

## Topic: Interpreting Distance – Time Graphs

Year Group: 1<sup>st</sup> Year (Mixed Ability)

For the lesson on 25<sup>th</sup> January 2017  
At St. Killian's College, New Inn, First Year  
Teacher: Sinead Holland

Lesson plan developed by: Sinead Holland, Helen Van Eesbeck & Marie Glynn

### 1. **Title of the Lesson:** Home and Away: Interpreting Distance – Time Graphs

### 2. **Brief description of the lesson**

Students associate real world motion scenarios with their corresponding distance – time graphs by relating the slopes of line segments with speed and direction of motion.

### 3. **Aims of the Lesson<sup>1</sup>**

#### **Short – Term Goals – We wish for the students to be able to:**

- connect verbal interpretations with graphs considering crucial features of the graphs,
- see a connection between speed and slope and direction of motion (moving towards or away from home), [Key Skill of Junior Cycle: Being Numerate – Expressing ideas mathematically]
- work with their peers, engage with their partners' explanations and thus gain confidence in presenting their shared view to the whole class. [Differentiated learning, Key Skill of Junior Cycle: Working with others]

#### **Long – Term Goals -We would like our students to be able to:**

- draw their own graphs from verbal interpretations,
- justify their choices to their peers,
- constructively challenge other students reasoning,
- become independent learners while valuing the opinions of others, [Key Skills of Junior Cycle: Working with others and Communicating]
- transfer the skills learned in mathematics to their science classes [Cross-curricular links]

### 4. **Learning Outcomes**

Students will be able to:

- connect a real world motion scenario with a distance-time graph,
- discuss and justify their choices,
- recognise that if the object is not moving then the corresponding graph is a horizontal line,
- recognise that a negative slope means that the object is moving in the opposite direction,
- classify the distance-time graphs using a Venn diagram.

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<sup>1</sup> Please see “Literacy and numeracy for learning and life. The National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020 ” which encourages to foster the literacy skills (including speaking and listening) and numeracy skills (thinking and communicating quantitatively making sense of information available, applying mathematical concepts in real-life scenarios).

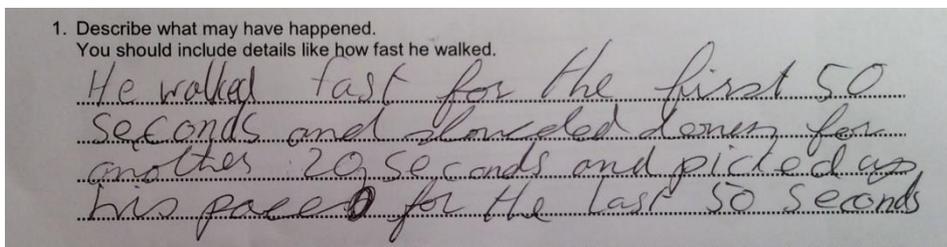
## 5. Background and Rationale

According to the Junior Cycle Mathematics Syllabus (4.2, page 28, sections 3.4, page 24, and section 3.5, page 34) students need to learn the following:

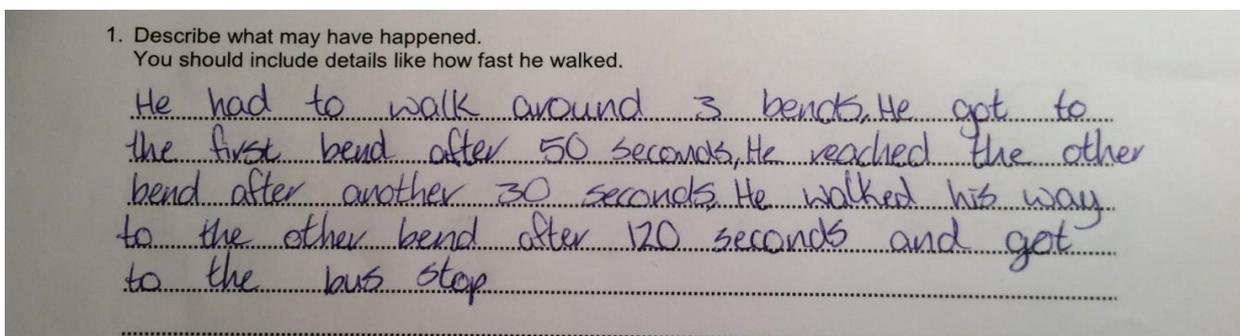
4.5 Relations without formulae	Using graphs to represent phenomena quantitatively	<input type="checkbox"/> Explore graphs of motion <input type="checkbox"/> Make sense of quantitative graphs & draw conclusions from them <input type="checkbox"/> Make connections between the shape of a graph & the story of a phenomenon <input type="checkbox"/> Describe both quantity & change of quantity on a graph
3.4 Applied Measure	Measure & Time	<input type="checkbox"/> Calculate, interpret & apply units of measure & time <input type="checkbox"/> Solve problems that involve average speed, distance & time
3.5 Sets (CIC)	Solving problems involving sets	<input type="checkbox"/> Illustrate sets using Venn diagrams

The major goal of this lesson is to assess how well 1<sup>st</sup> year students are able to interpret distance – time graphs and help students form a correct relationship between slope and speed and direction on motion. In November 2016 a group of 18 first year students were given a diagnostic task “Journey to the bus stop” (see Appendix 1) that revealed the kinds of difficulties the students might have with distance-time graphs (see examples of student work):

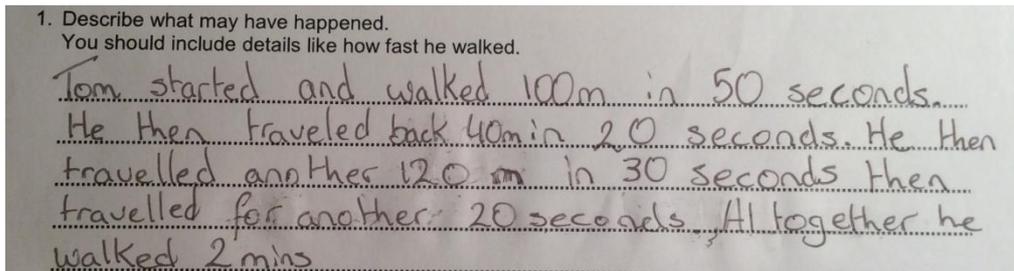
- Student interprets a negative slope as slowing down.



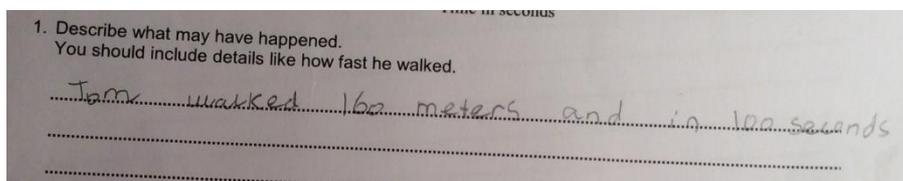
- Student interprets distance-time graph as a picture.



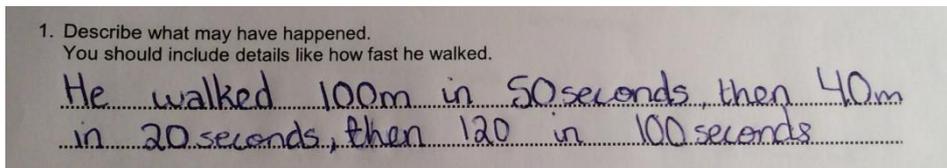
- Student fails to recognise that the horizontal line means the object had stopped moving.



- Student fails to add the going back distance to get the total distance covered.



- Student misinterprets the scale.



- Student does not explain why the graph is/is not realistic (7 out of 18 students).

We decided to address the first three misunderstandings in our research lesson.

Students often lack confidence in verbalising the reasoning behind their mathematical decisions. The purpose of doing the matching activity in pairs is to give students the opportunity to engage with their partners' explanations and then present their shared view to the whole class with greater confidence.

Mathematics has significant connections with Science where students are expected to be able to produce graphs and interpret them. We hope that students would be able to transfer the skills learned in mathematics to their science classes.

## 6. Research

The following resources have been used in the planning of the lesson:

- a) the Junior Certificate Mathematics Syllabus; the learning outcomes of the lesson are aligned with those of the syllabus as highlighted in section 5 above.

The research lesson aims at problem solving which, according to the JC Mathematics Syllabus (page 10), is integral part of teaching and means engaging in a task for which the solution is not immediately obvious.

The structured pair work and presentation of students' ideas support one of the objectives in the syllabus, *adaptive reasoning*, that is, capacity for logical explanation, justification & communication. It is also embedding Key Skills of Junior Cycle: Working with others, Communicating.

<http://www.juniorcycle.ie/Planning/Key-Skills>

- b) "Literacy and numeracy for learning and life. The National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020 "which encourages to foster the literacy skills (including speaking and listening) and numeracy skills (thinking and communicating quantitatively making sense of information available, applying mathematical concepts in real-life scenarios) in young learners.
- c) Teaching for Robust Understanding of Mathematics (**TRU Math**) – the Mathematics Assessment Project website  
<http://map.mathshell.org/lessons.php?unit=8225&collection=8>  
 The diagnostic task and card matching activity come from that website. The matching activity fosters students' interest and stimulates thinking & discussion around distance – time graphs.
- d) Maths Development Team's website – presentation "Understanding graphs" (activity "Archimedes is having a bath", homework activity – retell the story of the hare and the tortoise).  
<http://www.projectmaths.ie/workshops/workshop4/UnderstandingGraphs.pdf>

The lesson uses student centred approach to teaching and intends to promote peer - learning. Students are asked to discuss the graphs in pairs & throughout this activity they are encouraged to articulate their reasoning, justify their choices mathematically and question the choices put forward by others.

## 7. About the Unit and the Lesson

Learning Outcome of the Lesson	Lesson Design
<ul style="list-style-type: none"> <li><input type="checkbox"/> Connect a graph with a story</li> <li><input type="checkbox"/> Discuss &amp; justify findings</li> </ul>	<p>Pair Work: students match six different stories to six graphs. They have to write reasons beside each graph to justify their choice.</p> <p>Students will come up to the board to explain their thinking to the whole class.</p>
<ul style="list-style-type: none"> <li><input type="checkbox"/> Recognise that if the object is not moving then the corresponding graph is a horizontal line</li> </ul>	<p>The connection between the slope and speed and the direction of motion will be established through questioning.</p>

<input type="checkbox"/> Recognise that a negative slope means that the object is moving in the opposite direction.	
<input type="checkbox"/> Classify distance-time graphs using a Venn diagram	Students will place each card in an appropriate place on a Venn diagram with two sets “Journey starting at home” and “Journey finishing at home”.

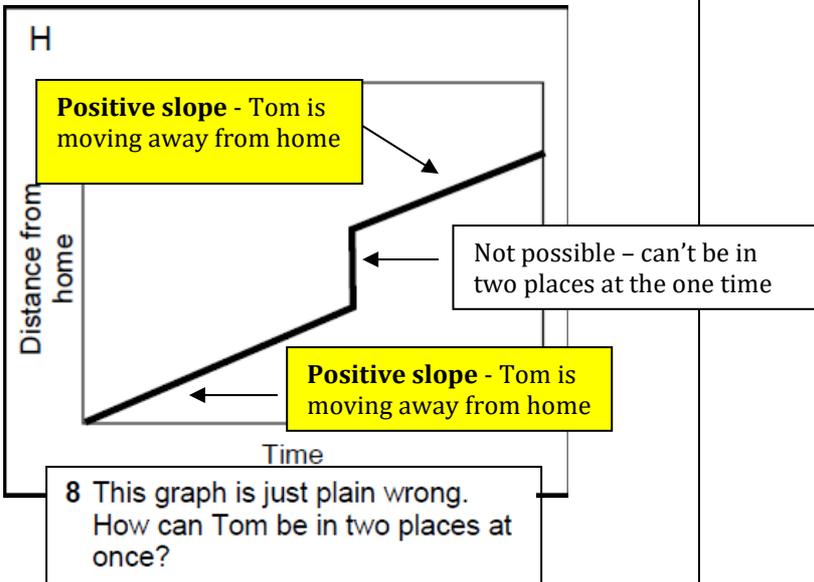
Prior to the research lesson, students will be introduced to co-ordinate geometry (plotting points, calculating slopes of lines). In the directly preceding lesson, students will learn to interpret slope as a rate of change of the amount of water in a bath with respect to time. The activity “Archimedes is having a bath” is supposed to prepare students for writing comments above each section of a distance-time graph during the research lesson. During the research lesson students apply the prior knowledge of slopes, speed and sets to interpret and classify distance-time graphs. They match the graphs and real-world scenarios, discuss the connection between the positive, zero and negative slope and moving away from home, standing still and moving towards home, respectively. When explaining the difference between walking and running students refer to the steepness of a line segment (the steeper the line segment the greater the speed of an object). Towards the end of the lesson students will have the opportunity to use a Venn diagram to classify graphs as those representing a journey starting at home and those representing a journey finishing at home. They will revise the concept of the intersection of two sets. In the following lesson, students will create a table showing possible times and distances for a selected distance-time graph. They will also draw their own distance-time graphs to show different stories presented to them.

## 8. Flow of the Unit

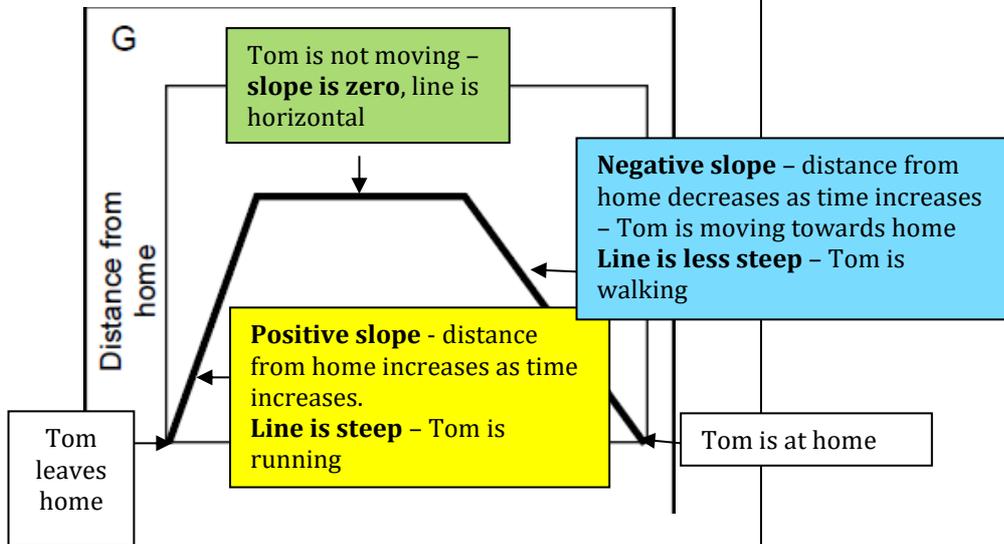
	Introduction to Co-ordinate Geometry	# of lesson periods
1	Diagnostic test (distance – time graph) to reveal the students’ current understandings & difficulties.	1 x 35 min.
2	Co-ordinating plane and locating points on the plane using coordinates  Finding the slope of a line using rise/run	4 x 35 min.
3	Interpreting slope (connecting slope and rate of change, volume of water in a bath when filling in the bath, emptying the bath, <i>Archimedes is having a bath</i> – matching a description to each part of the graph)	1 x 35 min.
4	Interpreting distance – time graphs	1 x 35 min. <b>(research lesson)</b>
5	Making up data for a distance-time graph and making graphs to	1 x 35 min.

	the given stories	
6	Completing a new copy of the diagnostic task (15 minutes)	1x35 min.

### 9. Flow of the Lesson

Teaching Activity	Points of Consideration
<p><b>1. Introduction</b></p> <p>Yesterday you learned that the slope of a line may illustrate how, for example, the amount of water in a bath changes as time goes on. Today you are going to continue exploring slopes of lines in the context of a journey.</p>	<p><b>Time: 2 minutes</b></p>
<p><b>2. Posing the Task</b></p> <p>You are given six graphs and six story cards (Appendix 2). Working with your partner take a story card and find a graph that goes with it. Each time you do this, explain your thinking carefully to each other and write your reasons for the match above each line segment on the graph.</p>	<p><b>Time: 2 minutes</b></p>
<p><b>3. Students work in pairs</b></p>	<p><b>Time: 10 minutes</b></p> <p>Observe how students explain and justify the matching of the cards to each other. Note any common mistakes.</p> <p>Having done the activity “Archimedes is having a bath” in the previous lesson students should be able to annotate their graphs appropriately.</p>
<p><b>3. Anticipated Student Responses</b></p> <p><b>Response 1</b></p> 	

**Response 2**

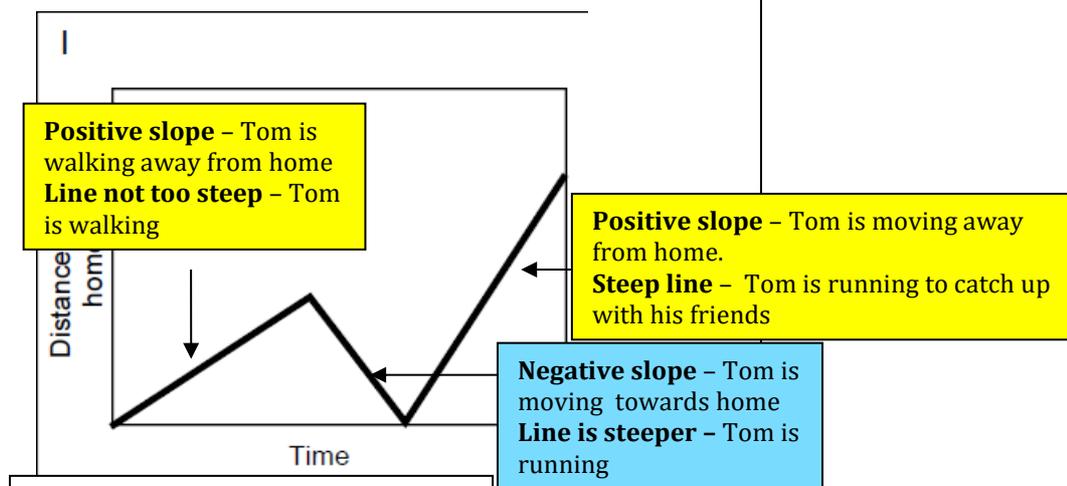


1 Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.

**T:** How does the distance covered in the third section compare with the distance covered in the first section of graph G?

**S:** They are the same (some students may think that distance covered in third section is longer; distance is the same but it takes longer to cover it in section 3 of the graph).

**Response 3**



7 Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.

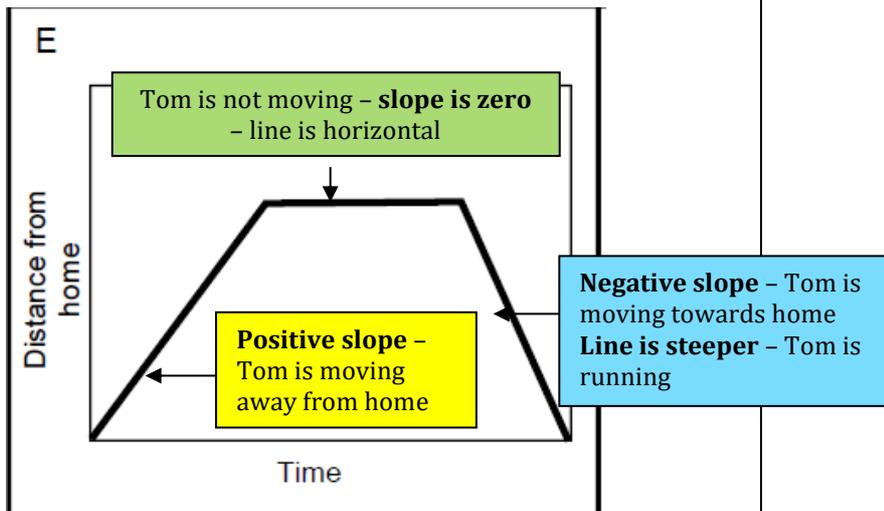
T: In which section of graph I is Tom moving fastest?

S: In the last section.

S: He is moving equally fast in sections two and three.

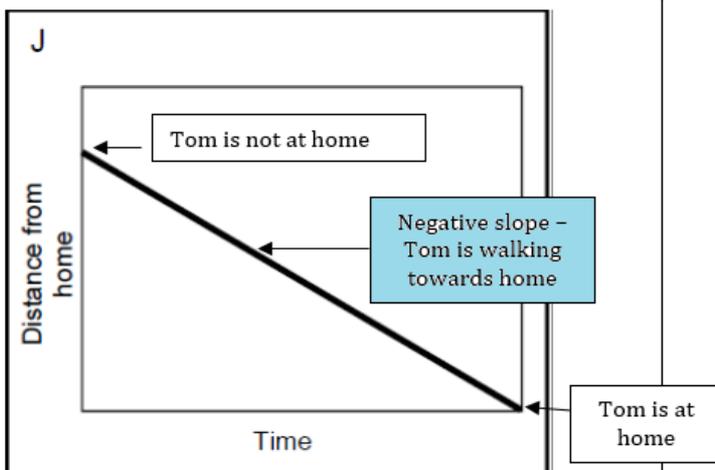
T: How could we determine which answer is correct? (Students could 1) use a protractor to measure the acute angles between the line segments and the x-axis, or 2) compare the rise over the same run.)

**Response 4**



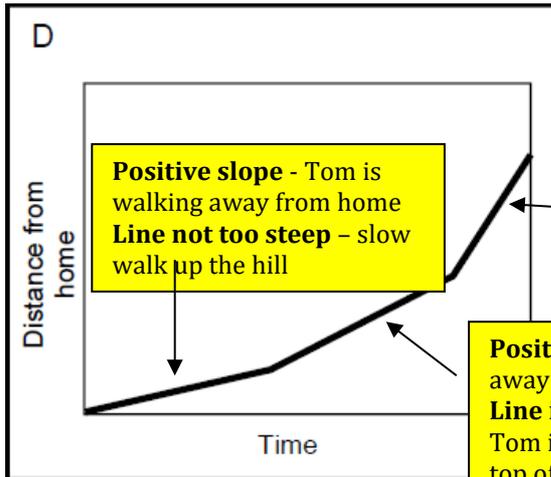
**6** Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.

**Response 5**



**9** After the party, Tom walked slowly all the way home.

**Response 6**



**Positive slope** - Tom is walking away from home  
**Line not too steep** - slow walk up the hill

**Positive slope** - Tom is still moving away from home  
**Steepest line** - Tom is running fast down the hill

**Positive slope** - Tom is still moving away from home  
**Line is steeper** than in 1<sup>st</sup> section - Tom is a bit faster walking across the top of the hill

2 Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.

**4. Presenting, Comparing and Discussing**

When students finish matching the cards, invite pairs of students to describe a pair of cards and the reasoning at the board.  
**Time: 12 minutes**

After the six pairs of cards have been explained at the board, ask students to compare all the graphs to see similarities.  
**Time: 8 minutes**

T: Can you see any groups of graphs that have something in common?

S: Graphs E&G

T: What is the same in both graphs?

S: They both start and finish at home.  
 S: They have positive, zero and negative slopes.

T: What is different in these two graphs?

S: G is steeper in the first section and less steep in the last section which means Tom was faster at the beginning and slower at the end of his journey. On the other hand, the first section in E is less steep than the last section meaning Tom was slower in the beginning and faster coming back home.

T: How can you tell from the graph if Tom is travelling away from home or towards home?  
 S: Line with positive slope tells that Tom is moving away from home. Distance from home increases as time increases. When the line has a negative slope, distance from home decreases as time increases, so Tom is moving towards home.  
 T: In which graphs does the journey start at home?

**Time: 20 minutes**

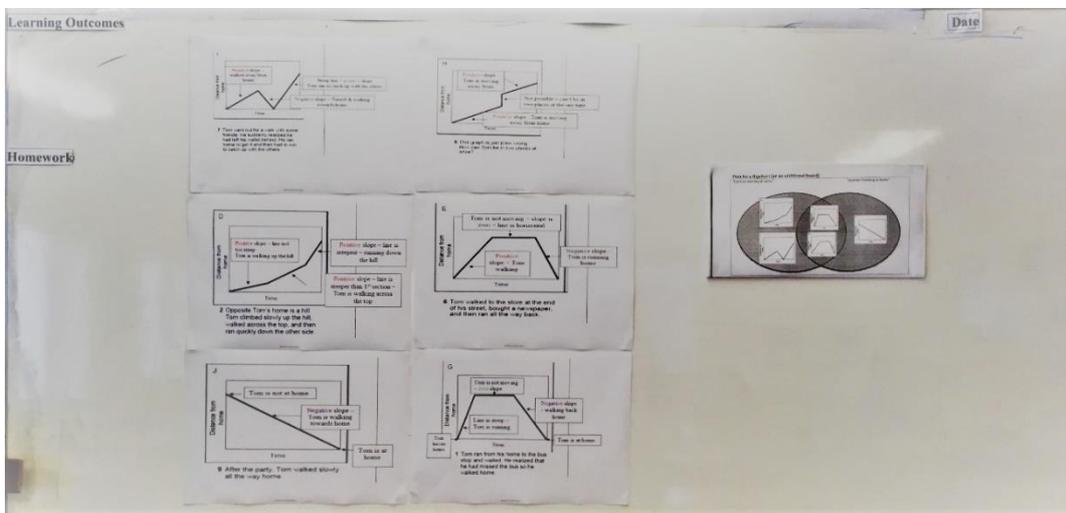
After a short discussion on the common features of the graphs a large Venn diagram is displayed (using a projector or a flipchart) with two overlapping circles "Journey starting at home" and "Journey finishing at home". Students come up to the board to place the graphs in the relevant parts of the Venn diagram. The graphs E and G belong to the intersection.

<p>T: Which graphs show a journey starting away from home?</p> <p>T: Which graphs show the journey ending at home?</p> <p>T: Place the graphs in the appropriate parts of the Venn diagram that illustrates two sets: "Journey starting at home" and "Journey finishing at home".</p>	
<p><b>5. Summing up</b></p> <p>Write down in your copies two things about speed and slope that you have learned today.</p> <p><b>Homework:</b> In your copies, retell the story of the hare &amp; the tortoise using the handout given.</p>	<p>The above questioning and filling in a Venn diagram serve as a summary and reinforcement of the main ideas of the lesson.</p> <p>As a short assessment task students are given homework "Retell the story of the hare and the tortoise" (see Appendix 3).</p>

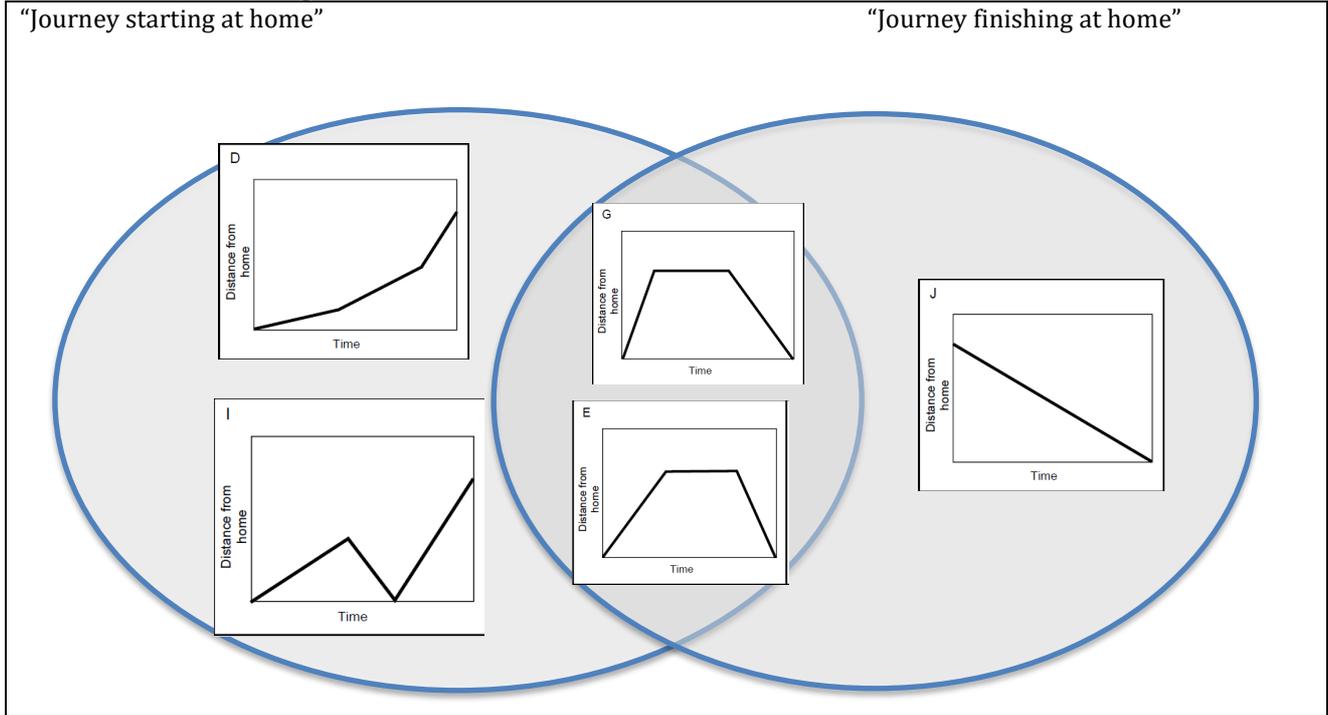
### 10. Evaluation

- There will be 3 maths teachers (equipped with seating charts) who will observe a class of 18 students (6 students per observer). One teacher will use Lesson Note during the duration of the class and will use her iPad to take pictures of the students' work. The other two teachers will use pen and paper to take notes.
- Students' work will be collected at the end of class.
- During the lesson, we will be interested in the following:
  1. How long do students spend on the task? How many cards are they able to match correctly?
  2. Can students justify why they matched a given story card with a given graph? How do they explain their choices to their partners? Do they use words such as *slope, steepness, speed, faster/slower, moving away/towards home etc.*?
  3. What kind of misconceptions arise?
  4. Are students confident when presenting their matches to the class at the board?
  5. How do students participate in the whole-class discussion? What questions do they ask?
  6. Are students able to place the graphs in the correct place on the Venn diagram?

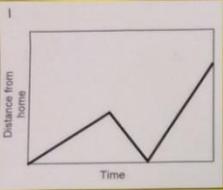
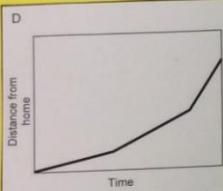
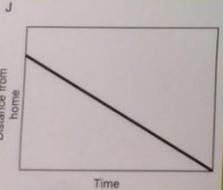
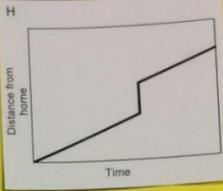
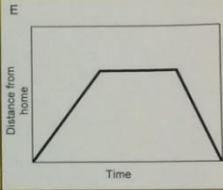
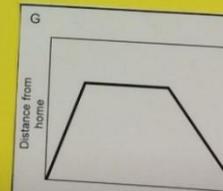
### 11. Board Plan



## Plan for a Venn diagram



## Card matching activity

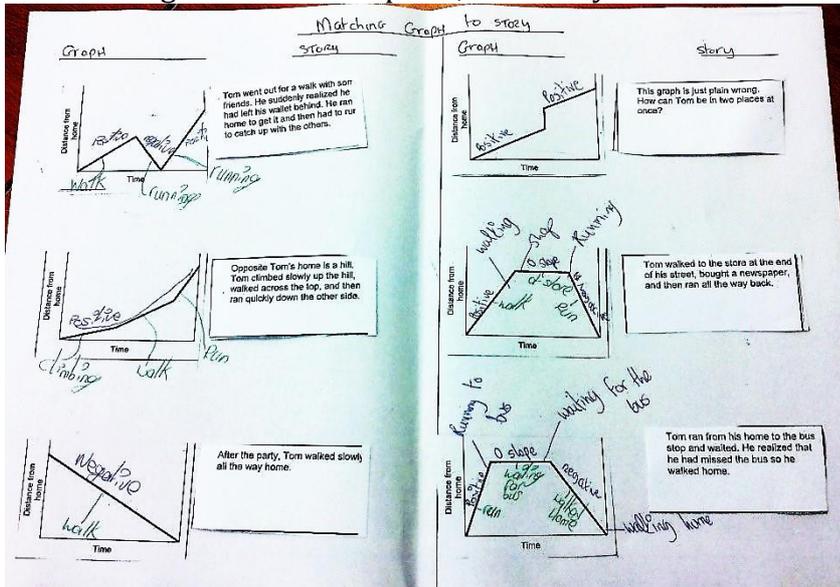
<div style="margin-bottom: 20px;">  </div> <div style="margin-bottom: 20px;">  </div> <div>  </div>	<div style="margin-bottom: 20px;"> <p>7 Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.</p> </div> <div style="margin-bottom: 20px;"> <p>2 Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.</p> </div> <div> <p>9 After the party, Tom walked slowly all the way home.</p> </div>
<div style="margin-bottom: 20px;">  </div> <div style="margin-bottom: 20px;">  </div> <div>  </div>	<div style="margin-bottom: 20px;"> <p>8 This graph is just plain wrong. How can Tom be in two places at once?</p> </div> <div style="margin-bottom: 20px;"> <p>6 Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.</p> </div> <div> <p>1 Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.</p> </div>

## 12. Post-lesson reflection

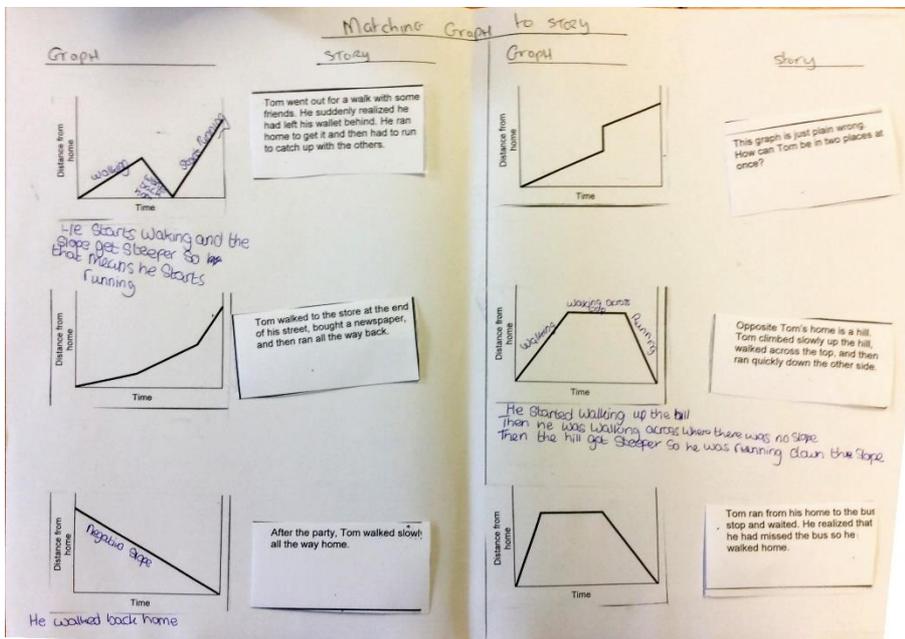
### The major patterns and tendencies in the evidence - key observations of student learning and thinking

Reflecting on the lesson, students did well overall on the tasks. Most groups matched the stories to the correct graph with no trouble. The students at the start were reluctant to write their interpretations on the graphs, they were more interested in matching the story given.

Out of the 3 groups each teacher observed, there was a pattern of 1 out of 3 to have difficulty with writing in beside each point, what they meant.



Example of a good groups work - a lot of information was written on the graph & also colour coded walking, running etc.



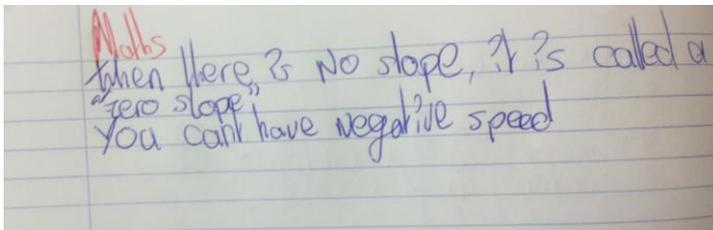
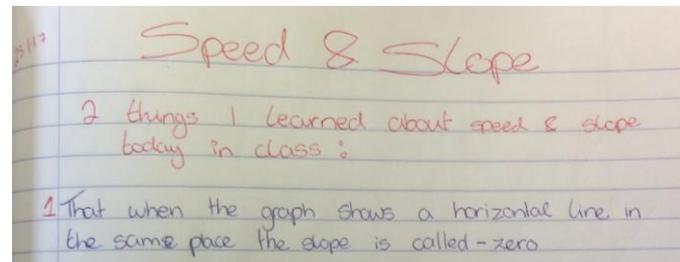
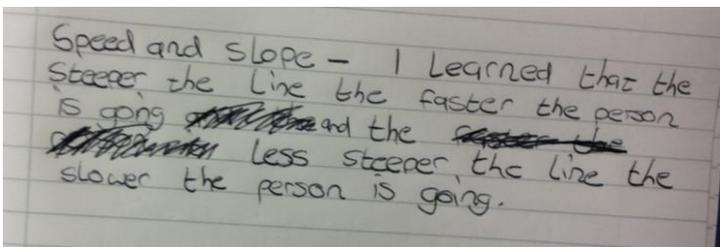
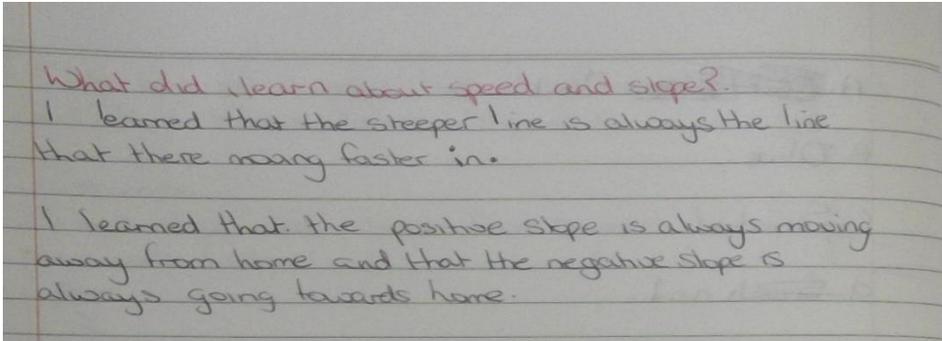
Misconception – mixed up D & E as they interpreted graph E as a picture of a hill

Students tended to be quiet when talking to their peers about the tasks. Nonetheless, explanations such as "He was climbing slowly so it's not as steep, here he was running that's why the line is steeper" had been observed.

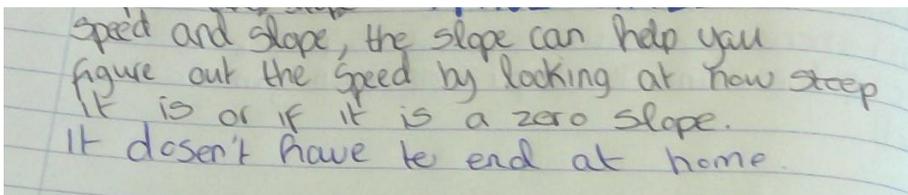
The groups that came up to the board were very confident about the difference between the positive slopes as they compared their steepness, they recognised negative slope well and a slope of 0.

Some students spoke straight to the teacher instead of speaking out to the class.

It was clear that all students learned from this task. The following are samples of what the students wrote down at the end of the lesson in their reflections.



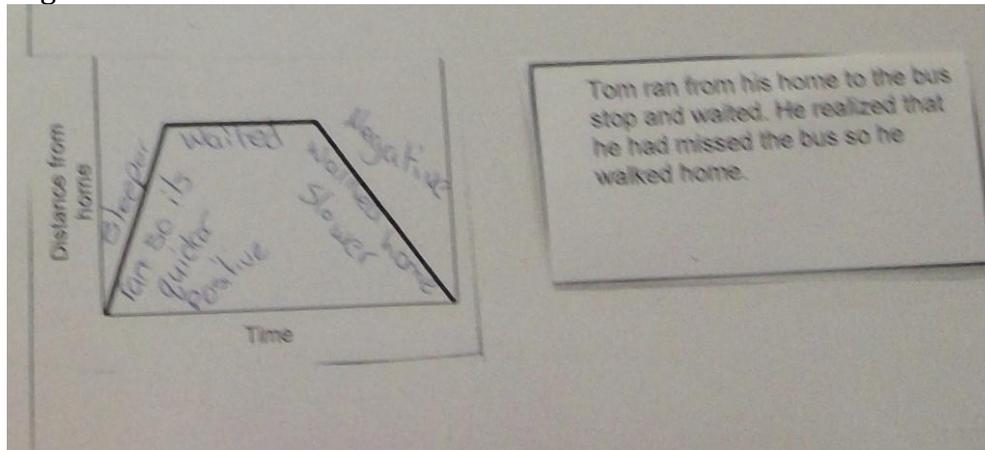
Students need to be encouraged to say "slope is zero" instead of "there is no slope".



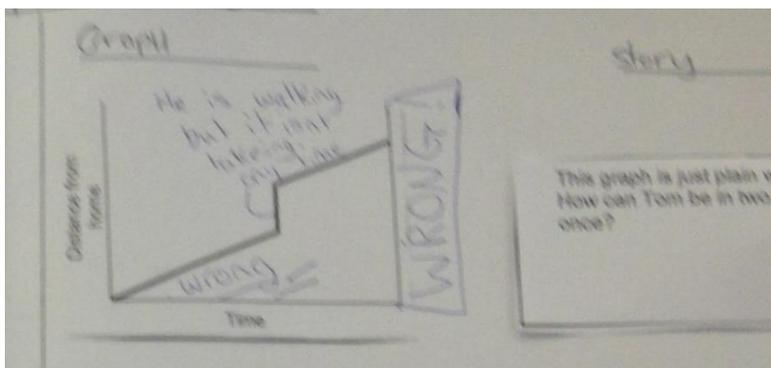
**What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?**

- Approx. one third of the students had difficulty with graphs E & D interpreting graph E as an actual hill. This was clarified when a pair of students presenting the correct match between cards D and hill walking at the board was asked by the teacher: *If he is running down the hill should the line not go down?* The presenters answered: *No because he's still moving away from home.* The pair presenting cards E and 6 knew that although the line segment in the first section of the graph is longer than the line segment in 3<sup>rd</sup> section, the person walked the same distance in both sections.

- The majority of students were good at interpreting the meaning of the graphs & had a good understanding of positive & negative slopes. They were also able to relate the steepness of the line to speed and hence they were able to say if Tom was running or walking.



- All the students got graphs J, H & I. This was surprising as they didn't say that Tom went down the hill in I and were able to interpret the fact that Tom couldn't be in two places at the one time in H.

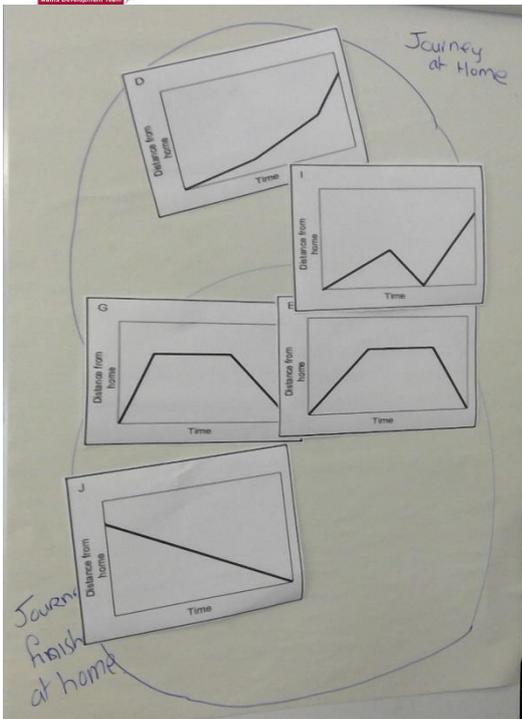


*"He is walking but it isn't taking any time. Wrong."*

### In what ways did students achieve or not achieve the learning goals?

The students achieved the learning goals as they were able to connect the graphs with the stories showing that they could see a connection between speed & slope & direction of motion. The majority of students recognised that the horizontal line in graphs E & G meant that Tom was at the shop / bus stop & therefore not moving. The students also knew that the slope of a horizontal line is zero. They were able to work with their peers in discussing the graphs but they need to communicate better, especially when sharing their views with their class. They were very shy & quiet when answering.

Students had no problem with classifying the graphs using a Venn diagram.



Asked about similarities among all the graphs students identified E and G immediately: *The slopes are positive, zero, negative. They start at home and finish at home.* Teacher's question: *How were we able to match the stories if the graphs are so similar?* was answered by the class "steeper line means faster".

### Based on your analysis, how would you change or revise the lesson?

- Pre-lesson activity "Archie in the bath" worked well and was a good lead into the research lesson.
- We would include an extension task to challenge the better able students, where maybe they wouldn't be given a story to a particular graph and they would have to themselves come up with a story.
- Blackboard – instead of students matching positive and negative slopes to graphs we would put up a preprepared sheet with all the solutions marked in – this would have led to greater time for the students to describe each graph to their peers – to ensure that all misconceptions were clarified.
- The Venn diagram worked well.
- Students were not very confident at the board in presenting their shared view to the class so this is something that needs practice.

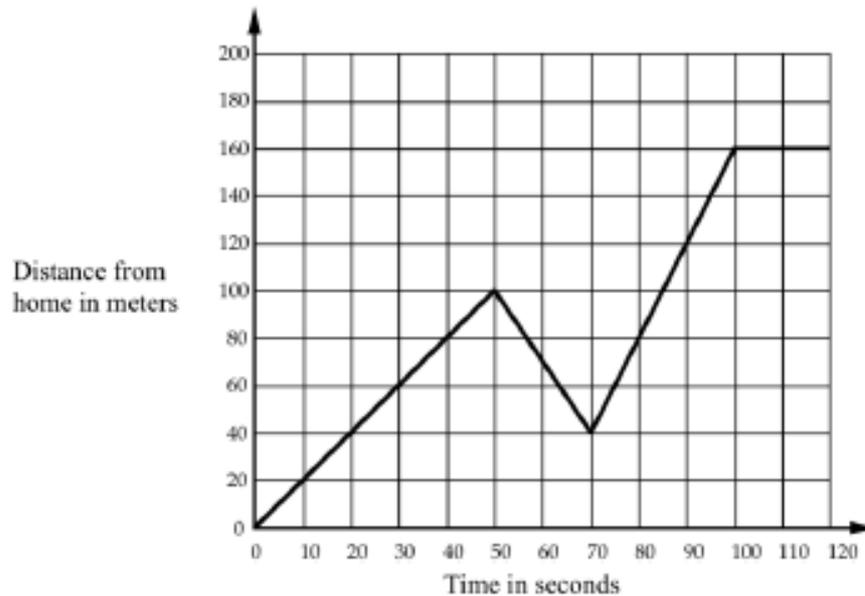
### What are the implications for teaching in your field?

- To encourage students to verbalise their answers more in class, to encourage more peer work where students have to discuss problems with each other
- To hold back as a teacher with the answer – let them do the talking
- It was a good idea to use a diagnostic task before and after the lesson; this provided us with evidence that learning had taken place for all students.

**Appendix 1.** Diagnostic task (accessed at <http://map.mathshell.org/lessons.php?unit=8225&collection=8>)

## Journey to the Bus Stop

Every morning Tom walks along a straight road from his home to a bus stop, a distance of 160 meters. The graph shows his journey on one particular day.



- Describe what may have happened. You should include details like how fast he walked.

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- Are all sections of the graph realistic? Fully explain your answer.

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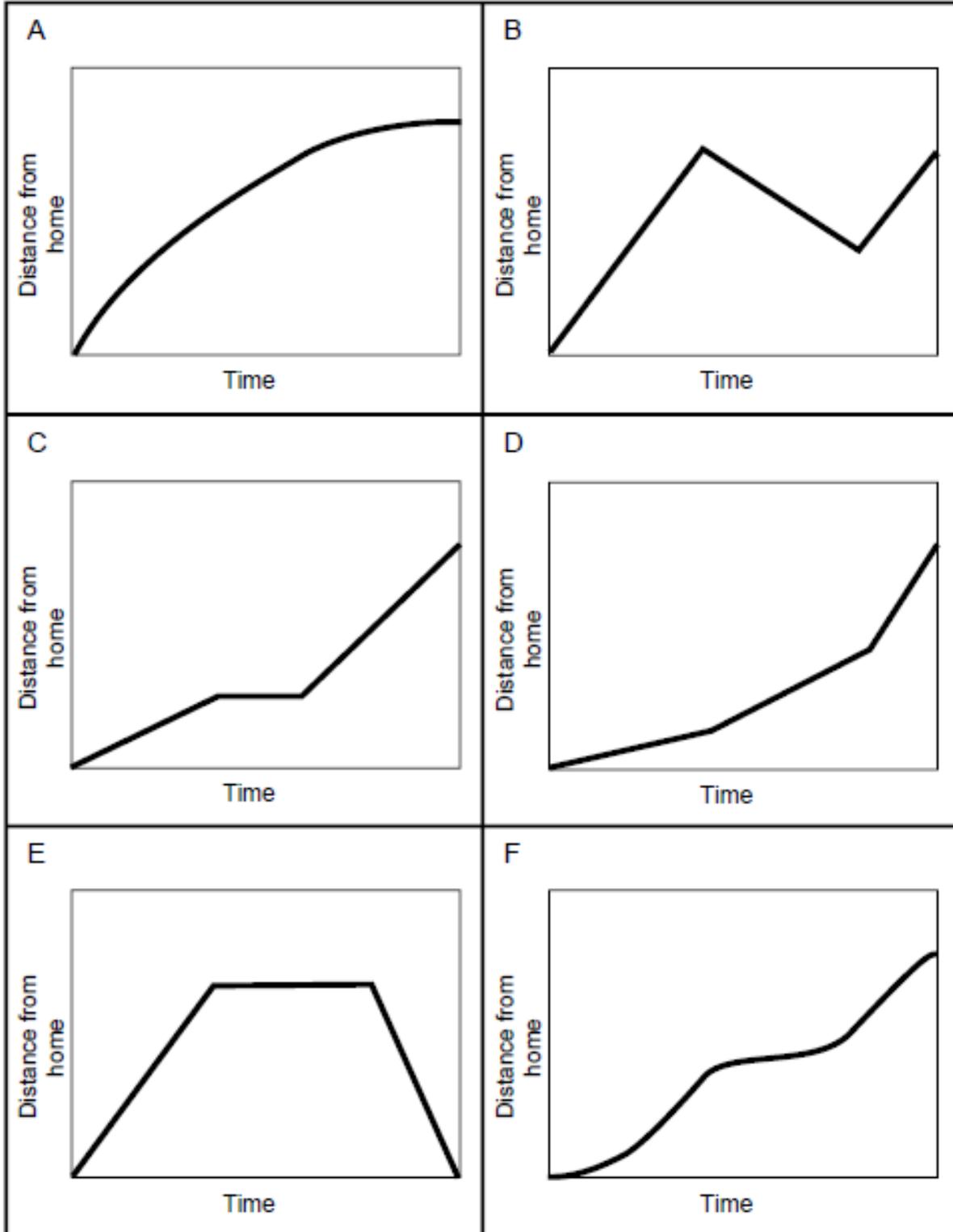
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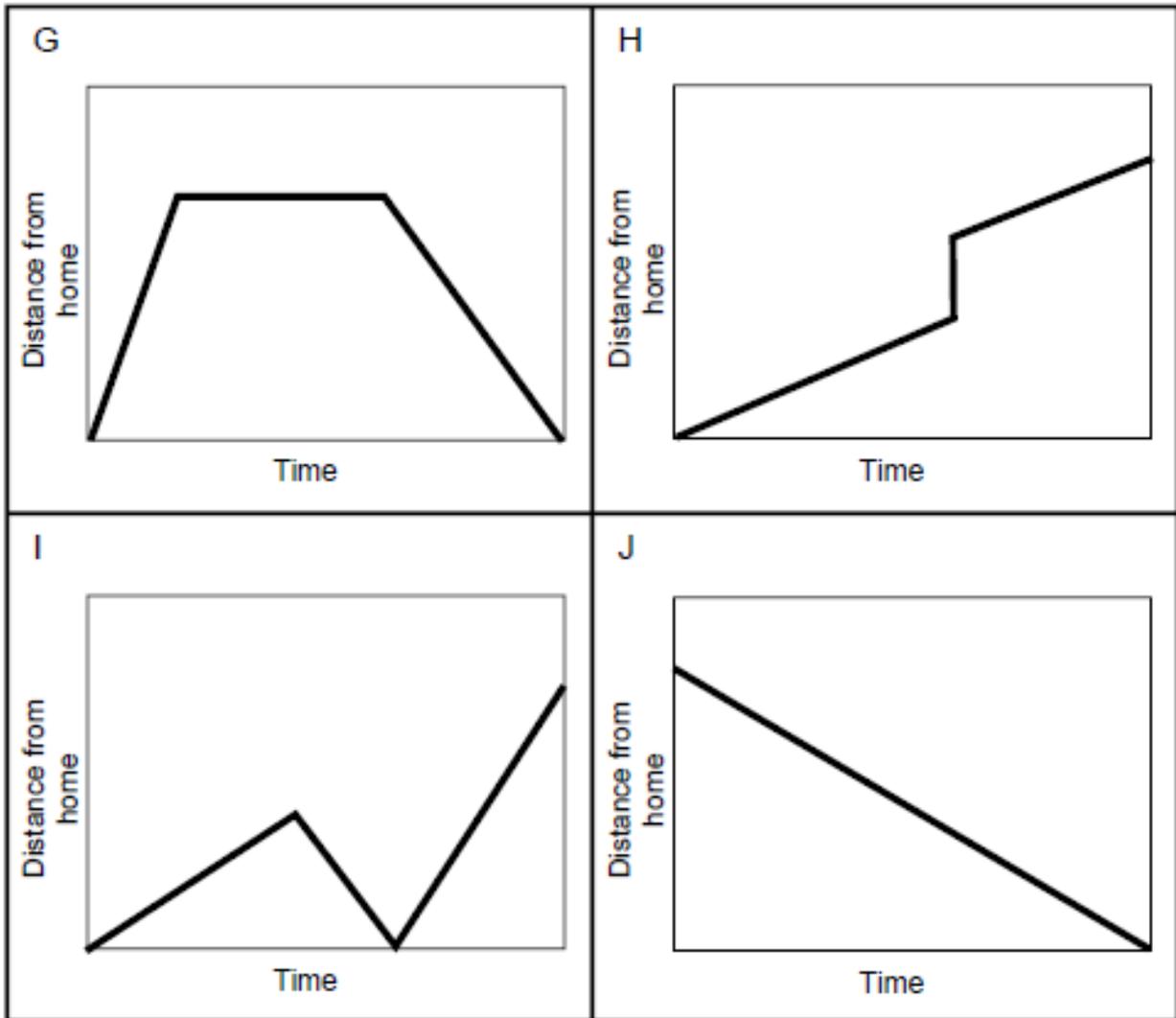
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**Appendix 2.** Cards for the matching activity. The following cards had been used in the research lesson: D, E, G, H, I, J in Set A and 1, 2, 6, 7, 8, 9 in Set B.

### Card Set A: Distance–Time Graphs



## Card Set A: Distance–Time Graphs (continued)



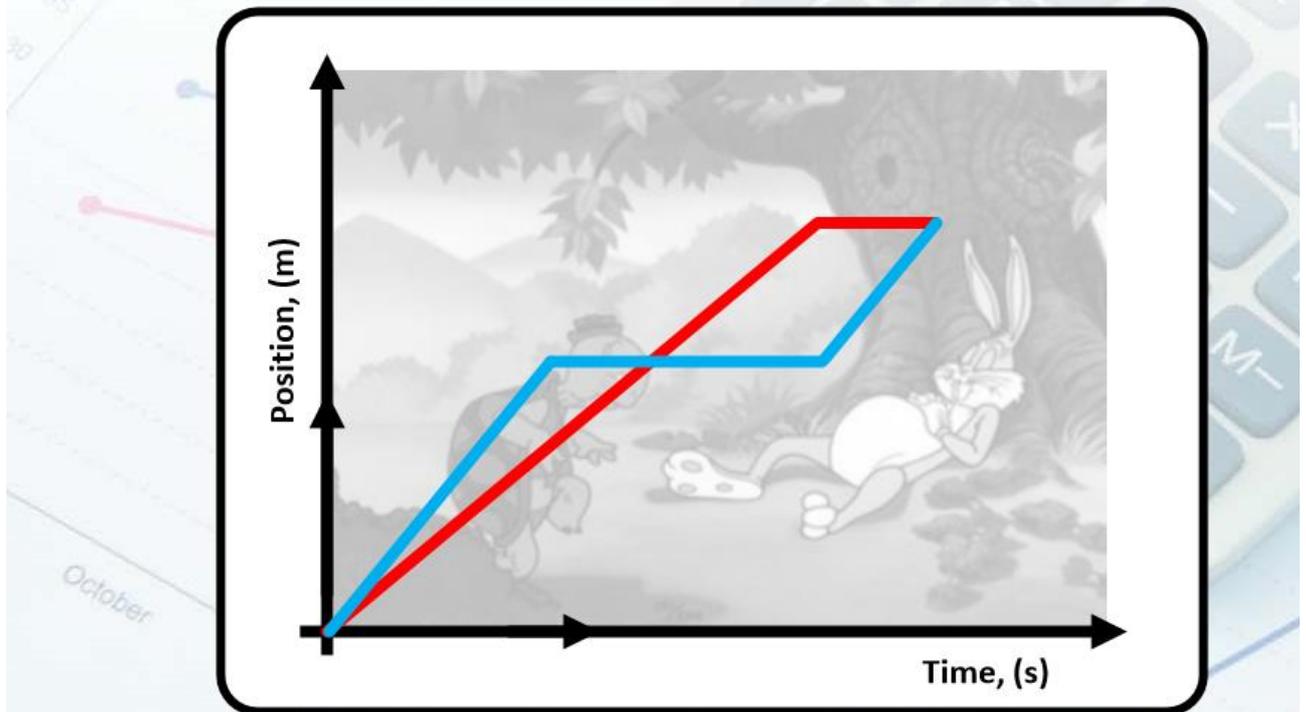
## Card Set B: Interpretations

<p><b>1</b> Tom ran from his home to the bus stop and waited. He realized that he had missed the bus so he walked home.</p>	<p><b>2</b> Opposite Tom's home is a hill. Tom climbed slowly up the hill, walked across the top, and then ran quickly down the other side.</p>
<p><b>3</b> Tom skateboarded from his house, gradually building up speed. He slowed down to avoid some rough ground, but then speeded up again.</p>	<p><b>4</b> Tom walked slowly along the road, stopped to look at his watch, realized he was late, and then started running.</p>
<p><b>5</b> Tom left his home for a run, but he was unfit and gradually came to a stop!</p>	<p><b>6</b> Tom walked to the store at the end of his street, bought a newspaper, and then ran all the way back.</p>
<p><b>7</b> Tom went out for a walk with some friends. He suddenly realized he had left his wallet behind. He ran home to get it and then had to run to catch up with the others.</p>	<p><b>8</b> This graph is just plain wrong. How can Tom be in two places at once?</p>
<p><b>9</b> After the party, Tom walked slowly all the way home.</p>	<p><b>10</b> Make up your own story!</p>

### Appendix 3. Homework

1. Write down two things about slope that you learned today.
2. Story telling

*Do you remember the story of the hare and the tortoise?  
Use this graph to re-tell the story:*



Explain what each line segment represents in the context of the story using what you learned in class today.