Lesson Research Proposal for 2nd Year Higher Level
Problem solving involving the sum of internal/external angles in polygons

For the lesson on 23rd of January
At Coláiste Iognáid,
Barbara O Riordan's class
Instructor: Barbara O'Riordan
Lesson plan developed by: Lorraine Kelly, Mary Scahill, Barbara O Riordan

1. **Title of the Lesson:** ***The EX Factor***

2. **Brief description of the lesson**

Through a problem-solving approach, students find the sum of exterior angles of a triangle in as many ways as possible. Individual students will describe their method and through discussion all students will learn that there are various approaches to solving the same problem

3. **Research Theme**

Referring to *Looking at our School 2016 – A Quality Framework for Post-Primary Schools* we identified the following goals as a priority for the improvement of teaching and learning:

a) We want our students to grow as learners through respectful interactions and experiences that are challenging and supportive
b) As teachers, we want to engage in professional development and professional collaboration to enhance our teaching skills

As mathematics teachers engaging in Lesson Study, we will realise these goals by:

- Designing a suitably challenging problem which will encourage our students to think insightfully and creatively
- Providing our students with opportunities to come up with their own approaches and methods to finding a solution, thereby developing their critical thinking and problem-solving skills
- Facilitating our students in developing their confidence, competence and communication skills, through the expression of their ideas and methods to their peers in their own words. This should also enhance the comprehension of other students in the classroom
- Promoting a consistent department-wide approach to the teaching of mathematics in our school with a view to improving mathematics teaching and learning and students' experiences in the mathematics classroom
- Encouraging an increased emphasis on the links between the five strands of the mathematics syllabi and across other subjects on the junior cycle curriculum

4. **Background & Rationale**

a) Why we chose the topic

Our junior cycle students are not used to learning through structured problem-solving, as they are usually exposed to a more didactic teaching approach. We would like to improve our teaching by introducing our 2nd year students to structured problem solving through geometry. We decided to focus our lesson on geometry because we feel that students appreciate geometry in so far as it moves away from the abstract nature of mathematics into an area that is tangible to them.

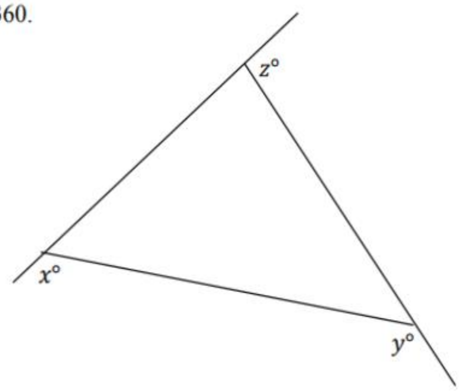
We chose the topic of angles in triangles which will be extended to quadrilaterals and n-sided polygons. Students seem to remember that the sum of the angles in a quadrilateral is 360^{o}, but they often lack an understanding of why this is the case. We would like our students to understand why the sum of the internal angles in a triangle is 180°, and then allow them to develop their thinking further, to determine the sum of internal angles in quadrilaterals, pentagons and n-sided polygons in general. This will enable students to link geometry with algebra through the creation of mathematical expressions and pattern recognition.

Furthermore, students will be given an opportunity to use what they learn from this activity, to figure out the sum of the external angles in triangles and n-sided polygons in general. This topic will cater for different learning styles and different levels of ability, since the process of reasoning can be illustrated visually in many ways. We would like to give our students an opportunity to engage in non-procedural, active learning in geometry where they can discover the answers for themselves according to their own abilities.

b) Our research findings

JC exams tend to assess students' problem-solving skills by setting unseen problems (see example below from 2012 JC Higher Level (Paper 2, Q15)).

(a) Prove that $x + y + z = 360$.



Mathematics Chief Examiner's Report for JC 2015 (accessed at https://www.examinations.ie/misc-doc/EN-EN-25073660.pdf) states that, "In terms of skills, the new syllabus has an increased emphasis on problem-solving, as well as on the skills of explanation, justification, and communication". Having regard to this report and, considering the nature of the problems posed on the junior cycle mathematics examination papers over the past number of years, we appreciate the need for students to enhance their problem-solving and deductive reasoning skills if they are to succeed at mathematics. The Chief Examiner's report also refers to the difficulty many students have when required to apply their knowledge of geometric results and theorems in deducing an unfamiliar proof. Moreover, the report recommends that students should learn to tackle problems using a variety of different methods, so that they "…build up their arsenal of techniques on familiar problems to help them to tackle unfamiliar ones" (C.E.R. 2015 pg.34). We hope that this lesson will mitigate the difficulties students have when faced with unfamiliar problems, by increasing their confidence in applying their mathematical knowledge to find solutions and justify their methods.

As teachers, we acknowledge that:

- We must cater for different types of learners since people view problems in different ways
- Providing an opportunity for students to engage in non-procedural, active-learning in geometry allows them to work things out by visualizing
- Prior learning and the scaffolding it provides, when moving through mathematics syllabi at second level, cannot be underestimated
- Students struggle when presented with abstract topics which they cannot relate to real-life
- Taking a constructivist approach to teaching mathematics, enables students to find their own solutions to problems thereby enhancing their problem solving and critical thinking skills

- Mathematics lessons should facilitate the development of students' investigative skills and deductive reasoning skills
- A discovery approach provides the opportunity for students to encounter formal geometrical results through a process of investigation and self-led discovery, in accordance with the NCCA's recommendation (Geometry for Post-Primary School Mathematics)
- It is essential to develop students' confidence in using their skills and knowledge to attempt a variety of mathematical problems
- In the mathematics classroom we should encourage students to approach mathematical problems in different ways and limit their dependence on the use of formulae and standard procedures
- Students need to feel confident in their own mathematical ability if they are to be comfortable in justifying and describing their methods.

5. Relationship of the Unit to the Syllabus

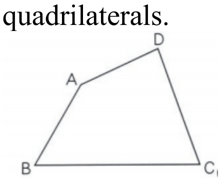
Related prior learning outcomes	Learning outcomes for this unit	Related later learning outcomes
In 5th and 6th Class students learn to make informal deductions about 2D shapes and their properties. Use angle and line properties to classify and describe triangles and quadrilaterals. Use 2D shapes and properties to solve problems. Explore the sum of the angles of a triangle. Measure the angles in a variety of triangles using a protractor. Explore the sum of angles in a quadrilateral. **In First Year students:** Link formal geometric results to their study of shapes in primary school. Investigate and understand that a straight angle is 180 degrees. Investigate and understand alternate angles by example and measuring. Theorem 4: the angles of any triangle add to 180 degrees. Theorem 6: each exterior angle is equal to the sum of the interior opposite angles.	All students will be able to identify the different types of angles and their measures Students will understand that angles in a triangle sum to 180^o and will be able to prove this geometric fact (Theorem 4) Students will deduce and prove that the exterior angle in a triangle is equal to the sum of the two interior opposite angles. (Theorem 6) Students will apply their knowledge of theorem 4 and 6 to deduce that the sum of three exterior angles of a triangle is 360^o Students will apply their knowledge to deduce that the sum of the exterior angles of a quadrilateral is 360^o Students will begin to appreciate the need to take logical steps when establishing a geometric proof Students will be able to establish specific properties and characteristics of geometric shapes Students will apply their knowledge to find the sum of the exterior angles of different polygons	Students should be able to analyse and process information presented in an unfamiliar context Students may deduce that the sum of the exterior angles of any polygon is 360^o Students should be able to apply their deductive reasoning to problem solving with theorems, trigonometry and co-ordinate geometry

6. Goals of the Unit

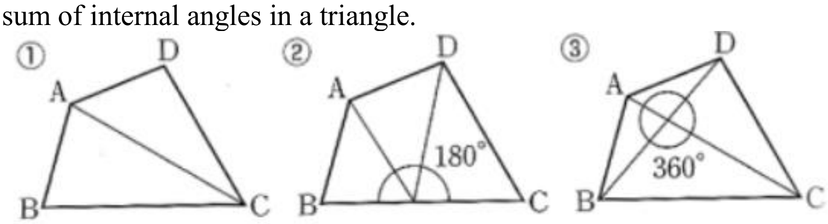
- Students will be familiar with straight angle, alternate angles and vertically opposite angles (Theorem 3 and 5).
- Students will understand that the angles in a triangle sum to $180°$ and be able to justify this (Theorem 4).
- Students will deduce and prove that the exterior angle in a triangle is equal to the sum of the two interior angles (Theorem 6).
- Apply their knowledge of Theorems 4 and 6 to find missing angles in different diagrams.
- Students develop strategies to show that the sum of the four interior angles of a quadrilateral is $360°$.
- Students deduce that the sum of the five interior angles of a pentagon is $540°$.
- Students understand the concept of a polygon.
- Students create a mathematical expression $(n-2)\times 180°$ that illustrates the relationship between the sum of the interior angles and the number of sides of a polygon.
- Students extend their knowledge to work out the sum of the exterior angles of a triangle, quadrilateral, and pentagon.
- Students deduce that the exterior angles of any polygon sum to $360°$.
- Students apply their knowledge to find the unknowns angle measures of both exterior and interior angles of different polygons.
- Students deduce that the sum of the exterior angles of any polygon is a full rotation of a circle.

7. Unit Plan

<table>
<tr><th>Lesson</th><th>Learning goal(s) and tasks</th></tr>
<tr><td>1</td><td>Review lesson on parallel lines and angles.</td></tr>
<tr><td>2</td><td>Use a suitable activity to prove Theorems 4 and 6.</td></tr>
<tr><td>3</td><td>Practice lesson, find the missing angles.</td></tr>
<tr><td>4</td><td>Through problem solving approach students find the sum of the interior angles of arbitrary quadrilaterals.



Students understand that the sum of the internal angles in a quadrilateral can be calculated using the sum of internal angles in a triangle.



$2\times 180° = 360°$, $3\times 180° - 180° = 360°$, $4\times 180° - 360° = 360°$</td></tr>
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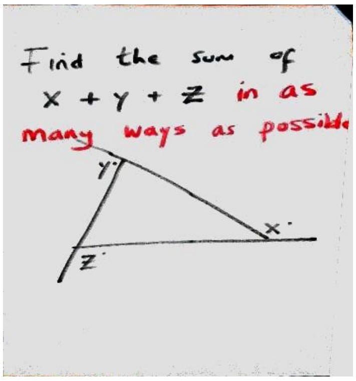
	Students' learning will be reinforced through homework tasks by extending to find the sum of the interior angles of arbitrary pentagons and hexagons.
5	Introduce the concept of a polygon. Understand that the sum of the internal angles of a polygon can be calculated by dividing the polygon into triangles. Populate table to illustrate the relationship between the sum of the interior angles and the number of sides of a polygon. Write algebraic expressions for the sum of interior angles in a polygon. $(n-2) \times 180$, $(n-1) \times 180° - 180°$, $n \times 180° - 360°$
6 Research lesson	Through problem solving approach students find the sum of exterior angles of a triangle.  Students' learning will be reinforced through homework tasks by extending to find the sum of the exterior angles of arbitrary quadrilaterals and pentagons.
7	Through discussion of their findings from the research lessons and homework students should discover that the exterior angles of any polygon sum to 360 degrees. Use suitable activities to allow students apply their knowledge to find the unknown angle measures of both exterior and interior angles of different polygons. Students may deduce that the sum of the exterior angles of any polygon is a full rotation of a circle.

8. Goals of the Research Lesson:

a) Mathematical Goals

Students will:

- Using prior knowledge of angles, triangles, parallel lines and angles students discover that the sum of the exterior angles of a triangle is 360°.
- Understand that geometric characteristics of external angles of a triangle are not specific to the particular types of triangles worked on during the lesson.
- Select a suitable approach developed in the lesson to determine the sum of the exterior angles of a quadrilateral is 360°.

b) Key Skills and Statements of Learning

In preparation for implementing the Junior Cycle Specification for Mathematics our maths department have begun to integrate the development of Key Skills into our maths lessons.

This lesson will address the following Key Skills:

1. Being Numerate: By engaging in suitable tasks, students will develop a positive attitude towards investigating, reasoning and problem solving.
2. Managing information and thinking: Students will be encouraged to think creatively and

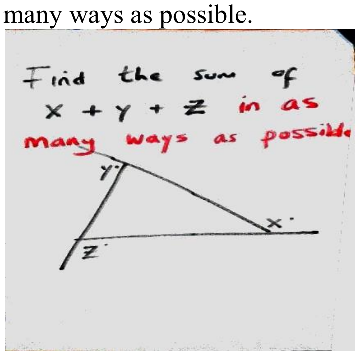
critically and record their results.

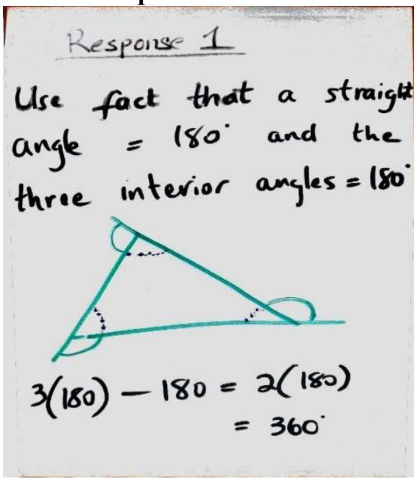
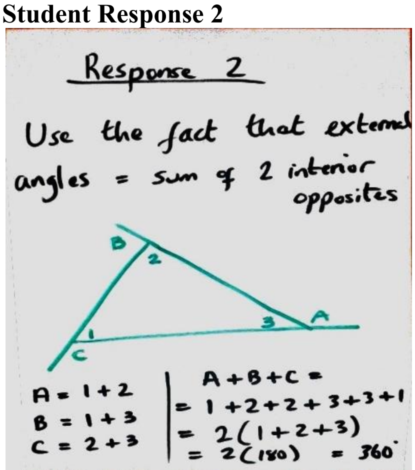
3. Being Creative: Students will explore options and alternatives as they actively participate in the construction of knowledge.
4. Communicating: During the lesson, students will present and discuss their mathematical thinking.
5. Working with Others: Students will learn with and from each other by discussing different approaches to solving the problem.
6. Staying Well: By engaging in tasks which are appropriate to their abilities, students' confidence and positive disposition to learning will be promoted.

This lesson also meets the following JC Statements of Learning:

15. The student recognises the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
16. The students describes, illustrates, interprets, predicts and explains patterns and relationships.
17. The students devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction [10 minutes] Before we start today's problem, I want to review some maths we have learnt to date. **What did you learn about triangles?** • The three angles of a triangle add up to 180 degrees. • Exterior angle equals the sum of the two opposite interior angles. Up to now we have looked at the interior angles of polygons. Today we will be finding the sum of exterior angles.	Open questioning of the students	Can all students recall their prior learning of the following; • Straight angles • Alternate angles • Corresponding angles • Vertically opposite angles • Sum of angles in a triangle • Exterior angle in a triangle equal to two opposite interior angles
Posing the task (5 minutes) Find the sum of angles x, y, z in the diagram below in as many ways as possible. 	Place the diagram on the board and distribute a copy of the problem among students. Use seating chart to record the approach used by each student. Note the order in which individual students will be	Do students understand the task?

	asked to come to the board during Ceardaíocht. If students are stuck, guide them by asking appropriate open questions to encourage them to persevere with the problem	
Student Individual Work (15 Minutes) **Student Response 1**  **Student Response 2** 	Use your seating chart to record the approach used by each student. Note the order in which you will call each student up during Ceardaíocht. If students are stuck, help them by asking appropriate questions. What do we know about straight angles? What does the exterior angle equal to?	Are students able to tackle the problem? Do students understand that this problem is about calculating the sum of the exterior angles of a triangle?

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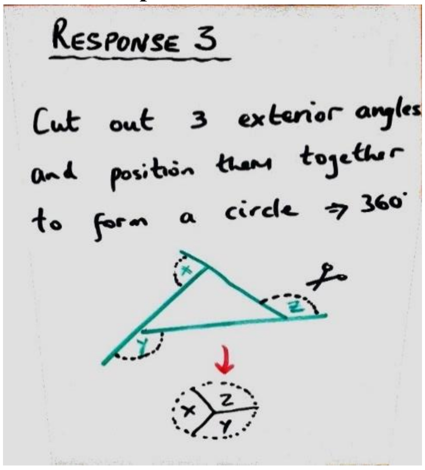
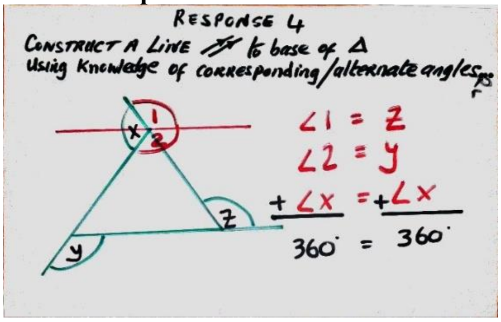
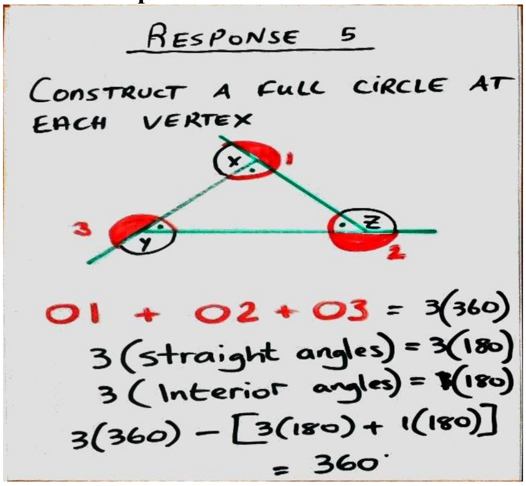
Student Response 3



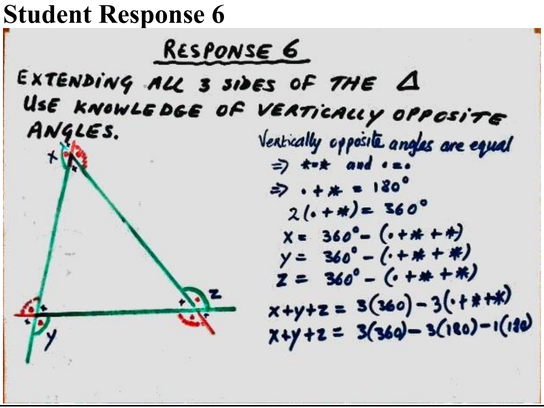
Student Response 4



Student Response 5



Student Response 6



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When the three angles in a triangle are summed, what is the result?

What do we know about corresponding angles?

What do we know about alternate angles?

What can we deduce from this?

What size is an angle of full rotation?
How many full rotations do we have around a triangle?

What do we know about vertically opposite angles?
What can we deduce from this?

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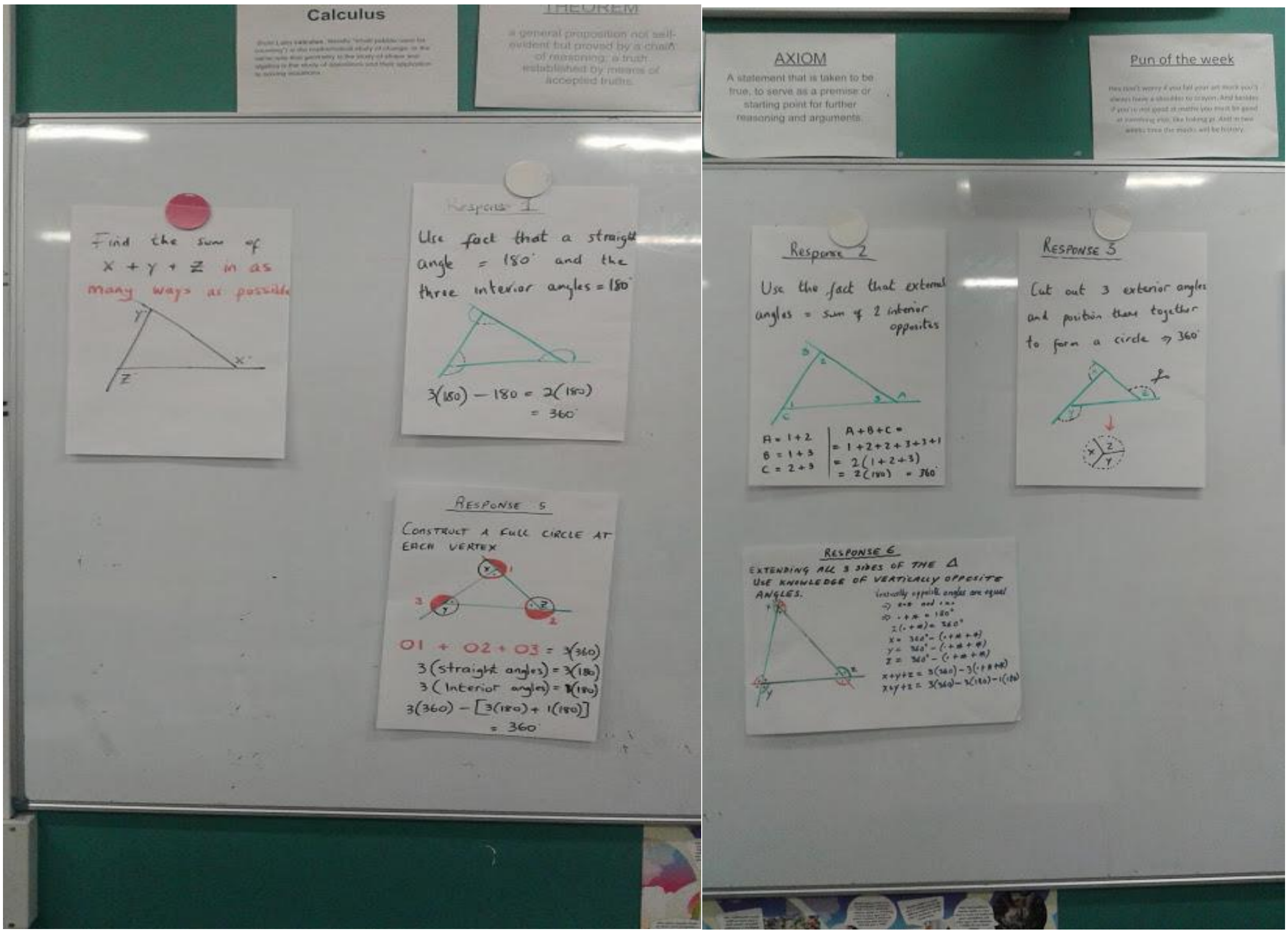
Ceardaíocht/ Comparing and Discussing (20 minutes) Ask specific students to come to the board and explain how they calculated the sum of the exterior angles. • Student Response 1: Using the fact straight angle equal 180 degrees and the three interior angles equal 180 degrees. • Student Response 2: Use the fact that exterior angle equals the sum of the two interior opposites. • Student Response 3: Cut out three exterior angles and position them together to form a circle equals 360 degrees. • Student Response 4: Construct a line parallel to base of triangle. Using knowledge of corresponding/alternate angles. Indicate <1=Z <2=Y <X=<X They both sum to 360 degrees. • Student response 5: Construct a full circle at each vertex. Circle 1+2+3= 3(360)- 3(180)- 3(interior angles of triangle)=360 degrees. • Student response 6: Extending all three sides of the triangle. Use knowledge of vertically opposite angles	When student presents work at the board make sure to attach their name to it. Ask students to raise their hands if they used this method. Did anybody use a different approach? Explain to students that there are several ways to find the sum of the exterior angles of a triangle.	Can students explain their approach? Do students recognize similarities/differences between their approach and that presented on the board? Do students offer alternative approaches to solving the problem?
Summing up & Reflection (10 minutes) We learned that: • Ask students to write a reflection. Distribution of homework task		Do the students' reflections represent the teacher's view of the lesson?

10. Board Plan



11. Evaluation

The general consensus was that the lesson was very successful, with the goals of the lesson achieved. All of the students were very positive about the whole lesson. Students became more comfortable with the process of the lesson as the lesson progressed. Students were very engaged and challenged by the task. No student was totally confused or lost; any misconceptions were addressed by their own discussion.

It was found that all students came up with at least two ways of solving the problem. Many of the students used the protractor at first to solve the problem. With gentle encouragement students were able to branch out and find other methods of solving the problem. When presented with the quadrilateral shape in the extension task they automatically measured the angles and needed to be encouraged to try other methods. By the end of the lesson it was found that all students did understand that the sum of the exterior angle sums to 360^{o} by using prior knowledge of angles, triangles, parallel lines and angles.

Understand that geometric characteristics of external angles of a triangle are not specific to the particular types of triangles worked on during the lesson was achieved by all.

12. Reflection

It was agreed by all that the task posed was straightforward and engaging for all students. Some methods used by students to solve this problem included using the protractor to measure the angles. Some cut out the angles and placed them together to form a circle of 360^{o}. Many used their knowledge of a straight angle and knowledge that a circle sums to 360^{o} to solve the problem.
One student used their knowledge that the exterior angle equals the sum of the two opposite interior angles

There were some misconceptions upon reflection including, when some students found the measure of the angles with the protractor they carried these values forward into their next diagram and this limited their opportunities for coming up with more solutions to the problem**.** Some assumed that the diagram presented was an equilateral triangle**.** After finding a solution, students needed to be encouraged to attempt other approaches. Some students noticed relationships and make connections across the strands of geometry and algebra was achieved by some students but not all.

Some comments by students included could they cut out the angles? Could they measure the angles? One commented that she had learned that she could figure out the size of the angles without using a protractor. All worked out that all exterior angles, no matter the shape, add up to360^{o}. After they found the sum of the exterior angle of the quadrilateral students understanding changed. When given the extension work students had no problems completing the task. In general, the discussion of student's solutions at the board, did promote thinking and learning.

Recommendations
Major points raised during the post-lesson discussion, and the team's own opinions;

- The students' work board would be populated with the 'expected response headings' in advance of lesson.
- Students relied on the concrete tools and numerical values rather more than hoped. Going forward we would suggest students should become more familiar dealing with abstract problems.
- Students should be allowed to explore concrete methods to cater for different learning styles and abilities.
- Students should be encouraged to try other methods to solve the problem.

Ideas for future study

- The hands on discovery approach we used for this lesson can be used across all the strands in mathematics.
- This lessons could be extended any polygon.

Research Lesson extension: Find the missing angles