

Topic: Volume of a cylinder
Year Group: Second Year

Lesson plan taught: January 20th 2017
At St Aidan's Community College, 2nd Year Higher level class
Teacher: Úna Hegarty
Lesson plan developed by: Úna Hegarty, Seamus Murphy, Áine McCarthy

- 1. Title of the Lesson:** Discovering the formula for the volume of a cylinder
- 2. Brief description of the lesson:** Students use tins of beans, rulers, calculators, worksheet, etc to calculate the volume of the tin of beans.
- 3. Aims of the Lesson:**

Long-range/thematic goals:

- I'd like to foster my students to become independent learners
- I'd like my students to become more creative when devising approaches and methods to solve problems
- I'd like my students to experience meaningful mathematics i.e. that they see a need for what they are studying
- I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities

Short-term goals:

For students to understand :

- The relationship between the base of a shape, its height and its volume
- To be able to use the formula to get the volume of a cylinder

4. Learning Outcomes:

As a result of studying this topic students will be able to:

- Develop an understanding of the relationship between the area of the base of a solid, its height and its volume.
- Get the volume of a cylinder.

5. Background and Rationale

- a. According to the syllabus students are required to calculate the volume of a cylinder and apply to problem solving questions.
- b. Students are very competent at substituting into the formula to compute correct volume, however have difficulty manipulating the formula to less straight forward questions. A lack of understanding of how the formula is derived is at the root of this difficulty.
- c. To enable to make a connection between the area of the base of a shape and its volume.

6. Research

Texts books . The Maths Development website and JC Maths syllabus. We also considered the key skills of junior cycle i.e that each student can become able to express their own ideas, compare and discuss ideas, establish a form of learning by comparing and taking good ideas from others and experience that maths is fun.

7. About the Unit and the Lesson

Page 24 of Junior Certificate Syllabus:

Topic	Description of topic Students learn about	Learning outcomes Students should be able to
3.4 Applied measure	<p>Measure and time.</p> <p>2D shapes and 3D solids, including nets of solids (two-dimensional representations of three-dimensional objects).</p> <p>Using nets to analyse figures and to distinguish between surface area and volume.</p> <p>Problems involving perimeter, surface area and volume.</p> <p>Modelling real-world situations and solve a variety of problems (including multi-step problems) involving surface areas, and volumes of cylinders and prisms. The circle and develop an understanding of the relationship between its circumference, diameter and π.</p>	<ul style="list-style-type: none"> – calculate, interpret and apply units of measure and time – solve problems that involve calculating average speed, distance and time – investigate the nets of rectangular solids – find the volume of rectangular solids and cylinders – find the surface area of rectangular solids – identify the necessary information to solve a problem – select and use suitable strategies to find length of the perimeter and the area of the following plane figures: disc, triangle, rectangle, square, and figures made from combinations of these – draw and interpret scaled diagrams – investigate nets of prisms (polygonal bases) cylinders and cones – solve problems involving surface area of triangular base prisms (right angle, isosceles, equilateral), cylinders and cones – solve problems involving curved surface area of cylinders, cones and spheres – perform calculations to solve problems involving the volume of rectangular solids, cylinders, cones, triangular base prisms (right angle, isosceles, equilateral), spheres and combinations of these

8. Topic Description of topic Students learn about Learning outcomes

9.

Lesson		# of lesson periods
1	<ul style="list-style-type: none"> • Review of perimeter and area of squares, rectangles and triangles. 	1 x 40 min.
2	<ul style="list-style-type: none"> • Area of a parallelogram 	1 x 40 min.
3	<ul style="list-style-type: none"> • Area and Circumference of a circle. 	3 x 40 min.
4	<ul style="list-style-type: none"> • Surface area of a rectangular solid. 	1x 40 min.
5	<ul style="list-style-type: none"> • Volume of a rectangular solid 	2 x 40 min.

6	<ul style="list-style-type: none"> Discovering formula for volume of a cylinder 	1 x 40 min (research lesson)
7	<ul style="list-style-type: none"> Volume of a cylinder 	1 x40 mins
8	<ul style="list-style-type: none"> Volume of a prism 	1x 40 mins

10. Flow of the Lesson

Teaching Activity	Points of Consideration
<p>1. Introduction (2-3mins) Review of area of a circle and review of volume of a rectangular solid.</p>	<p>Can anyone remember how we get the area of a circle? Units? What does 'r' stand for? What is π? (Both the number and that it is circumference divided by diameter) How do you get the volume of this solid? (Rectangular) Units?</p>
<p>2. Posing the Task (10 mins) Question: You have applied for a job at a baked bean factory ,as part of the application you must find the volume of this tin of beans.</p> <p>Students will have worksheet, tin of beans, ruler, calculator.</p> <p>See Appendix 3 & 4</p>	<p>They start working on the task, discussing possible ways of doing it. Ask students if they are clear about what they need to do and if they have any questions. Teacher to explain to students that they will have 10 minutes to solve the problem and that it is important that they understand the task as the teacher won't be answering any questions during that time. At the end of the 10 minutes, students will be called up to the board to explain their methods. Everybody must write down their own work.</p>
<p>3. Anticipated Student Responses (20 mins) R1 – students may read weight in grams from tin. R2 – Students may get the area of a rectangular prism which encloses the area of the base of the cylinder R3 –. Students may multiply $l \times w \times h$ R4 – Students find the area of the base and multiply by the height. R5 – Students may look up the formula in the Maths tables</p>	<p>Students who finish early will be encouraged to check their solution by using a different method.</p>
<p>4. Comparing and Discussing</p> <ol style="list-style-type: none"> As above Are the methods similar / different? 	<p>How many other students did it this way? Any other methods? Which method do you prefer? Did all the methods give you the same answer? The focus of the discussion will be on the relationship between the area of the base and the height of the cylinder. And focus on units used. We will know that they are benefiting from the discussion if they are paying attention to other groups' solutions, contributing their own ideas, able to justify their thinking and extending their own solutions in light of other students work.</p>
<p>5. Summing up Teacher reinforces that the volume was found by multiplying the area of the base by the height of the cylinder.</p>	

Teacher moves from words to $V=\pi r^2 h$ and shows students where to locate it in the Maths tables.
 Teacher gives homework from textbook - finding the volume of various cylinders.

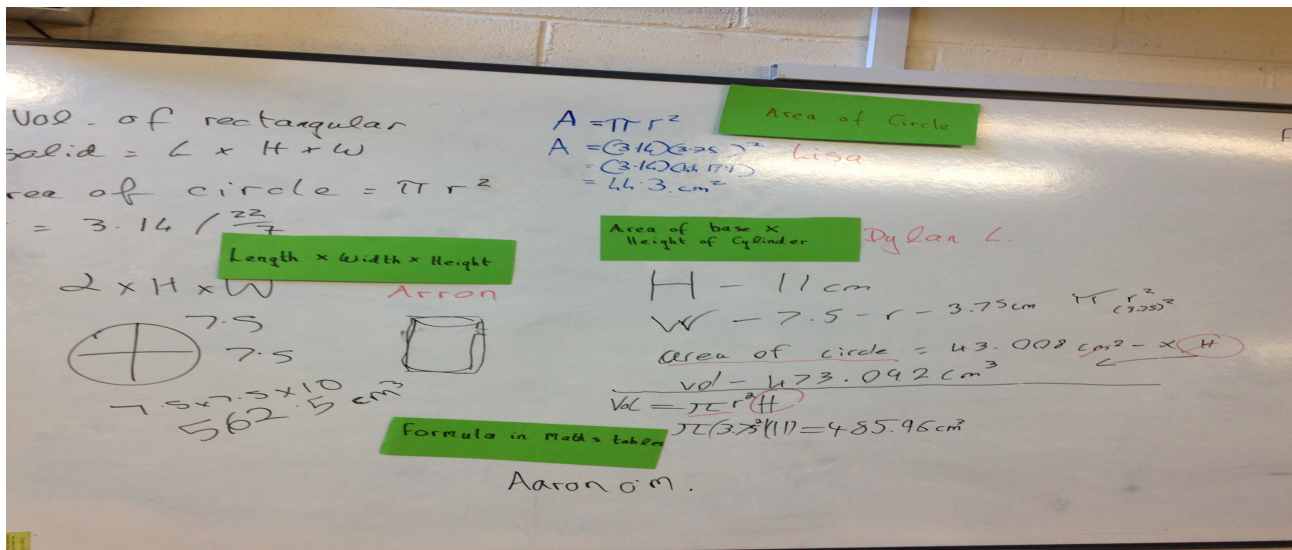
11. Evaluation

We divided ourselves so each teacher observed 3-5 students each. We used the student record observation sheet. We met after the lesson and compared observations. We divided student work into three different categories-poor-good- very good.

12. Board Plan

The whiteboard contains the following handwritten work:

- Prior Knowledge** (written in blue ink):
 - Area of Circle: πr^2
 - Volume of a Rectangular Solid = $L \times W \times H$
 - $\pi = 3.14$
 - Length-cm, Area-cm², Volume-cm³
- Tom's work** (green sticky note: "Length x Width x Height"):
 - $7.4 \times 7.5 \times 10.8 = 599.4 \text{ cm}^3$
- Aaron's work** (green sticky note: "Volume of Rectangular Solid"):
 - $8 \text{ cm} \times 8 \text{ cm} = 64 \text{ cm}^2$
 - $64 \text{ cm}^2 \times 11 \text{ cm} = 704 \text{ cm}^3$
- Mary's work** (green sticky note: "Weight on tin"):
 - 420g.
- Katie's work** (green sticky note: "Area of base x Height of Cylinder"):
 - Area of base = $\pi r^2 = (3.14)(4)^2 = (3.14)(16) = 50.24 \text{ cm}^2$
 - $50.24 \times 10.5 \text{ cm} = 527.52 \text{ cm}^3$
- Timmy's work** (green sticky note: "Formula in Maths table"):
 - $V = \pi r^2 h$
 - $V = 3.14 \times 3.5^2 \times 10.6$
 - Volume = 407.729 cm^3



13. Post-lesson reflection

- What are the major patterns and tendencies in the evidence ?**
 A lot of students got the area of the circle but some struggled after that. Other students used $L \times W \times H$. Very few of the students used the 1cm square grid provided. Other students got the formula from the maths tables provided. Some students misunderstood the perimeter of circle as length.
- What are the key observations or representative examples of student learning and thinking ?**
 We felt that the students were very enthusiastic and had a very positive relationship with the teacher. We divided student work into three groups. The first group struggled to make progress with little or no attempt. The second group managed to get the area of a circle but failed to progress subsequently. The final group produced work of a high standard and arrived at the volume through various methods.
- What does the evidence suggest about student thinking such as their misconceptions, difficulties, confusion, insights, surprising ideas, etc ?**
 The main difficulty which we hadn't anticipated was some students misunderstood the perimeter of circle as length. On a positive note we were both surprised and pleased with the students' willingness to stick to their task and offer each other help.
- In what ways did students achieve or not achieve the learning goals ?**
 Many students used their prior knowledge to come up with the formula for the volume of a cylinder and see the connection between area and volume. Those who struggled to see the connection were guided to this conclusion through the teacher's final summation of students' work from board.
- Based on your analysis, how would you change or revise the lesson ?**
 Timing wise we felt the lesson went very well. We also felt students were very engaged with the problem. In terms of change, we discussed placing the cylinder in a cuboid. However we felt it would be difficult to find an everyday household object that the tin of beans would fit into. We concluded we would change very little about the lesson.

What are the implications for teaching in your field We felt the student enthusiasm reinforced the positives of discovery based learning. We feel that this will really help them with problem solving skills for the junior cert. While this type of teaching requires additional preparation we feel the rewards justify the extra effort. As teachers we felt it was good to step back and let the students discover their own solutions. The students were very attentive to other students' work and this allowed the teacher to extract the merit in all efforts.

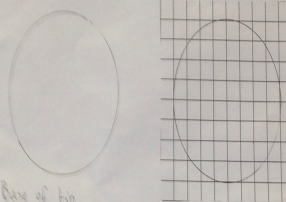
Appendix 1

Worksheet - Volume of a Tin of Beans

Arron Callanan

You have applied for a job with a baked bean manufacturer. As part of the interview, you have been asked to find the volume of this tin of beans.

You are given the tin of beans, this worksheet, ruler, calculator, etc.



Base of tin

<p>Attempt 1</p> <p>Circle 20 radius 10</p> <p>$20 \times 5 = 100$</p> <p>Area = πr^2 $(10 \times 3.14) \times 2$ 62.8</p> <p>Volume = $100 \times 5 = 500$</p>	<p>Attempt 2</p>
<p>Attempt 3</p> <p>$10 \times 7.5 \times 10 = 602.9$</p> <p>Volume = 562.5</p>	<p>Attempt 4</p>

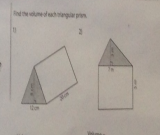
Reflection:

What did you learn from doing this task?

Which is your preferred method to find the volume of the tin of beans?

Volume of cylinder = Area of the base x Height of Cylinder

Find the volume of each triangular prism.



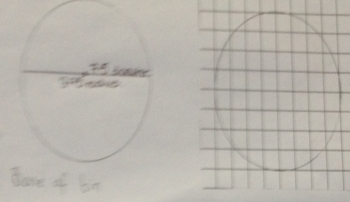
Volume = _____

Appendix 2

Worksheet - Volume of a Tin of Beans

You have applied for a job with a baked bean manufacturer. As part of the interview, you have been asked to find the volume of this tin of beans.

You are given the tin of beans, this worksheet, ruler, calculator, etc.



Base of tin

<p>Attempt 1</p> <p>$r = 7.5 \text{ cm}$ $w = 7.5 \text{ cm}$ Area of circle = πr^2 Circumference = $2\pi r$ $2\pi \times 7.5 = 47.1$ $10 \times 50 = 500$ $500 - 47.1 = 452.9$</p>	<p>Attempt 2</p> <p>$r = 7.5$ $7.5 \times 10 \times 7.5 = 562.5$</p> <p>$r = 7.5$ $w = 7.5$ $r = 7.5$ Area of circle = πr^2 Volume = $177.5 \times 2 \times 10 = 3550$</p>
<p>Attempt 3</p> <p>$r = 7.5$ $2 \times \pi \times 7.5 \times 10 = 471$ $500 - 471 = 29$</p>	<p>Attempt 4</p> <p>177.5×2 $177.5 \times 2 \times 10$ 3550</p>

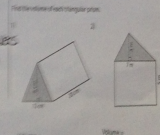
Reflection:

What did you learn from doing this task?

Which is your preferred method to find the volume of the tin of beans?

Find area of circle x height

Find the volume of each triangular prism.



Volume = _____

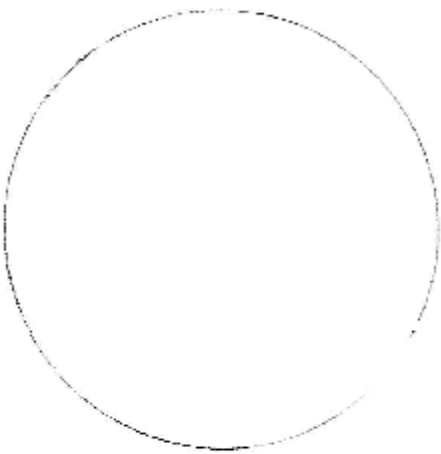
Appendix 3

Worksheet – Volume of a Tin of Beans

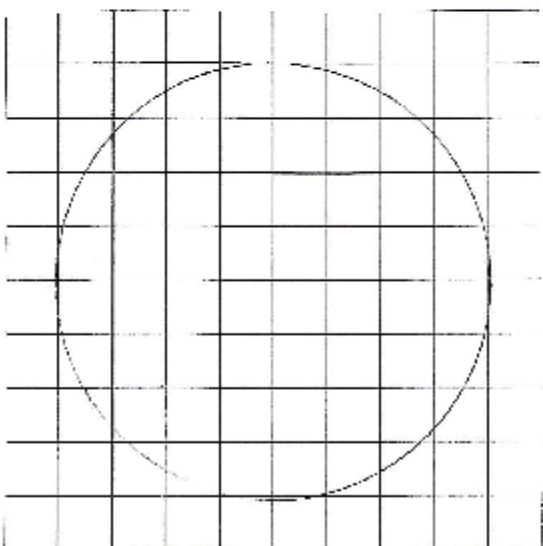


You have applied for a job with a baked bean manufacturer. As part of the interview, you have been asked to find the volume of this tin of beans.

You can use **the** tin of beans, this worksheet, ruler, calculator, etc.



Base of tin



<p>Attempt 3</p>	<p>attempt 4</p>
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Reflection:

What did you learn from doing this task?

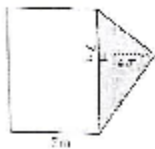
Which is your preferred method to find the volume of the tin of beans?

Find the volume of each tin (underneath).

1)



2)



Volume =

Volume =

Appendix 3: Student Observation Record

5. Beginning of Lesson: Observe understanding of prior knowledge and of the task								
	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7	Student 8
(i) Questions on the formula								
(ii) Misconceptions								
(iii) Wording of the task, e.g. "Width"								
(iv) Do students ask for measurements?								
(v) Questions asked by students								
Other Observations								

(iii) Identify if and when a student used a practical approach								

6. During Lesson: Observe student engagement and note progress								
	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7	Student 8
(i) Questions asked to teacher								
(ii) Questions asked to other group members								
(iii) Identify if and when a student used a practical approach How long did they spend? Did they go back?								
(iv) Identify if and when a student used a written approach How long did they spend?								
(v) Did students persist with the task? Or give up? Were prompts required?								
(vi) Rate student understanding of the practical element of the task. 1 = poor 2 = some understanding 3 = competent								
(vii) Rate student understanding of the written element of the task. 1 = poor 2 = some understanding 3 = competent								
Other observations								

7. During Discussion:

Observe student engagement.

	Student 1	Student 2	Student 3	Student 4	Student 5	Student 6	Student 7	Student 8
(i) Are the students attentive to what's happening on the board?								
(ii) Presenters: are clarifications or prompts needed to their board work?								
(iii) Did the presenting students' presentation and discussion promote their teaching and learning?								
(iv) During oral questioning at the end of the lesson, was there evidence of understanding and learning?								
Other observations								
Issues that need to be addressed in the next class								
Recommended changes to the lesson plan								