5^{th} Year HL Interpreting Quadratic Functions

For the lesson on: 7/12/16
At Maynooth Education Campus, Michelle Kelly's class
Teacher: Michelle Kelly
Lesson plan developed by: Michelle Kelly, Alice Mooney & Gráinne O' Rourke/ Gillian Russell

1. **Title of the Lesson: The problem with a water rocket.**

2. **Brief description of the lesson:**
Using students' prior knowledge of solving quadratics algebraically and geometrically to apply it to a real-life situation.

3. **Aims of the Lesson:**
I would like my students to connect and review the concepts that we have studied already. These relate to "managing information and thinking", "being creative" and "being literate" key skills at Junior Cycle.
I would like to foster my students to take ownership of their own learning. These relate to the managing myself key skills at Junior Cycle.
I would like to foster my students work with others in pairs or groups in the ethos of working with others and communicating key skills at Junior Cycle.
I would like my students to become more aware of the application of Maths in real life. These relate to being numerate key skills at Junior Cycle.
I would like to emphasise to students that a problem can have several equally valid solutions.

4. **Learning Outcomes:**
<u>1st level:</u>
Every student will be able to evaluate the mathematical problem and select a minimum of one suitable method to answer the question.
<u>2nd level:</u>
Most students will be able to evaluate the mathematical problem, select a minimum of one method to answer the question and solve it.
<u>3rd level:</u>
A few students will be able to evaluate the mathematical problem, select least two methods to answer the question, solve it and how the solution relates to the question asked.

5. **Background and Rationale**

Using different methods to interpreting knowledge of quadratic functions to solving real life problem:

Determine when $f(x) = 0$

- Maximum height
- Turning points
- Y-intercept

- X-intercept

Methods

- Substitution
- -b formula
- Square form
- Plotting/sketching the function.

The rationale for this activity is to further develop students' problem solving capacity by applying their knowledge of algebra and functions to this real-life problem

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6. Research

The resources that we used in creating this lesson are as follows:
We looked at several resources

- A range of Leaving Certificate Text books
- NCCA Syllabus
- Exam Papers
- Scheme of work

7. About the Unit and the Lesson

As the Leaving Certificate syllabus outlines (p 42):
At each syllabus level students, should be able to:

- Explore patterns and formulate conjectures
- Explain findings
- Justify conclusions
- Communicate mathematics verbally and translate it into mathematical form
- Apply their knowledge and skills to solve problems in familiar and unfamiliar contexts
- Analyse information presented verbally and translate it into mathematical form
- Devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions

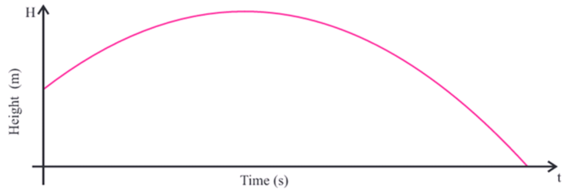
In the proposed lesson students, will draw on their knowledge of Algebra and Functions, previously studied. (Section 5.1 and Section 4.2) Through discussions with fellow students they will explore the use of different approaches to solve this real-life problem.

8. Flow of the Unit:

Lesson	Topic	# of lesson periods
1	• Introduction to Functions	2 x 1hr.
2	• Composite functions	1 x 1hr.
3	• Types of functions (Injective, surjective & bijective)	1 x 1hr.
4	• Inverse functions.	2 x 1hr.
5	• Plotting and sketching functions.	3 x 1hr. (research lesson)
6	• Transformation of functions	1 x 1hr.

9. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction (5 minutes) Today we will have a problem-solving class. Worksheet with the problem-solving question will be distributed and key words (e.g. rocket, model, launches, determines) will be identified.	This lesson is designed to challenge students to solve a real-life question using their prior knowledge of Algebra and Functions. Students are accustomed to participating in problem solving classes. However, to reduce the chance of limiting their approach, no prerequisite knowledge will be highlighted at the start of the class.
2. Posing and Completing the Task (25 - 30 minutes) On the worksheet, you are given a problem-solving question. Your task is to find as many possible solutions for each part of the question. A mathematics student builds a water rocket and launches it from the roof of the school. It is determined that the height above the ground from the moment it is launched can be modelled by the function: , which is sketched below:	- Ensure in preceding lessons that the students are comfortable with the iPad app. - iPad app will be used to record their thinking visually and orally. - Will draw attention to the fact the rocket was launched from the school roof. - Think, pair, share will be employed. - Teachers circulate the room assessing students' work to plan how to orchestrate



(a) Determine the height of the school roof.
(b) Determine the rockets maximum height above ground.
(c) Determine the time the rocket was in the air for, correct to the nearest second.

the presentation of students' work on the board and class discussion.

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3. Anticipated Student Responses

Q1 (a) Find the height of the school roof.
Option 1 Algebraic Method
The y-intercept of the graph represents its initial starting height! This is found when $t = 0$.

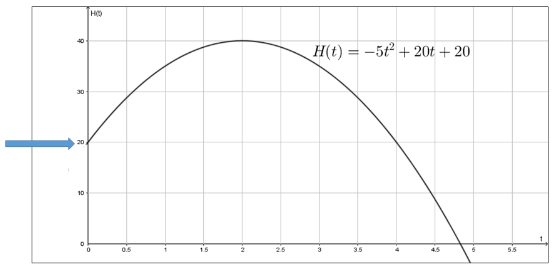
$H(t) = -5t^2 + 20t + 20$ $H(0) = -5(0)^2 + 20(0) + 20$ $H(0) = 20$	**Possible Errors:** • Finding when $H(t) = 0$ • Calculator errors • Not knowing where/ how to start

The height of the school building is $H(t) = 20$ metres

Option 2: Graphical method
Using the function operation on the calculator create a table and then graph the function using suitable scales.

Time t	Height $H(t) = -5t^2 + 20t + 20$	Height $H(t) = -5t^2 + 20t + 20$
0	$H(0) = -5(0)^2 + 20(0) + 20$	$H(0) = 20$
1	$H(1) = -5(1)^2 + 20(1) + 20$	$H(1) = 35$
2	$H(2) = -5(2)^2 + 20(2) + 20$	$H(2) = 40$
3	$H(3) = -5(3)^2 + 20(3) + 20$	$H(3) = 35$
4	$H(4) = -5(4)^2 + 20(4) + 20$	$H(4) = 20$
5	$H(5) = -5(5)^2 + 20(5) + 20$	$H(5) = -5$

The height of the school roof is found from the graph when $t = 0$.



The height of the school building is $H(t) = 20$ m

Option 3: By comparison of the general quadratic function

$$y = ax^2 + bx + c$$

$$H(t) = -5t^2 + 20t + 20$$

Therefore, when $t = 0$ by comparison $c = 20$.
So the height of the school is $H(t) = 20$ metres

Possible Errors:

Slide 1

- **Finding when H(t)=0**
- **Calculator errors**
- **Not knowing where/ how to start**

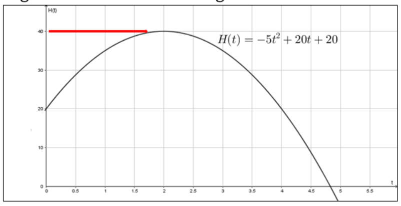
Slide 2/3

- Calculator errors
- Inputting the incorrect function into the calculator
- Graphing the function incorrectly
  - Reversing the co-ordinates
  - Plotting the coordinates incorrectly
  - Not getting a parabola
  - Not getting an "n" shaped function
  - Forgetting the minus sign in front of the $-5t^2$
  - Reversing the coordinates
  - Reversing the axes
  - Starting the graph of the function at (0, 0)
  - Incorrect scaling on either axis
- Reading off the incorrect value for the height

Q1(b) Determine the rocket's maximum height above the ground.

Option 1 Graphically

The y coordinate of the maximum turning point represents the maximum height of the rocket above ground.



The maximum height the rocket reaches is $H(2) = 40$ metres

Option 2 Complete the square form and find the turning point

Write the function in completed square form $f(x) = a(x-h)^2 + k$ and write down the y coordinate of the turning point (h, k).

$H(t) = -5t^2 + 20t + 20$

$H(t) = -5(t^2 - 4t - 4)$

$H(t) = -5[(t-2)^2 - 4 - 4]$

$H(t) = -5[(t-2)^2 - 8]$

$H(t) = -5(t-2)^2 + 40$

From the completed square form, the maximum turning point is the coordinate (2, 40). Therefore, the maximum height the rocket reaches is $H(2) = 40$ metres.

Option 3 Using trial and error

Using the given function: $H(t) = -5t^2 + 20t + 20$ substitute in different t values, until the maximum turning point is found.

Time t	Height $H(t) = -5t^2 + 20t + 20$	Height $H(t) = -5t^2 + 20t + 20$
0	$H(0) = -5(0)^2 + 20(0) + 20$	$H(0) = 20$
1	$H(1) = -5(1)^2 + 20(1) + 20$	$H(1) = 35$
2	$H(2) = -5(2)^2 + 20(2) + 20$	$H(2) = 40$
3	$H(3) = -5(3)^2 + 20(3) + 20$	$H(3) = 35$
4	$H(4) = -5(4)^2 + 20(4) + 20$	$H(4) = 20$
5	$H(5) = -5(5)^2 + 20(5) + 20$	$H(5) = -5$

Option 4 Algebraically using the fact that the maximum height is found when $t = \frac{-b}{2a}$.

$t = \frac{-b}{2a} = \frac{-20}{2(-5)} = 2 \Rightarrow$ this is the time when the rocket will reach its maximum height

Substitute $t = 2$ into the function $H(t) = -5t^2 + 20t + 20$

$H(2) = -5(2)^2 + 20(2) + 20$

$H(2) = 40m$

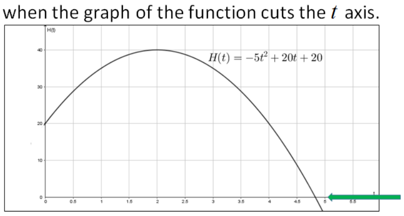
Therefore, the maximum height the rocket reaches is 40 metres.

Q1(c) Determine the time the rocket was in the air for correct to the nearest second.

Option 1 Graphically

From the graph of the function find the total time taken for the rocket to reach he ground.

Note: this is when the graph of the function cuts the t axis.



$t \approx 4.8$ seconds, to the nearest second $t = 5$ seconds

Slide 4

- Reading off the wrong value

Slide 5

- Not getting a parabola
- Not getting an "n" shaped function
- Forgetting the minus sign in front of the -5t^2
- Reversing the coordinates
- Reversing the axes
- Starting the graph of the function at (0, 0)
- Incorrect scaling on either axis
- Reading off the wrong value

Slide 6

- Any algebraic error when finding the completed square form
- Not including errors
- Not writing down the maximum height from the turning point coordinates

Slide 7

- Not writing down the maximum height from the turning point coordinates

Slide 8

- Incorrect substitution of a and b values into the formula
- Calculation errors

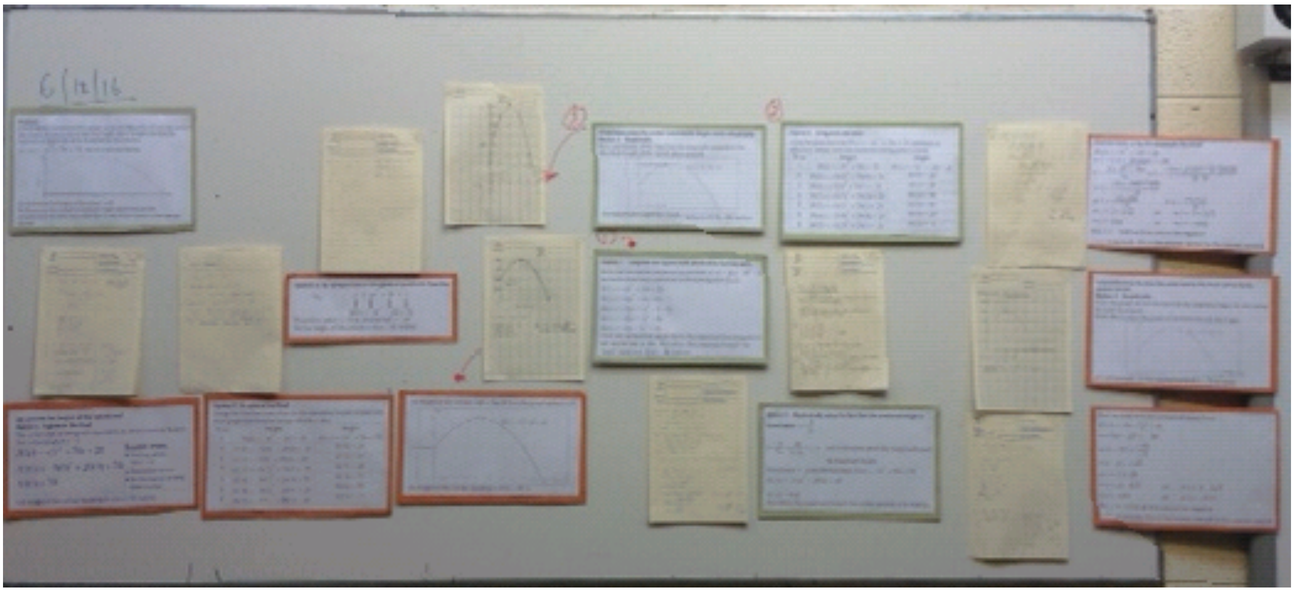
Option 2 Algebraically Find the roots of the function using: • the completed square form, or $$f(x) = a(x-h)^2 + k$$ • the quadratic formula. $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ Hence, identify the positive value for t, as this is the time taken for the rocket to reach the ground. Find the roots from the completed square form: $H(t) = -5(t-2)^2 + 40$ $\Rightarrow -5(t-2)^2 = -40$ $\Rightarrow (t-2)^2 = \frac{-40}{-5}$ $\Rightarrow t - 2 = \pm\sqrt{8}$ $\Rightarrow t = 2 \pm \sqrt{8}$ $\Rightarrow t = 2 + 2\sqrt{2}$ or $\Rightarrow t = 2 - 2\sqrt{2}$ $\Rightarrow t = 4.83$ or $\Rightarrow t = -0.83$ But, $t \neq -0.83$ as time cannot be negative $\therefore t = 5$ seconds, this is the answer correct to the nearest second. Find the roots using the quadratic formula: $H(t) = -5t^2 + 20t + 20$ $a = -5, b = 20 \; and \; c = 20$ $t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow t = \frac{-20 \pm \sqrt{(20)^2 - 4(-5)(20)}}{2(-5)}$ $\Rightarrow t = \frac{-20 \pm \sqrt{400 + 400}}{-10}$ $\Rightarrow t = \frac{-20 \pm \sqrt{800}}{-10}$ $\Rightarrow t = \frac{20 \pm 20\sqrt{2}}{10}$ $\Rightarrow t = 2 + 2\sqrt{2}$ or $\Rightarrow t = 2 - 2\sqrt{2}$ $\Rightarrow t = 4.83$ or $\Rightarrow t = -0.83$ But, $t \neq -0.83$ as time cannot be negative $\therefore t = 5$ seconds, this is the answer correct to the nearest second.	**Slide 9** • Not reading off the value accurately from the graph. • Reading the incorrect value form the graph. • Reversing the coordinates **Slide 10/ 11/ 12** • Not identifying that the values of the roots are required. • Writing down the incorrect quadratic formulae • Attempting to factorise the quadratic and getting the wrong factors • Not identifying that the positive value of t is only required, as the time taken cannot be negative • Writing down both values of t and finding the time between the 2 points • Finding the maximum height
4. Comparing and Discussing (20minutes) The mostly likely approach for each part is given above (see options per part).	Teacher picks a student who tried the most common method in the class. When the student has presented his/her reason, the teacher places a poster on the board.
5. Summing up (5minutes) Using post-it notes surveys students will write down one thing that they thought they did well today in class and one thing they need to improve on.	

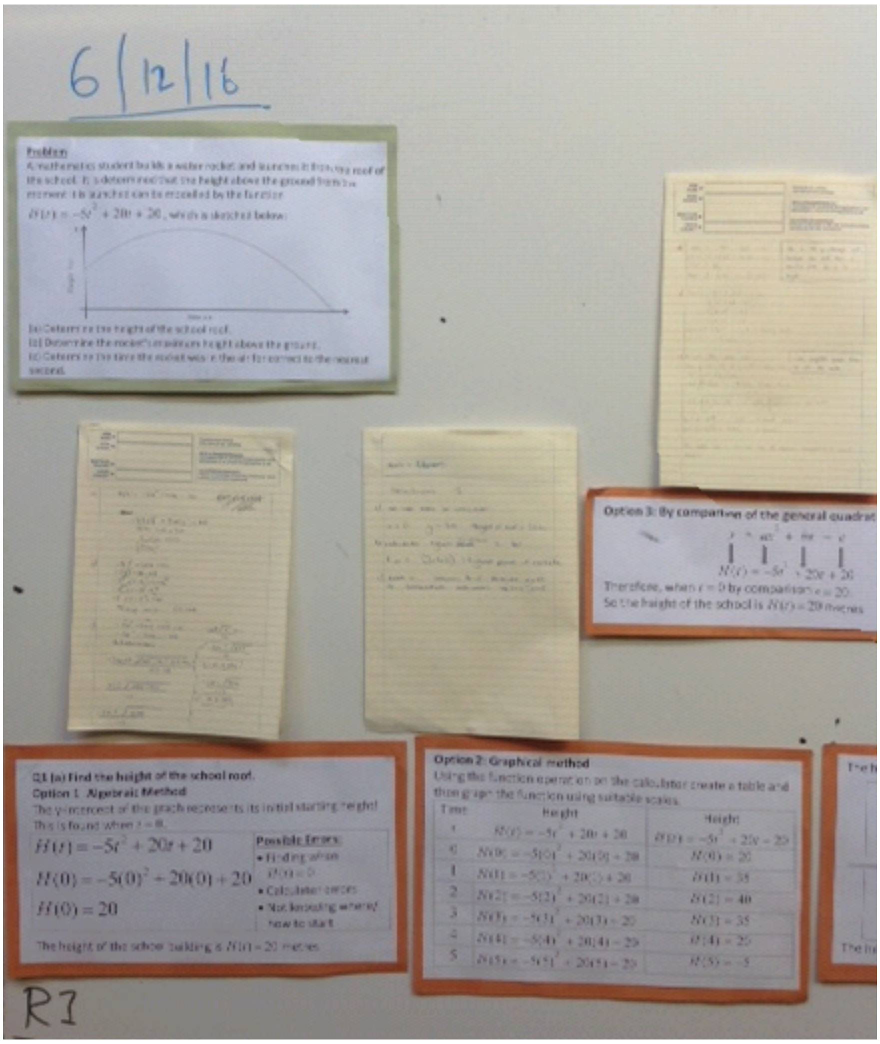
10. Evaluation

- A seating plan provided by the teacher.
- Three observers will circulate around the room and take note of students' approaches and thinking for each part of the question.
- Types of student thinking will focus on:

Introduction/Posing the task	Was the wording of the questions clear? Do students understand the key words in the questions? Can students recall their prior knowledge on Algebra and Functions?
Individual Group Work	Are prompts required? What strategies do they employ? Are they able to determine the minimum information needed? How long do students spend on the task? What kind of questions do students ask? Do they persist with the task?
Discussion	Are students attentive to what is happening on the board? Are classifications needed to presenters' board work? Did the discussion promote student learning?

11. Board Plan

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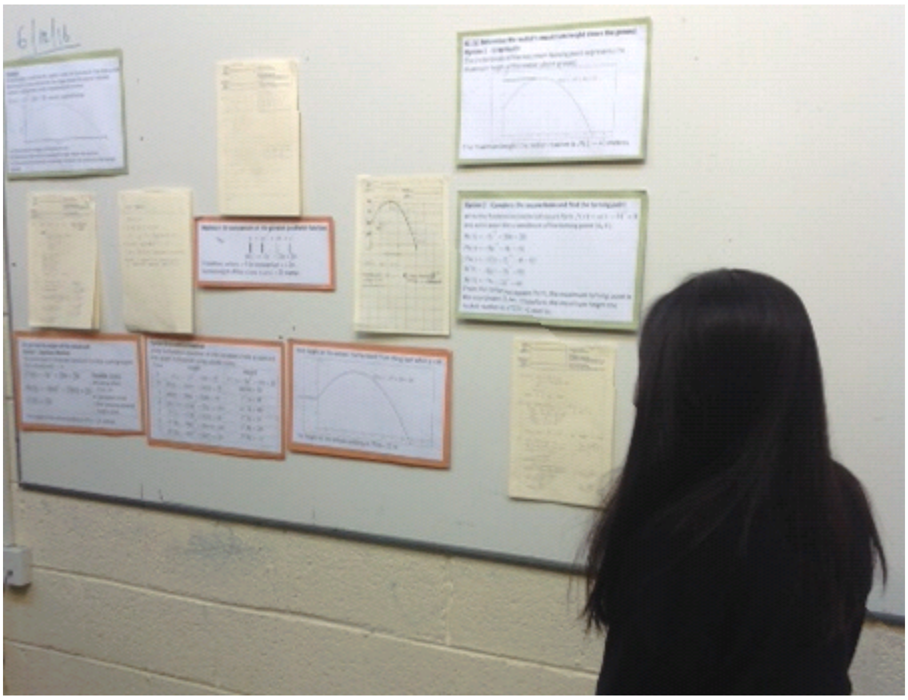
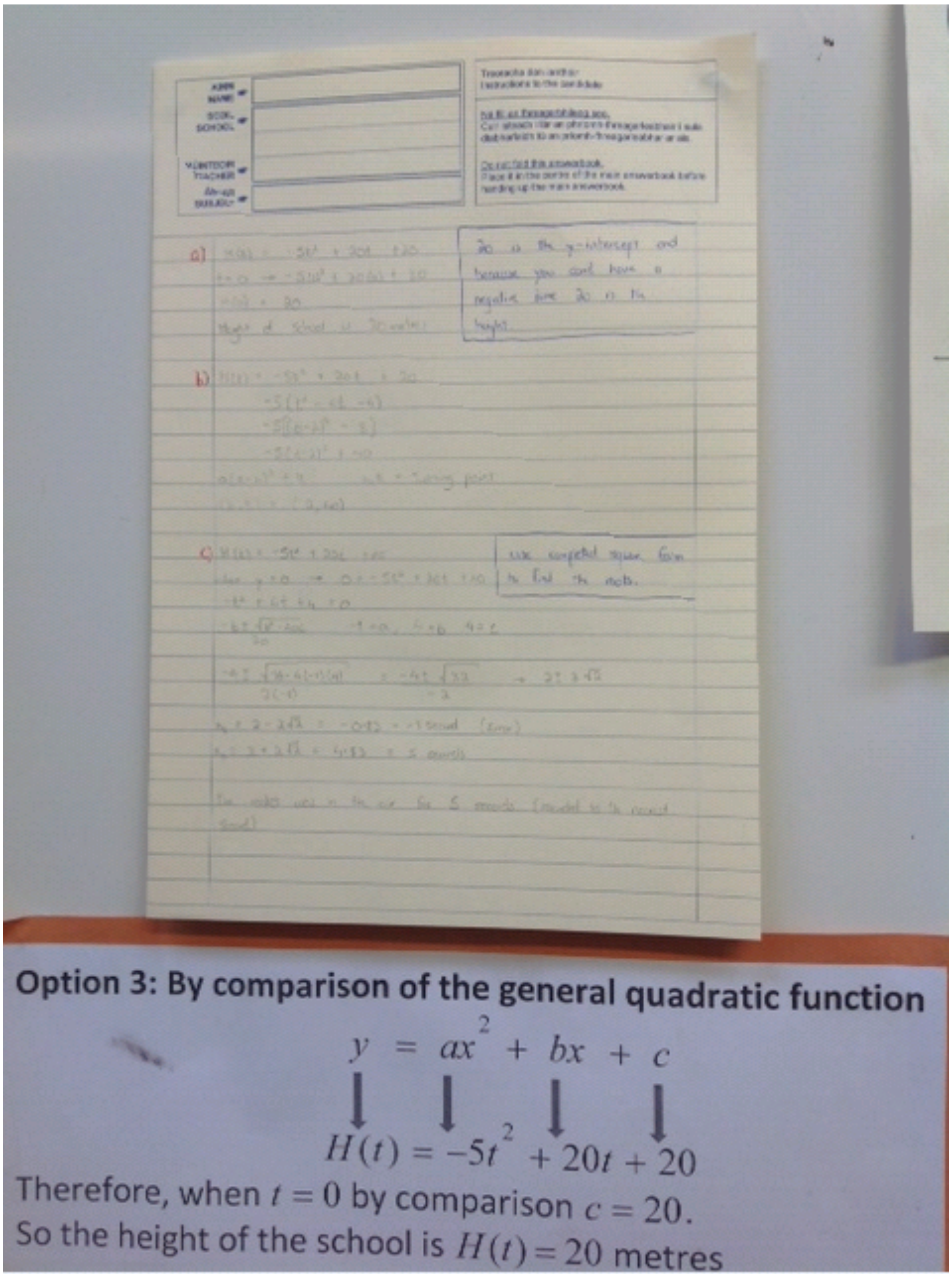
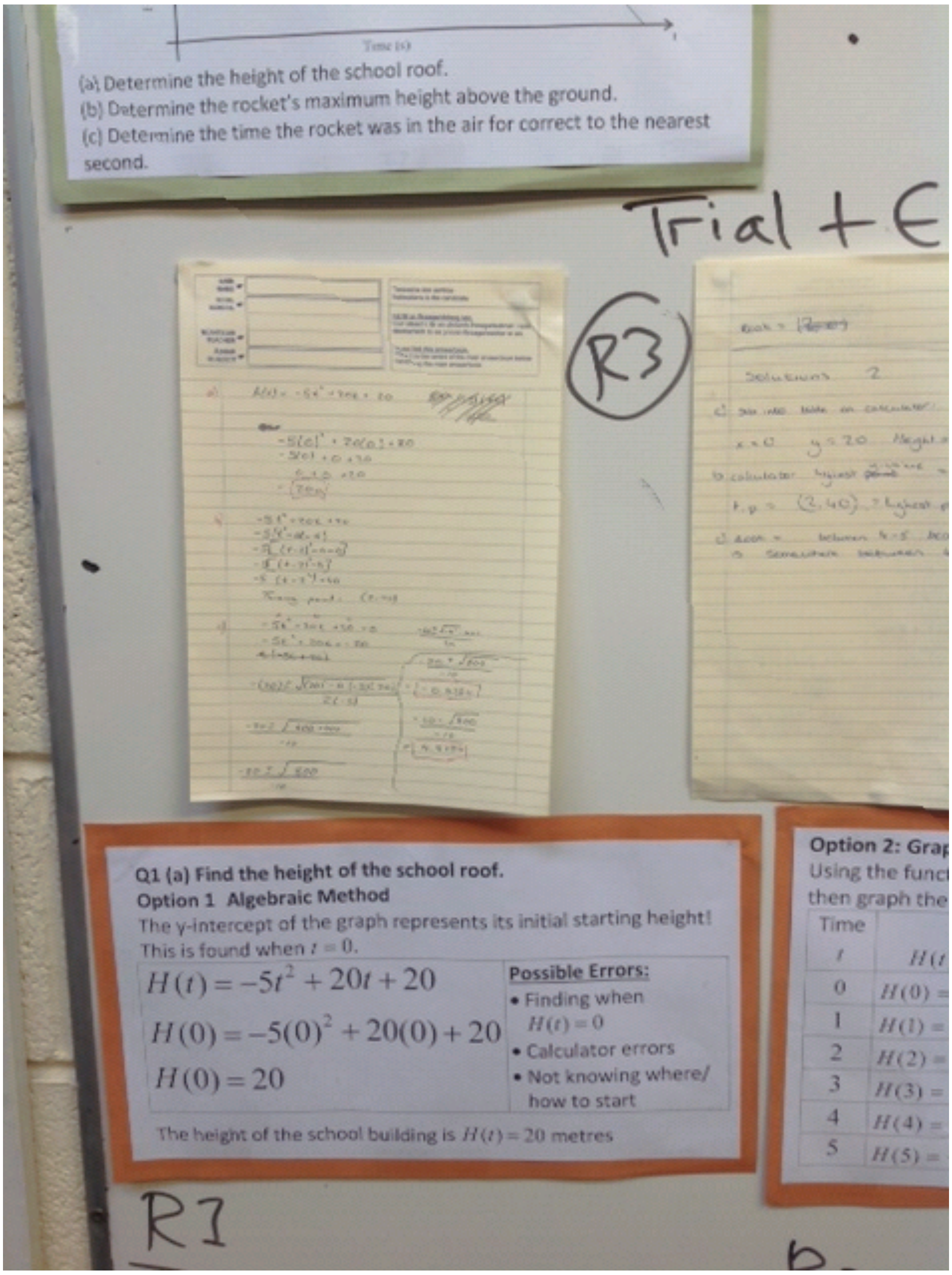
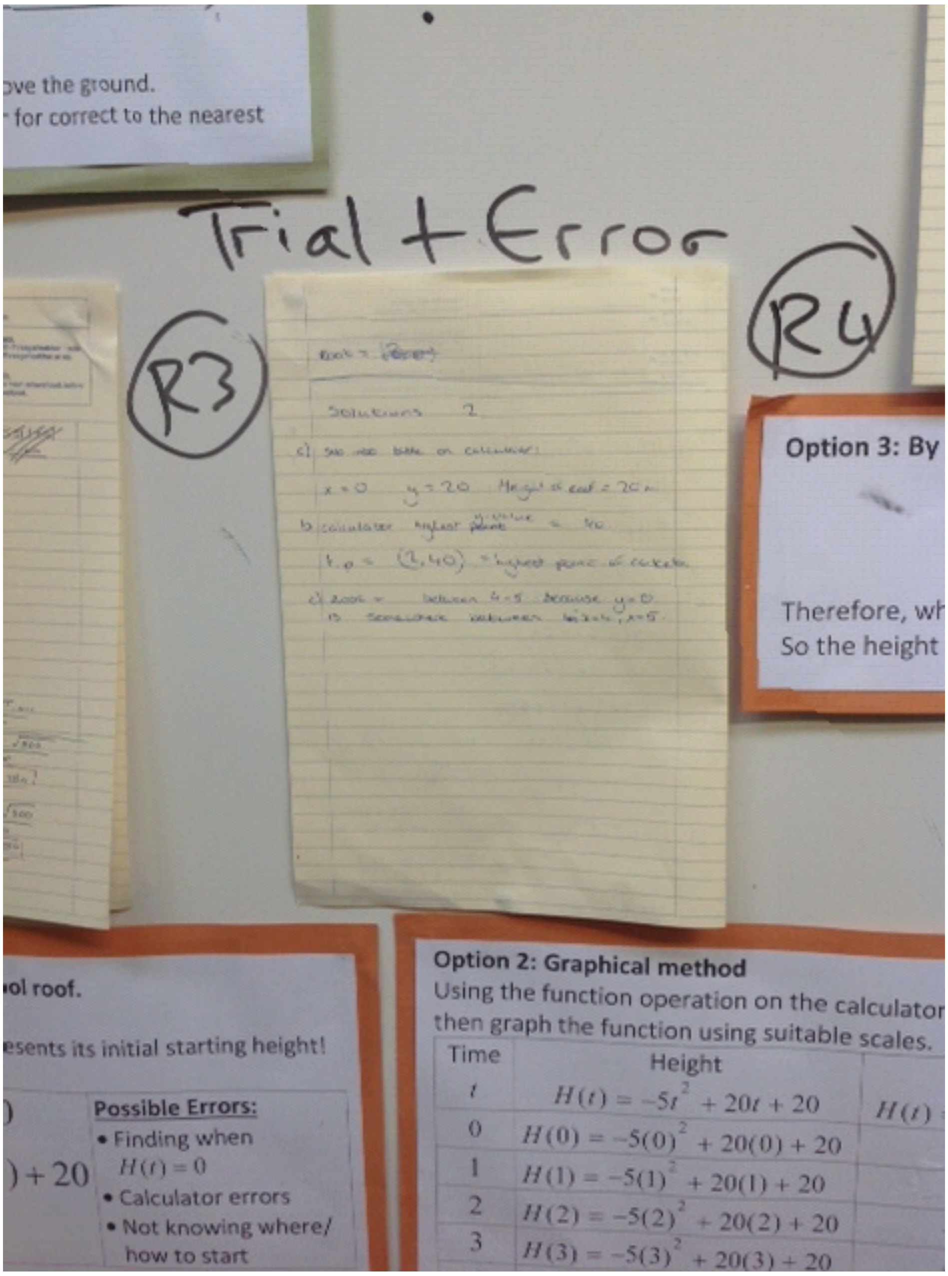
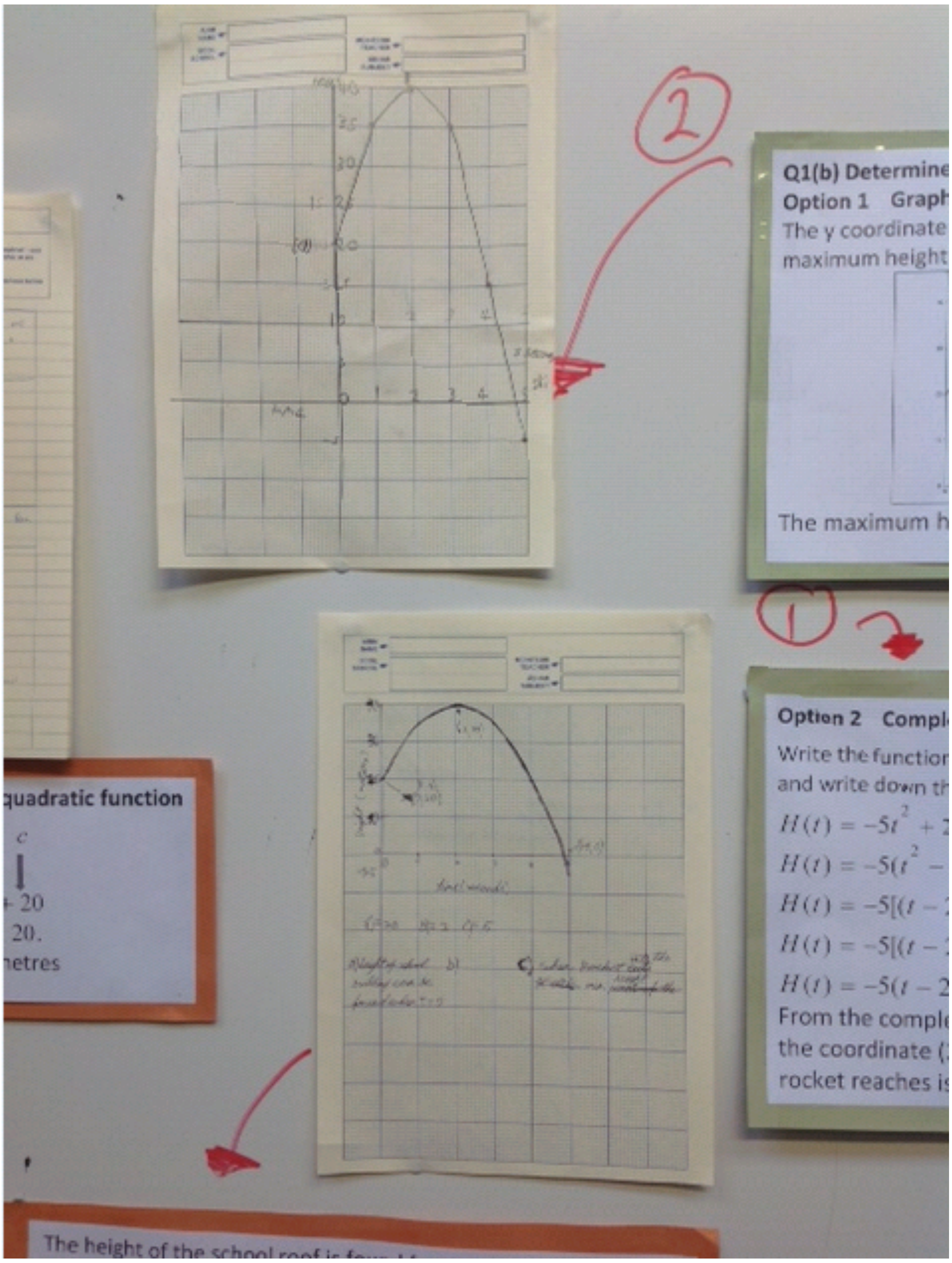
Problem
Option 3: By comparison of the general quadratic
Q1 (a) Find the height of the school roof.
Option 1 Algebraic Method
H(t) = −5t² + 20t + 20
H(0) = −5(0)² + 20(0) + 20
H(0) = 20
Possible Errors:
Option 2: Graphical method
R1

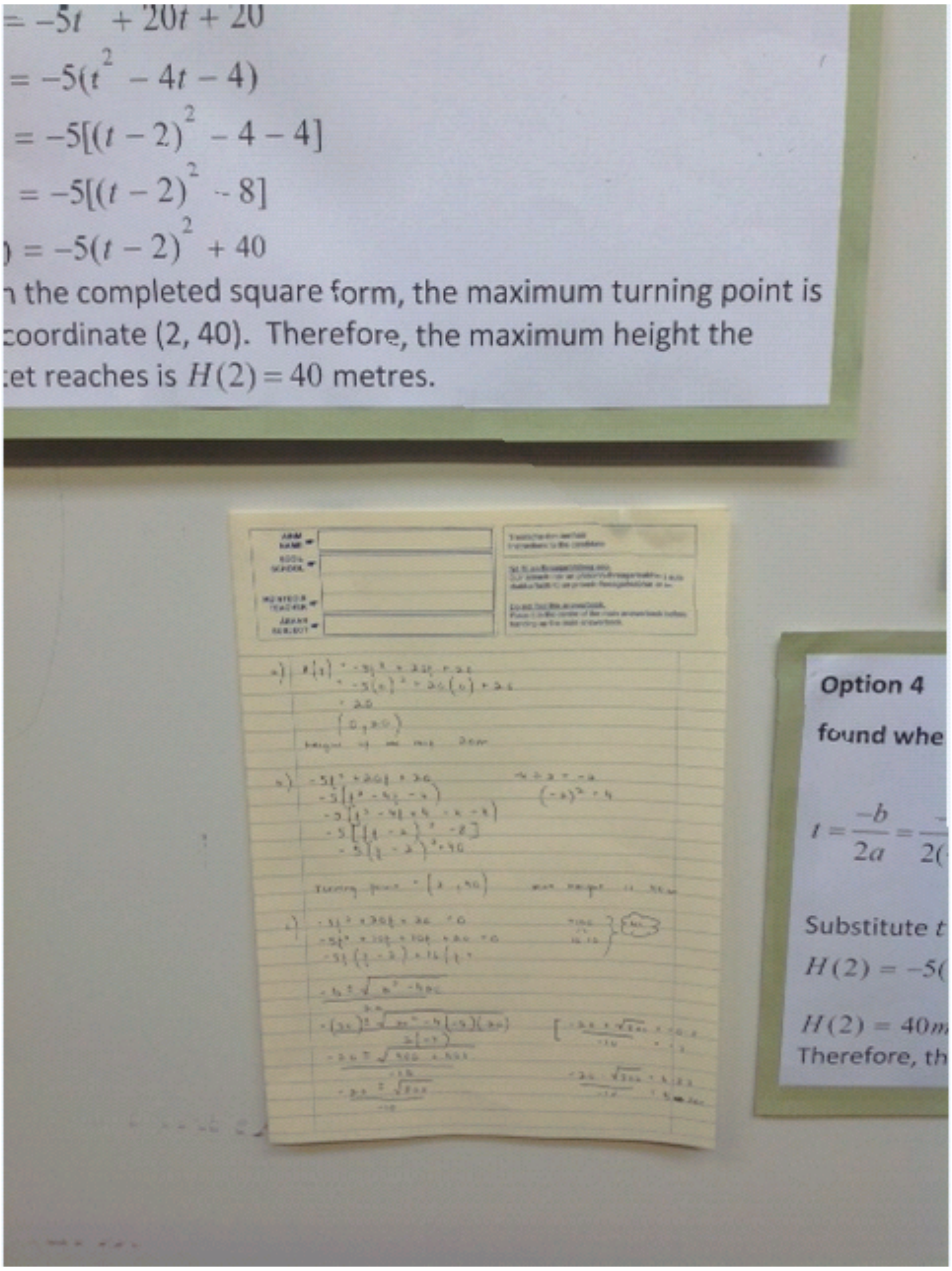


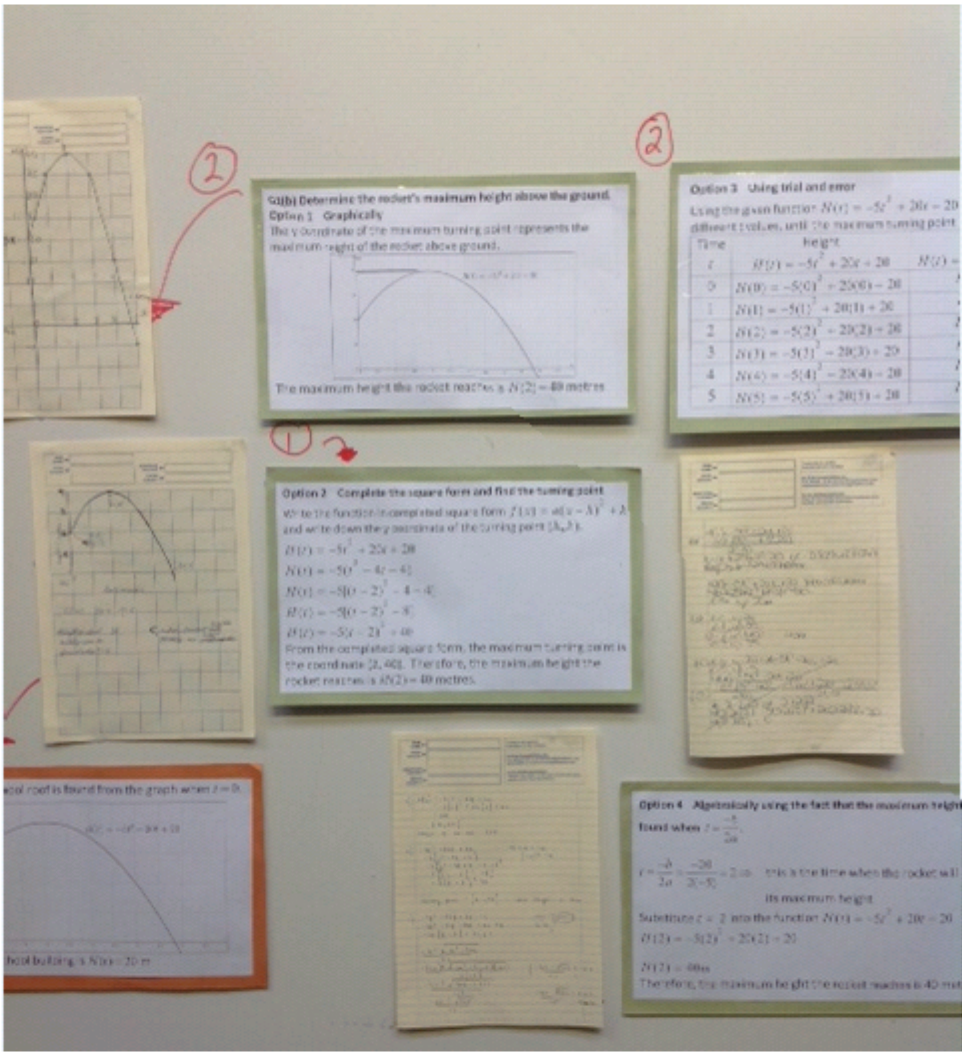

Option 3: By comparison of the general quadratic function
$y = ax^2 + bx + c$
$H(t) = -5t^2 + 20t + 20$
Therefore, when $t = 0$ by comparison $c = 20$.
So the height of the school is $H(t) = 20$ metres

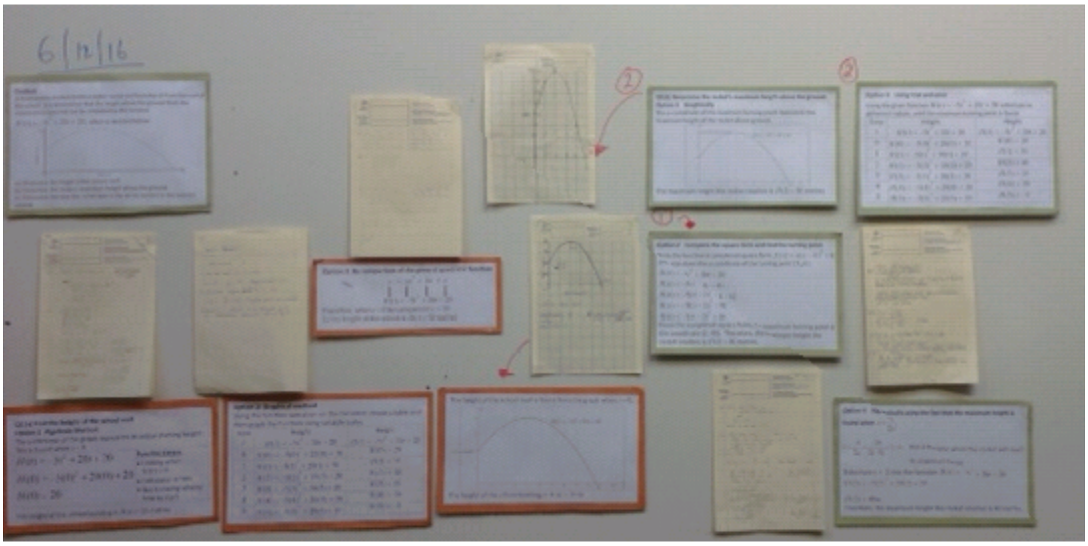

Time (s)
(a) Determine the height of the school roof.
(b) Determine the rocket's maximum height above the ground.
(c) Determine the time the rocket was in the air for correct to the nearest second.
Trial + E
R3
Q1 (a) Find the height of the school roof.
Option 1 Algebraic Method
The y-intercept of the graph represents its initial starting height!
This is found when $t = 0$.
$H(t) = -5t^2 + 20t + 20$
$H(0) = -5(0)^2 + 20(0) + 20$
$H(0) = 20$
Possible Errors:
• Finding when $H(t) = 0$
• Calculator errors
• Not knowing where/ how to start
The height of the school building is $H(t) = 20$ metres
Option 2: Grap
Using the func
then graph the
Time
t
$H(t$
0 $H(0) =$
1 $H(1) =$
2 $H(2) =$
3 $H(3) =$
4 $H(4) =$
5 $H(5) =$
R1


ove the ground.
for correct to the nearest
Trial + Error
R3
R4
Option 3: By
Therefore, wh
So the height
ol roof.
esents its initial starting height!
Possible Errors:
• Finding when
H(t) = 0
• Calculator errors
• Not knowing where/
how to start
Option 2: Graphical method
Using the function operation on the calculator
then graph the function using suitable scales.
Time
Height
t
H(t) = −5t² + 20t + 20
0
H(0) = −5(0)² + 20(0) + 20
1
H(1) = −5(1)² + 20(1) + 20
2
H(2) = −5(2)² + 20(2) + 20
3
H(3) = −5(3)² + 20(3) + 20


Q1(b) Determine
Option 1 Graph
The y coordinate
maximum height
The maximum h
Option 2 Compl
Write the functior
and write down th
From the comple
the coordinate (
rocket reaches is
quadratic function
20.
netres
The height of the school roof is


= −5(t² − 4t − 4)
= −5[(t − 2)² − 4 − 4]
= −5[(t − 2)² − 8]
= −5(t − 2)² + 40
the completed square form, the maximum turning point is
coordinate (2, 40). Therefore, the maximum height the
reaches is H(2) = 40 metres.
Option 4


Q3(b) Determine the rocket's maximum height above the ground.
Option 1 Graphically
The y coordinate of the maximum turning point represents the maximum height of the rocket above ground.
Option 3 Using trial and error
Option 2 Complete the square form and find the turning point
From the completed square form, the maximum turning point is the coordinate (2, 40). Therefore, the maximum height the rocket reaches is H(2) = 40 metres.
Option 4 Algebraically using the fact that the maximum height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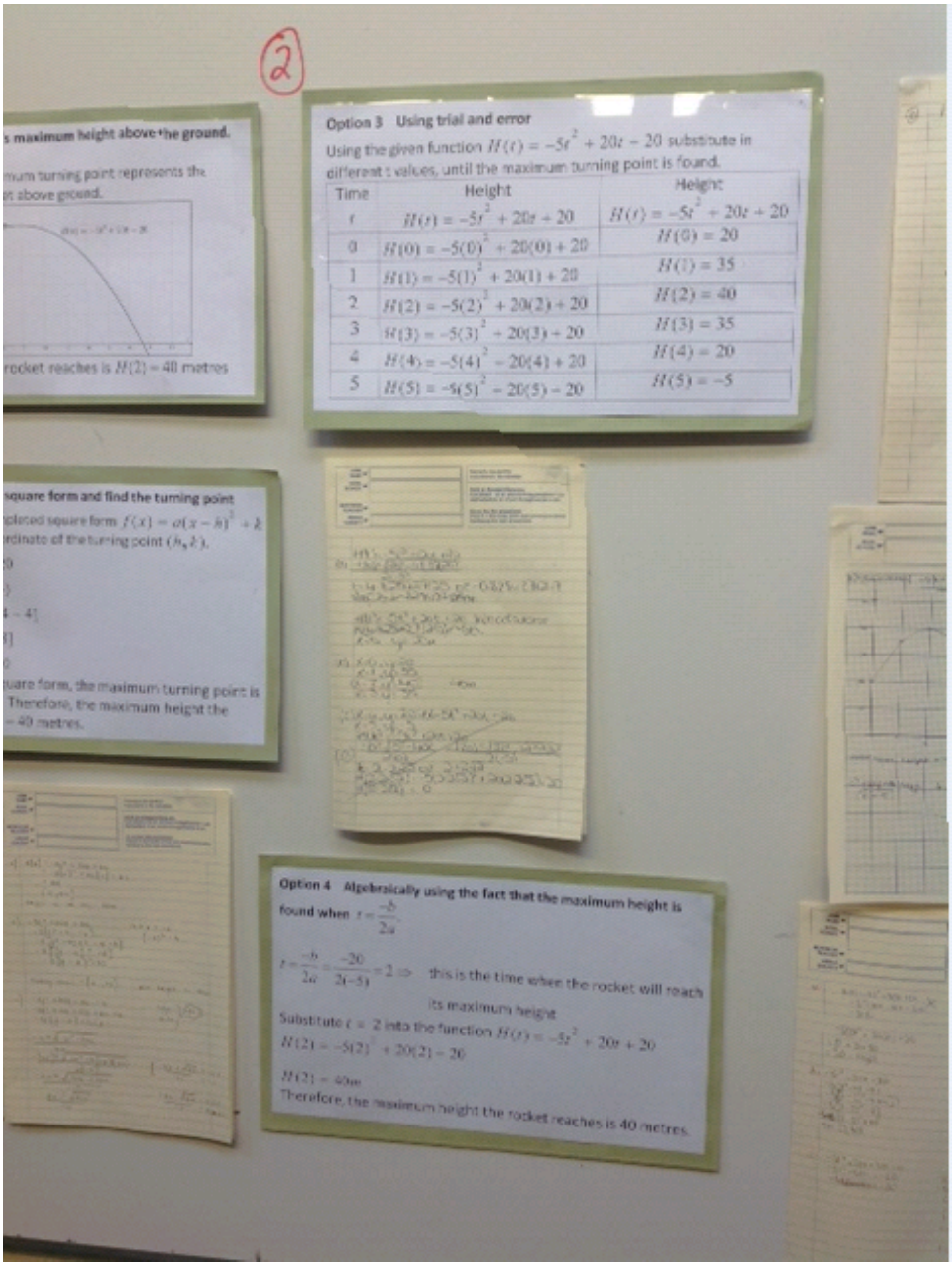
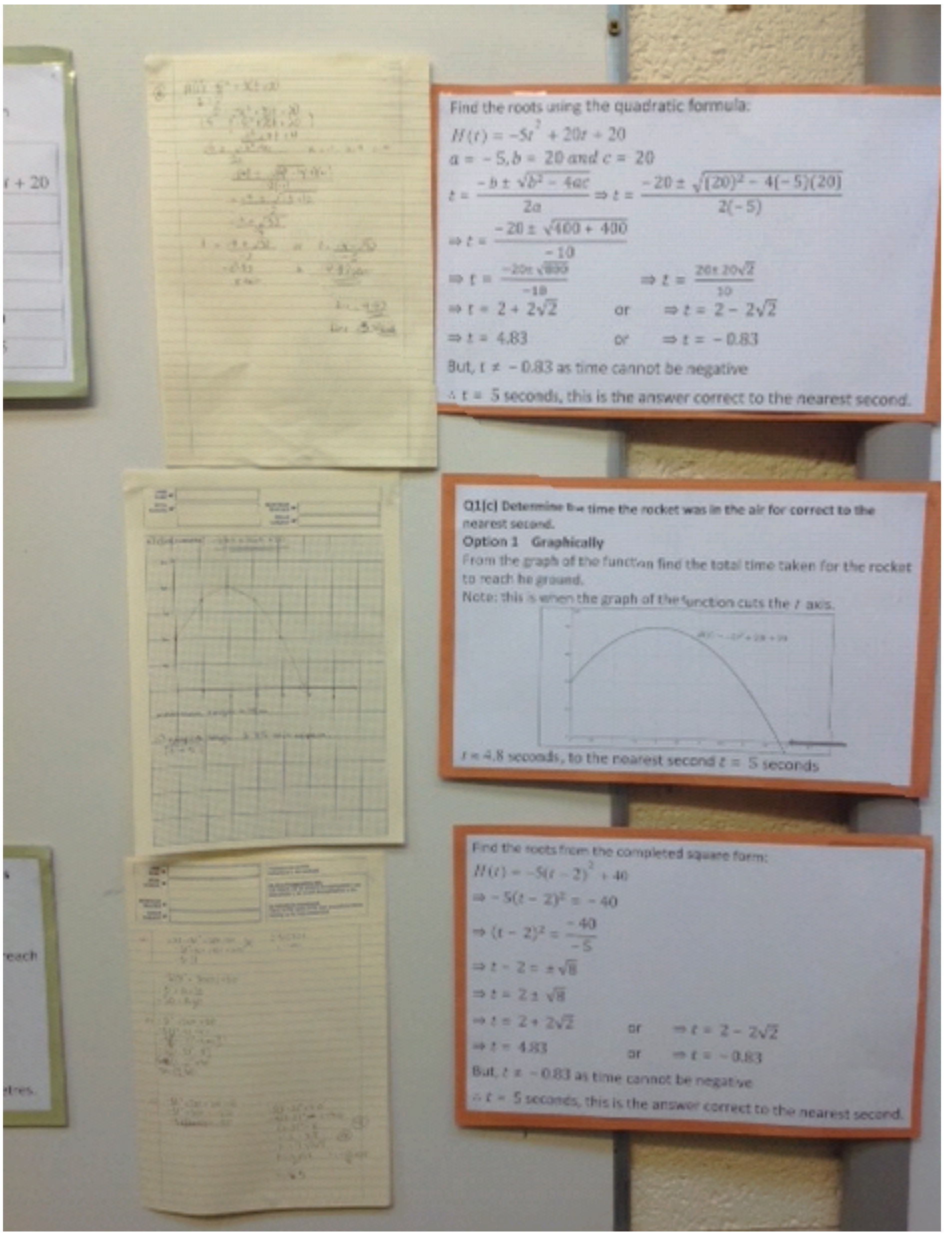
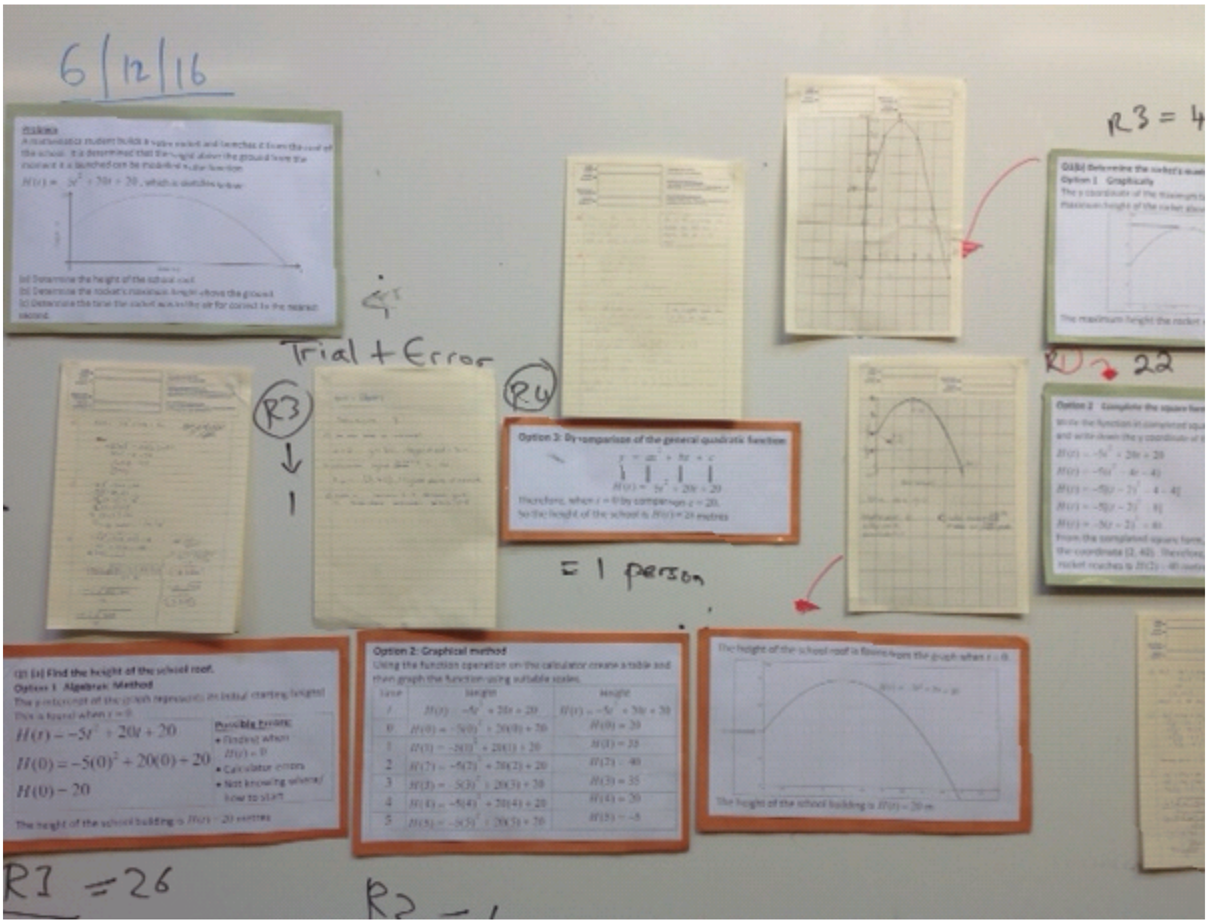


2
Option 3 Using trial and error
Using the given function $H(t) = -5t^2 + 20t + 20$ substitute in different t values, until the maximum turning point is found.
Time t | Height $H(t) = -5t^2 + 20t + 20$ | Height $H(t) = -5t^2 + 20t + 20$
0 | $H(0) = -5(0)^2 + 20(0) + 20$ | $H(0) = 20$
1 | $H(1) = -5(1)^2 + 20(1) + 20$ | $H(1) = 35$
2 | $H(2) = -5(2)^2 + 20(2) + 20$ | $H(2) = 40$
3 | $H(3) = -5(3)^2 + 20(3) + 20$ | $H(3) = 35$
4 | $H(4) = -5(4)^2 + 20(4) + 20$ | $H(4) = 20$
5 | $H(5) = -5(5)^2 + 20(5) + 20$ | $H(5) = -5$
Option 4 Algebraically using the fact that the maximum height is found when $t = \frac{-b}{2a}$
$t = \frac{-b}{2a} = \frac{-20}{2(-5)} = 2 \Rightarrow$ this is the time when the rocket will reach its maximum height
Substitute $t = 2$ into the function $H(t) = -5t^2 + 20t + 20$
$H(2) = -5(2)^2 + 20(2) + 20$
$H(2) = 40m$
Therefore, the maximum height the rocket reaches is 40 metres.
s maximum height above the ground.
rocket reaches is $H(2) = 40$ metres
square form and find the turning point
uare form, the maximum turning point is
Therefore, the maximum height the


Find the roots using the quadratic formula:
H(t) = −5t² + 20t + 20
a = −5, b = 20 and c = 20
⇒ t = 2 + 2√2 or ⇒ t = 2 − 2√2
⇒ t = 4.83 or ⇒ t = −0.83
But, t ≠ −0.83 as time cannot be negative
∴ t = 5 seconds, this is the answer correct to the nearest second.
Option 1 Graphically
From the graph of the function find the total time taken for the rocket to reach he ground.
Note: this is when the graph of the function cuts the t axis.
t ≈ 4.8 seconds, to the nearest second t = 5 seconds
Find the roots from the completed square form:
H(t) = −5(t − 2)² + 40
⇒ −5(t − 2)² = −40
⇒ t − 2 = ±√8
⇒ t = 2 ± √8
⇒ t = 2 + 2√2 or ⇒ t = 2 − 2√2
⇒ t = 4.83 or ⇒ t = −0.83
But, t ≠ −0.83 as time cannot be negative
∴ t = 5 seconds, this is the answer correct to the nearest second.


Trial + Error
R3
R4
= 1 person
Q3 (a) Find the height of the school roof.
Option 1 Algebraic Method
$H(t) = -5t^2 + 20t + 20$
$H(0) = -5(0)^2 + 20(0) + 20$
$H(0) = 20$
R2 = 1
R1 = 26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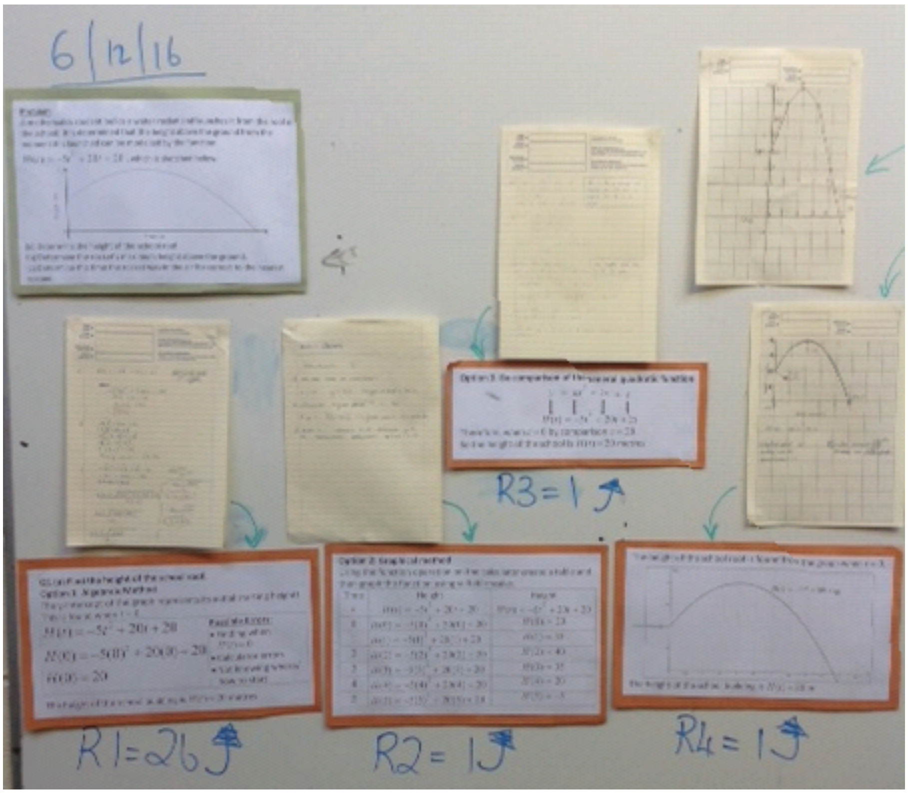
Trial + Error
R3
R4
= 1 person
R3 = 4
R1 = 22
Option 3: By comparison of the general quadratic function
Option 2: Graphical method
Q3 (a) Find the height of the school roof.
Option 1 Algebraic Method
$H(t) = -5t^2 + 20t + 20$
$H(0) = -5(0)^2 + 20(0) + 20$
$H(0) = 20$
R1 = 26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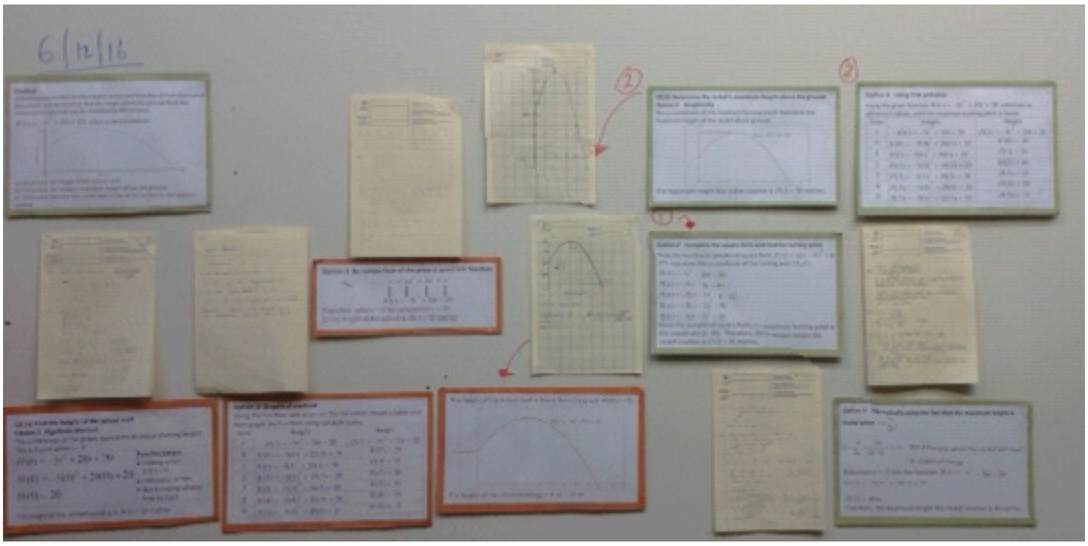
R3=1
R1=26
R2=1
R4=1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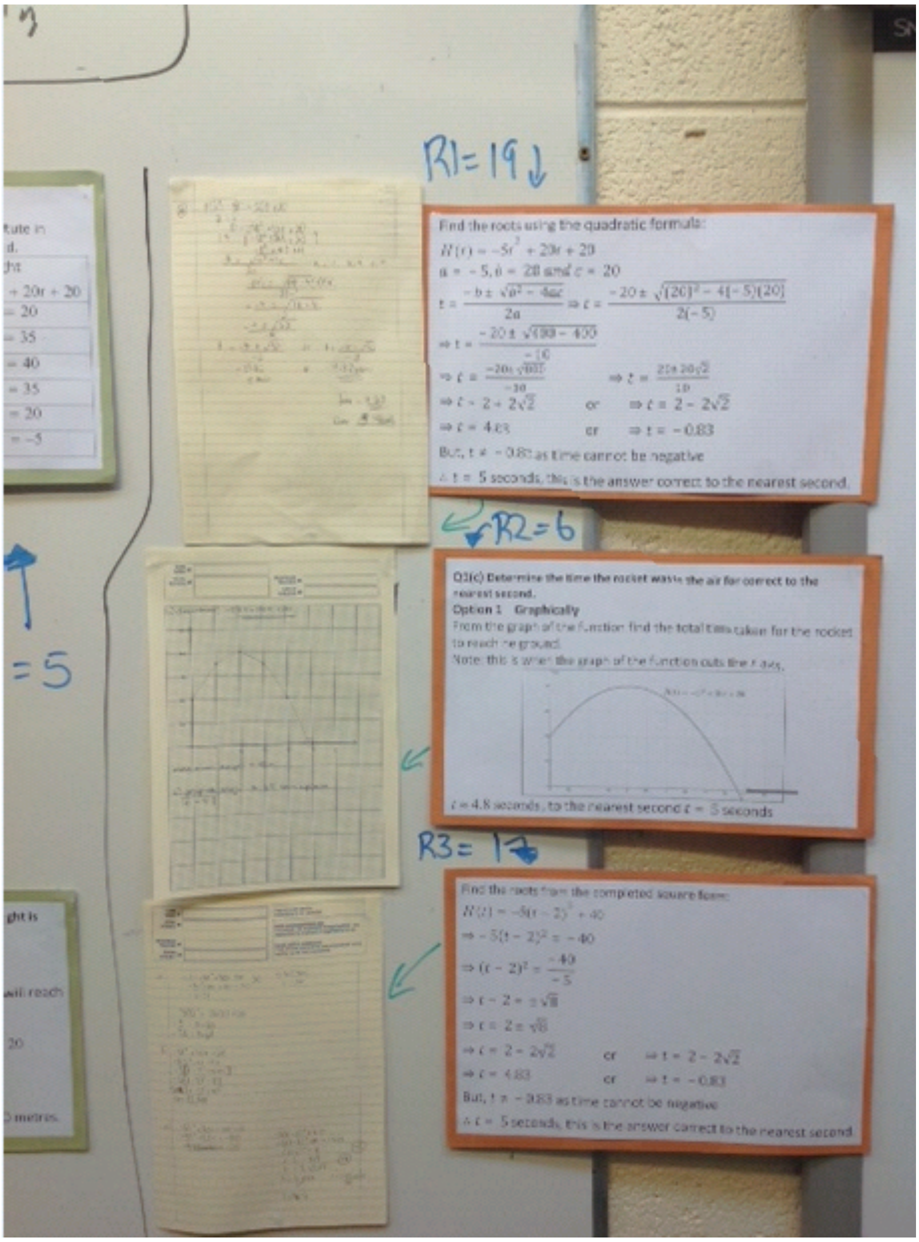
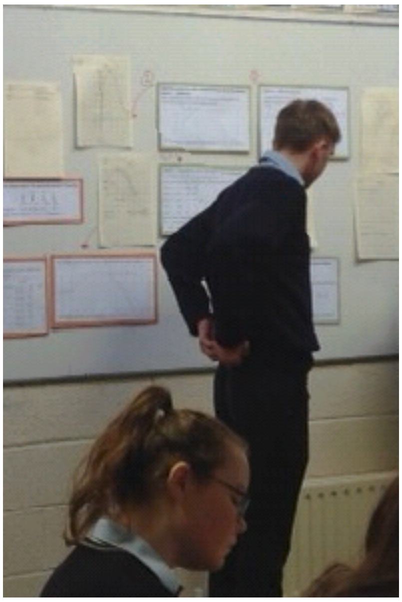
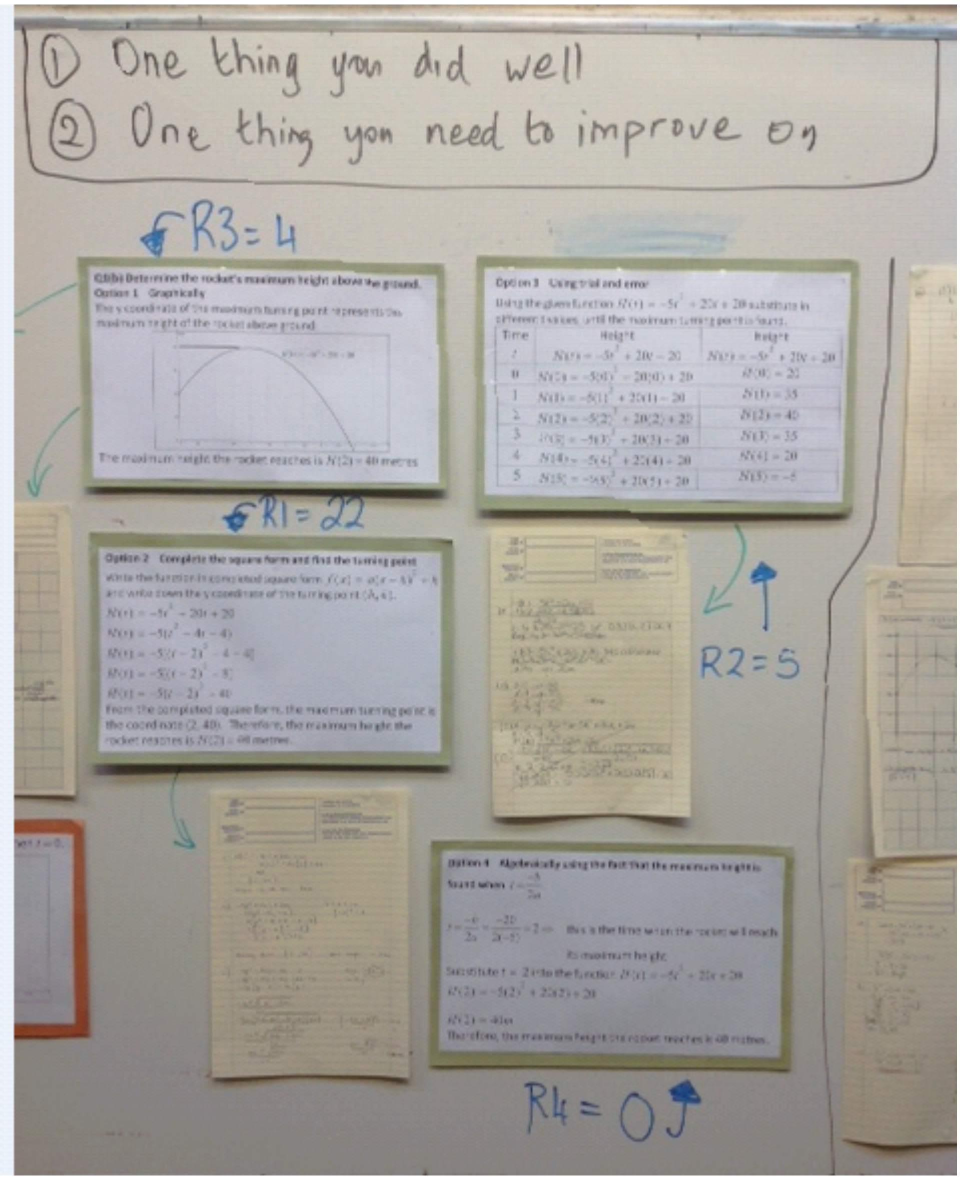
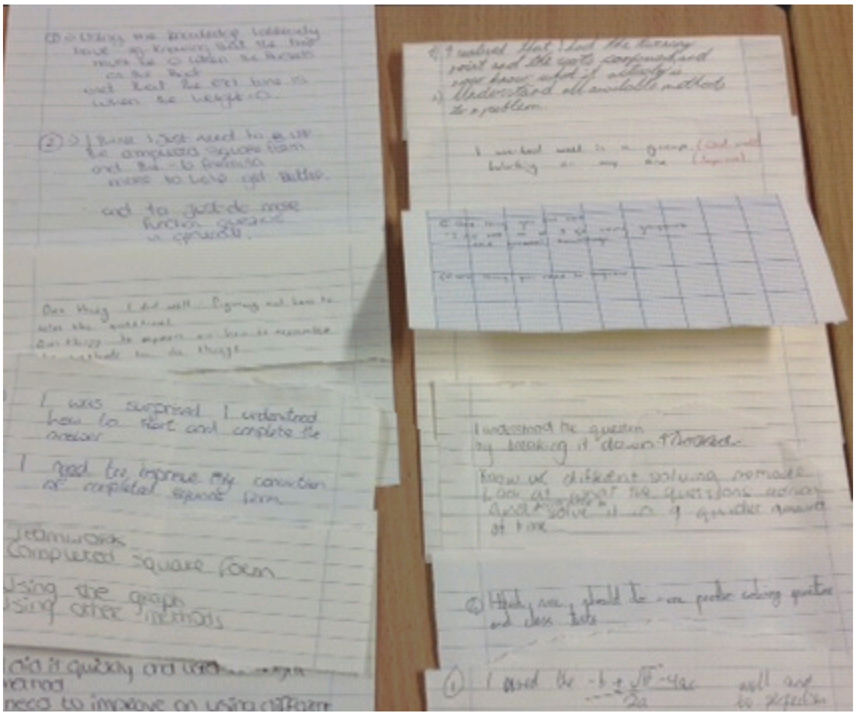

R1=19
Find the roots using the quadratic formula:
But, t ≠ −0.83 as time cannot be negative
∴ t = 5 seconds, this is the answer correct to the nearest second.
R2=6
Q1(c) Determine the time the rocket was in the air for correct to the nearest second.
Option 1 Graphically
From the graph of the function find the total time taken for the rocket to reach the ground.
t = 4.8 seconds, to the nearest second t = 5 seconds
R3=
Find the roots from the completed square form:
But, t ≠ −0.83 as time cannot be negative
∴ t = 5 seconds, this is the answer correct to the nearest second.
=5


Modelled
Shown or demonstrated by
Velocity



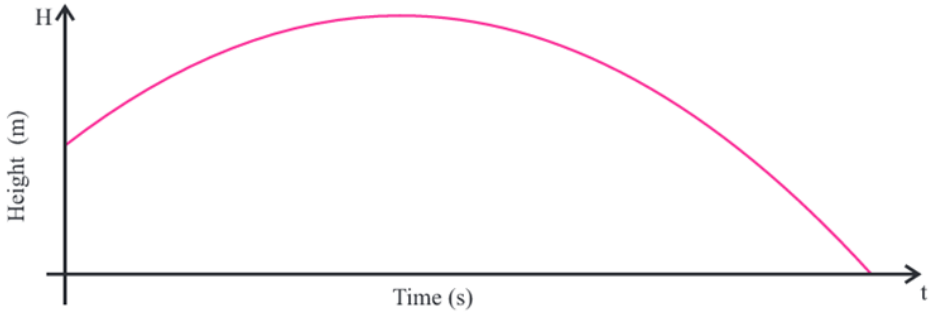

① One thing you did well
② One thing you need to improve on
R3 = 4
R1 = 22
R2 = 5
R4 = 0


I was surprised I understood how to start and complete the answer
I need to improve my conversion of completed square form
Teamwork
Completed square form
Using the graph
Using other methods
I understood the question by breaking it down
Know all different solving methods

Problem

A mathematics student builds a water rocket and launches it from the roof of the school. It is determined that the height above the ground from the moment it is launched can be modelled by the function

$H(t) = -5t^2 + 20t + 20$, which is sketched below:



(a) Determine the height of the school roof.

(b) Determine the rocket's maximum height above the ground.

(c) Determine the time the rocket was in the air for correct to the nearest second.

Q1 (a) Find the height of the school roof.

Option 1 Algebraic Method

The y-intercept of the graph represents its initial starting height! This is found when $t = 0$.

$H(t) = -5t^2 + 20t + 20$ $H(0) = -5(0)^2 + 20(0) + 20$ $H(0) = 20$	**Possible Errors:** • Finding when $H(t) = 0$ • Calculator errors • Not knowing where/ how to start

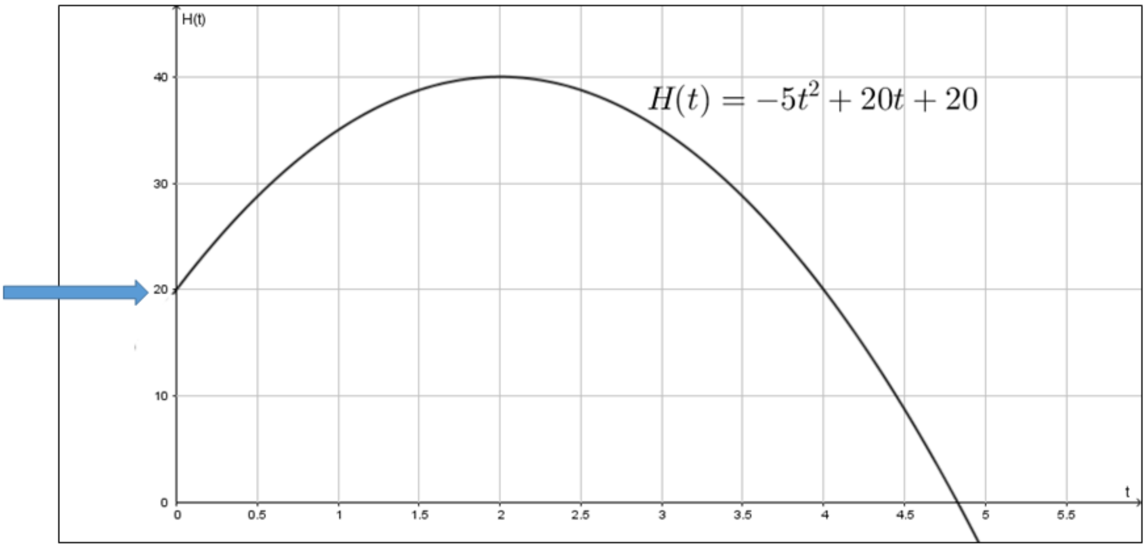
The height of the school building is $H(t) = 20$ metres

Option 2: Graphical method

Using the function operation on the calculator create a table and then graph the function using suitable scales.

Time t	Height $H(t) = -5t^2 + 20t + 20$	Height $H(t) = -5t^2 + 20t + 20$
0	$H(0) = -5(0)^2 + 20(0) + 20$	$H(0) = 20$
1	$H(1) = -5(1)^2 + 20(1) + 20$	$H(1) = 35$
2	$H(2) = -5(2)^2 + 20(2) + 20$	$H(2) = 40$
3	$H(3) = -5(3)^2 + 20(3) + 20$	$H(3) = 35$
4	$H(4) = -5(4)^2 + 20(4) + 20$	$H(4) = 20$
5	$H(5) = -5(5)^2 + 20(5) + 20$	$H(5) = -5$

The height of the school roof is found from the graph when $t = 0$.



The height of the school building is $H(t) = 20$ m

Option 3: By comparison of the general quadratic function

$$y = ax^2 + bx + c$$

$$H(t) = -5t^2 + 20t + 20$$

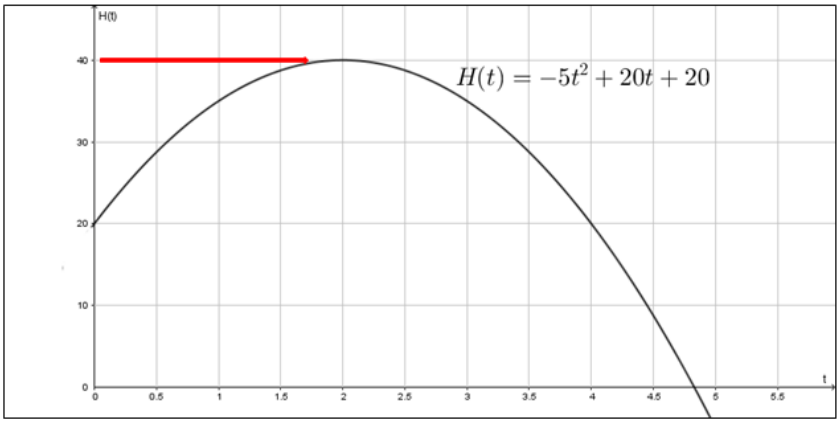
Therefore, when $t = 0$ by comparison $c = 20$.

So the height of the school is $H(t) = 20$ metres

Q1(b) Determine the rocket's maximum height above the ground.

Option 1 Graphically

The y coordinate of the maximum turning point represents the maximum height of the rocket above ground.



The maximum height the rocket reaches is $H(2) = 40$ metres

Option 2 Complete the square form and find the turning point

Write the function in completed square form $f(x) = a(x-h)^2 + k$ and write down the y coordinate of the turning point (h, k).

$$H(t) = -5t^2 + 20t + 20$$

$$H(t) = -5(t^2 - 4t - 4)$$

$$H(t) = -5[(t-2)^2 - 4 - 4]$$

$$H(t) = -5[(t-2)^2 - 8]$$

$$H(t) = -5(t-2)^2 + 40$$

From the completed square form, the maximum turning point is the coordinate (2, 40). Therefore, the maximum height the rocket reaches is $H(2) = 40$ metres.

Option 3 Using trial and error

Using the given function: $H(t) = -5t^2 + 20t + 20$ substitute in different t values, until the maximum turning point is found.

Time t	Height $H(t) = -5t^2 + 20t + 20$	Height $H(t) = -5t^2 + 20t + 20$
0	$H(0) = -5(0)^2 + 20(0) + 20$	$H(0) = 20$
1	$H(1) = -5(1)^2 + 20(1) + 20$	$H(1) = 35$
2	$H(2) = -5(2)^2 + 20(2) + 20$	$H(2) = 40$
3	$H(3) = -5(3)^2 + 20(3) + 20$	$H(3) = 35$
4	$H(4) = -5(4)^2 + 20(4) + 20$	$H(4) = 20$
5	$H(5) = -5(5)^2 + 20(5) + 20$	$H(5) = -5$

Option 4 Algebraically using the fact that the maximum height is found when $t = \frac{-b}{2a}$.

$t = \frac{-b}{2a} = \frac{-20}{2(-5)} = 2 \Rightarrow$ this is the time when the rocket will reach its maximum height

Substitute $t = 2$ into the function $H(t) = -5t^2 + 20t + 20$

$H(2) = -5(2)^2 + 20(2) + 20$

$H(2) = 40m$

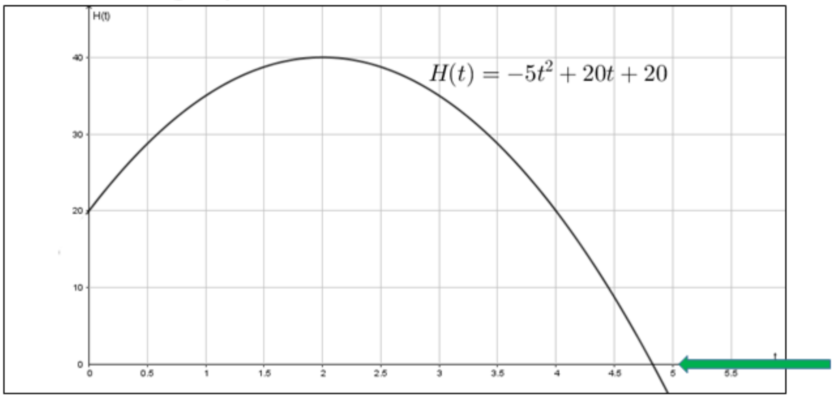
Therefore, the maximum height the rocket reaches is 40 metres.

Q1(c) Determine the time the rocket was in the air for correct to the nearest second.

Option 1 Graphically

From the graph of the function find the total time taken for the rocket to reach he ground.

Note: this is when the graph of the function cuts the t axis.



$t \approx 4.8$ seconds, to the nearest second $t = 5$ seconds

Option 2 Algebraically

Find the roots of the function using:

- the completed square form, or

$$f(x) = a(x-h)^2 + k$$

- the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Hence, identify the positive value for t, as this is the time taken for the rocket to reach the ground.

Find the roots from the completed square form:

$H(t) = -5(t-2)^2 + 40$

$\Rightarrow -5(t-2)^2 = -40$

$\Rightarrow (t-2)^2 = \dfrac{-40}{-5}$

$\Rightarrow t - 2 = \pm\sqrt{8}$

$\Rightarrow t = 2 \pm \sqrt{8}$

$\Rightarrow t = 2 + 2\sqrt{2}$ or $\Rightarrow t = 2 - 2\sqrt{2}$

$\Rightarrow t = 4.83$ or $\Rightarrow t = -0.83$

But, $t \neq -0.83$ as time cannot be negative

$\therefore\ t = 5$ seconds, this is the answer correct to the nearest second.

Find the roots using the quadratic formula:

$$H(t) = -5t^2 + 20t + 20$$

$$a = -5, b = 20 \; and \; c = 20$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow t = \frac{-20 \pm \sqrt{(20)^2 - 4(-5)(20)}}{2(-5)}$$

$$\Rightarrow t = \frac{-20 \pm \sqrt{400 + 400}}{-10}$$

$$\Rightarrow t = \frac{-20 \pm \sqrt{800}}{-10} \qquad \Rightarrow t = \frac{20 \pm 20\sqrt{2}}{10}$$

$$\Rightarrow t = 2 + 2\sqrt{2} \quad \text{or} \quad \Rightarrow t = 2 - 2\sqrt{2}$$

$$\Rightarrow t = 4.83 \quad \text{or} \quad \Rightarrow t = -0.83$$

But, $t \neq -0.83$ as time cannot be negative

$\therefore t = 5$ seconds, this is the answer correct to the nearest second.

12. Post-lesson reflection

Teacher's Lesson Observations

Expected:

- Some students would find it difficult at the start. They don't understand the question- Questions need to be scaffolded.
- Anticipated some students would have more than one possible solution.
- Expected students to mainly work in groups.
- Some would have issues going to the board
- 4 students are visual learners but most are kinaesthetic.

Not Expected:

- Didn't expect everyone to get a solution for each part.
- Surprised who actually volunteered to go to the board and presented their work really well.
- Surprised by one student using completing the square to find roots.
- All students completed all parts in the time.
- Good confidence with other teachers in the room.

Lesson Aims:

- Yes, everyone met the learning intentions (Level 2)

Other Observations

- Good timing of the lesson.
- One student left out of group work was well able to work by himself once he got a bit of assistance.

- Surprised by students with learning difficulties that made a great start to each part of the questions without any prompts.
- A few students were reluctant to try find a second method.
- Students working in a competitive grouping pushed themselves to find another method.
- Some students disregarded the graph given completely.
- Students who made mathematical errors in (b) in completing the square struggled with part (c).
- Maybe part (c) does not make sense to include the rounded up answer – one student raised this issue.
- Students' ability to attempt each part is attributed to prior lessons where students had been exposed to a visual for each algebraic method.
- Teacher highlighted the x-axis being relabelled as t.
- One student (oral learner) at the end of the lesson approached the teacher and explained another method for part (c).
- Students themselves didn't realise they how well they would do in this activity.
- Ordering of Solutions by team was not expected.
- Initially, students who were unsure asked the teacher as she circulated.

Flow of the Lesson

The right amount of time given to each part and right amount of teacher input.
Good summary at the end.

Student Feedback (from post-its)

One thing I did well:

1. I worked well in a group.
2. Finding methods faster.
3. I was surprised I knew how to start the questions.
4. Attempted all questions.

Any Changes to the Lesson

- Remove correct 'to the nearest second' for part (c) - facilitate group discussion.
- Could adapt lesson to 6th Year Maths class after completing Calculus.

Did the lesson identify any errors students made that need to be address?

All anticipated errors indicated in solutions occurred. Surprisingly, students scaled their graphs correctly and did not mix up the x- and y- axis.

Aside from the lesson, the process- is it valuable?

Yes, all teachers and students involved in this process found it valuable. Working with other teachers and discussing students' approaches was beneficial for both teachers and students involved in a reflective capacity. It facilitated:

- Learning intentions
- Literacy/ Maths terminology
- Numeracy

- AFL
- AOL
- Working with others: Team work/ Pair work & communication
- Respecting Differences
- Higher order thinking
- Predicting correct values for solutions
- Identifying Patterns
- Representing data
- A broad review of several strands: strand 3, 4 & 5.
- Mathematical conversations amongst colleagues and students
- Students' understanding that there are multiple solutions and methods to solve a problem
- Confidence building
-

What needs to change to the process to make it more widespread?

To make this whole process more widespread, teachers must buy into this and see it as a valuable learning experience. To facilitate this, lesson study must be incorporated into Subject planning hours.

Problem

A mathematics student builds a water rocket and launches it from the roof of the school. It is determined that the height above the ground from the moment it is launched can be modelled by the function

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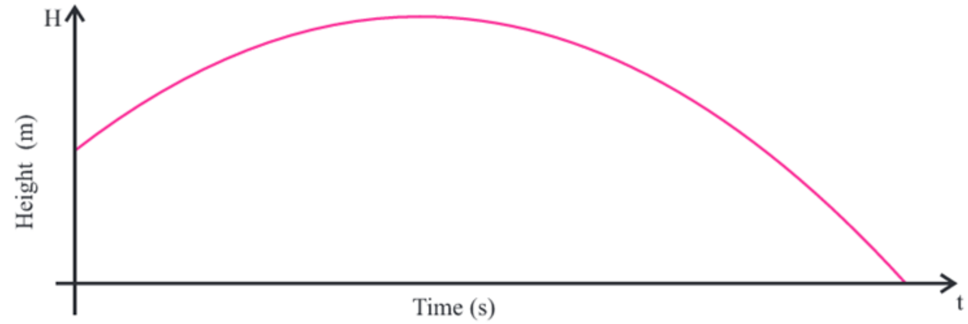

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