The Window Maker

For the lesson on 26 January 2017
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Teacher: Amiee Doyle
Second Year Higher Level
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1. Title of the Lesson: The Window Maker

2. Brief description of the lesson:

Students are presented with a task. There are several routes to the successful solution. Drawing on their prior knowledge of linear equations in one variable, students are tasked with generating a pair linear equations in two variables. Students are given the opportunity to share and explain their work with their peers at the whiteboard.

3. Aims of the Lesson:

Short Term Aims:

- Develop equations in two variables.
- Begin the process of solving a pair of simultaneous equations.
- For students to be able to approach problem solving with more confidence.

Long Term Aims:

- I'd like my students to appreciate that algebra is a tool for making sense of certain situations.
- I'd like my students to appreciate that mathematics can be used to communicate thinking effectively. **(Key Skill: Communicating and Working With Others)**
- I'd like to emphasise to students that a problem can have several equally valid solutions.
- I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities. **(Key Skill: Being Numerate)**
- I would like to support our students in developing their literacy and numeracy skills through discussing ideas. [1]

[1] This Lesson Proposal illustrates a number of strategies to support the implementation of Literacy and Numeracy for Learning and Life: the National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020 (Department of Education & Skills 2011).

4. Learning Outcomes:

As a result of studying this topic students will be able to:

- Calculate area and perimeter using their prior knowledge.
- Form algebraic equations in two variables to represent given information.
- Prepare to solve a pair of simultaneous equation.

5. Background and Rationale

An ability to solve equations with two variables is a key skill for exam success in the Leaving Certificate and Junior Certificate. Rigorous drilling of the procedure for solving simultaneous equations has made students' adequately proficient for solving out of context, purely algebraic questions; however, contextualised problems involving simultaneous equations are often poorly attempted or answered.

Possible reasons for this include:

- A poor understanding of two variable equations despite their solid grounding in single variable functions and equations.
- Disjointed pedagogy, whereby the skill of solving a pair of simultaneous equations is initially taught; followed by worded problems for the 'real-life context' several months later.

This lesson proposal looks to remediate the above using a problem solving lesson that builds on students' understanding of single variable equations, broadens their knowledge to double variable equations and set the stage for a meaningful exploration of the skill of solving two variable simultaneous equations. Ultimately, this should make it easier for students to relate to the topic and may cut out the "What are we going to use this for?" question.

6. Research

- *Junior Certificate Guidelines for Teachers* (DES 2002, Government Publications Sales Office).
- First Year Handbook (MDT).
- Second Year Handbook (MDT).
- Third Year Handbook (MDT).
- *Junior Certificate Mathematics Syllabus* (DES 2016, Government Publications Sales Office).
- www.projectmaths.ie
- www.nrich.maths.org
- Chief Examiners Report on Junior Certificate Mathematics 2006 (SEC 2006).

- Chief Examiners Report on Junior Certificate Mathematics 2016 (SEC 2016).
- *Literacy and Numeracy for Learning and Life* (DES 2011).

7. About the Unit and the Lesson

The Junior Certificate Syllabus outlines material that is required to be studied during the three years of junior cycle education. The syllabus outlines the material initially in strands, of which there are five, listed below:

- Statistics and Probability
- Geometry and Trigonometry
- Number
- Algebra
- Functions

Each strand in sub-divided into topics where a description of the topic is given (what the student learns about) and learning outcomes are detailed (what the student should be able to do). Page 26 of the syllabus notes that learners 'will use real life problems as vehicles to motivate the use of algebra and algebraic thinking'.

Section 4.2 outlines several learning outcomes that are addressed through this lesson.

4.2 Representing situations with tables, diagrams and graphs	Relations derived from some kind of context – familiar, everyday situations, imaginary contexts or arrangements of tiles or blocks. Students look at various patterns and make predictions about what comes next.	– use tables, diagrams and graphs as tools for representing and analysing linear, quadratic and exponential patterns and relations (exponential relations limited to doubling and tripling) – develop and use their own generalising strategies and ideas and consider those of others – present and interpret solutions, explaining and justifying methods, inferences and reasoning

Although the task is not based on a linear repeating pattern, it does provide an opportunity for teachers to weave the syllabus' problem solving skills (see below) with other elements of the syllabus.

Students should learn about	Students should be able to
4.8 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

8. Flow of the Unit:

Lesson		# of lesson periods
1	• Simple problems leading to linear equations.	3 x 40 min.
2	• Form linear equations.	1 x 40 min.
3	• Forming equations with two variables.	**1 x 40 min. Research Lesson**
4	• Solving simultaneous equations.	3 x 40 min.

9. Flow of the Lesson

Teaching Activity	Points of Consideration
1. Introduction **Prior knowledge:** • Area of a rectangle. • Perimeter of a rectangle. • Forming equations using a single variable. • Solving linear equations.	• Teacher sticks a poster on the board from previous lesson(s).
2. Posing the Task	• The teacher asks students if they understand the problem. • The teacher asks students to read through the problem on their own (1 minute). • The teacher asks if there are any clarifications needed before individual work begins on the task.
3. Anticipated Student Responses R1: Students may calculate the area of each of the four boxes. Students may calculate the perimeter of each of the four boxes. Students may divide the cost by either the area or	R1: Advise students that they have correctly found the area of glass and the length of the perimeter but that dividing the cost by the area or the perimeter will not give the same answer for each of the four windows.

perimeter.

R2:
Students may calculate the area of each of the four boxes.
Students may calculate the perimeter of each of the four boxes.
Students may add the answers together and divide the cost by the answer.

R3:
Students may form an equation using the perimeter and area and letting it equal the cost of the window.

R4:
Students may use the equation formed and use trial and error to calculate the cost of the glass and the frame.

R5:
Students may form two equations and solve these equations to find the cost of the glass and the frame.

R2:
Advise students that they have correctly found the area of glass and the length of the perimeter but that dividing the cost by the area and perimeter combined will not give the same answer for each of the four windows.

R3:
Advise students that they have correctly found the area and the perimeter and that their equation have been correctly formed but highlight the fact that they have two unknown variables and ask students what else they would need.

R4:
Advise students that they have found the correct answer but ask them how they could do it more efficiently.

R5:

4. Comparing and Discussing

The table below outlines the costs, areas and perimeters of the different windows. It will be used to identify links between the different variables.

Cost	**# Panes**	**# Lengths**
150	9	12
140	8	12
380	24	28
290	18	22

Solution 1

Cost	**# Panes**	**# Lengths**
150	9	12
140	8	12
380	24	28
290	18	22

$$150 = 9p + 12l$$
$$140 = 8p + 12l$$

Solution 2

Cost	**# Panes**	**# Lengths**
150	9	12
140	8	12
380	24	28
290	18	22

$$150 = 9p + 12l$$
$$290 = 18p + 22l$$

	Solution 3 **Cost** | **# Panes** | **# Lengths** 150 | 9 | 12 140 | 8 | 12 380 | 24 | 28 290 | 18 | 22 $140 = 8p + 12l$ $380 = 24p + 28l$ Solution 4 **Cost** | **# Panes** | **# Lengths** 150 | 9 | 12 140 | 8 | 12 380 | 24 | 28 290 | 18 | 22 $150 = 9p + 12l$ $140 = 8p + 12l$ $290 = 17p + 24l$ Now solve $290 = 17p + 24l$ $290 = 18p + 22l$ $p = 10$...€10 per small pane $l = 5$...€5 per small length
5. Summing up What did you learn today? Which solution did you find to be the best? How will what you have learned today help you in the future?	Two separate variables which need to separate names when forming an equation. Students will give feedback to the teacher on their learning using sticky notes. The sticky notes will have been distributed to the students at the beginning of the lesson.

10. Evaluation

There will be three observing teachers in the room. The room is divided into zones and each observing teacher has been assigned a zone. Samples of student work will be recorded using iPads. One observing teacher will use LessonNote to record interactions during the lesson. A seating plan will be given to observing teachers in advance and can be used to record relevant information and classroom interactions.

Observing teachers will pay particular attention to the following:

- Students who are thinking and not writing as an indication of their lack of understanding of the task.
- Approaches involving trial and error.
- Approaches involving the construction of a table.
- Any misconceptions in student knowledge.

A formative assessment component has been built into the lesson to allow for student feedback. The class teacher will collect the worksheets from the students after the class.

11. Board Plan

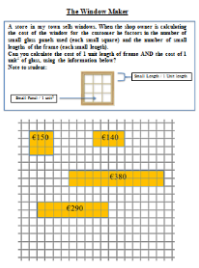
Prior Knowledge

- Perimeter
- Area
- Problem solving strategies

Table

Cost	# Panes	# Lengths
150	9	12
140	8	12
380	24	28
290	18	22

Task



Solution 1

Cost	# Panes	# Lengths
150	9	12
140	8	12
380	24	28
290	18	22

$$150 = 9p + 12l$$
$$140 = 8p + 12l$$

Solution 2

Cost	# Panes	# Lengths
150	9	12
140	8	12
380	24	28
290	18	22

$$150 = 9p + 12l$$
$$290 = 18p + 22l$$

Solution 3

Cost	# Panes	# Lengths
150	9	12
140	8	12
380	24	28
290	18	22

$$140 = 8p + 12l$$
$$380 = 24p + 28l$$

Solution 4

Cost	# Panes	# Lengths
150	9	12
140	8	12
380	24	28
290	18	22

$$150 = 9p + 12l$$
$$140 = 8p + 12l$$

$$290 = 17p + 24l$$

Now solve

$$290 = 17p + 24l$$
$$290 = 18p + 22l$$

$p = 10$...€10 per small pane
$l = 5$...€5 per small length

12. Post-lesson reflection

- What are the major patterns and tendencies in the evidence?
  - Students had a good command of the prior knowledge of perimeter and area of rectangles.
  - Students are reasonably comfortable coming to the board to explain their workings on the task.
  - The majority of students used the problem solving strategy of method trial and error.

- What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc.?
  - Students tended to link a single variable, either the perimeter or the area, to the cost. They struggled to see the relationship between the two variables and the cost.

- In what ways did students achieve or not achieve the learning goals?
  - Many of the long term lesson aims were achieved. For instance, students appreciate that algebra is a tool for making sense of certain situations and many of the Key Skills have been reinforced by the lesson (Communication and Being Numerate).
  - It is noted that further lessons are required to develop mathematical fluency when working with or forming simultaneous equations; however, student feedback indicates that the short term goals of the lesson were achieved.

    "I learned how to form simultaneous equations"

    "I learned how to put equations together"

    "I thought it was good because I learned about solving simultaneous equations"

    "I learned that if you look further into the question, there are many methods to solving"

    "I learned that if you have a problem try another way to figure it out and that there is not always one solution"

    "I know that I need to write down the information that I know for future questions"

- Based on your analysis, how would you change or revise the lesson?
  - The task may be too text heavy. Rewording may be appropriate. If the task is to be used in its current format, students must be given adequate time to engage with the task.
  - To use the lesson as an introduction to simultaneous equations the number of window panes should be reduced, thereby reducing the number of possible solutions to the task.
  - If the lesson is to be used in its current format it should be used at the end of a sequence of problem solving lessons.

The Window Maker

A store in my town sells windows. When the shop owner is calculating the cost of the window for the customer he factors in the number of small glass panels used (each small square) and the number of small lengths of the frame (each small length).
Can you calculate the cost of 1 unit length of frame AND the cost of 1 unit^2 of glass, using the information below?
Note to student: