Lesson Research Proposal for 2nd Year Higher Level Mathematics
Finding Solutions to Quadratic Equations

For the lesson on Tuesday, 16^{th} of January 2018
At St. Declan's College, Ms. Rebecca Strain's class
Instructor: Ms. Rebecca Strain
Lesson plan developed by: Ms. Rebecca Strain, Ms. Deirdre Coyle, Mr. Niall Nolan

1. Title of the Lesson: Diving for Roots

2. Brief description of the lesson

This lesson has been designed to allow students to solve quadratic equations using a Functions approach. Once students find those solutions they make further connections to the algebraic method(s) for factorising and solving. They are given the context of a seagull diving for fish into the sea and are asked to find the points where the seagull enters and leaves the water.

3. Research Theme

At St. Declan's College we want teachers to:
collectively develop and implement consistent and dependable formative and summative assessment practices.

As Maths teachers we are implementing this in a number of ways:
Formative Assessment:

- In the coming academic we will introduce and implement structured problem-solving approaches to introduce new concepts to students and assess prior knowledge.
- Key questioning within classes: This is to draw on students' prior knowledge, engage them in their learnings and improve their use of Mathematical keywords.
- Engineering discussion: We plan for and implement group work and paired work in our classes.
- Student led board work: When reviewing homework, for example, students are invited to share their approach to the problem(s) given. Peers offer their ideas and support.
- Random selection: Students are chosen at random using lollipop sticks, random number generators etc. This encourages balanced and fair participation in class.

Summative Assessment:

- Common exams for subject year and level. The Maths Department in our school write common exams so that students in the same year group and doing the same level all do the same exam. This is to maintain standards within our school. It also helps us create comparable data.
- First year students are assessed formally more frequently. This is to comprehensively inform teachers of progress throughout the year so that fair and good decisions can be made when it comes to dividing students into Higher and Ordinary Level for 2^{nd} year onwards.

4. Background & Rationale

a) Why you chose the topic

We chose this because we find students learn solving quadratic equations as a procedure and have no real understanding of its association with quadratic functions or real-life applications. They perceive quadratic functions and quadratic equations to be completely isolated topics. Therefore, they do not identify future links between intersecting functions, the equality of equations and simultaneous equations. We want to enable them to solve word problems which involve quadratic equations. We want to create links between Strand 4 (Algebra) and Strand 5 (Functions). We do not want them to see these as isolated topics but rather interconnected concepts. Students should be able to interpret their answers rather than just produce them (what does x = 1 mean? What does y = 5 mean?), so that when presented with a real-life problem they can interpret it.

b) Your research findings

Historically this topic has taught using examples followed by consolidation work. Emphasis has been on procedure as opposed to creative problem solving. Students become focused on the answer as opposed to the rationale behind what they are doing and why. There is a reliance on text books to guide our teaching and learning. Functions and Algebra are featured in separate chapters and, in many cases, dealt with as completely isolated ideas. They do not capitalize on the opportunities to make the many connections between these topics. In one of our schools we found that students had to be retaught these topics again at Senior Cycle as they had very poor retention of it from Junior Cycle.

“They struggled noticeably with questions that involved substantial amounts of algebra”
Chief Examiners Report, Junior Certificate Mathematics, 2015

“The objectives listed in the syllabus ….. to develop: the ability to recall relevant mathematical facts; instrumental understanding (“knowing how”); relational understanding (“knowing why”); the ability to apply mathematical knowledge and skills to solve problems; analytical and creative powers in mathematics; and an appreciation of and positive disposition towards mathematics.”
Chief Examiners Report, Junior Certificate Mathematics, 2015

“Candidates also struggled in some reasonably standard algebra questions on this paper, for example … find the irrational roots of a quadratic equation (Question 9(b)).”
Chief Examiners Report, Junior Certificate Mathematics, 2015

“Most candidates struggled with factorising (Question 9), and in particular when asked to factorise a quadratic expression (part (c)). Some students showed an awareness of the form the answer should take in this part, but were unable to find the actual answer.
Most candidates also had difficulty solving equations (Question 11(a) and (c)).”
Chief Examiners Report, Junior Certificate Mathematics, 2015

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
In 6th class students were enabled to: - identify relationships and record symbolic rules for number patterns deduce and record rules for given number patterns - explore the concept of a variable in the context of simple patterns, tables and	Revision of evaluating expressions, multiplying expressions together, factorizing trinomial expressions. − show that relations have features that can be represented in a variety of ways − distinguish those features	**At Leaving Certificate Level** Students working at OL will be expected to: **<u>4.2</u>** select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: • f(x) = g(x), with f(x) = ax+b, g(x) = cx+d where a, b, c, d ∈ Q

simple formulae and substitute values for variables In 1st Year students were enabled to: The use of tables, graphs, diagrams and manipulatives to represent and analyse patterns (e.g. using unifix cubes) and introduce concepts of variables and constants A relationship as that which involves a set of inputs, a set of outputs and a correspondence from each input to each output Relations derived from some kind of context – familiar, everyday situations, imaginary contexts or arrangements of tiles or blocks. How to develop and use their own generalising strategies and ideas and consider those of others How to present and interpret solutions, explaining and justifying methods, inferences and reasoning How to generalise and explain patterns and relationships in words and numbers In 2nd Year they have covered: − evaluate linear quadratic and cubic expressions − add and subtract simple algebraic expressions. − use the associative and distributive property to simplify such expressions	that are especially useful to identify and point out how those features appear in different representations: in tables, graphs, physical models, and formulas expressed in words, and algebraically − use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others − recognise that a distinguishing feature of quadratic relations is the way change varies - Consolidate their understanding of the concept of equality - solve quadratic equations of the form $x^2 + bx + c = 0$ where $b, c \in Z$ and $x^2 + bx + c$ is factorisable $ax^2 + bx + c = 0$ where $a, b, c \in Q$ $x \in R$ − explore patterns and formulate conjectures − explain findings − justify conclusions − communicate mathematics verbally and in written form − apply their knowledge and skills to solve problems in familiar and unfamiliar contexts - devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw	• $f(x) = g(x)$ with $f(x) = g(x) =$ where a, b, c, e, f, p, q, r ∈ Z • $f(x) = k$ with $f(x) = ax2 + bx + c$ (and not necessarily factorisable) where a, b, c ∈ Q and interpret the results **4.3:** select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form: • $g(x) \leq k$, $g(x) \geq k$, • $g(x) < k$, $g(x) > k$, where $g(x) = ax + b$ and a, b, k ∈ Q **5.1:** use graphical methods to find approximate solutions to • $f(x) = 0$ • $f(x) = k$ • $f(x) = g(x)$ where f(x) and g(x) are of the above form, or where graphs of f(x) and g(x) are provided Students working at HL will be expected to **4.2:** Select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to equations of the form: $f(x) = g(x)$ with $f(x) =$; $g(x) = k$ where a, b, c, d, e, f, q, r ∈ Z select and use suitable strategies (graphic, numeric, algebraic and mental) for finding solutions to • cubic equations with at least one integer root • simultaneous linear equations with three unknowns • one linear equation and one equation of order 2 with two unknowns and interpret the results **4.3:** select and use suitable

solve first degree equations in one or two variables, with coefficients elements of Z and solutions also elements of Z	relevant conclusions. <u>Functions</u> - Interpreting and representing linear, quadratic and exponential functions in graphical form. − interpret simple graphs − plot points and lines − draw graphs of the following functions and interpret equations of the form f(x) = g(x) as a comparison of functions • f(x) = ax + b, where a, b ∈ Z • $f(x) = ax^2 + bx + c$, where a ∈ N; b, c ∈ Z; x ∈ R • $f(x) = ax^2 + bx + c$, where a, b, c ∈ Z, x ∈ R	strategies (graphic, numeric, algebraic, mental) for finding solutions to inequalities of the form: • g(x) ≤ k, g(x) ≥ k; • g(x) < k, g(x) > k, with g(x) = ax2+bx+c or g(x) = and a, b, c, d, k ∈ Q, x ∈ R • | x – a | < b, | x – a | > b and combinations of these, with a, b, ∈ Q, x ∈ R

6. Goals of the Unit

a) Students will be able to see patterns, trends and relationships.
b) Students will form a positive disposition towards investigating, reasoning and problem solving. The unit, not just the research lesson creates lots of opportunities for this.
c) Students can, using their own initiative, transfer and apply skills learned in one context to another context.
d) Students will be able to devise, illustrate, interpret, predict and explain patterns and relationships with regards to quadratic expressions, equations and graphs.
e) Student will devise and evaluate strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.
f) Students will be able to recall prior knowledge of algebra procedures (expansions of brackets, factorizing, evaluating expressions).
g) Students will gain an appreciation that x can have multiple values and is therefore called a variable. Changing the values of x results in different y values. These will be defined as couples.
h) Students can apply their prior knowledge from coordinate geometry to plot couples on a graph.
i) Students will discover that the couples formed from a linear function creates a line when plotted. Students will be able to plot a number of different linear functions where the coefficient of x is an integer.
j) Students will discover that the couples formed from a quadratic function creates a parabola when plotted. Students will be able to plot a number of different quadratic functions where the coefficient of x^2 is an integer. They should observe and predict the inversion of the graph when there is a negative coefficient of x^2.
k) Students will be able to form connections between the factors, roots and the expression. This forms the basis for future connections to be made with the further study of functions.

l) Students will be able to use the roots of a quadratic equations to sketch the graph of a quadratic function.
m) Students will be able to use quadratic equations and graphs to solve problems.

7. Unit Plan

Lesson	Learning goal(s) and tasks
1	Recap - Expanding brackets - Factorising – HCF, Grouping, Difference of two squares, Trinomials.
2	Recap - Evaluating expressions including negative values, squares etc. Input/Output- recalling that x is a variable. Substituting a number of values for x into the same function.
3	Presenting the idea of the relationship between the input and the output. Putting the input and output together as a couple. Students should make the recognize the couple as a coordinate. Using the coordinates to plot a Linear Function, coefficient of x is an integer.
4	Graphing Quadratics Coefficient of x squared will be an integer.
5	Research lesson.
6	Solving quadratic equations algebraically. Using the solutions to sketch the graph.
7	Solving problems using quadratic equations. Appropriate problem.

8. Goals of the Research Lesson:

The Goals of the lesson should refer to:

a) <u>Mathematical Goals</u>

Students will:
- use prior knowledge from both functions and algebra to solve a quadratic equation.
- make connections between factors and roots.
- identify that the points at which the graph crosses the x axis form the roots of the equation.

b) <u>Key Skills and Statements of Learning</u>

This lesson will address the following Key Skills.

This lesson will implement and promote JC Key Skills in the following ways:
1. Being Literate: Students will have the opportunity to express their ideas clearly and accurately.
2. Being Numerate: It will develop a positive disposition towards problem solving.
3. Staying Well: Students' confidence and positive disposition to learning will be promoted.
4. Communicating: Students will present and discuss their mathematical thinking.
5. Being Creative: Students' will explore options and alternatives as they actively participate in the construction of knowledge.
6. Working with Others: Students will learn with and from each other.
7. Managing information and thinking: Students will be encouraged to think creatively and critically.

This lesson is also designed to meet the following JC Statements of Learning in particular:

15. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
16. The students describes, illustrates, interprets, predicts and explains patterns and relationships.
17. The students devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

9. Flow of the Research Lesson:

<table>
<tr><th>Steps, Learning Activities
Teacher's Questions and Expected Student Reactions</th><th>Teacher Support</th><th>Assessment</th></tr>
<tr><td>Introduction
Today we are going to use what we have learned so far this year to solve a problem. When we are finished working on the problem we will come go through our solutions together on the board and learn from each other.</td><td></td><td></td></tr>
<tr><td>Posing the Task

A seagull leaves its nest to dive into the sea to catch a fish.

The dive from start to finish lasts less than 8 seconds.

The function below gives us the height of the bird for every second, t, of the dive.

At what time in seconds will the bird enter and leave the water?

Use as many different ways as you can to show your answers.

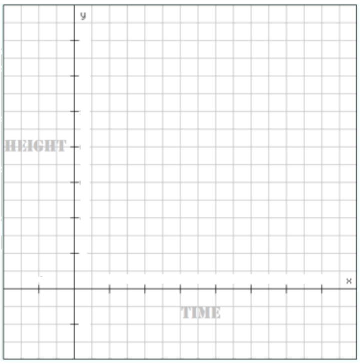
$h(t) = (t-2)(t-6)$

Clarifying the problem:
"Does everybody know what sea level is?"
"What is the height of the bird when it enters the water?"

We have left some materials on your desks if you want them. You have 10 minutes to come up with a solution in as many ways as you can.</td><td>The problem will be displayed on the board. The students will also be given handouts of the problem.

On each desk there will be a variety of resources that the students may want to use e.g. tables, axes (not numbered), graph paper, blank paper.

Students will have their own calculators, rulers, pencils etc.


</td><td>Students in this cohort are generally confident to ask questions to improve their understanding. We will therefore know if and when they are ready to proceed with the problem.

As observers we will look out for students that will look for clarification from their peers rather than their teacher and record any questions asked.</td></tr>
<tr><td>Student Individual Work
We have anticipated a number of responses as follows:

Response A:</td><td>Prompting questions
"What is h?"</td><td>Students are asked to so this individually. Are</td></tr>
</table>

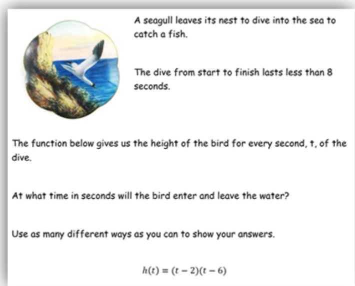
Using trial and improvement students substitute a number of values into the given expression until they are find one or more solutions. **Response B:** Students expand the brackets but may stop here. $t^2 - 8t + 12$ **Response C:** Students expand the brackets and use this trinomial to don Trial and Improvement. **Response D:** Using a functions approach. Substituting values between the 0 and 8 (inclusive) to systematically try a variety of inputs in a given domain using the expression $t^2 - 8t + 12$ **Response E:** Using a functions approach. Substituting values between the 0 and 8 (inclusive) to systematically try a variety of inputs in a given domain using the expression $t^2 - 8t + 12$ **Response F:** Identifying a pattern. **Response G:** Using the table to record inputs/outputs. This demonstrates a greater level of organization. Allows us to compare outputs easily. **Response H:** Drawing the parabola using the axes given (or axes they have drawn themselves. **Response I:** Identifying that the solution ($t = 2$ and $t = 6$) is connected to the factors given but not necessarily being able to explain why this is the case. **Response J:** Setting up an equality relationship. $t^2 - 8t + 12 = 0$ **Response K:** Setting up an equality relationship. $(t - 2)(t - 6) = 0$ **Response L:** Knowing that if the product of two numbers is zero then one or both of those numbers must equal zero. $(t - 2) = 0$ and/or $(t - 6) = 0$ **Response M:** Being able to solve the quadratic equation algebraically.	"What is t?" "Is this similar to anything you've done before? What did you do?" – This question is in regards to have factors and might encourage them to expand those brackets. "What times can't the bird be in the water?" i.e. $t > 8$ seconds. "So what values could you look at instead?" "How would you find out what height the bird is at the start?" – this prompt should lead to students realizing $t = 0$ at the start. "Have you used any of the resources?" This may prompt students to fill out a table of values or draw the graph. "Have you solved equations before? Could you use an equation for this?" This question may encourage students to let the expression equal to zero.	they inclined to look around and seek reassurance or ideas from others? Are students inclined to rub out or cross out their work if they are not sure? Is their value to be gained from examining these discarded workings? Do students notice the similarities between the methods A-C, D-E and A – E? Do students expand the brackets? Do students go from Trial and improvement to a more formal functions, input/output method? Do students use the resources? Do students know how to use the resources? Do the students use the resources in a way we do not anticipate e.g. leaving out a column of the table and just recording inputs/outputs, drawing their own axes instead of the ones used? Do students recognize that they can use an algebraic approach? Can students link their answer back to the question in any way? Do students equate the expression to zero?
Ceardaíocht /Comparing and Discussing **Response A:** Using trial and improvement students substitute a number of values into the given expression until they are find one or more solutions.	Did anyone else do it this way?	Students will not have substituted into an expression presented like

Response B: Students expand the brackets but may stop here. $t^2 - 8t + 12$ **Response C:** Students expand the brackets and use this trinomial to don Trial and Improvement. **Response D:** Using a functions approach. Substituting values between the 0 and 8 (inclusive) to systematically try a variety of inputs in a given domain using the expression $t^2 - 8t + 12$ **Response E:** Using a functions approach. Substituting values between the 0 and 8 (inclusive) to systematically try a variety of inputs in a given domain using the expression $t^2 - 8t + 12$ **Response G:** Using the table to record inputs/outputs. This demonstrates a greater level of organization. Allows us to compare outputs easily. **Response H:** Drawing the parabola using the axes given (or axes they have drawn themselves. **Response I:** Identifying that the solution ($t = 2$ and $t = 6$) is connected to the factors given but not necessarily being able to explain why this is the case. **Response J:** Setting up an equality relationship. $t^2 - 8t + 12 = 0$ **Response K:** Setting up an equality relationship. $(t - 2)(t - 6) = 0$ **Response L:** Knowing that if the product of two numbers is zero then one or both of those numbers must equal zero. $(t - 2) = 0$ and/or $(t - 6) = 0$ **Response M:** Being able to solve the quadratic equation algebraically.	Why did you multiply the expressions together? Did anyone else do this and why? Did anyone do it this way? Can anyone see any similarities and differences between these methods? Did they get the same result? Why is that? Why did you use those values? Have we seen this method before? What values could we have used? What values couldn't we use? Did anyone else use this approach? Does anyone see any similarities between this one and the previous approach? Can we similarities between this and all the methods on the board? Why did you decide to use the table? Are there any advantages for using the table? Why did you decide to use the graph? Can we notice anything from the graph? Where is the height equal to zero? For what values of t is h=0. Can we find the solution from looking at the graph? I: You noticed a connection? What was it? How did you notice it? Did anyone else do this? Can we figure out why there is a connection between the expression and the solutions. Why did you decide to write an equation? Why did you let it equal to zero? Have we seen something like this before? Can we use the same method to solve this as a linear equation? Why? If we multiply two numbers together and get an answer of 0, what does that tell us about the numbers? When is $x - 2 = 0$? When is $x - 6 = 0$? Why we end up with two solutions instead of just one? Which method is probably the	this before. Did they have any difficulties? Do students realise that $(t - 2)(t - 6)$ and $t^2 - 8t + 12$ are equivalent expressions and that substituting the same values for t into either expression will lead to the same outcome. Are students communicating their ideas well? Are students making connections between their peers' work on the board? Are students sharing their observations of other peoples' work? Did students identify the relationships: • Between the solution and the expression • Between the horizontal intercepts and the solution. • Other relationships e.g. the y intercept and the height when $t = 0$. Have students thought to set up an equation relationship. Having set up an equation do students have any ideas how to proceed? Do students know that id the product of two numbers/expressions is zero then one/both of those numbers/expressions must be zero? Do students realise that $t - 2 = 0$ when $t = 2$ because $2 - 2 = 0$, and t –

	quickest? Which method would you use in future? How could we describe this method to someone else.	6 = 0 when t = 6 because 6 – 6 = 0. Do students know that students could use methods for solving linear equations to solve for t when given t – 2 = 0 or t – 6 = 0.
Summing up & Reflection Today we have learned that there are multiple ways to find a solution to a problem. We have discussed and used Trial and Improvement, Functions, Tables and Graphs. We have discovered that we can set a quadratic equation and solve it algebraically. We have learned that when a quadratic equations, (such that the expression is equal to zero) is written using factors we can use those factors to easily find the solution by letting each factor equal zero. Snowball Activity. **Homework:** Students will be given one equations to solve for homework. The question will include a trinomial expression. Students are asked to find solutions to the equation in as many ways as they can.	Teacher will use the board work to briefly review the Trial and Improvement/Functions approaches. The teacher will dwell on the method for solving quadratic equations algebraically. Teacher will give students the opportunity to reflect on their learning experience this lesson. "What did you learn? How did you learn? What did you enjoy? What did you not enjoy?" Teacher will provide paper for student reflection. Students will through comments at their teacher.	Have students made the connections we wanted them to? Do students: 1. Know that the value(s) at which the graph crosses the horizontal axis give the solution to the equation. 2. Understand the method for solving quadratic equations algebraically and can they use it. Did students enjoy the lesson? What about this lesson did the students enjoy? Did they learn?

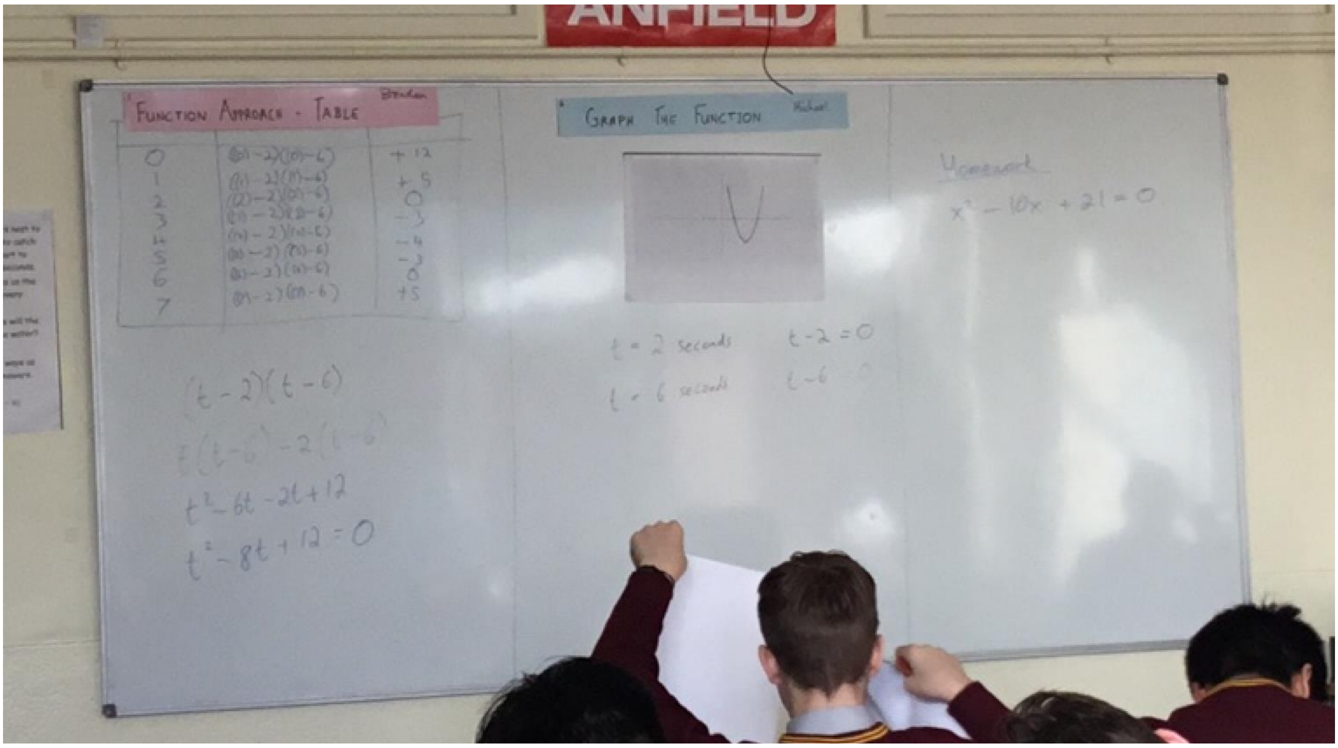
10. Board Plan

Question on poster adjacent to whiteboard



Anticipated layout of student work

Trial and Error	Functions (input/output) (t-2)(t-6) | t^2 - 8t + 12	Equating the Function to zero
Multiplying Out Brackets	Functions Table (t-2)(t-6) | t^2 - 8t + 12	Equating the factors to zero
Trial and Improvement with the Trinomial	Graphing the function	Solving the equation to get roots.



11. Evaluation

a. Did students understand the question?
The teacher used good questioning to assess whether the students understood the question and prompted students to clarify what was meant by sea level, the height of sea level, the height of the bird at sea level is h = 0, etc. During the observation it became clear that some students included t = 8 and others did not. During Ceardaíocht the class discussed this in detail and various students tried to put forward convincing arguments. The class eventually agreed that because ‘less than 8 seconds’ was mentioned in the question instead of ‘less than or equal to 8 seconds’ that 8 should not be included. Some students did not consider t = 0 as part of their domain. The teacher and students discussed what the coordinate t = 0 represents, in this case the bird’s nest. This enhanced students’ understanding of the y-intercept.

b. Did the materials given work well with the lesson?
The students were given a broad selection of resources. Students were more inclined to use the resources that they were more familiar with. They adapted the table to include an extra column as they were more accustomed to using a table with 4 columns, to accommodate couples at the end. Those that drew the curve used the graph paper as opposed to the labelled axes given.

c. Did the timing of the lesson work out?
We all agreed that 10 minutes was not long enough to see the variety of responses we anticipated but it was an appropriate amount of time to focus their efforts and be able to contribute to the Ceardaíocht. The Ceardaíocht was very well timed. The teacher brought several students to the board to show their individual responses. The teacher used appropriate questioning to draw out students’ knowledge and understanding in order to allow them to make new insightful connections. As a result of the teacher guiding the students through the Ceardaíocht students were able to instruct each other and learn from each other. Students were brought to the board and given the opportunity to explain their new taught processes. The Ceardaíocht was well timed which gave the teacher an opportunity to set appropriate homework and get student evaluations (Snowball).

d. Did the students come up with the anticipated answers?
Of the anticipated answers outlined previously only two were approached by the students. We are of the opinion that given more time students may have achieved more of the responses. We were surprised that students didn't try a Trial and Improvement method but we put this down to recently learning functions. Had this question been given to a different class there would likely have been more Trial and Improvement responses.
We also found it quite interesting that no student expanded the brackets to get a trinomial expression which they would have been more used to seeing in functions questions. This was later discussed during Ceardaoicht and students made strong connections between the solutions, the factors and the trinomials.

e. Is there anything about the lesson that could be improved on?
We think there is greater scope to emphasise the y-intercept by including it as a part of the question or as an extension to the question. It was discussed during Ceardaíocht but could have been mentioned in the problem.
Although it did not cause a problem or any confusion in the class we are aware that no unit of measurement for height was indicated in the question.

12. Reflection

After the research lesson, the team should write a reflection, which will normally include:

a) What we had hoped to observe during the lesson:
We hoped the students would make connections between functions and quadratic expressions. We didn't want students to see these as isolated topics and to team them. We believed that this would be achieved through medium term planning and that this research lesson would the culmination of that planning.
We wanted the students to see a connection between the factors and roots.

b) What we observed during the lesson:
The students all started by finding couples and drawing the graph. No student found the solutions from the table alone, which was possible. Although they understood the problem from the start they got too engrossed in the procedure of filling out the table and drawing the graph to relate back to the initial question. During Ceardaíocht their attention was brought back to the table and many, not all, were then able to see the answer there.
Timing worked well. Students contributed well by either coming to the board or by asking/answering good questions. Students enjoyed going to the board to demonstrate their knowledge and understanding. They also enjoyed learning from each other as evidenced in the feedback at the end of the lesson. Some students commented that they felt challenged and that they enjoyed that challenge, "What I liked about today is I was challenged".

c) major points raised during the post-lesson discussion, and the team's own opinions;
The teacher was very clear with the students and the team all agreed that the intentions of the lesson were all achieved. We were impressed with how the students engaged with the lesson, how they contributed to lesson, their ability to pose well thought out questions and to answer each other's questions in a group discussion. We noted that students were very good at articulating their ideas using the correct mathematical terminology.
We discussed the many ways that the context of the lesson could be further exploited in future lessons e.g. inequalities, maxima, minima, inequalities, completing the square etc.
We think that the students found this lesson memorable. The teacher has since found the context of the question useful in class. She can often refer back to the seagull when explaining

something to her students and it aids their understanding.

d) ideas for future study.
The problem can be adapted in a number of ways to explore other areas of functions, algebra and perhaps calculus.

Although this class was designed specifically for a 2^{nd} Year Higher Level class we believe there is scope for it to be adapted for an Ordinary Leaving Certificate Class.

13. Benefits of Participating in Lesson Study

Participant A
"The Lesson Study was of great benefit both to me and my class. It helped me think of alternative ways to teach a commonly misinterpreted topic, and the students having approached it from a functions perspective have a much more meaningful understanding of both quadratics and functions, and understand that they are related to each other, not two separate topics.

I will definitely be using the lesson again with both Junior and Senior students as I think both the lesson and the preparations before (and indeed after) are very understandable and easy to use. It applies "real life quadratics" for the students, which at the end of the day is what we want them to take home from it."

Participant B
"What I really got from the lesson study was the following:

- A clearer understanding of the terminology associated with the new Junior Cert Program.
- Good ideas about approaches to teaching through collaboration with my colleagues.
- An insight into a new approach to how to teach problem solving which allows the students to draw their own conclusions and which allows them to make connections themselves throughout the course of the lesson.
- How a lesson based entirely on learner outcomes really works."

Appendix 1

Quality Framework for Post-Primary Schools – Teaching & Learning

Learner outcomes	Students enjoy their learning, are motivated to learn, and expect to achieve as learners	☐
	Students have the necessary knowledge and skills to understand themselves and their relationships	☐
	Students demonstrate the knowledge, skills and understanding required by the post-primary curriculum	☐
	Students attain the stated learning outcome for each subject, course and programme	☐
Learner experiences	Students engage purposefully in meaningful learning activities	☐
	Students grow as learners through respectful interactions and experiences that are challenging and supportive	☐
	Students reflect on their progress as learners and develop a sense of ownership of and responsibility for their learning	☐
	Students experience opportunities to develop the skills and attitudes necessary for lifelong learning	☐
Teachers' individual practice	The teacher has the requisite subject knowledge, pedagogical knowledge and classroom management skills	☐
	The teacher selects and uses planning, preparation and assessment practices that progress students' learning	☐
	The teacher selects and uses teaching approaches appropriate to the learning intention and the students' learning needs	☐
	The teacher responds to individual learning needs and differentiates teaching and learning activities as necessary	☐
Teachers' collective / collaborative practice	Teachers value and engage in professional development and professional collaboration	☐
	Teachers work together to devise learning opportunities for students across and beyond the curriculum	☐
	Teachers collectively develop and implement consistent and dependable formative and summative assessment practices	☐
	Teachers contribute to building whole-staff capacity by sharing their expertise	☐

Junior Cycle Key Skills and Statements of Learning

Key Skills

KS1	Managing myself
KS2	Staying well
KS3	Monitoring information & thinking
KS4	Being numerate
KS5	Being creative
KS6	Working with others
KS7	Communicating
KS8	Being literate

Statements of Learning

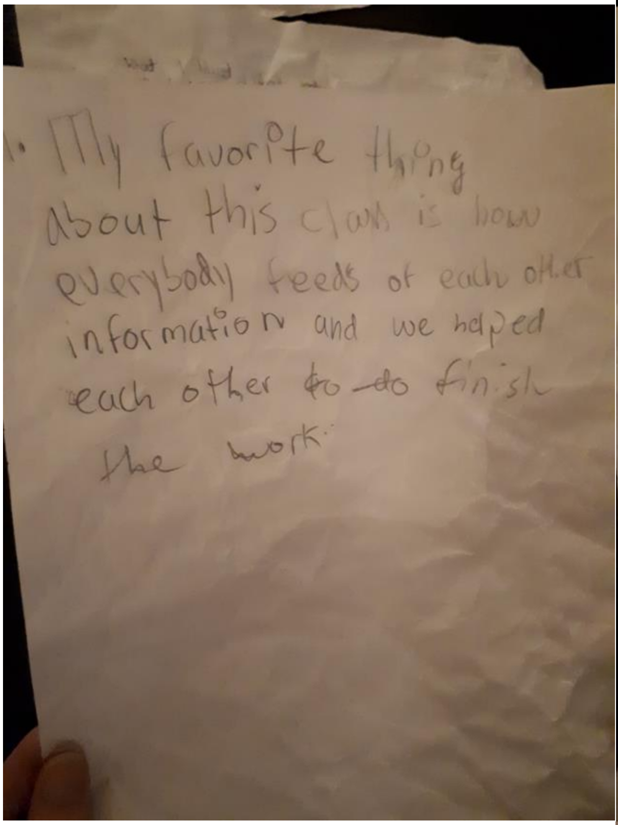
	The student
SL1	communicates effectively using a variety of means in a range of contexts in L1*
SL2	listens, speaks, reads and writes in L2* and one other language at a level of proficiency that is appropriate to her or his ability
SL3	creates, appreciates and critically interprets a wide range of texts
SL4	creates and presents artistic works and appreciates the process and skills involved
SL5	has an awareness of personal values and an understanding of the process of moral decision making
SL6	appreciates and respects how diverse values, beliefs and traditions have contributed to the communities and culture in which she/he lives
SL7	values what it means to be an active citizen, with rights and responsibilities in local and wider contexts
SL8	values local, national and international heritage, understands the importance of the relationship between past and current events and the forces that drive change
SL9	understands the origins and impacts of social, economic, and environmental aspects of the world around her/him
SL10	has the awareness, knowledge, skills, values and motivation to live sustainably
SL11	takes action to safeguard and promote her/his wellbeing and that of others
SL12	is a confident and competent participant in physical activity and is motivated to be physically active
SL13	understands the importance of food and diet in making healthy lifestyle choices
SL14	makes informed financial decisions and develops good consumer skills
SL15	recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning
SL16	describes, illustrates, interprets, predicts and explains patterns and relationships
SL17	devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills
SL18	observes and evaluates empirical events and processes and draws valid deductions and conclusions
SL19	values the role and contribution of science and technology to society, and their personal, social and global importance

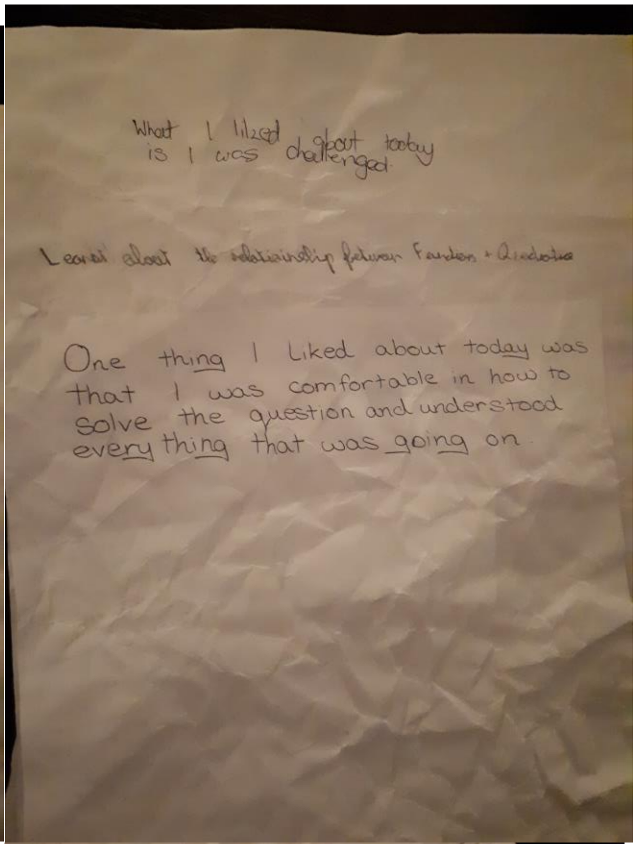
SL20	uses appropriate technologies in meeting a design challenge
SL21	applies practical skills as she/he develop models and products using a variety of materials and technologies
SL22	takes initiative, is innovative and develops entrepreneurial skills
SL23	brings an idea from conception to realisation
SL24	uses technology and digital media tools to learn, communicate, work and think collaboratively and creatively in a responsible and ethical manner

L1 is the language medium of the school (Irish in Irish-medium schools). L2 is the second language (English in Irish-medium schools).

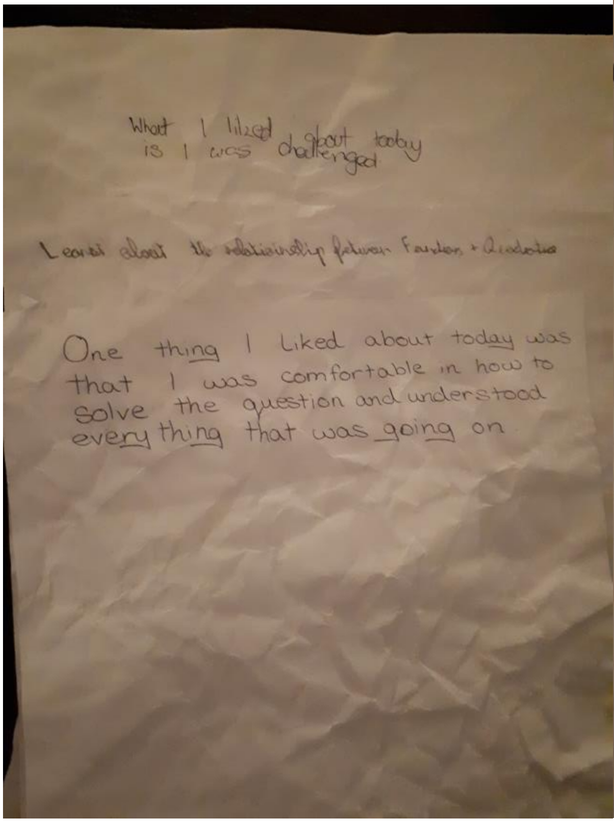
Appendix 2

Student Feedback

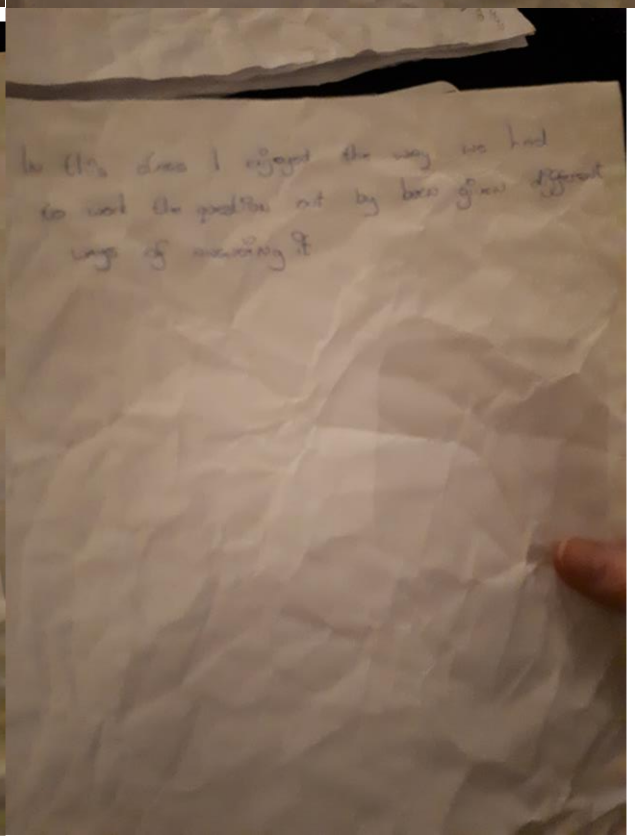

1. My favorite thing about this class is how everybody feeds of each other information and we helped each other to ~~do~~ finish the work.

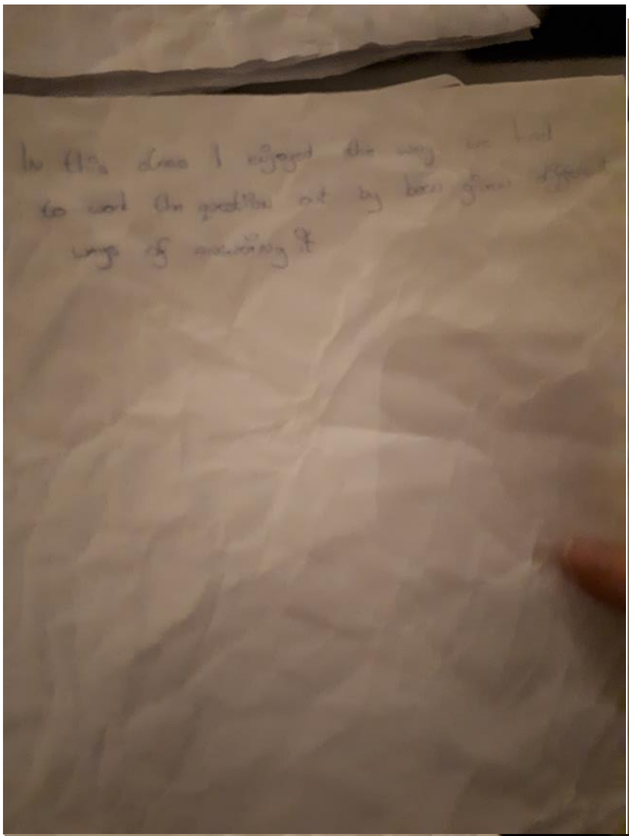

What I liked about today is I was challenged

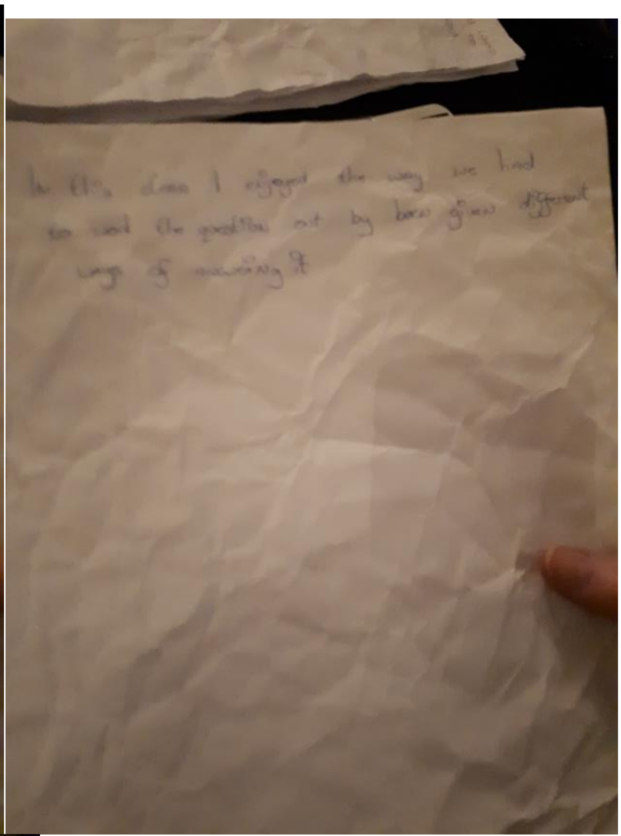
One thing I Liked about today was that I was comfortable in how to solve the question and understood everything that was going on.

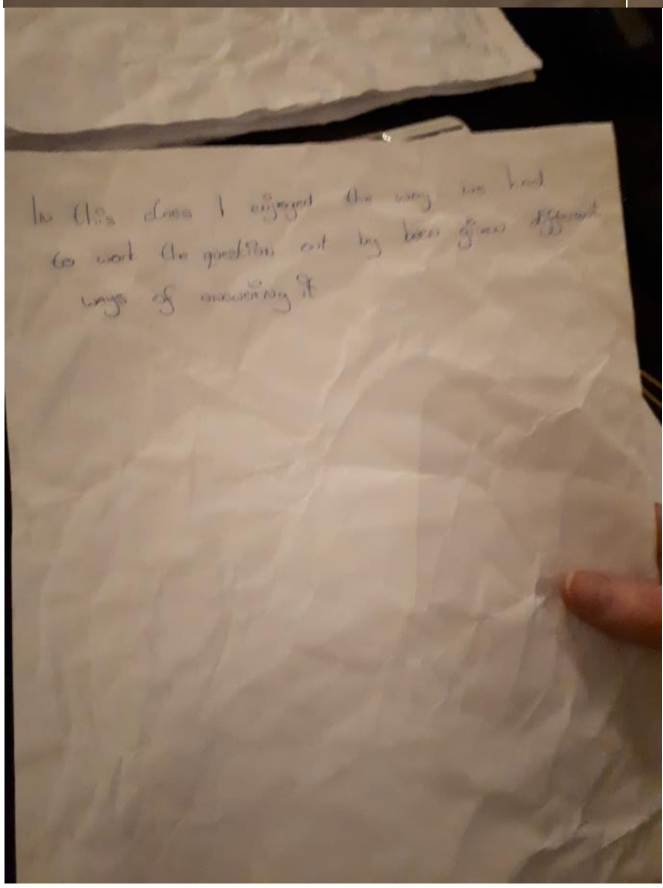

What I liked about today is I was challenged

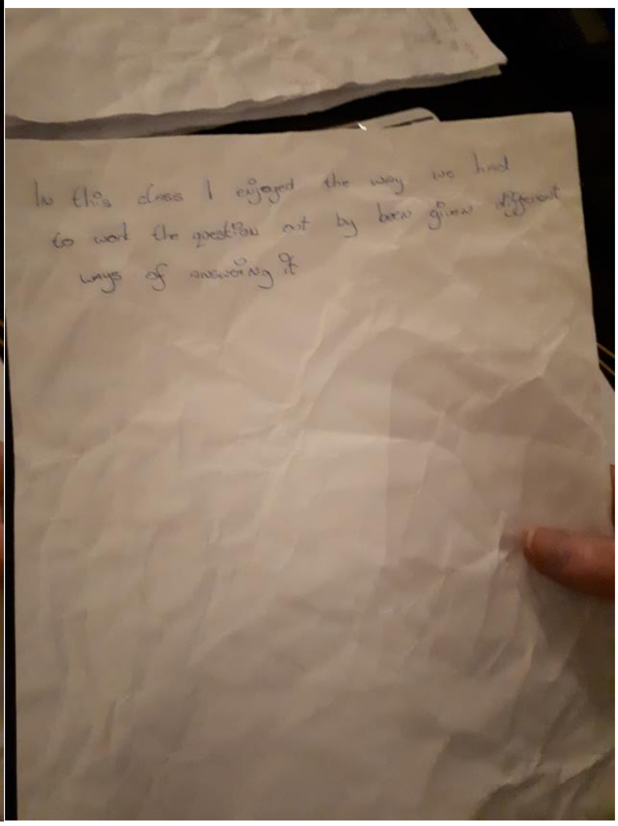
One thing I Liked about today was that I was comfortable in how to solve the question and understood everything that was going on


In this class I enjoyed the way we had to work the question out by being given different ways of answering it


In this class I enjoyed the way we had
to work the question out by been given different
ways of answering it

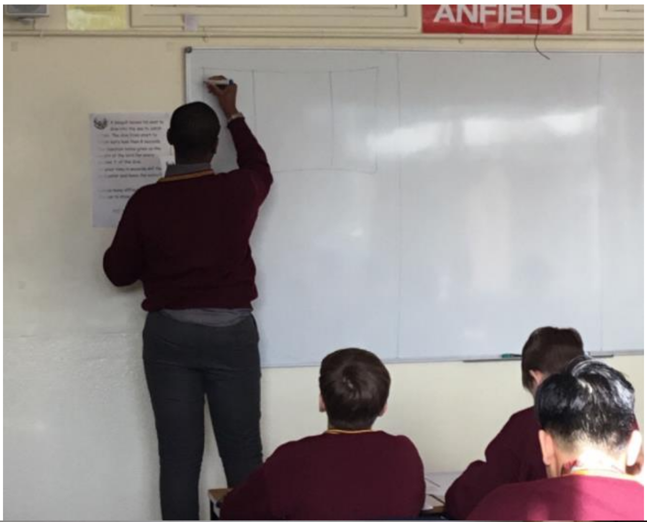

In this class I enjoyed the way we had
to work the question out by been given different
ways of answering it

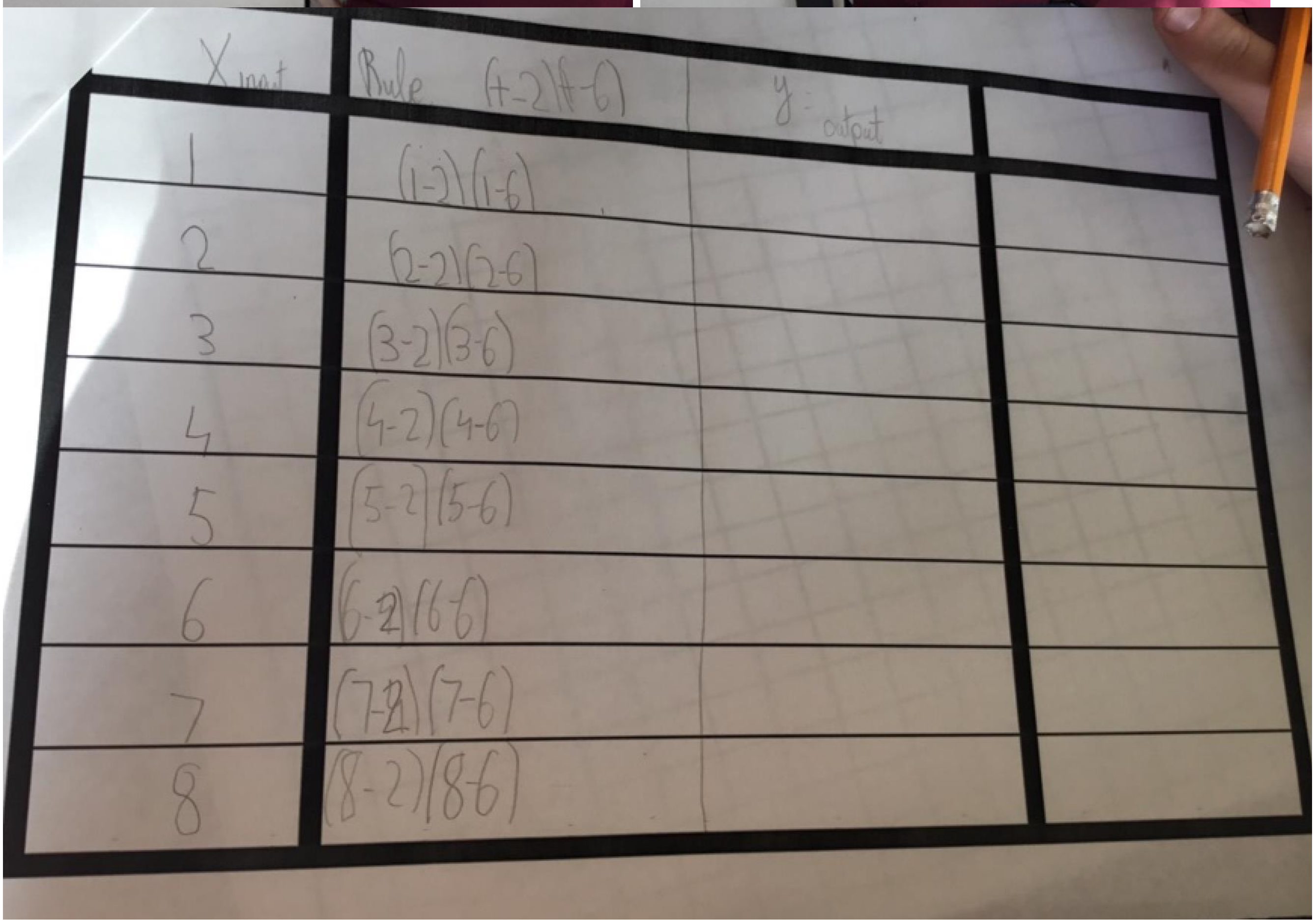

In this class I enjoyed the way we had
to work the question out by been given different
ways of answering it

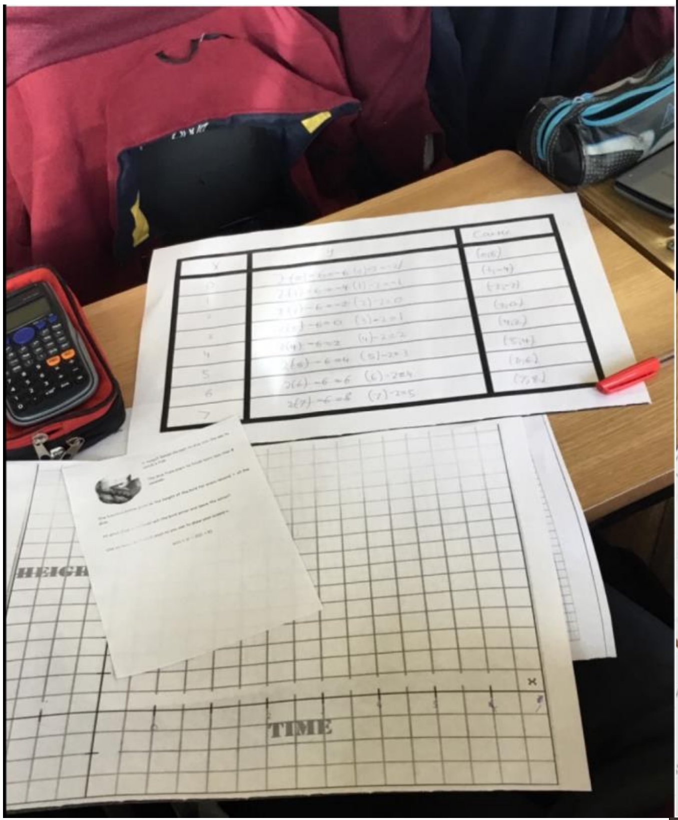

In this class I enjoyed the way we had
to work the question out by been given different
ways of answering it

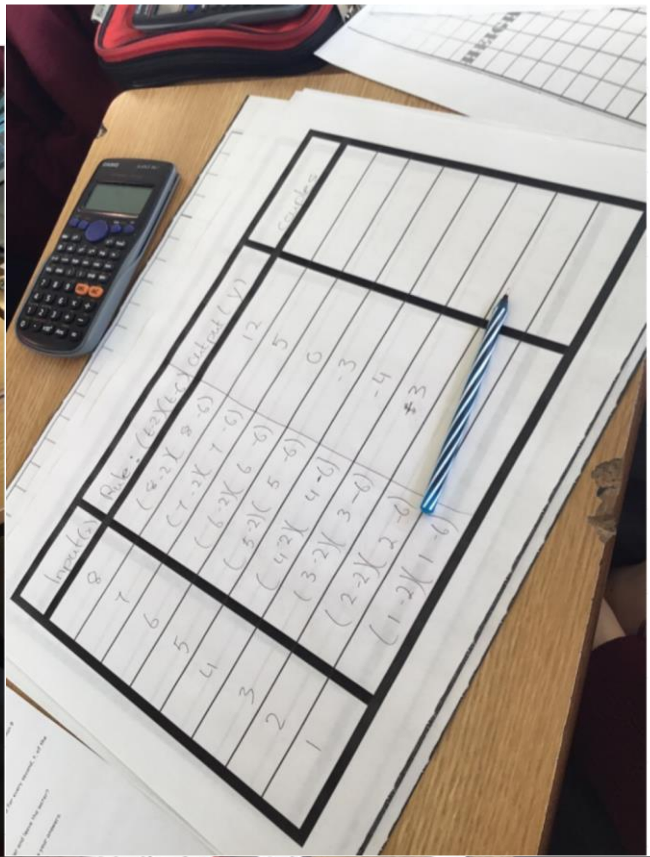
Appendix 3

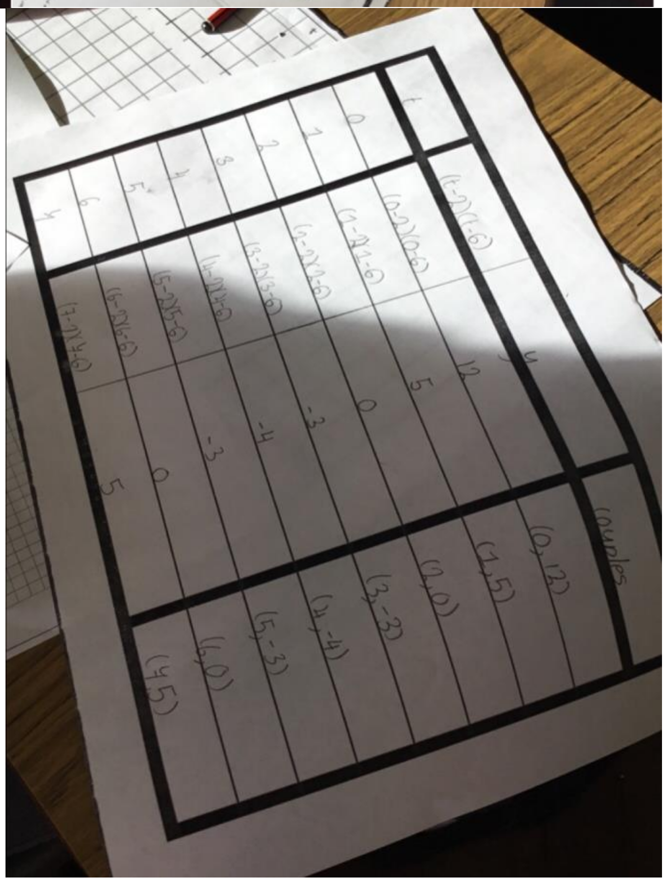
In Class Photos

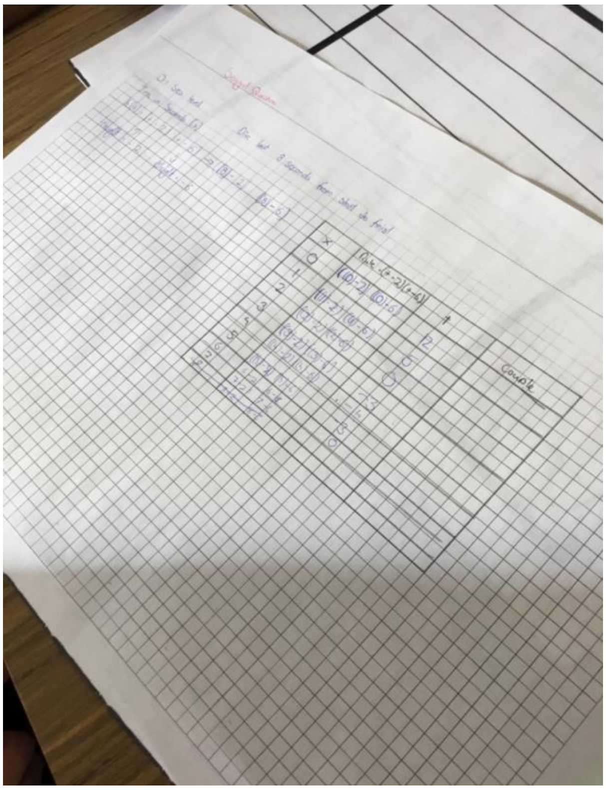




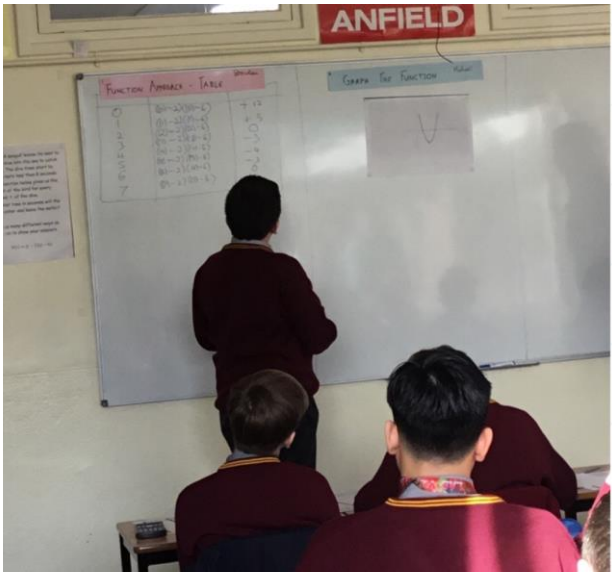


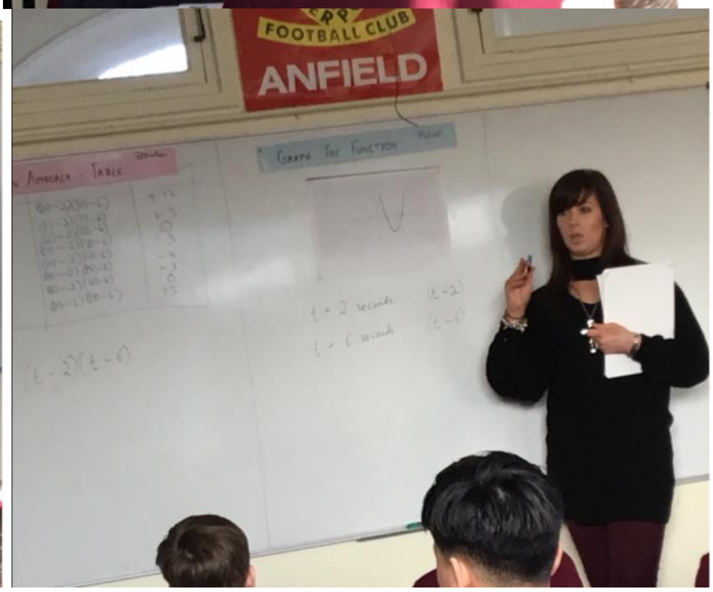


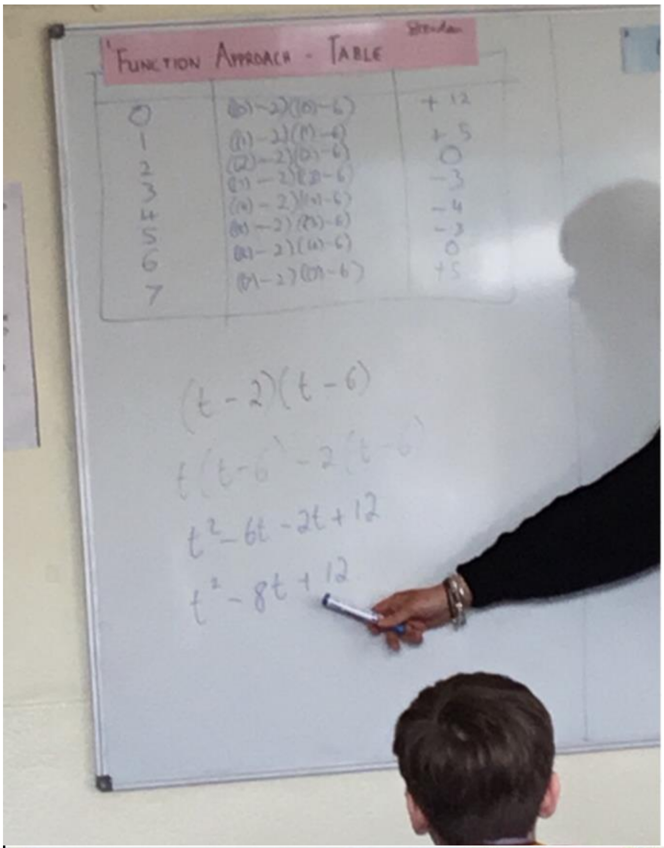


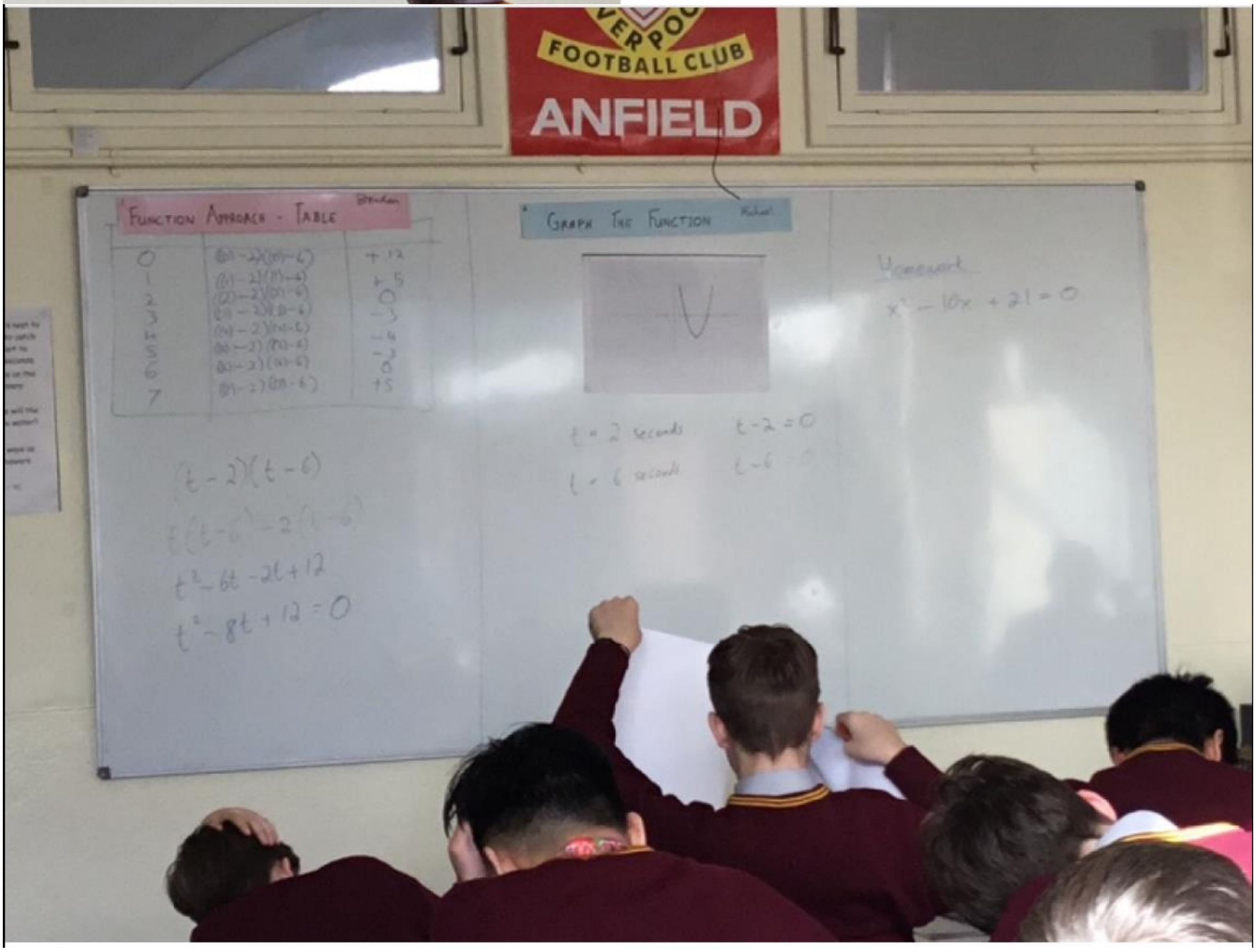



FOOTBALL CLUB
ANFIELD


ANFIELD


FOOTBALL CLUB
ANFIELD


FUNCTION APPROACH - TABLE
$(t-2)(t-6)$
$t^2-6t-2t+12$
$t^2-8t+12$


FOOTBALL CLUB
ANFIELD
FUNCTION APPROACH - TABLE
GRAPH THE FUNCTION
Homework
$x^2-10x+21=0$
t = 2 seconds
t - 2 = 0
$t^2-8t+12=0$