A Lesson Research Proposal for 2nd Year Quadratic Equations/Functions

For the lesson on date
At St. Mary's Holy Faith Glasnevin,
Instructor: Ms. M. Rochford
Lesson plan developed by: Darren Murphy, David Murphy, Rebecca Fitzgerald, Deirdre Hayden, Eilish Coleman & Maeve Rochford.

1. Title of the Lesson: Murling and The Little Function

2. Brief description of the lesson

Students will work with a real world problem involving establishing the distance from the origin and roots of a quadratic function. Students will use many methods in trying to establish the most efficient method to establish these values. Students will then transfer this knowledge to a slightly different situation to cement the learning.

3. Research Theme

Through the teaching of this unit we want students to:

a) Enjoy their learning, are motivated to learn and expect to achieve as learners.
b) Work both independently and collaboratively in a very purposeful and productive manner.

As mathematics teachers, we endeavor to achieve these aims by paying attention to the following entry points in our everyday classes:

a) Having an increased focus on the depth of the problems chosen in classes.
We will attempt to create and use problems that have a greater number of possible routes to arrive at a solution rather than the current practice of a one dimensional one solution problem. We aim to create problems that a solution of some level is achievable for students regardless of his/her abilities.

b) Create opportunities to allow for a combination of independent and collaborative work.
By ensuring that the problems created have enough scope for all students to arrive at some solution, we aim to allow students to explain these varying approaches to one another and then also to critique one another's work through peer assessment. We will also aim to allow sufficient time for students to work and think to allow for maximum productivity.

4. Background & Rationale

a) Why you chose the topic:
Quadratic Equations using their connection to functions. The chief examiner's report remarks on how students "struggled at times when more involved understanding was required" and more importantly candidates "had more difficulty with questions that required them to draw on multiple strands of the syllabus at once". A point we felt was true in the context of our schools as students failed to see the links between many topics but in particular one of this magnitude.

b) Your research findings: *class tests cat4 JC exams*
In researching this issue on a micro scale, we saw that in our schools, questions involving more than one topic of mathematics were answered poorly compared to isolated questions. In an attempt to address this is the past we have tried to pinpoint the link. However, this approach does not seem to be working. A point reiterated in the Chief Examiner's Report 2015 on Junior Certificate

Mathematics whereby it was noted that HL students have great difficulty connecting functions to the various parts of the syllabus.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
In 4th Class students learn to: - *explore, recognize and record patterns & sequences.* In 5th and 6th Class students learn to: - *Explore the context of a variable in the context of simple patterns, tables and simple formulae and substitute values for variables.* In 1st year students learn to: - Use graphs as tools for representing and analyzing linear, quadratic and exponential patterns. In 2nd students learn to: - Draw graphs of linear functions.	- Use representations to reason about the situation from which the relationship is derived and communicate their thinking to others. - Describe both quantity and change of quantity on a graph. - Solve quadratic equations. - Estimate accurately the solution to a quadratic equation based on its function graph. - Find approximate solutions using graphical methods for f(x)=g(x) and interpret the results.	Later in Junior Cycle we aim to extend the content of this unit in the following ways: - Form quadratic equations given whole number roots. -

6. Goals of the Unit

At the end of the unit we need students to be able to:

- understand the link between quadratic equations and quadratic functions
- have increased their motivation and willingness to attempt further problems due to experiencing success
- verify the solutions to a quadratic equation in more than one way
- increase their confidence to work independently and productively
- represent a quadratic equation graphically
- use the correct terminology to describe their findings

7. Unit Plan

Lesson	Learning goal(s) and tasks
1 The Research Lesson	Introduce quadratic functions in a problem solving method This lesson involves: -Graphing quadratic functions -Making a link between quadratic equations and quadratic functions -A willingness to investigate multiples approaches to solving the problem
2	Using the link discovered in lesson 1, use a graph of a quadratic function to solve a quadratic equation
3	Solving quadratic functions with non-integer roots
4	Using graphs, find the point of intersection between two functions

8. Goals of the Research Lesson:

The Goals of the lesson should refer to:

a) Mathematical Goals

Students will:
-be able to graph a quadratic function.
-understand the link between quadratic functions and quadratic equations.

b) Key Skills and Statements of Learning

- Being Numerate : Seeing patterns, trends and relationships,
- Managing Information and Thinking : Thinking creatively and critically, being curious
- Being Creative : exploring options and alternatives
- Working with Others: respecting differences
- Communicating: Using number
- Being Literate : Expressing ideas clearly and accurately
- Managing Myself: Being able to reflect on my own learning
- Staying Well: Being confident

Statements of learning:

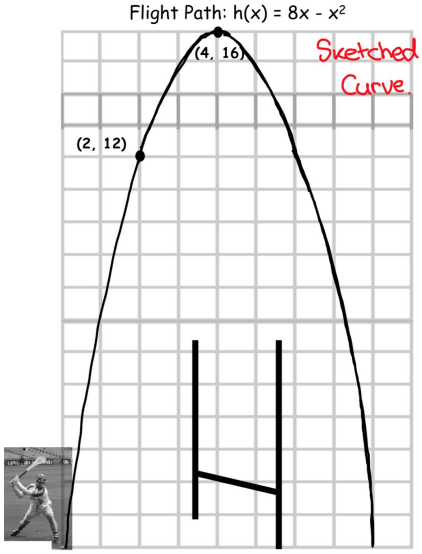
1)communicates effectively using a variety of means on a range of context in L1
15) recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning
16) describes, illustrates ,interpretes, predicts and explains patterns and relationships

17) devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction: - Recap prior knowledge regarding linear equations. - Show the students an image of a hurling player taking a sideline ball and ask the students to trace the flight path of a ball either on the whiteboards.. - Float the idea can we use mathematics to determine the flight path of the ball and establish how far away from the player the ball will hit the ground?		The purpose of this introduction is to highlight to the students the possibility of a connection between mathematics and the real world. Through tracing the path we hope to kickstart the students minds into thinking about the quadratic shape (unbeknownst to them).
Posing the Task: Posing of our task will come in three parts: 1. Verbal Instructions - 'The diagram shows a player taking a sideline puc. The ball travels along this flight path. At 2m away from the player the sliother is 10m above the ground. The ball has travelled a further 2m and is now at its highest vertical position. 2. Worksheet 3. Question posed on the board	The support for the students here is the task will be posted onto the board "How far from where Liam strikes the ball will it hit the ground?" The students will also be given the worksheet "Murling 5.0".	
Student Individual Work: **Solution 1: Sketching/Tracing the curve**	The teacher will walk around the room assessing the students work and keep the students engaged in the problem. Students will receive praise and encouragement throughout the process . **Sketching/Tracing the curve/Graphing** The teacher may encourage the student to extend their drawing of the curve if they have only filled in part of the curve.	

MATHS + HURLING = MURLING

Flight Path: $h(x) = 8x - x^2$

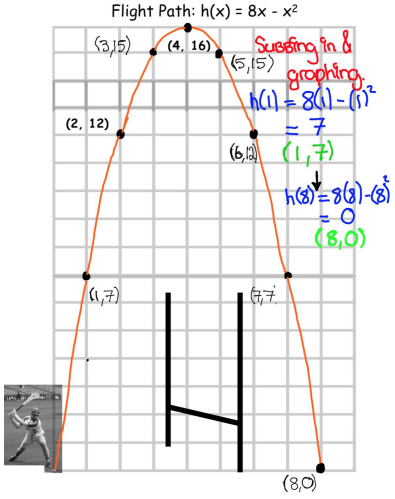


Subbing in + Graphing
The teacher may ask the students to think of more numbers to sub into the function they have only a limited number of answers
The teacher may point to the ball on the graph and ask the student what is the time and the height at the beginning of the balls flightpath.

Solution 2: Subbing in + Graphing

MATHS + HURLING = MURLING

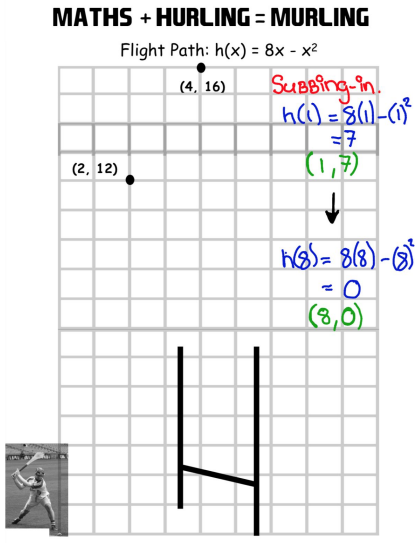
Flight Path: $h(x) = 8x - x^2$



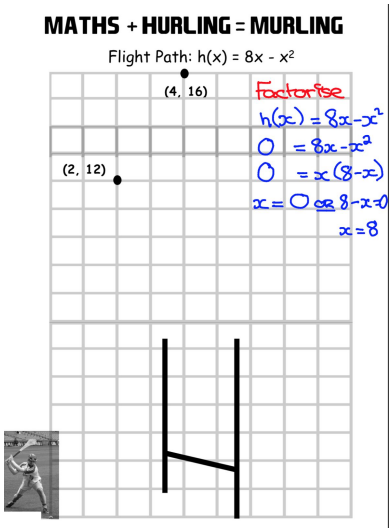
Subbing In ONLY
Students will be probed on what could you do with all of those couples that you have worked out, drawing on previous knowledge of linear couples.

Solution 3: Subbing in ONLY

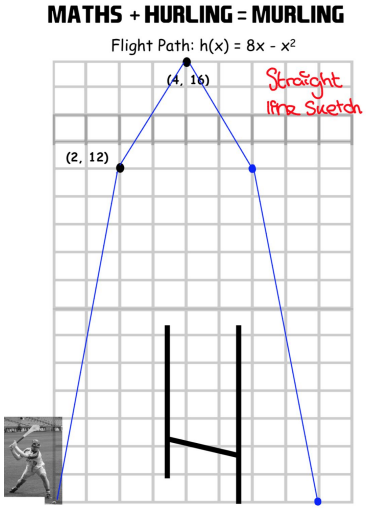
Factorising
The teacher may ask the student if they can find a second value for t if only one is given



Solution 4: Factorising

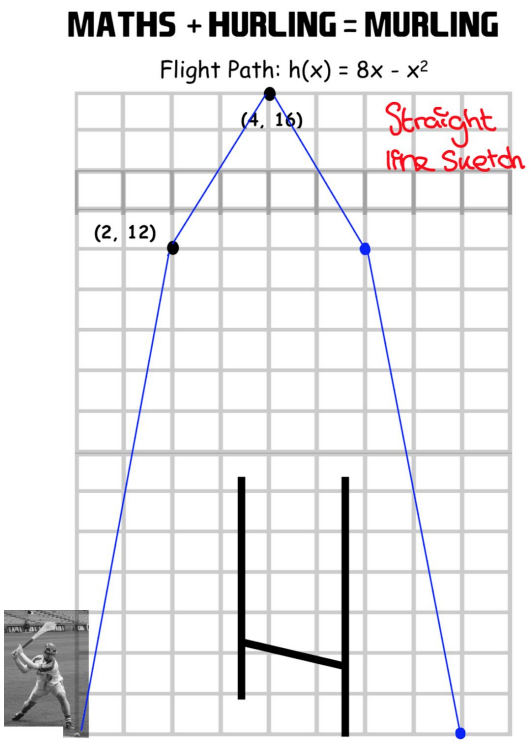


Solution 5: Graphing (Symmetry with Lines)



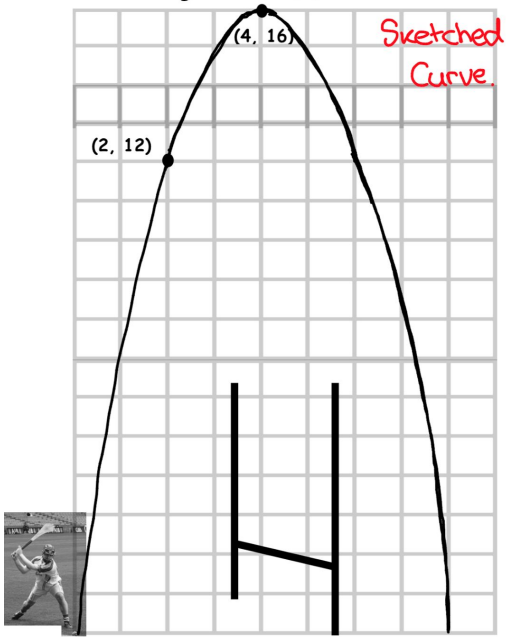
Graphing (Symmetry with Lines)

Here we will attempt to draw the students attention to the idea of is that how you think the ball flies?

Ceardaíocht /Comparing and Discussing 1) Symmetry (Straight Lines) or Curve sketching  2) Graphing (Curved Sketch)	1) This method is more to do with a disconnect to reality. If the students join points using straight lines, draw their attention back to the tracing of the flight path in the introduction to clarify. 2) The biggest issue with this solution is accuracy. so while the student has done some correct work pose the question of how could you be sure it goes through here etc.	The purpose here is to get the students to identify the steps to improving one another solutions and identify which of these methods is the most efficient and visually appealing.

MATHS + HURLING = MURLING

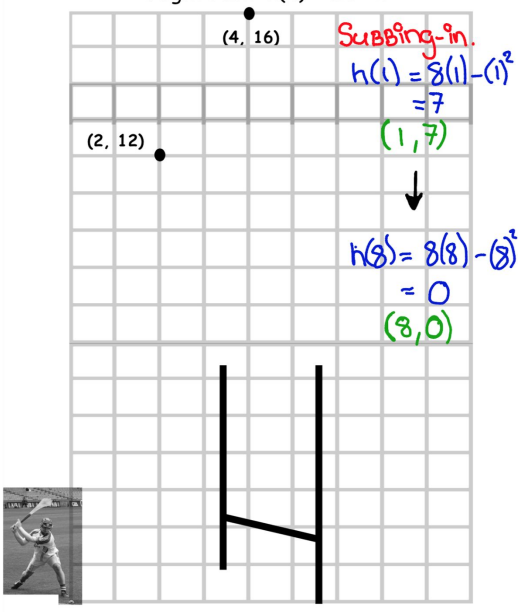
Flight Path: $h(x) = 8x - x^2$



3) Subbing in ONLY

MATHS + HURLING = MURLING

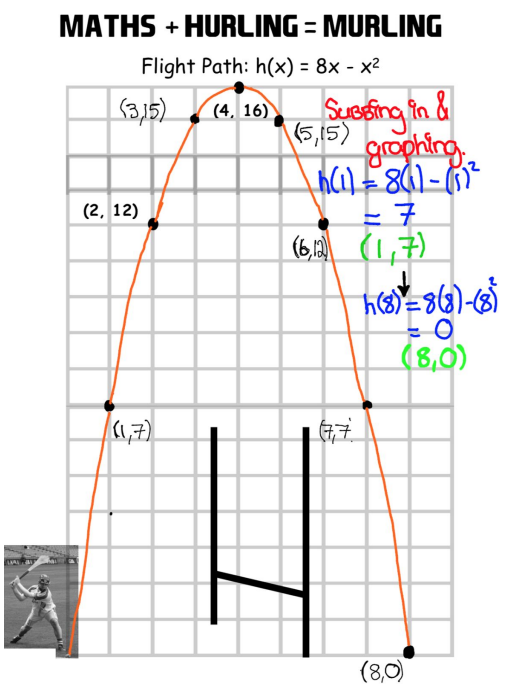
Flight Path: $h(x) = 8x - x^2$



4) Subbing in + Graphing

3) This approach while being the most accurate is missing a visual aid and thus not satisfying our goals with this lesson. Try to point out to the student the absence of an alternative view or what would be a more appealing image to show the ball on the ground.

4) This is the new strategy we are chasing. So we will highlight this to the class at this point.



Summing up & Reflection

The most efficient method to solve this problem is to sub in the x values into the function and graph the solutions. secondly students should be able to recognise that a function containing a variable squared has a curved graph.

reflection
we asked the students:

- what they understood
- what they noticed
- what questions they have
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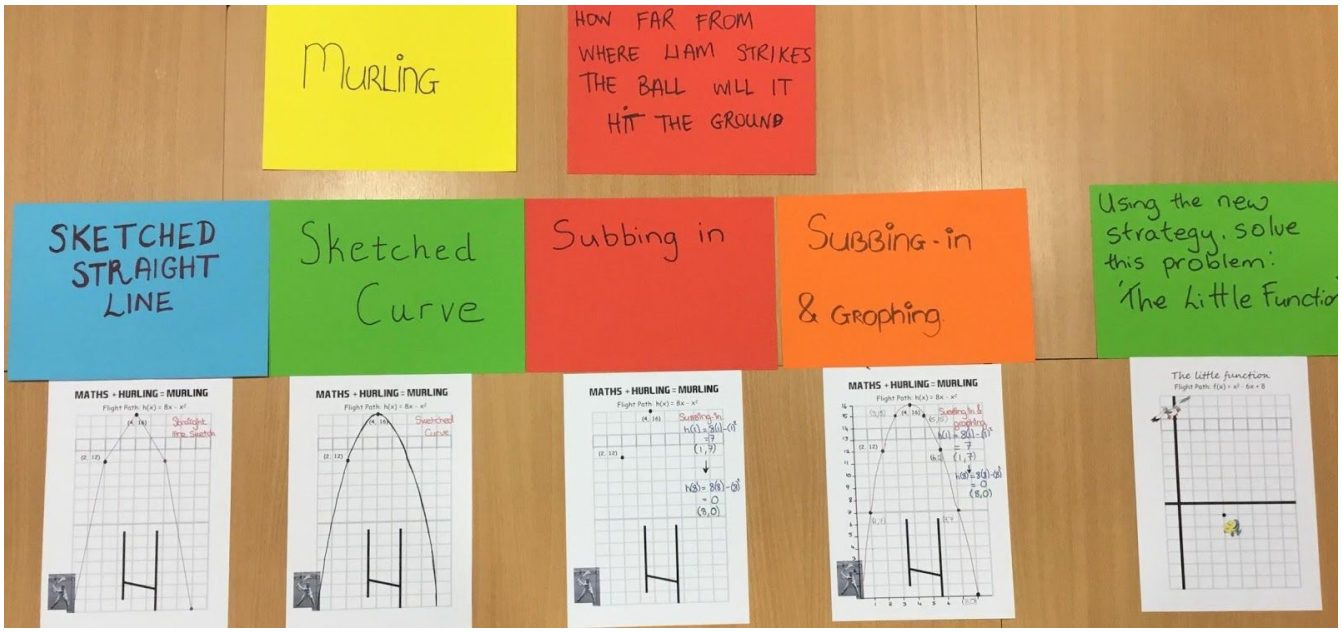
Extension Question:
Students are given a positive quadratic function and asked to use subbing in and graphing to solve a quadratic equation using the worksheet entitled "The Little Function"

9. Board Plan



10. Evaluation

In our post lesson discussion, it is hoped that we may address some of the key components on the research lesson under the heading our research theme. Our first point under research theme was that "students enjoy their learning" and can "expect to achieve as learners". We will also reflect upon the second of our research themes, "students working independently and collaboratively in an purposeful and productive manner". Below is how we planned to observe the class:

- The classroom was divided up into 4 sections (2 rows per observer) and one observer was looking after gathering the photographic evidence of the work during the lesson just in case students were erasing some of their thought process. Each observer had a seating plan and took notes regarding approaches taken, obvious misconceptions and any "Eureka" moments. The observer gathering the photographic evidence was using the iOS application LessonNote to record their notes and photographs.

- As student expectations of achievement and enjoyment of learning were a key theme for the lesson, observers were taking notes based on sign of frustration, enjoyment, confusion etc. Observers were looking for signs such as body language, scribbles on paper, lack of engagement etc.
- Working independently was also under observation, and the students ability to do so. This was particularly important with this class group as the teacher felt that this is an area in which this class were seriously lacking.

- Our final key point for observation was, did the students achieve the mathematical goals of the lesson, in other words could they "graph a quadratic function" and secondly did they "understand the link between a quadratic function and quadratic equation".

11. Reflection

i) In terms of engagement and enjoyment of the lesson, all observers noted that every single student engaged with the task for the allotted time and some even asked for more.
ii) By the extension task all students were able to at least sketch a quadratic curve, with most able to plot it accurately. As for the link between quadratic functions and quadratic equations, the students understood that they are connected in some way but were not quite clear on the exact link (this was to be developed on in the next lesson by the classroom teacher).
iii) The teacher remarked that a lot of the students were quite anxious with the prospect of working on their own and some were very slow to get started on the task. The teacher explained that normally this group would be very dependent on peer discussion to kick-start solving a problem.
iv) The observers noted that the students were hesitant in writing down a possible solution unless they had prior assurance that it was the correct way to go.
v) Students had previously covered linear functions in class and the equation of a line but they had not looked at quadratic functions. This led to an over dependence on attempting to calculate the slope of the line between the two given points. Although initially thought of as a problem, the group remarked on the possibility of using this concept to introduce calculus at senior cycle.
vi) Students self confidence also improved as one student remarked "I feel so smart for being called up to the board". This was the student with the first solution.
vii) The teacher remarked on how the more technically able students depended on their memorised mathematical steps and ignored common sense.
vii) One misconception we hadn't accounted for was that students associate a more sophisticated answer with using a ruler to draw straight lines and therefor some of the students thought the curve was less sophisticated.
viii) When presented with the extension problem, there was notably less time taken to get started as students appeared more confident.
ix) The most significant point from all observers was the ability of this style of lesson to allow all students to experience some form of success in the lesson. Students that used straight lines in task 1 were no drawing curves, students that were just sketching curves were no using a table of values to plot a graph. The observers felt that each student in their area moved forward with their learning.
x) The decision to use an hour long class for this lesson was mentioned as very important. It was felt that the standard 40 mins would make achieving any goal of the lesson almost impossible.
xi) Ideas for future study:

The group felt that had we taught the students linear fucntions in a similar manner then they would have felt more comfortable working on their own, thus allowing the lesson to progress faster and creating the opportunity to discuss the link between functions and equations. In other words, get the the students to use the two values that they found for where the ball hits the ground with the equation and explore any potential findings.