

Topic :An introduction to Algebraic & Word Equations

Year Group :1stYearJCSP

For the lesson on **6th December 2016**
At **Moyne College, Ballina, Co.Mayo**
Ms. Attracta Glavey's class
Teacher: **Attracta Glavey**
Lesson plan developed by: **Attracta Glavey**
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Title of the Lesson:

An introduction to Algebraic & Word Equations

Brief description of the lesson:

This lesson proposes to build on the prior knowledge of the students with regard to money and arithmetic to develop an understanding of word equations and their uses.

Aims of the Lesson:

From the teacher's perspective:

I'd like my students to appreciate that mathematics can be used to solve real world problems and that they can use letters to represent numeric values. By using letters to represent 'the value of' an item, they will see that mathematics can be used to solve problems where otherwise they may have guessed or estimated a solution.

I'd like my students to appreciate that mathematics can be used to communicate thinking effectively and appreciate that algebra is a tool for making sense of certain situations.

I'd like to emphasise to students that a problem can have several equally valid solutions and hopefully build my students' enthusiasm for the subject by engaging them with stimulating activities.

I'd like a gradual approach to algebra, moving from real to abstract at ease.

I'd like my students to understand that:

- *many real life problems can be described many ways mathematically.*
- *maths can be checked and verified*
- *there are many different methods of solving the same problem*

Learning Outcomes:

As a result of studying this topic students will be able to:

- 1. Interpret problems with a fresh outlook*
- 2. see the relevance of forming word equations*
- 3. add like terms*
- 4. convert word equations into algebraic expressions*
- 4. substitute values for variables within an algebraic expression*

Background and Rationale

The syllabus requires that students learn how to form & understand both word equations & algebraic expressions. It also requires that students can substitute values for variables, evaluate & interpret solutions. The syllabus asks us to improve students' enthusiasm and understanding by adopting a more hands on approach.

In the past, students have had many problems with this topic. Some have an aversion to the word 'Algebra'! This may stem from a dislike of the topic by an older sibling or even a parent. We believe that this approach of using 'money' problems to introduce algebra may help, as most students are familiar with money concepts and enjoy problems of a monetary nature. This may also assist to combat that perceived aversion!

Students often don't realise that X is the same as $1X$ - they often misunderstand that different variables may not be summed e.g. $3X + 2Y \neq 5XY$ - they may also have difficulty recognising that $0X$ is in fact 0. We try to address these and other misconceptions during this lesson.

We would hope that at the end of this lesson that students should realise that there may be more than one approach to solving a problem. They will be able to add like terms.

This lesson also promotes mathematical literacy and approaches algebra in a practical way, ensuring that all students in the class, regardless of ability, will have a good grasp of the topic after the forty minutes.

As some students can problem solve better with the aid of a diagram - we actively promote the use of diagrams and use real life everyday problems as part of our activity to stimulate a more meaningful understanding of algebra for students.

Research

In preparing this lesson we used the following:

- Junior Certificate Mathematics Guidelines for Teachers – Dept. of Education and Science 2002*
- First year Teachers Handbook from Project Maths website*
- NCCA(2010): 'Common Introductory Course for Junior Cycle Mathematics'*

- Department of Education and Skills(2011). 'Literacy and numeracy for learning and life'

About the Unit and the Lesson

- Strand 3 Number systems Page 22 Learning Outcomes
 Ref 3.1 -use the equivalence of fractions, decimals and percentages
 -Consolidate their understanding and learning of factors in N
 -Check a result by working the problem backwards.
- Strand 4 Algebra Page 27 Learning Outcomes
 Ref 4.4 -use the representations to reason about the situation
- Page 28 Learning Outcomes
 Ref 4.6 -Using letters to represent quantities that are variable.
 Arithmetic operations on applications to real life contexts.
 Substituting and factoring.

Flow of the Unit:

Lesson		# of lesson periods
1	<ul style="list-style-type: none"> • Factoring of natural numbers – this class promotes the use of factors as a means of providing shorter routes to solving problems • Decimalisation – working with decimals – we practice adding & subtracting decimals, multiplying decimals by whole numbers, moving the decimal place left and right. • Basic Arithmetic operations with real numbers – we <u>remind</u> them of the usefulness of BIMDAS. 	3 x 40 min.
2	<ul style="list-style-type: none"> • An introduction to Algebraic & Word Equations 	1 x 40 min. (research lesson)
3	<ul style="list-style-type: none"> • Substitution of values for variables 	2 x 40 min.
4	<ul style="list-style-type: none"> • Forming Algebraic equations as we did in our lesson. • Working with algebraic equations working backwards. 	2 x 40 min.
5	<ul style="list-style-type: none"> • Solving simple real life problems using diagrams & algebra. 	2 x 40 min.

Flow of the Lesson

Teaching Activity	Points of Consideration
<i>The pupils are welcomed, a Roll call is taken, and the guidelines for this class are re-enforced.</i>	<i>The class seating plan was pre-arranged and the pupils sat in their allocated seats.</i>
<p>Introduction (3minutes including above) <i>Prior knowledge on multiplying decimals by a whole number is covered. Pupils are reminded to be mindful of the position of the decimal place. (5 minutes). Pupils asked to have their calculators and a pen on their desk.</i></p>	<i>Students are reminded to ‘line up’ digits beneath each other carefully – to make adding and subtracting all the easier.</i>
<p>Posing the first task(3 minutes) <i>Students are told that they have won a €10 cinema voucher from a radio show. They are told that the voucher will cover the €8 cost of entry to the cinema and are allowed to spend the rest of the voucher in the cinema shop. The cinema shop has four items on sale only Fanta €1.40, Tayto €0.80, Pop corn €0.40 and Chop bars€0.30. They are asked how many ways can they spend their money. The display on their sheets is mirrored on the Promethean board. They are given five minutes to work on the problem. Following which will follow a 6 minute review.</i></p>	<p><i>The students are given a copy of the Voucher. The teacher explains to them that they may spend the remainder of the voucher once they have paid for entry to the cinema. The remainder of the voucher is spent in the cinema shop. They are given a copy of the shop price list with pictures. They must deduce how many Fanta, Tayto’s, Chomp bars and/or Pop corns they may purchase for the remaining €2.</i></p>
<p>Anticipated Student Responses</p> <p><i>R1: .Students may sum all four products and get €2.90 –they may then look to deduct items to make up €2 One fanta + one tayto + 1 chomp + 1 popcorn= €2.90 Take away 1Popcorn then take away 1Chomp</i></p> <p><i>R2: .Students purchase one Fanta and determine how much change remains from €. They then decide how to spend this change by inspection of the Shop Price List. i.e. If they buy a single Fanta, they will have €0.60 left So 1 Fanta + 2 chomps = €2.00</i></p> <p><i>R3: .Students may use factoring and see that they could purchase five Pop corns i.e 5Popcorns = €2</i></p>	<p><i>If a pupil is stuck, the pupil may be asked to consider one or more than one Pop corn and see what else they may be able to buy. This would ensure that the pupil uses a method of deduction by subtracting the value of P from €2 etc. If pupils finish early, they will be encouraged to seek all solutions.</i></p>
<p>Comparing and Discussing(6 minutes) <i>The teacher asks the first student to come to the board and fill out their answer using a pre-defined format. This format is similar to the one presented to the students during the task. The student is asked to explain his/her answer.R1</i></p> <p><i>The class is then asked if they got this answer and if it was the same method used.</i></p>	<p><u>Solutions</u> <i>One Fanta and 2 Chomp bars Two Tayto and 1 Popcorn One Tayto and 4 Chomp bar One Tayto and 3 Popcorn 5 Popcorn 2 Popcorn and 4 chomp bars</i></p> <p><i>There are six unique solutions.</i></p>

<p>The next student is asked to the board to consider what happens when they purchased one tayto etc. Again the student explains their method & again the class is tested for a response.R2</p> <p>The next student is brought to the board and shows the class his factoring response. R3</p> <p>These responses will be limited on time as they merely lead as an introduction to the second task.</p>	
<p>Posing the Second task(3 minutes)</p> <p>Students are told that they have won a €10 cinema voucher from a radio show. They are told that the voucher will cover the €8 cost of entry to the cinema and are allowed to spend the rest of the voucher in the cinema shop. They are now told that:</p> <p>'F' is the cost of a Fanta : $F=€1.40$ 'T' is the cost of a Tayto : $F=€0.80$ 'C' is the cost of a Chomp : $F=€0.40$ 'P' is the cost of a Popcorn : $F=€0.30$</p> <p>They are asked how many ways can they spend their money. They are asked to write their answers using the letters F,T,C and/or P.</p> <p>They are given eight minutes to work on the problem. Following which will follow a 9 minute review.</p>	<p>The students are given a copy of the Voucher. The teacher explains to them that they may spend the remainder of the voucher once they have paid for entry to the cinema. The remainder of the voucher is spent in the cinema shop.</p> <p>This time it is explained to the that :</p> <p>'F' represents the cost of one Fanta = €1.40 'T' represents the cost of one Tayto = €0.80 'C' represents the cost of one Chomp = €0.40 'P' represents the cost of one Popcorn = €0.30</p> <p>They consider one of the previous solutions in Task1 and asked if the same answer could be represented using the letters F, T, C and/or P.</p>
<p>Comparing and Discussing(9 minutes)</p> <p>The teacher asks the first student to come to the board and fill out their answer using a pre-defined format. This format is similar to the one presented to the students during the task. The student is asked to explain his/her answer.R1</p> <p>The class is then asked if they got this answer and if it was the same method used.</p> <p>The next student is asked to the board to consider what happens if they purchased one tayto etc. Again the student explains their method & again the class is tested for a response.R2</p> <p>The next student is brought to the board and shows the class his factoring response. R3</p> <p>A fourth student may be asked to the board to consider if they purchased two Tayto's etc. Again the student explains their method & again the class is tested for a response.R4</p> <p>A fifth student may be asked to the board to explain how they considered two chomps etc.R5</p> <p>Finally a sixth student might elicit a response regarding four chomps.R5</p> <p>The teacher compares all the answers and asks if any answer has been put down more than once.</p> <p>Another student is prompted to the board to show that</p> <p>$1P + 2T$ is the same as $2T + 1P$</p>	<p>The pupils become comfortable replacing letters with numbers.</p> <p>The students see that there are a finite few solutions and should feel good in producing their efforts.</p> <p>Solutions</p> <p>$1F + 2C = €2$ ✓ $2T + 1P = €2$ ✓ $1T + 4C = €2$ ✓ $1T + 3P = €2$ ✓ $2T + 1P = €2$ R $1P + 2T = €2$ R $3P + 1T = €2$ R $5P = €2$ ✓ $2P + 4C = €2$ ✓ $4C + 2P = €2$ R $4C + 2P = €2$ R $4C + 1T = €2$ R</p> <p style="border: 1px solid black; padding: 2px; display: inline-block;">R = repeating solution</p> <p>There are six unique solutions.</p>
<p>Summing up – (3 minutes)</p> <p>The Teacher asks which answer if any is the best.</p> <p>Finally the teacher lists all twelve solutions on the board</p>	

and asks the students how many Fanta, Tayto, Pop corn and chomp bars were represented in the twelve solutions. [Even though there are only six unique solutions]

Homework task(1 minute)

It is as designed in the pane below.

Homework 1

You have won a prize from a radio show! The prize includes a 'Free Entry' to the cinema for **two people** as well as a €6 voucher to the cinema shop. A copy of the voucher is printed here along with the pricelist for the cinema shop. If **both people** are to get the **same items** from the shop, how many items can you buy? Write down as many possible solutions as you can.

This ticket Admits 2 people to cinema and €6 voucher to cinema shop [No change given]

F = cost of buying a Fanta = €1.40
T = cost of buying a Tayto = €0.80
C = cost of buying a Chomp = €0.40
P = cost of buying a Popcorn = €1.40

1.  +  +  +  = €6.00

2.  +  +  +  = €6.00

3.  +  +  +  = €6.00

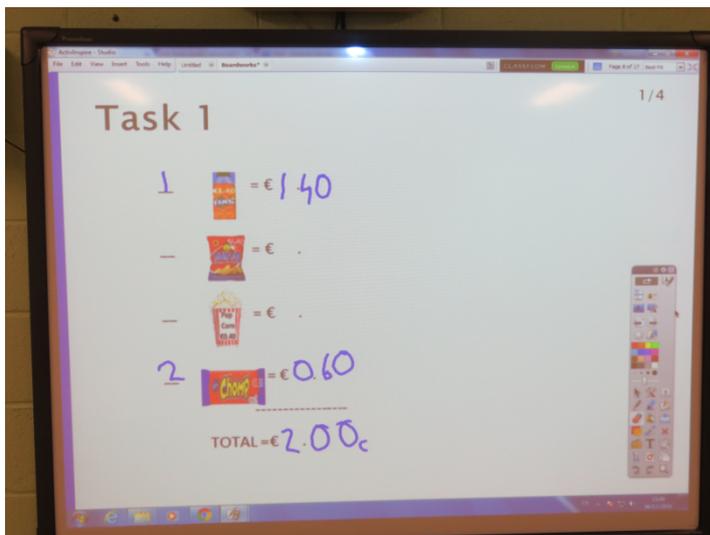
4.  +  +  +  = €6.00

Evaluation

- Attracta showed previous work on the Promethean board and asked her class to recall the previous lesson on multiplying and adding decimals. They recalled that it was necessary to 'line-up' the digits carefully and this would make the problem easier to do.
- Attracta moved through the class –Brendan seated at far back corner of class.
- Attracta recorded students' preparatory work and decided on the order she would bring students to board.
- Brendan, armed with a seating plan, recorded (on paper) who seemed to be working, struggling or needed help focussing on the task.
- Attracta brought four pupils to board – Brendan recorded that they completed their tasks successfully. The pupils needed no encouragement – they enjoyed their trips to the Promethean board. They were comfortable at the board & required very little prompting. All of their work was saved on the computer – so that we could inspect it post-lesson.
- The learning outcomes and the aims of the lesson were definitely achieved. The first task was a 'taster' for the main event. The pupils understood that all money was to be spent and that there were a lot of different solutions. The fact that there was more than one solution made the task all the more enjoyable for the pupils as they wanted to search for other possibilities.

- In the second task, they were asked to write their solutions and any other new solutions using *letters*. It was clearly explained to them that the individual letters had a value associated with them.
- Attracta brought four more pupils to board – Brendan recorded that they completed their tasks quite successfully. Again, the pupils required no encouragement – they enjoyed their trips to the Promethean board. They offered good explanations along with their work and required very little prompting. All student board work was stored on file.
- Attracta collected all their hard copy work and then summed up & gave homework assignment. She thanked them for allowing Brendan to visit.

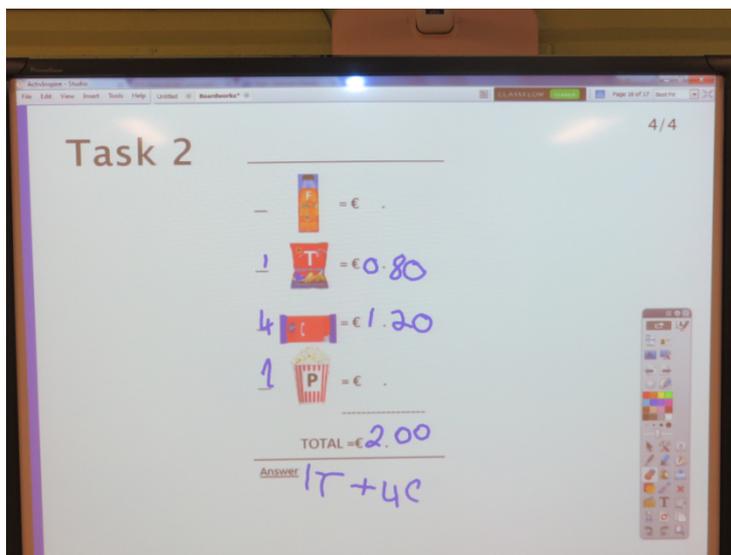
Board Work



Each pupil was given a sheet at the start of the task with room for four solutions.

The sheet was arranged similar to a shop receipt. We thought that they would be familiar with this concept.

When brought to the board, the pupils filled in a value on the left side of the product they bought and the total on the other side. They summed the total to €2.



In the second task, the word describing the product was replaced with a single letter. They were given a sheet with room for four possible solutions.

When brought to the board, the pupils filled in a value on the left side of the product letter they bought and the total on the other side. They summed the total to €2.

They concluded by showing their answer as

Task 2

1		= €	1.40
-		= €	.
2		= €	0.80
-		= €	.
TOTAL = €			.
Answer $1F + C2 = 2.00$			

Interesting.

Post-lesson reflection

The class were asked how many ways they could spend their change from their Voucher, once entry to the cinema was paid.

Some questions came from the floor as they were progressing with their work - :

1. Is there more than one solution Miss?
2. Does that mean we have €2 to spend?
3. Do we have to spend all of the €2?

Although, most of the class understood the task fully – it was natural to see that one or two pupils would seek the comfort of their Teachers’ attention to ensure that they were on the right track.

Attracta handled it very well and said “*If I had that voucher in the cinema, I would want to spend it all – wouldn’t you?*” She left it at that & the class were happy with her answer.

This brought to mind, the need for problem solving to be ‘bred’ into our pupils. While we need to encourage and cajole our pupils, we also need to breed into them a desire to work independently and to follow the path their solution leads them to, without any fear or anxiety.

Key observations

The pupils were familiar with a shopping bill and receipt. They showed little or no hesitation in working with the problem. They adapted to the problem quite easily once the notion of *letters* was introduced in the second task. The pupils explained at the board that $1F + C2 = €2$.

When a pupil was asked ‘*what did C2*’ mean, he said it was ‘two chomp bars’ and then the pupil suggested that $2C$ might be a better solution. When pushed further the student explained that $2C$ was in fact €0.80 – indicating that he understood that ‘C’ was the cost of a chomp bar.

Perceived difficulties

What stood out the most was the desire of the class to be correct – they wanted to please their teacher. They worked well and enjoy the attention of their peers. While they were working on the first task, some strived to gain attention. This first task was a great ice-breaker and a great lead-in to the main task – the second task. Once the second task commenced, you could feel the relief in the air as they became more comfortable with their own self directed work as they ploughed through various solutions. There was also a greater level of satisfaction around the class once the second task was complete.

Achieving the learning goals

I was delighted to see that the pupils came to terms with the objectives of the class. They were able to construct a solution and see that it was not unique. They could converse with Attracta and form small algebraic equations at the end of the class. Attracta asked them how they enjoyed the class & one responded that it was different – using letters instead of numbers. All in all they reported that they enjoyed it and wished to do more.

Lesson Revision

We discussed this point thoroughly after the lesson.
The short answer was – we wouldn't change anything!

If we were to look at the questions posed during the first task – particularly – 'Does that mean we have €2 to spend'? We felt that we should leave the description of the problem as it was & allow the student to experience a real world problem. This was opposed to asking the pupil "*How many items in a shop can you purchase for €2, where you must spend all of your money!*"

The plan works well for a forty minute class – it might be a stretch for a 35 minute class – you would have to trim content somewhere. The review work time was, in my opinion, more important than the work time.

This particular lesson could work very well as a team-teaching exercise.

*Brendan Doherty
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December 8th 2016*