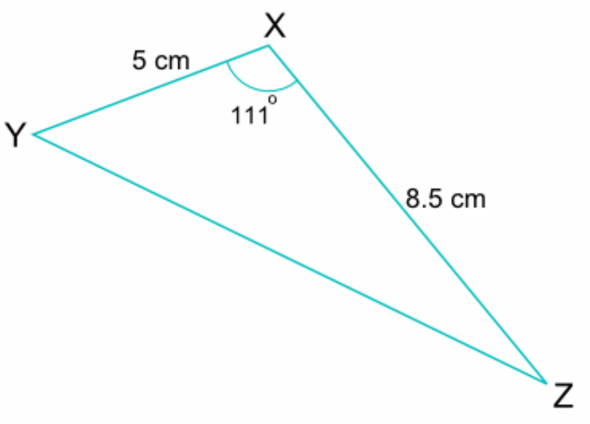
Problem: How could Frieda accurately calculate the area of the field?

Appendix 3

- Write a line or two reflection on what you have learned today.
- What problems did you encounter?
- How did other students' help you solve the problem of Freida's field?

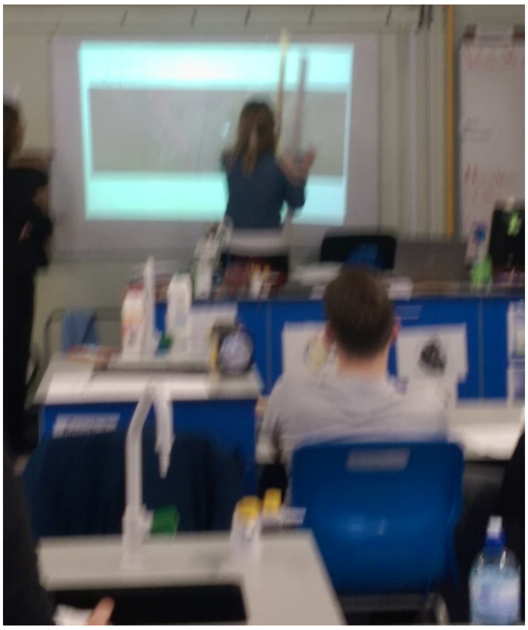
Also,

Use the new formula *(area of a triangle = ½ abSinC)* to answer the following question*:*



What is the area of the triangle in cm^2 ?

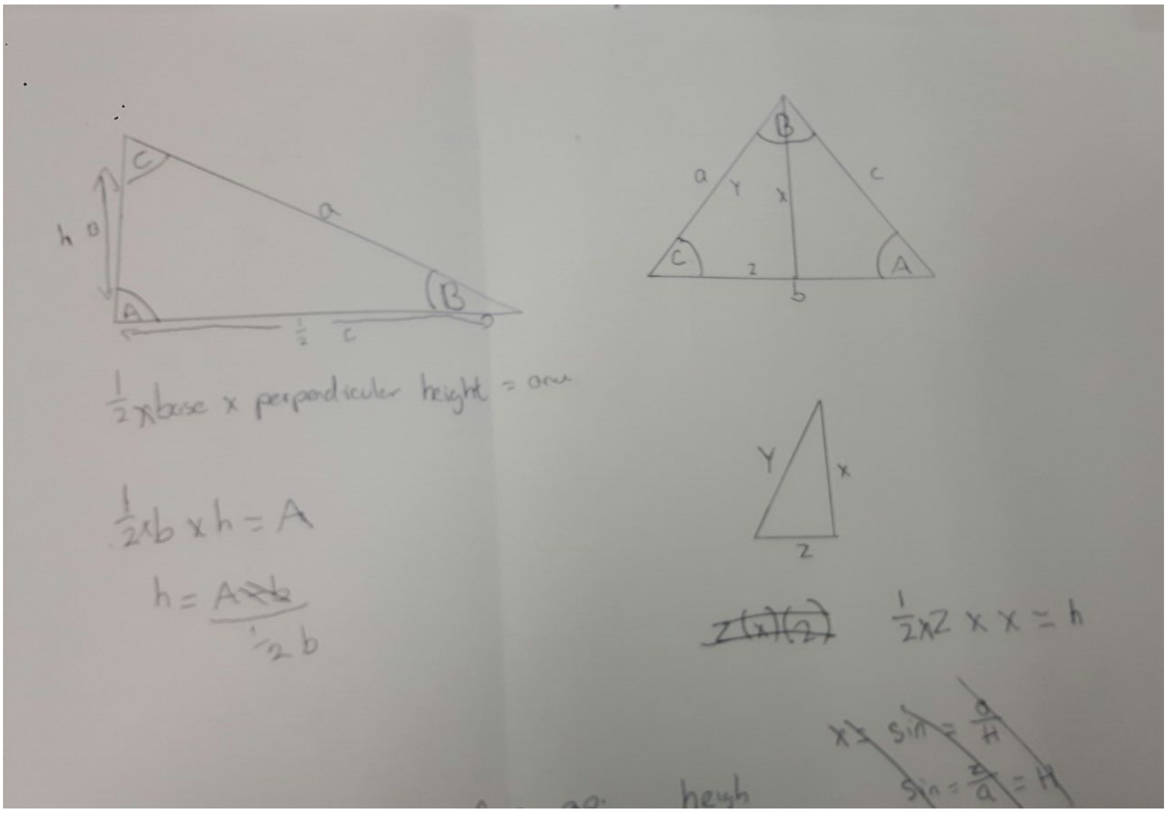
Appendix 4



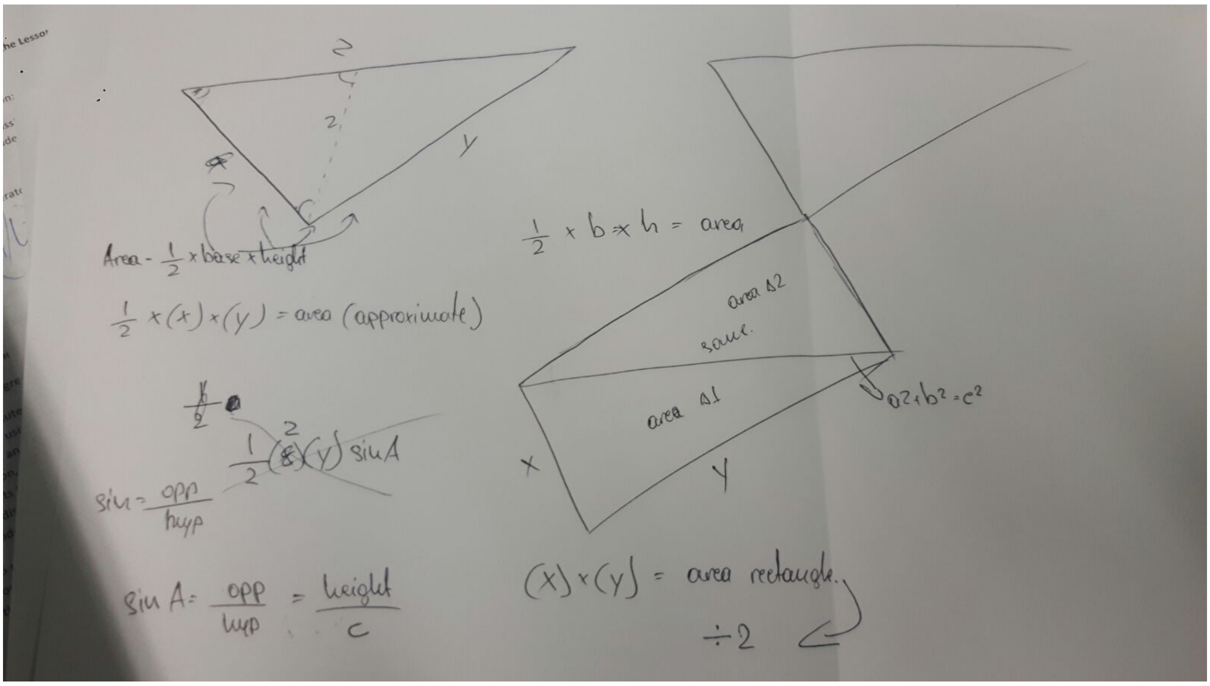
Student drawing perpendicular height

Student labelling angles in the triangle

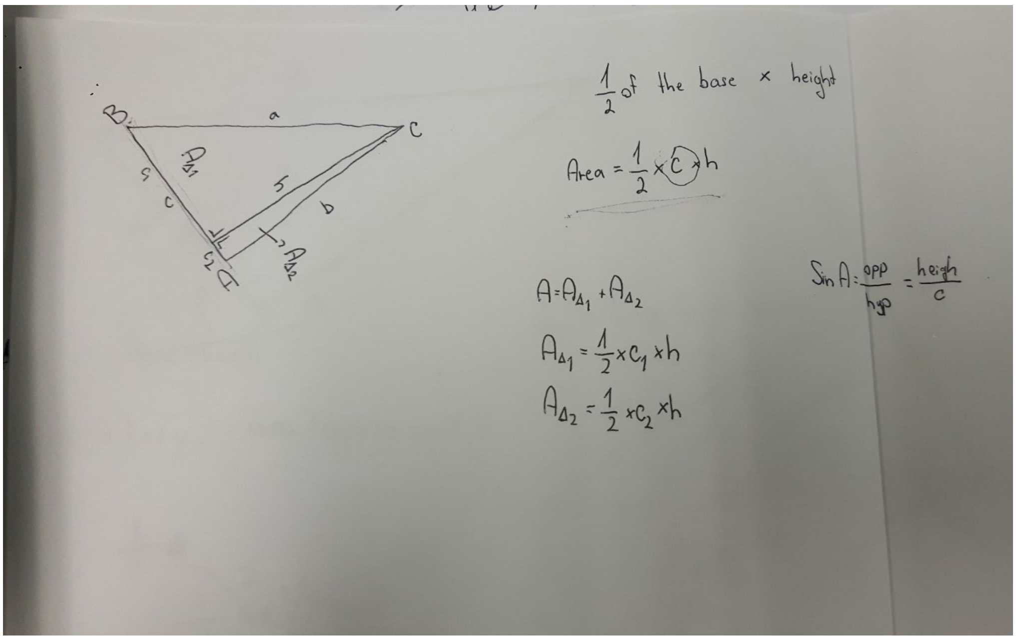
Samples of students' work



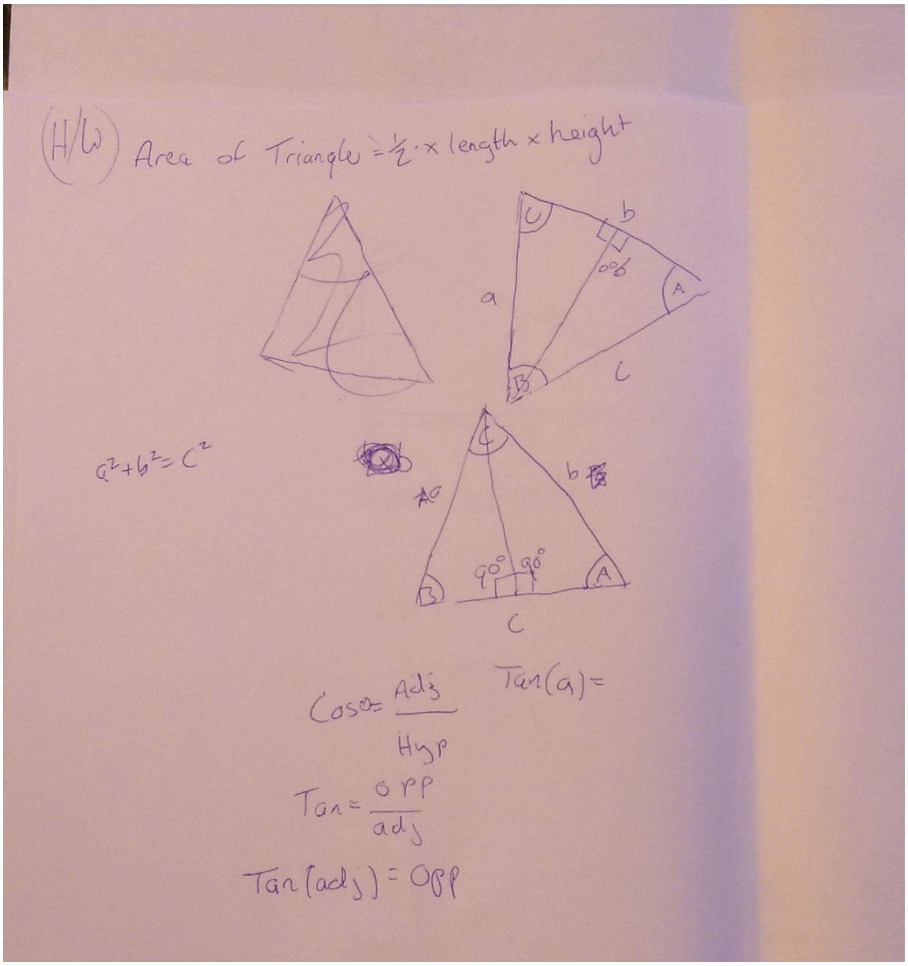
Perpendicular height drawn and triangle correctly labelled



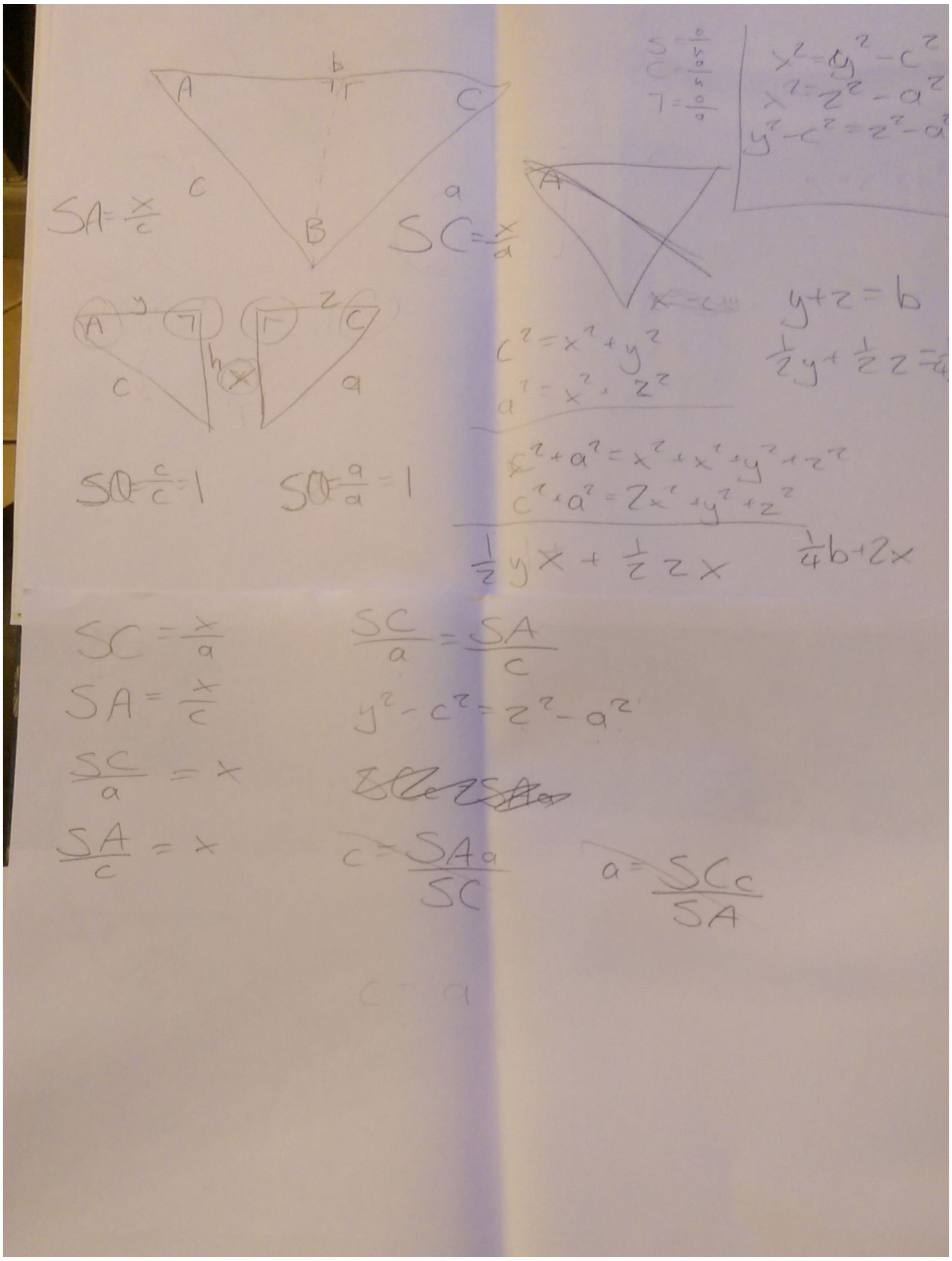
Doubled area – formed a rectangle



Divided into two separate triangles



Trigonometric ratios mentioned, but correct labelling lacking



Focus on Sine (SC) and manipulation of formula but failing to substitute