

2nd Year Mixed Level More Pizza for Me?

For the lesson on 24th Jan 2017
St. Marks Community School, 2nd year Mathematics
Teacher: Nicola Doyle

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1. Title of the Lesson: More Pizza for Me?

2. **Brief description of the lesson:** Students have to decide between two parties: Sharon has invited nine people and is ordering nine twelve-inch pizzas. Claire has invited nine people and she is ordering four eighteen-inch pizzas. Which will provide each person with the most pizza? Students, using their mathematical knowledge of area, proportion and the circle, they will arrive at a logical answer to the problem proposed.

3. Aims of the Lesson:

I'd like my students to:

- Appreciate that mathematics can be used to solve real world problems.
- Appreciate that mathematics can be used to communicate thinking effectively.
- Encourage independent learning.
- Promote creativity and imagination when solving mathematical problems.
- Emphasise to students that a problem can have several equally valid solutions.
- Build my students' enthusiasm for the subject by engaging them with stimulating activities.
- Use prior knowledge the concepts that we have studied already.

For students to understand: We want students to use mathematical concepts from across the Mathematical strands such as fractions and area to understand proportion and how mathematics can be used to solve real world problems.

4. Learning Outcomes:

As a result of students participating in this lesson they will be able to:

- Apply a number of different mathematical methods to discover the concept of equivalency.
- Use previous knowledge on the area and sector of a circle.
- To appreciate where ratio and proportion can be used to solve an everyday problem.

5. Background and Rationale

Link to the Syllabus:

Strand 3:

3.1. Number Systems

3.3. Ratio and Proportion

3.4. Applied Measure 1

3.4 Applied Measure 2- Area and Volume

In the past students have had difficulty with:

- Understanding that fractions are a part of a whole object.
- Understanding the concept of the denominator.
- Linking different strands of the course to problem solve
- Interpreting questions
- Demonstrating that fractions can be equivalent.

6. Research

- Project Maths Website.
- Folens Active Maths Book.
- Mathematical Post Primary Syllabus.

7. About the Unit and the Lesson

Junior Cycle Key Skills:

The multiple problem solving approach to the question can decrease mathematical anxiety that students may have in relation to the subject. Solutions can range from the basic to elaborate, this ensures that students of all levels and abilities are included in the class. This aspect of the lesson links in with the Junior cycle key skill such as: Staying well, managing myself, working with others and being creative.

Student also work on their Communication skills when working as part of a group and when explaining their solution to the class at the board. Managing Information and Thinking is also targeted when students are displaying the multiple solutions to the class. This allows all students to observe, process and appreciate the variety of different methods to solving the same question.

8. Flow of the Unit:

Handbooks would be useful here

Lesson		# of lesson periods
1	● Discovering π	1 x 1hr
2	● Area of a circle	1 x 1hr
3	● Research Lesson	1hr (research lesson)
4	● Problem solving involving area and proportion	1 x 1hr
5	● Problem solving involving area and proportion	1 x 1hr

9. Flow of the Lesson

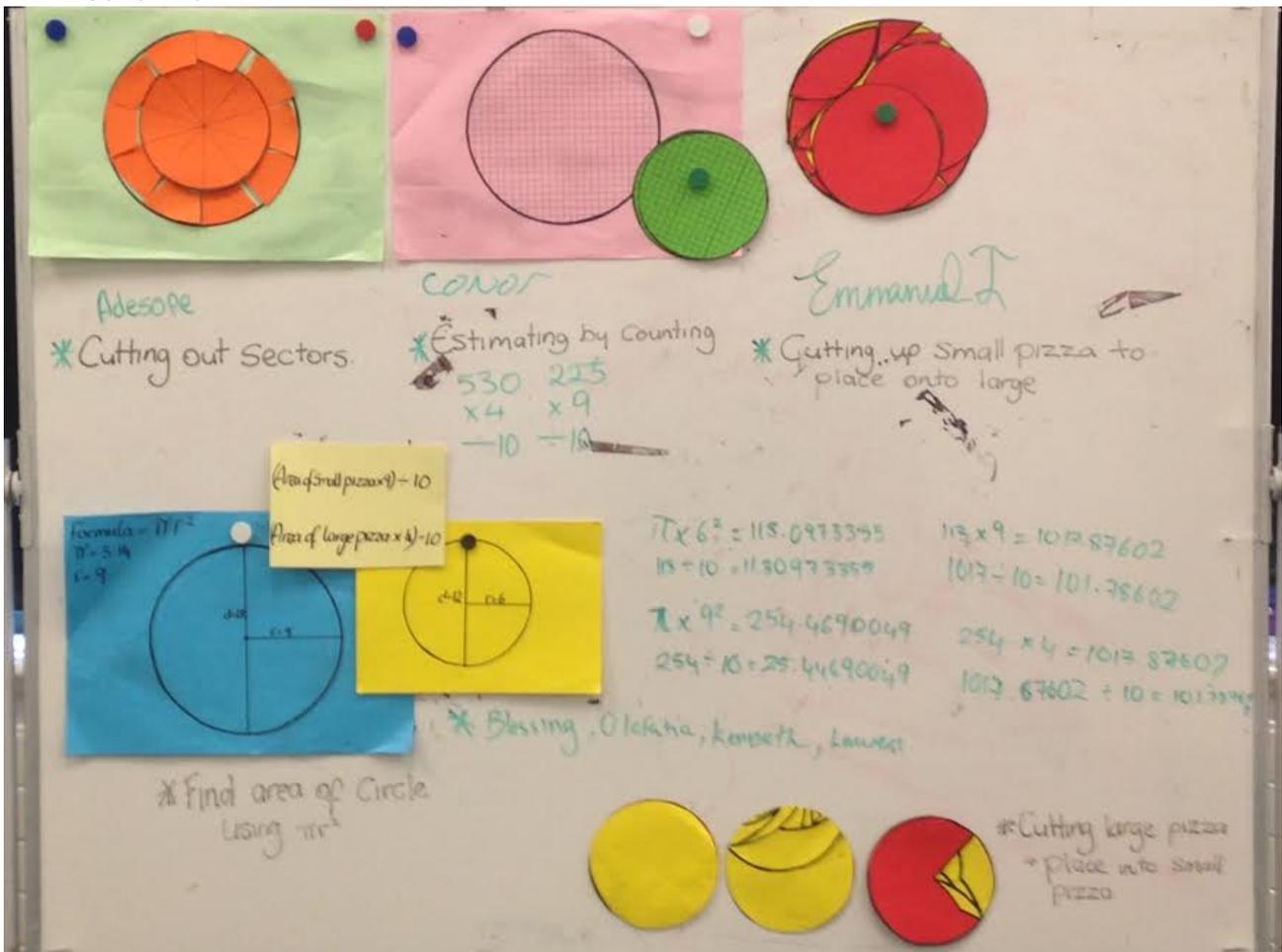
Teaching Activity	Points of Consideration
<p>1. Introduction</p> <ul style="list-style-type: none"> Place students in groups being mindful of ability levels. Recap on previous lesson 	<ul style="list-style-type: none"> What is a denominator? What is a numerator? How would you find half of a number? How do you divide by a fraction? What is the area of the circle formula?
<p>2. Posing the Task</p> <p>You have been invited to two parties on the same night. You have to decide which party to go to? Sharon has invited 9 people and is ordering 9 12 inch pizzas. Claire has invited 9 people but she is only ordering 4 18 inch pizzas. Which party would you decide to go to?</p>	<p>What would be the deciding factor for you when picking what party to go to?</p> <p>How can we mathematically back up and verify our decision?</p>
<p>3. Anticipated Student Responses</p> <ul style="list-style-type: none"> Students divide the circles into fractions. Students count the squares in each sector. Students count the squares in the full circle. Students cut out circles and place on large circle. Students cut of sectors and fit into full circle. Some use the area formula to find area of full circle or sector. Protractors could be used. Students estimate wrongly by visual misconception. Students use diameter instead of radius. Students think dimension is the circumference instead of diameter. Students might not include the person who is holding the party and share among 8 people. 	<p>While circulating the classroom the lead teacher asked questions about their work and encouraged them to discuss findings with their group to make sure they were correct.</p> <p>Early finishers were encouraged to try solve the problem in as many ways as possible this done to then encouraged to help other groups.</p>
<p>4. Comparing and Discussing</p> <ul style="list-style-type: none"> Cutting and placing circles onto one another Counting squares per sector Counting squares per circle Finding area of a circle using formula Finding area of a sector 	<p>Peer teaching, peer learning and discovery.</p>
<p>5. Summing up</p>	

Teacher calls students to the board to display their answer starting from the most naïve. Teacher then discusses the solutions with the class focusing on the idea of proportion and equivalence. The distribution of area will also be discussed in detail. The question will be linked to everyday life scenarios and students will appreciate the use of mathematics in the world around them.

10. Evaluation

- Two teachers will observe the lesson taking notes
- Teachers will record any misconceptions
- Teachers will record how students approach the problem
- Teachers will record the student’s names which they will call to the board
- Students solutions were shared with the class
- Pictures of the student’s work will be taken when the teachers are observing

11. Board Plan



The whiteboard displays several student solutions:

- Adesofe:** *Cutting out sectors. Diagram shows a large orange circle with several smaller orange circles cut out from its perimeter.
- Conor:** *Estimating by counting. Includes a multiplication table:

$$\begin{array}{r} 530 \\ \times 4 \\ \hline -10 \\ \hline 2120 \end{array}$$
 and another:

$$\begin{array}{r} 215 \\ \times 9 \\ \hline -10 \\ \hline 1935 \end{array}$$
- Emmanuel I:** *Cutting up small pizza to place onto large. Diagram shows a large red circle with several smaller red circles inside it.
- Area Calculations:**
 - Area of small pizza = 10
 - Area of large pizza = 10
 - Area of large pizza = πr^2 , with $r = 3.14$ and $r = 9$.
 - Area of large pizza = πr^2 , with $r = 4$ and $r = 6$.
- Blasing, Olofaha, Kenneth, Lawrence:** *Find area of Circle using πr^2 . Includes calculations:

$$\pi \times 6^2 = 113.0973355$$

$$113 = 10 = 1130973355$$

$$\pi \times 9^2 = 254.4690049$$

$$254 = 10 = 2544690049$$
- Other Calculations:**
 - $113 \times 9 = 1013.87602$
 - $1013 = 10 = 101387602$
 - $254 \times 4 = 1017.87602$
 - $1017.87602 = 10 = 101787602$
- Bottom Right:** *Cutting large pizza to place into small pizza. Diagram shows a large yellow circle, a medium yellow circle, and a small red circle.

12. Post-lesson reflection

What are the major patterns and tendencies in the evidence? Discuss.

We realised that higher levels students are more inclined to look for a more direct and mathematical root to solve the problem.

The lower level students were more inclined to explore different solving options and were more at ease when presented with practical worksheets.

What are the key observations or representative examples of student learning and thinking?

Higher level students were less likely to begin solving the problem using the cutting methods. These students seemed confused by the number of worksheets provided and instead looked for a formula or a more mathematical method to solve the task.

In contrast the ordinary level students were content with the number of worksheet they had to solve the problem. They were more adventurous and explored a number of options to find the answer.

What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.

We were surprised by what methods students were initially drawn to when beginning the problem solving process. We noticed that student didn't opt to solve the problem using, what we considered, the most naïve method of solving.

For example, students didn't opt to use the 'counting of squares, method to find the area of the large and small pizzas. We considered this one of the less difficulty method to find the area of the circles. Also this 'counting of squares' method was revised in the prior knowledge and still student didn't think to use this method of problem solving. On review we considered that the squares may have been too small and off putting for the students. And this method required a lot of time and accuracy to derive at the correct solution and students wanted to get to the end result quicker. The students who did actually use this method, surprised us as they came up if a quick and efficient method of count the squares. They counted one quarter of the squares in larger circle and multiplies it by four to get the number of squares in entire circle and followed this procedure for the smaller circle.

A number of misconceptions were

1. Mistaking the diameter by the area.

A number of students mistook the diameter for the pizza as the area of the pizza. For example, Area of all 12 circles: $12 \times 9 = 108$ rather than using the area of a circle formula.

2. Mistaking the ratio that there was 4:12 large is to small pizzas as 1:1

Some students would find how many times the area of the small pizza would fit into the area of one larger pizza and forget that there was in fact four large pizzas.

3. Comparing the sector of the 18" to the sector of the 12" instead of considering that there was 4 of the larger and 9 of the other.

In what ways did students achieve or not achieve the learning goals?

Students did achieve their learning goals. At the end of the lesson students were aware that there were a number of methods that could be applied to derive the correct answer. Students solved the problem that they were given and could logically back up their answer. Most of the students understood that there are a number of methods to find the area of a circle.

Using rectangular pizzas had also been suggested when designing the lesson, in case some students were too challenged at this stage by the circles. It turned out that a few struggled with the circles.

It became apparent also that certain simplifying assumptions are required when mathematically modelling a real world situation. In this problem, for example, students had to simplify by assuming each person ate the same amount of pizza.

Based on your analysis, how would you change or revise the lesson?

Based on our analysis we would consider changing some aspects of the lesson. We would increase the area of the squares in the circles to try and entice more students to use the counting squares method of solving the problem. Along with this, we would put less emphasis on fractions in the prior knowledge section of the lesson so as not to minimize other approaches that students might choose.

As suggested above, rectangular pizzas could be an option if some students are struggling, as the goal of using some prior knowledge in new contexts would still be maintained and they would be kept on task.

Finally, we could tell students that there are 10 at the party to make it clear that Claire and Nicola include themselves.

What are the implications for teaching in your field?

This lesson was very student centred. Students worked together to solve a given problem with little assistance from the teacher. In the past we all feel that we lacked this approach to teaching and instead planned more teacher centred lessons. Teaching this Pizza problem has showed us that many students are more capable problem solvers, when the lesson is carefully constructed, than we initially gave them credit for, this is illustrated as students were all successful in solving the task. As a result, we will try to plan a more holistic lesson in our future teaching that focuses on student centred learning.

We taught the lesson to a mixed ability class. We noted that all students were successful with the task. However, we felt that higher level students opted to solve the problem using higher order mathematical thinking. We also observed that weaker students were more at ease and willing to use the worksheets and extra material to derive an answer. Beside that fact that students approached the problem solving from different perspectives all students were successful at getting a correct answer. This illustrates that this approach to teaching mathematics caters for a diverse classroom and differentiation between students' individual abilities.