

Third Year Higher Level- How many ways can you prove it?

At Colaiste Choilm Tullamore, Co. Offaly.

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Lesson plan developed by: Patricia Tyrell, Eilish Kinsella, Ann Molloy.

- 1. Title of the Lesson:** How many ways can you prove it?
- 2. Brief description of the lesson:** The students will construct the bisector of an angle and using investigative methods will verify the construction.
- 3. Aims of the Lesson:**

Long-range goals:

- That students appreciate that Mathematics can be used to communicate thinking effectively.
(JC Key Skill: Managing Information & Thinking)
- For students to become more creative when devising approaches and methods to solve problems.
(JC Key Skill: Being Creative)
- To emphasise to students that a problem can have several equally valid solutions.
(JC Key Skill: Being Numerate)
- For students to connect and review the concepts that we have studied already.
(JC Key Skill: Managing Information and Thinking)

Short-term goals:

- The ability to use prior knowledge to construct and prove the bisection of an angle using a wide variety of methods.
(JC Key Skill: Managing Information and Thinking)

4. Learning Outcomes:

- The student will appreciate the value of proof and the value of never assuming.
- The students will see many approaches to verifying the construction using prior knowledge and investigative methods.

5. Background and Rationale:

The students have already met:

- [Ruler axiom] The properties of the distance between points.
- [Protractor Axiom] The properties of the degree measure of an angle.
- Congruent triangles (SAS, ASA and SSS).

- Bisector of a given angle, using only compass and straight edge.

Difficulties students have had in the past:

Students struggle to understand the Geometry behind the Proofs and hence fail to make a connection between the construction and a formal Proof. They resort to Rote learning of Proofs without the genuine understanding, which means that they find it difficult to go on to apply and use the Proof to solve problems.

Syllabus

- The geometrical results should be first encountered through discovery and investigation.

6. Research

Syllabus

www.examinations.ie

Maths Development Team Website - Tutorials for Geometry

The Van Hiele Theory of Geometric thought

Project Maths Junior Certificate handbooks.

7. About the Unit and the Lesson

Discovery learning is used throughout the flow of the lesson to ensure a deeper understanding of the construction that also ensures the students retain the information better since they discover it themselves and learn from each other.

8. Flow of the Unit:

Lesson		# of lesson periods
1	<ul style="list-style-type: none"> • Congruent triangles (SAS, ASA, SSS and RHS). 	1 hour.
2	<ul style="list-style-type: none"> • Triangle Theorems: Similar Triangles, A line parallel to one side of a triangle divides the other 2 sides into the same ratio. 	2 hour.
3	<ul style="list-style-type: none"> • Circle Theorems: Angle in a semi-circle, the angle at the centre of a circle is twice the angle at the circumference standing on the same arc and Angles at the circumference standing on the same arc are equal. 	2 hour.
4	<ul style="list-style-type: none"> • Structured Problem solving approach to prove the bisection 	Research Lesson 1 hour.

	of an angle	
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One Hour lessons in research lesson school.

9. Flow of the Lesson

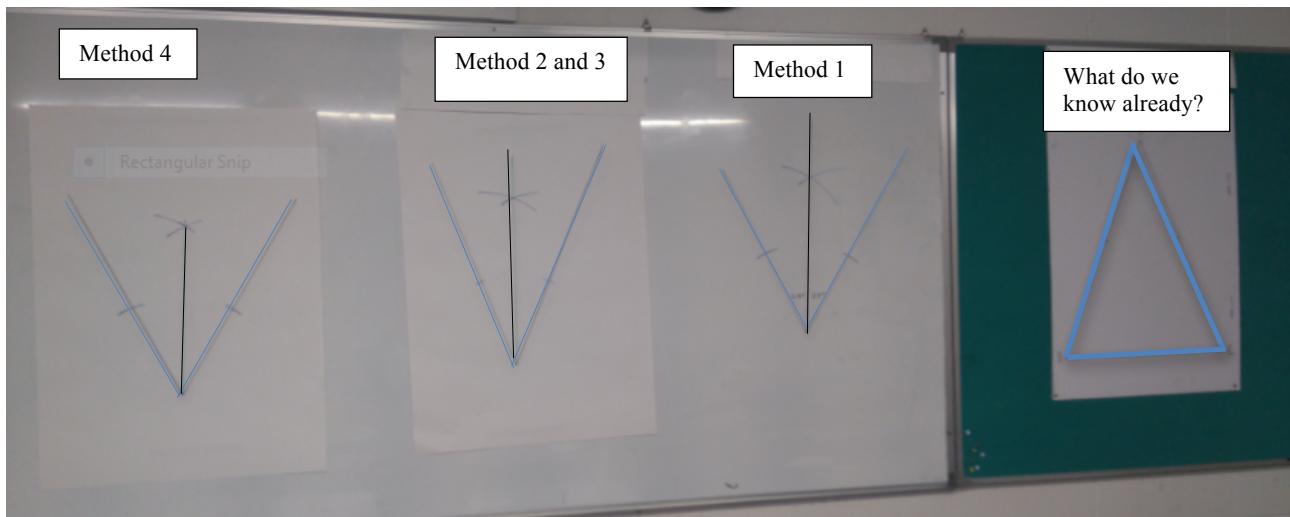
Teaching Activity	Points of Consideration
<p>1. Introduction (10 mins) What type of a triangle is this? (Isosceles triangle)</p> <p>What do we call the points where the arms of the angles meet? (vertex)</p> <p>Can you name the three angles of the triangle in two different ways?</p> <p>If I asked you to bisect this triangle, what does it mean? (Cut it in half)</p> <p>Where will I draw a line to bisect this triangle? (Through A)</p> <p>Is there any other line I can draw which would bisect the triangle? (No)</p> <p>What has happened to the original triangle? (It has been split into two congruent triangles)</p> <p>Why are the two triangles congruent? (Side Side Side, Angle Side Angle, Side Angle Side, Right Angle Hypotenuse Side)</p>	<p>Teacher displays poster of isosceles triangle on the board</p> <p>Ensure in the preceding lessons that students are comfortable with the words arc, bisector, vertex, labelling angles, congruent.</p>
<p>2. Posing the Task (15 mins) Problem for today: To construct and to prove the bisector of an angle</p> <p>Task 1:</p> <p>Your task is to draw an acute angle ABC and bisect this angle.</p> <p>Task 2: Prove that the line drawn is the bisector of the angle ABC using at least 3 different methods.</p>	<p>Students use a compass and a ruler to draw and bisect the angle.</p>

<p>3. Anticipated Student Responses</p> <p>Proof 1: Use of protractor to measure the angles</p> <p>Proof 2: Students will cut out the angle and fold along the bisector</p> <p>Proof 3: Students will cut out triangles and place them on top of each other.</p> <p>Proof 4: Students will use congruent triangles to show the 2 triangles are congruent by Side, Side, Side.</p>	<p>Students will complete this task individually.</p> <p>I will ask leading questions to prompt.</p>
<p>4. Comparing and Discussing</p> <p>Teacher will select students to present their work ranging from the most basic proof to the most sophisticated proof (congruent triangles)</p> <p>Students will present their proof to the class and students will record all methods in their copies.</p>	<p>Students can use their construction to prove the angle has been bisected</p> <p>Teacher emphasizes the importance of connecting the construction to the prior knowledge</p>
<p>5. Summing up</p> <p>Extension Task for homework: Construct a parallelogram and prove the opposite sides and the opposite angles are equal in measure.</p> <p>Now I would like you to write down two sentences in your copies about what you have learned in today's lesson.</p>	

10. Evaluation

- 3 observers who will observe a third of the class each.
- Each observer will have a copy of the seating plan the following will be observed;
 1. First Method student came up with to prove bisection of angle. (Did they go for cutting/folding or theoretical proof)
 2. How long did it take them to construct 2 triangles?
 3. Misconceptions
 4. Were they comfortable with the prior knowledge?
 5. How far in to the student task was congruency suggested.
 6. Did they make assumptions without backing up with proof?
 7. What questions did they ask?

11. Board Plan



12. Post-lesson reflection

What are the major patterns and tendencies in the evidence?

From the start, all students were clear of what was involved in the task. There was a mix of methods with some students opting for theoretical proof first but the majority opted to measure with protractor. We were surprised that the folding and cutting was the last method and the teacher had to prompt to use the scissors.

What are the key observations or representative examples of student learning and thinking?

The entire class were very comfortable with prior knowledge. During the 10 min introduction, congruency was identified very quickly and individual students identified the cases of congruency. At about five minutes in, the majority of students had moved on to constructing triangles. We were surprised that instead of forming 2 congruent triangles the majority formed one Isosceles triangle from the constructed angle. We were surprised that similar triangles were used as a method as this was not mentioned in the prior knowledge. It took up to 15 minutes for the first construction of two congruent triangles.

What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?

Students were clear that any statements or assumptions they made had to be backed up with a reason. However, a small number of students assumed that the angle was bisected and used this as part of their proof.

In what ways did students achieve or not achieve the learning goals?

The students achieved the learning Goals of the lesson in that all students used the construction to come up with at least three methods to prove the bisection of the angle.

Based on your analysis, how would you change or revise the lesson?

When each student was presenting their method to the class, there was very little student questioning. In future, as teachers, we would recommend probing the students who did not come up with a particular method to measure their level of understanding.

What are the implications for teaching in your field?

After going through this process, we would change our methods of teaching formal proofs on the JC HL course. We would use a more structured problem solving approach when teaching these proofs.

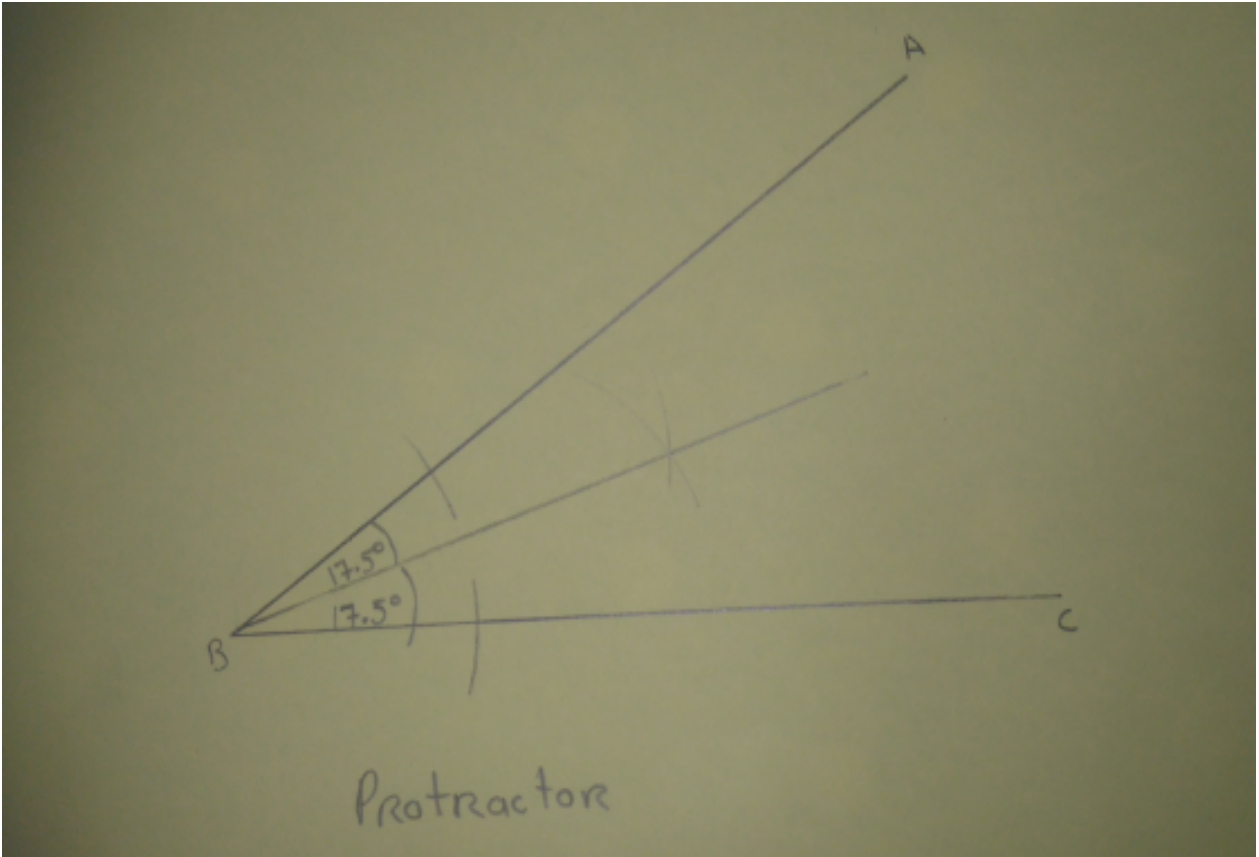
Students came up with the following six methods.

- **Proof 1:** Use of protractor to measure the angles (Case File 1)
- **Proof 2:** Students cut out the angle and folded along the bisector (Case File 2)
- **Proof 3:** Students cut out triangles and placed them on top of each other (Case File 3)
- **Proof 4:** Students used congruent triangles to show the 2 triangles were congruent by Side, Side, Side (Case File 4)
- **Proof 5:** Students proved that they were congruent by forming an Isosceles triangle and then using RHS (Case File 5)
- **Proof 6:** Students used similar triangles (Case File 6)

Final Thoughts:

We feel that this lesson proves that students are capable of constructing proofs using prior knowledge and as teachers we need to move towards investigative methods to teach formal proofs.

Case file 1



Case file 2

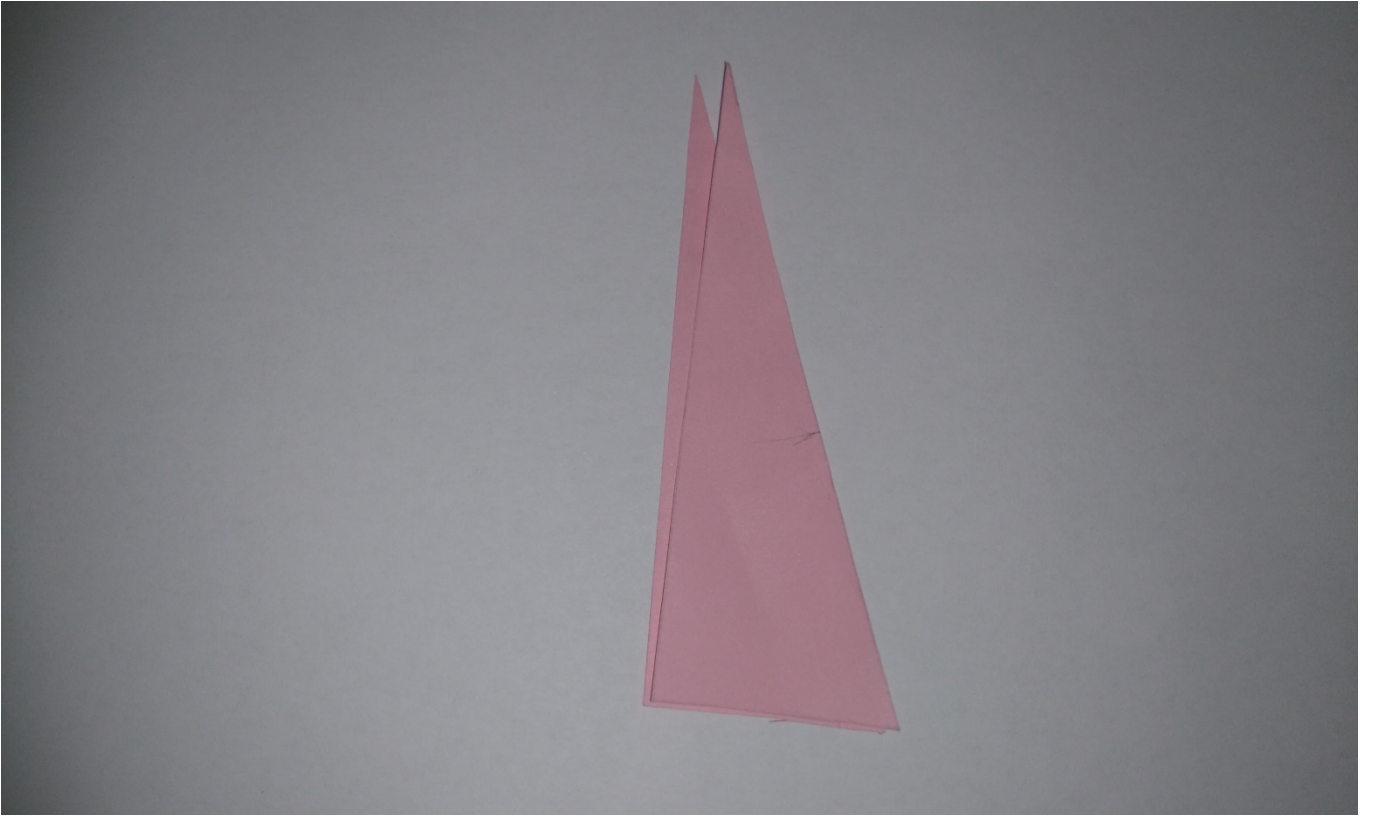


cut by cutting

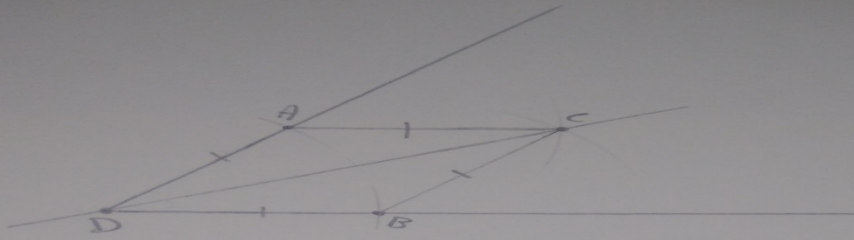
$\triangle ABJ$ folds on top of $\triangle ACJ$ perfectly

so \therefore the $\angle AJC$ = the bisector

Case File 3



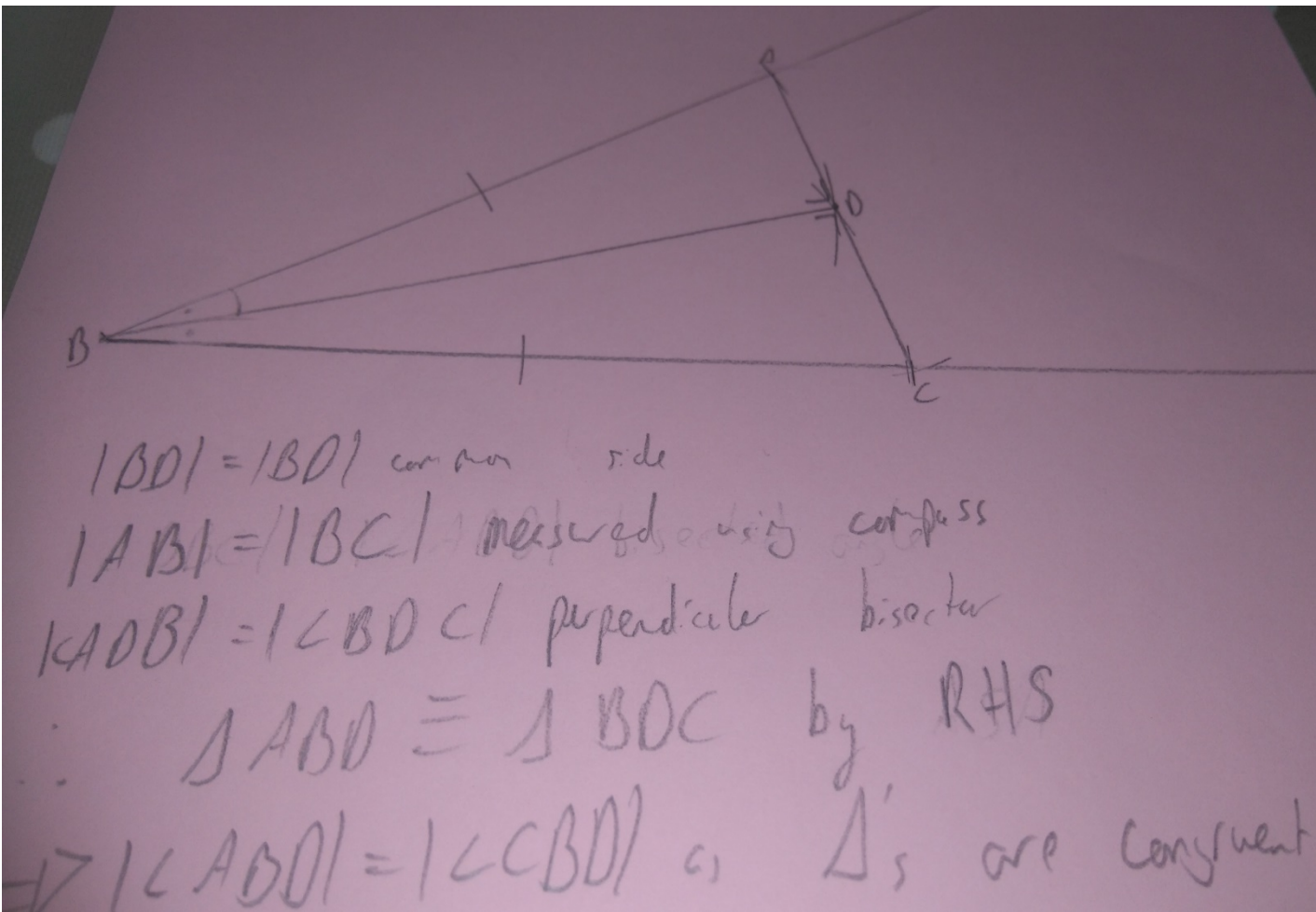
Case File 4



$|AD| = |DB|$ (length on the compass hasn't changed)
 $|AC| = |CB|$ (length on the compass hasn't changed)
 $|DC| = |DC|$ (common side)
 $\therefore \triangle ADC \cong \triangle DBC$ by SSS
 $\Rightarrow |\angle ADC| = |\angle CDB|$

Student Number = 5

Case File 5



Case File 6

Method 4. Prove they are similar

~~ABD~~

$$\frac{|AB|}{|BC|} = \frac{|AD|}{|DC|} = \frac{|BD|}{|BD|}$$

$$\therefore \angle ABD = \angle DBC$$

