

Lesson Research Proposal for First Years – Multiplying Messi’s Millions

For the lesson on Wednesday 24th January 2018

At St. Mary’s Academy, Ms. N. Merrick’s class

Instructor: Ms. Merrick

Lesson plan developed by: Ms. Merrick, Ms. Fenelon & Ms. Maguire

1. Title of the Lesson: *Multiplying Messi’s Millions.*

2. Brief description of the lesson

In this lesson students will investigate models such as the number line (directed numbers) including the accumulation of groups of equal size approach, investigate the properties of arithmetic (commutative, associative & distributive), and use the concept of a “thought experiment” in real life contexts to illustrate the operation of multiplication in \mathbb{Z} . In particular students will try to make sense of the law that governs the operation of multiplication of a negative number by a negative number by revisiting previous learning on number and number operations in an effort to develop their understanding of different number types and their properties. Students will learn that the multiplication of a negative number by a negative number results in a positive number.

3. Research Theme

At St. Mary’s Academy, St. Mary’s College & Ardscoil na Tríonoide we want our teachers to:

- a) Have the requisite subject knowledge, pedagogical knowledge and classroom management skills. In particular in daily classroom teaching we want our teachers to feel confident that they can cater for different abilities by providing **all** students with effective learning and teaching that meets their progress in learning through the provision of an inclusive learning environment.
- b) Value and engage in professional development and professional collaboration.

As a Mathematics department, we will actively support the achievement of these goals in the following ways:

a) **Subject Knowledge**

It is expected that all teachers will have a deep understanding of all subject material they are responsible for teaching. Teachers are expected to promote student recognition and understanding of links between topics to students and to highlight everyday uses and examples of each mathematical topic in everyday life. Where required, teachers are expected to deepen their knowledge of certain concepts and ideas using personal research and collaboration with teachers both

internal and external to their subject department (ie. Lesson Study).

b) Pedagogical Knowledge

While good in depth knowledge of one's subject is central to effective learning and teaching, our Mathematics Department also recognize that knowledge of a wide variety of methodologies are essential to allowing the subject of mathematics to appeal to all students. Teachers are encouraged to engage in CPD to familiarize themselves with current and developing methodologies with a view to increasing student interest in and engagement with the subject, and in turn, appealing to students of all abilities.

c) Common Methodologies

It is agreed that in the coming academic year we will use structured problem solving to introduce new concepts to students. We aim to encourage students to develop resilience and creativity through discovery learning. The use of problem solving to discover key mathematical concepts and skills aims to promote a deeper understanding of these ideas among students, as well as to promote long term learning of mathematical skills.

4. Background & Rationale

a) Why we chose this topic

This lesson is aimed at first/second year students. Over the past few years significant changes have occurred in the learning and teaching of mathematics in our second level schools. Greater emphasis is now being placed on developing a conceptual understanding of mathematics.

Yet for many the learning and teaching of the operations of addition, subtraction, multiplication and division in Z remain abstract in nature. This situation is exacerbated through the use of drill and practice exercises and the rote learning of 'sign rules'. As students' progress through the mathematics syllabus (i.e. algebra) deficiencies in their conceptual understanding of the operations of addition, subtraction, multiplication and division in Z become more evident and ultimately problematic.

b) Our research findings

Our discussions in this area have led us to conclude that student's interactions with the operations of addition, subtraction, multiplication and division in Z are very traditional, teacher-led and imbalanced through the use of drill and practice exercises of 'sign rules'. It is rare that students are presented with opportunities to fully understand or interact with concept of addition, subtraction, multiplication and division in Z and the type of situation in which it arises.

To address this we plan to introduce the operation of multiplication in Z in the context of a problem, where students can use their prior knowledge of the properties of arithmetic, together with their knowledge of the number line and arising from their work in the area of 'thought experiments', develop a deeper understanding of the operation of multiplication in Z with a particular emphasis on multiplying a negative number by a negative number. It is hoped that by developing a deeper understanding of the underlying concepts behind the operation of multiplication in Z , students will retain this information for longer, making less memorization errors and be flexible enough in their thinking to apply their knowledge across the entire mathematics syllabus as required.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
At primary level, students are introduced to the concept of positive and negative numbers. The notation encountered by students at primary level is different to the standard notation used at post-primary level. For example, $+5 + -7$, is read as positive 5 and negative 7. It may be an idea to start students with this notation initially when dealing with integers. For example $-6 + -7$ (read as negative 6 plus negative 7) gives -13 (read as negative 13). (Teaching and Learning Plans Integers, MDT).	Investigate models such as the number line, (directed numbers), investigate the properties of arithmetic (commutative, associative & distributive) and use the concept of a “thought experiment” in real life contexts to illustrate the operation of addition, subtraction, multiplication and division in Z .	Application of the operations of addition, subtraction, multiplication and division in Z throughout the mathematics syllabus, and in particular, the application of the Distributive Law in algebra e.g. Factorization of quadratics.

6. Goals of the Unit

- Students will understand that some types of problems give rise to mathematical sentences with both positive and negative numbers.
- Students will understand the operations of addition, subtraction, multiplication and division in Z .
- Students will be apply the operations of addition, subtraction, multiplication and division in Z to solve real life problems.
- Students will verbalize their understanding of the operations of addition, subtraction, multiplication and division in Z .

7. Unit Plan

Lesson	Learning goal(s) and tasks
1	Operations of addition and subtraction in Z
2	Operation of multiplication in Z Positive x positive; positive x negative; negative x positive
3 Research Lesson	Use a suitable problem to illustrate the Operation of multiplication in Z Negative x negative
4	Operation of division in Z
5	Order of operations
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6	Overview & Application
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8. Goals of the Research Lesson:

The Goals of the lesson should refer to:

a) Mathematical Goals

Students will:

- a) Understand that some types of problems give rise to mathematical sentences necessitating the operation of multiplication of negative by negative in Z .
- b) Understand that the operation of multiplication of negative by negative in $Z =$ positive.
- c) Understand that models such as the number line (directed numbers), investigation of the properties of arithmetic (commutative, associative & distributive) and use the concept of a “thought experiment” in real life contexts to illustrate the operation of multiplication of negative by negative in Z .

b) Key Skills and Statements of Learning

In preparation for implementing the Junior Cycle Specifications for Mathematics our mathematics department have begun to integrate the development of key skills into mathematics lessons.

This lesson will address the following key skills:

1. Being Literate: through Ceardaiocht, students will have the opportunity to express their ideas clearly and accurately.
2. Being Numerate: by engaging in suitable tasks, students will develop a positive attitude towards investigating, reasoning and problem solving.
3. Managing myself: student s will have the opportunity to reflect on their own learning when the teacher asks them to write a reflection at the end of the lesson.
4. Staying well: by engaging in tasks that are appropriate to their abilities, students’ confidence and positive disposition to learning will be promoted.
5. Being creative: students will explore options and alternatives as they actively participate in the construction of knowledge.
6. Working with others: student s will learn with and from each other by discussing different approaches to solving problems.
7. Managing information and thinking: students will be encouraged to think creatively and critically.
8. Communicating: During Ceardaiocht, students will present and discuss their mathematical thinking.

This lesson also meets the following JC Statements of Learning:

1. The student communicates effectively using a variety of means in a range of contexts.
15. The student recognizes the potential use of mathematical knowledge, skills and understanding in all areas of learning.
17. The student devises and evaluates strategies for investigating and solving problems

using mathematical knowledge, reasoning and skills.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student reactions	Teacher Support	Assessment
<p>Introduction (5 mins)</p> <p>Before we start today's lesson, I would like to recap over some of the maths methods that we have learned using the thought experiment idea in the past few weeks. In particular: Using the number line, decomposition of numbers and accumulation of groups of equal size.</p> <p>What rules do we already know about multiplying integers?</p> <ul style="list-style-type: none"> - A positive number multiplied by a positive number will result in a positive number - A positive number multiplied by a negative number results in a negative number <p>Using your knowledge of the previous rules that we have discovered, let's revise these ideas with a simple problem.</p>	<p>Place an image for positive, negative and multiplication signs on the whiteboard.</p> <p>Support this by placing the rules on the board using symbolic language.</p>	<p>Can students verbalise their learning so far?</p>
<p>Posing the Task 1</p> <p>Adidas have made a deal with Lionel Messi. They will pay Messi €10,000 for every match that he wears their brand of football boots. He</p>	<p>Present a picture of Messi wearing the football boots, and a picture of a bank vault on the board.</p>	<p>Can students carry out a simple revision task on the laws of multiplying integers that they have learned so far?</p>

<p>wears the football boots for 6 matches. Given that the starting balance is €0, in as many different ways as possible can you show the effect these payments will have on Adidas' bank account?</p> <p>Represent the above information by compiling a mathematical expression describing the effect that this deal will have on Adidas' bank account.</p>	<p>Place a positive and a negative sign in the appropriate positions.</p>	
<p>Student Individual Work 1</p>	<p>Use your seating chart to record the model used by each student. Note the order in which you will call each student up during Ceardaiocht.</p> <p>If students are stuck, help them by asking appropriate questions e.g. Is money coming in or going out of Adidas' bank account?</p>	<p>Are students able to tackle the problem?</p> <p>Are students able to develop a mathematical expression for this problem?</p> <p>Do students understand that money being removed from the bank account is represented by a negative sign?</p> <p>Do students recognise that the repeated nature of this transaction is represented by multiplication?</p>
<p>Ceardaiocht/Comparing and Discussing 1</p> <p>There are multiple models that students could use to</p>	<p>When students present work at the board, make sure to attach their name to it.</p>	<p>Can students explain their approach?</p> <p>Do students recognise similarities/differences</p>

<p>calculate the effect on Adidas' bank account.</p> <p>Ask specific students to come to the board and explain how they calculated the effect on Adidas' bank account.</p> <p>Student Response 1 – Number line (This involves the idea of multiplication being a journey of x units)</p> <p>Student Response 2 – Distributive Law (This involves splitting the sum up into its positive and negative components)</p> <p>Student Response 3 – Accumulation of groups of equal size (This involves breaking the numbers up into smaller units and using the journey along the numberline method with these smaller units)</p>	<p>Ask the students to raise their hand if they used this method.</p> <p>Did anyone use a different approach?</p> <p>Any other methods used?</p>	<p>between their approach and that presented on the board?</p> <p>Do students offer alternative approaches to solving the problem?</p> <p>Can students verbalise their understanding of why a negative multiplied by a positive resulted in a negative?</p>
<p>Posing the Task 2</p> <p>During the seventh match, Messi wears Nike boots and this breaks his contract with Adidas. As a penalty, Adidas removes the €10,000 payment per match for the next seven matches, meaning that Adidas have seven less payments to make. Hint: Use what you know about how to write mathematical expressions to help you get started!</p>	<p>Place a picture of Messi wearing the wrong boots on the board.</p> <p>Place the symbols for positive, negative and multiplication on the board, along with €, 10,000 and 7. Ask the students to rearrange all symbols into a mathematical sentence to represent the problem posed.</p>	<p>Do students understand the task?</p>

<p>I want you to now work on coming up with your mathematical expression to describe the changes to Adidas' bank balance as a result of this deal (students work individually). After 5 minutes, we will come together as a class and use your work to help us in solving problem three. (This section focusses on students developing a sentence to describe what affect this will have on Adidas' bank balance)</p> <p>Try your best!</p>		
<p>Student Individual Work 2</p>	<p>Use your seating chart to record the mathematical sentence used by each student. Note the order in which you will call each student up during Ceardaiocht.</p> <p>If students are stuck, help them by asking appropriate questions e.g. What was happening before Messi broke the contract? What is happening now? In the past Adidas agreed to pay Messi €10,000 for each match (this was represented by a negative 10,000), but the fact that he has broken his contract removes their need to do this and seven less payments will be made.</p>	<p>Are students able to tackle the problem?</p> <p>Are students able to develop a mathematical expression for this problem?</p> <p>Do students understand that money being removed from the bank account is represented by a negative sign?</p> <p>Do students recognise that the repeated nature of this transaction is represented by multiplication?</p> <p>Are students able to develop and verbalise their understanding of the problem?</p>

<p>Problem 3</p> <p>In how many different ways can you show the effect that this will have on Adidas' bank account? (This section of the task focuses on the students actually solving the problem).</p> <p>Clarifying the problem</p> <p>Let's go through the problem to make sure that everyone understands it.</p> <p>How does this problem compare to Problem One?</p> <p>How is it similar?</p> <p>How is it different?</p> <p>I want you to now work on problem 3 individually. Try your best. If you get stuck, talk to the person beside you.</p>		<p>Do students understand how this problem is similar to the previous problem?</p> <p>Do students understand how this problem is different to the previous problem?</p>
<p>Student Individual Work 3</p>	<p>Use your seating chart to record the model used by each student. Note the order in which you will call each student up during Ceardaiocht.</p> <p>If students are stuck, help them by asking appropriate</p>	<p>Are students able to tackle the problem?</p> <p>Are students able to develop a mathematical expression for this problem?</p>

	<p>questions e.g. Is money coming in or going out of Adidas' bank account?</p>	<p>Do students understand that money being removed from the bank account is represented by a negative sign?</p> <p>Do students recognise that the repeated nature of this transaction is represented by multiplication?</p>
<p>Ceardaíocht/Comparing and Discussing 3</p> <p>There are multiple models that students could use to calculate the effect on Adidas' bank account.</p> <p>Ask specific students to come to the board and explain how they calculated the effect on Adidas' bank account.</p> <p>Student Response 1 – Number line (the idea of representing multiplication as a journey along a numberline)</p> <p>Student Response 2 – Distributive Law (This involves splitting the sum up into its positive and negative components)</p> <p>Student Response 3 – Accumulation of groups of equal size (using manipulatives such as Unifix</p>	<p>When students present work at the board, make sure to attach their name to it.</p> <p>Ask the students to raise their hand if they used this method.</p> <p>Did anyone use a different approach?</p> <p>Any other methods used?</p>	<p>Can students explain their approach?</p> <p>Do students recognise similarities/differences between their approach and that presented on the board?</p> <p>Do students offer alternative approaches to solving the problem?</p> <p>Can students verbalise their understanding of why a negative multiplied by a negative resulted in a positive?</p>

<p>cubes to represent multiplication)</p> <p>Student Response 4 – Thought Experiment (This involves students using verbal reasoning to solve the problem i.e. ‘I’m not doing nothing’ means ‘I’m doing something’)</p> <p>What do we notice from each of the methods that you used? What happens when we multiply a negative number by a negative number?</p> <p>Can you draw a conclusion about multiplying a negative number by a negative number from your solutions?</p>		
<p>Summing up and Reflecting</p> <p>We learned that:</p> <ul style="list-style-type: none"> • A negative multiplied by a negative results in a positive • Negative by negative can occur in real life situations <p>Can you think of where we might see a problem like this in everyday life? Where might we see a negative number multiplied by another negative number? (If required students may need to be prompted to consider the removal of debt repayments)</p>		<p>Do the student’s reflections represent the teacher’s view of the lesson?</p>

numbers together would occur e.g. The removal of a debt.

The students were very engaged right throughout the lesson. It is likely that this is due to the subject of the lesson being based on Lionel Messi and his football boots, an area in which the students had a keen interest. As a result of this interest, it was observed that students were more engaged in the lesson and in turn, more likely to participate fully with the task.

The students used a variety of methods to demonstrate the process of multiplication. The most common method employed by the students was the idea of 'a journey along a number line', and it was noted that more reluctant learners in the group used the 'thought experiment' method i.e. Reasoning out the problem using an English sentence rather than using more traditional forms of maths. We found that the Distributive Law method was used by very few students. One pair of students solved the problem using a trend graph, and despite being correct, this was a not a solution that we had anticipated.

The lesson could have been improved by providing more explicit explanation of the term mathematical expression. Student feedback during the lesson indicated that they were initially unsure of the full meaning of this term, however, once it was explained to them, they were able to complete the task quite easily.

It was also felt that the task that we posed to the student could have been rephrased to avoid some confusion. At the beginning of Task 2, we should have stated clearly that a) Messi was not paying Adidas any money, rather they were forgoing or 'losing their need' to pay Messi and b) to assume that Adidas' bank balance was to return to €0 at the beginning of Task 2. This led to some confusion, as students were linking their answer from Task 1 to Task 2 which is not what was intended. It was also noted that this lesson would have been more suited to a 1st Year group who were encountering these properties of multiplication for the first time. The effectiveness of the problem was limited due to the students already being aware of the rule 'negative by negative equals positive'.

Thirdly, the concept of the journey along the number line should have being more explicitly explained during the Ceardaiocht phase of the lesson. The students understood the idea, but found it tricky to apply to more abstract examples of negative by negative multiplication. More emphasis on the explanation of this concept would have consolidated the idea in the students' minds in a more meaningful way, and would have allowed them to carry it forward to aid their understanding of more abstract examples.

Finally, it was observed throughout all three tasks during the lesson that students were unlikely to try to attempt to find more than one solution to the problem. It is likely that this is due to the way that maths is taught in Ireland. There is a strong emphasis on obtaining the answer, and very little importance placed on the method with which this answer has been achieved. This could easily be corrected with a common approach that emphasizes the importance of the process not the product. This change in mindset is key to a more open approach to the effective learning and teaching of mathematics.

Appendix 1

Quality Framework for Post-Primary Schools – Teaching & Learning

Learner outcomes	<p>Students enjoy their learning, are motivated to learn, and expect to achieve as learners <input type="checkbox"/></p> <p>Students have the necessary knowledge and skills to understand themselves and their relationships <input type="checkbox"/></p> <p>Students demonstrate the knowledge, skills and understanding required by the post-primary curriculum <input type="checkbox"/></p> <p>Students attain the stated learning outcome for each subject, course and programme <input type="checkbox"/></p>
Learner experiences	<p>Students engage purposefully in meaningful learning activities <input type="checkbox"/></p> <p>Students grow as learners through respectful interactions and experiences that are challenging and supportive <input type="checkbox"/></p> <p>Students reflect on their progress as learners and develop a sense of ownership of and responsibility for their learning <input type="checkbox"/></p> <p>Students experience opportunities to develop the skills and attitudes necessary for lifelong learning <input type="checkbox"/></p>
Teachers' individual practice	<p>The teacher has the requisite subject knowledge, pedagogical knowledge and classroom management skills <input type="checkbox"/></p> <p>The teacher selects and uses planning, preparation and assessment practices that progress students' learning <input type="checkbox"/></p> <p>The teacher selects and uses teaching approaches appropriate to the learning intention and the students' learning needs <input type="checkbox"/></p> <p>The teacher responds to individual learning needs and differentiates teaching and learning activities as necessary <input type="checkbox"/></p>
Teachers' collective / collaborative practice	<p>Teachers value and engage in professional development and professional collaboration <input type="checkbox"/></p> <p>Teachers work together to devise learning opportunities for students across and beyond the curriculum <input type="checkbox"/></p> <p>Teachers collectively develop and implement consistent and dependable formative and summative assessment practices <input type="checkbox"/></p> <p>Teachers contribute to building whole-staff capacity by sharing their expertise <input type="checkbox"/></p>

Junior Cycle Key Skills and Statements of Learning

Key Skills

KS1	Managing myself
KS2	Staying well
KS3	Monitoring information & thinking
KS4	Being numerate
KS5	Being creative
KS6	Working with others
KS7	Communicating
KS8	Being literate

Statements of Learning

	The student
SL1	communicates effectively using a variety of means in a range of contexts in L1*
SL2	listens, speaks, reads and writes in L2* and one other language at a level of proficiency that is appropriate to her or his ability
SL3	creates, appreciates and critically interprets a wide range of texts
SL4	creates and presents artistic works and appreciates the process and skills involved
SL5	has an awareness of personal values and an understanding of the process of moral decision making
SL6	appreciates and respects how diverse values, beliefs and traditions have contributed to the communities and culture in which she/he lives
SL7	values what it means to be an active citizen, with rights and responsibilities in local and wider contexts
SL8	values local, national and international heritage, understands the importance of the relationship between past and current events and the forces that drive change
SL9	understands the origins and impacts of social, economic, and environmental aspects of the world around her/him
SL10	has the awareness, knowledge, skills, values and motivation to live sustainably
SL11	takes action to safeguard and promote her/his wellbeing and that of others
SL12	is a confident and competent participant in physical activity and is motivated to be physically active
SL13	understands the importance of food and diet in making healthy lifestyle choices
SL14	makes informed financial decisions and develops good consumer skills
SL15	recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning
SL16	describes, illustrates, interprets, predicts and explains patterns and relationships
SL17	devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills
SL18	observes and evaluates empirical events and processes and draws valid deductions and conclusions
SL19	values the role and contribution of science and technology to society, and their personal, social and global importance

SL20	uses appropriate technologies in meeting a design challenge
SL21	applies practical skills as she/he develop models and products using a variety of materials and technologies
SL22	takes initiative, is innovative and develops entrepreneurial skills
SL23	brings an idea from conception to realisation
SL24	uses technology and digital media tools to learn, communicate, work and think collaboratively and creatively in a responsible and ethical manner

L1 is the language medium of the school (Irish in Irish-medium schools). L2 is the second language (English in Irish-medium schools).