Lesson Research Proposal for 2nd Year HL Maths - Patterns

For the lesson on 7th February 2018
At Nenagh CBS & Nenagh College
Instructor: Ronan Heavey
Lesson plan developed by: Michelle Guinan, Audrey Clarke & Frank Macken.

1. Title of the Lesson: The Great Wall of Nenagh

2. Brief description of the lesson
Problem solving approach to link Patterns, Co-ordinate Geometry and Algebra.

3. Research Theme
In our Tipperary Schools, we want students to:

a) Experience opportunities to develop the skills and attitudes necessary for lifelong learning.
b) Engage purposefully in meaningful learning activities.

As mathematics teachers, we will actively support the achievement of these goals by:

a) Working together to devise learning opportunities for students across and beyond the curriculum.
   Teachers work together efficiently and effectively to enable and empower students to see learning in Maths as a lifelong endeavor and integrated across different strands and subjects.
b) Selects and uses planning, preparation and assessment practices that progress students' learning.
   Teachers identify and prepare resources tailored to match the learning intentions of each lesson. Teachers also use different assessment strategies to meet different learners' needs.

4. Background & Rationale

a) Why you chose the topic

This lesson is aimed at second-year higher level students. Patterns is quite a new topic to some teachers. It came in with the new Maths syllabus and was mainly a Leaving Certificate (optional) topic until then. As a result some teachers find it quite a difficult topic to teach as they would often have avoided it. Furthermore students often lack the verbal skills to describe what they are seeing and apply that to the Maths scenario. Also Patterns can have implications across various other topics such as Algebra, Functions & Graphs, Coordinate Geometry and Financial Maths.

b) Our research findings

Following discussions in our lesson study group, we have concluded that the teaching of Patterns can often be teacher-led and involve just going through different questions in the textbook. This leads to students not getting a deep understanding of the topic, its usefulness in other areas of Maths and how its formulae come to be derived.
As a result of these issues we have decided to try a different approach. We will use Patterns through a problem solving lesson while using prior knowledge of the students. By using a practical and structured problem solving question we hope that students will gain a deeper understanding of Patterns while also giving us the opportunity to link this knowledge to Co-ordinate Geometry and Algebra.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
In Primary School, as early as **First Class**, students are exposed to Patterns including odd and even numbers. As students progress through the Primary school years, this topic remains a key part of the Maths syllabus. By the time they get to **Fifth and Sixth Class**, students are expected to learn to: ➢ *identify relationships and record verbal and simple symbolic rules for number patterns* ➢ *identify relationships and record symbolic rules for number patterns* ➢ *explore the concept of a variable in the context of simple patterns, tables and simple formulae and substitute values for variables* In the Common Introductory Course in **First Year**, students examine Patterns and the rules that govern them. They learn to: ➢ *use tables and diagrams to represent a repeating-pattern situation* ➢ *generalise and explain patterns and relationships in words and numbers* ➢ *write arithmetic expressions for particular terms in a sequence*	In **Second Year**, students are expected to go into more general terms with Patterns. They also must deal with Patterns in context and examine their algebraic relationship. ➢ *use tables to represent a repeating-pattern situation* ➢ *generalise and explain patterns and relationships in words and numbers* ➢ *write arithmetic expressions for particular terms in a sequence* ➢ *use tables, diagrams and graphs as tools for representing and analysing linear, quadratic and exponential patterns and relations (exponential relations limited to doubling and tripling)* ➢ *develop and use their own generalising strategies and ideas and consider those of others* ➢ *present and interpret solutions, explaining and justifying methods, inferences and reasoning* ➢ *find the underlying formula written in words from which the data are derived (linear relations)* ➢ *find the underlying formula algebraically from which the data*	Later on into Leaving Certificate Maths students will further their understanding of Patterns. ➢ *explore graphs of motion* ➢ *make sense of quantitative graphs and draw conclusions from them* ➢ *make connections between the shape of a graph and the story of a phenomenon* ➢ *describe both quantity and change of quantity on a graph* ➢ *use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others* ➢ *discuss rate of change and the y-intercept; consider how these relate to the context from which the relationship is derived, and identify how they can appear in a table, in a graph and in a formula* ➢ *decide if two linear relationships have a common value* ➢ *recognise problems involving direct proportion and identify the necessary information to solve them*

	are derived (linear, quadratic relations) ➢ *show that relations have features that can be represented in a variety of ways* ➢ *distinguish those features that are especially useful to identify and point out how those features appear in different representations:in tables, graphs, physical models and formulas expressed in words, and algebraically use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others* ➢ *recognise that a distinguishing feature of quadratic relations is the way change varies*	

6. Goals of the Unit

a) Students will understand the relationship between a set of inputs and a set of outputs.
b) Students may then apply their prior knowledge and use tables, diagrams and graphs to represent these situations.
c) Students will understand that patterns can be explained in words and numbers.
d) Students will understand ways to express a general relationship of a pattern (Linear, Quadratic or Exponential).

7. Unit Plan

Lesson	Learning goal(s) and tasks
1 The Research Lesson	Introduce Patterns in a Problem-Solving context Use this problem to: Show the link between Algebra, Co-ordinate Geometry (slope) and Patterns
2	Further learning of students from the concrete to the abstract in Patterns
3	General term of Linear and Quadratic Sequences

8. Goals of the Research Lesson:

a) Mathematical Goals

Students will

- Understand the link between Patterns, Co-ordinate Geometry and Patterns
- Apply their knowledge to go from the concrete to the abstract in Patterns

b) Key Skills and Statements of Learning

In preparation for this lesson the Junior Cycle Key Skills and Statements of Learning have been taken into consideration.

In this lesson we will address the Key Skills in the following ways:

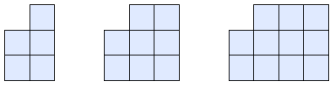
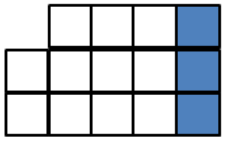
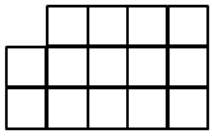
I. Managing Information & Thinking: Students will be encouraged to think creatively and critically.
II. Being Numerate: Students will see patterns, trends and relationships.
III. Being Creative: By being creative, students will explore options and implement their ideas to solve problems.
IV. Working With Others: As the teacher takes solutions from the class and presents them on the board, students will learn by working with others.
V. Communicating: Students will be encouraged to present their thinking and explain the rationale behind it.
VI. Being Literate: Students will express their ideas during Ceardaíocht
VII. Managing Myself: Students will have an opportunity to reflect on their learning when the lesson is over
VIII. Staying Well: By applying their own prior knowledge and being engaged in avtive learning students will be positive about their learning.

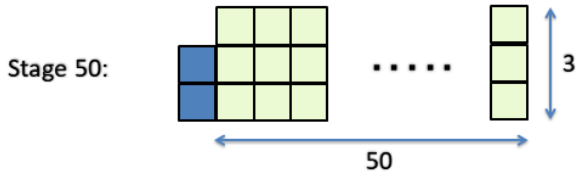
This lesson also meets the following Junior Cycle Statements of Learning:

1. The student communicates effectively using a variety of means in a range of contexts.
15. The student recognizes the potential uses of mathematical knowledge, skills and understanding in all areas of learning.

16. The student describes, illustrates, interprets, predicts and explains patterns and relationships.
17. The student devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

9. Flow of the Research Lesson:

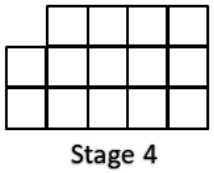
Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction Before we begin today's lesson, let's review what we have done over the last few weeks.	Place an image of a graph on the board. Check that students understand slope and y-intercept in relation to the graph and the equation.	Do students understand the question and are ready for the main problem?
Posing the Task Hand out several copies of the question to each student. ♦ - student response(s) Ask them to look at the pattern. Can they see the trend? What would the next stage look like? Explain your thinking. Clarify the problem and ensure that students understand what is being asked. What would the 50th stage be like? Can you write a general term for a stage?	Present an image of the problem from the data projector to the board. Also hand out the problem to students with space for solutions underneath.  Stage 1 Stage 2 Stage 3	Do students use their prior learning to examine and solve the task?
Student Individual Work ***Anticipated Response 1:***  Stage 4 A column of 3 extra squares is added each time. ***Anticipated Response 2:***  Stage 4 There are 2 at the start plus a Rectangle that is 3 squares high.	Use student seating chart to record the approach used by each student. Note the order in which you will call each student up during Ceardaíocht. Help students recall their prior learning.	Do students understand the problem? Are they able to tackle it? Do students use their prior learning in tackling the problem?



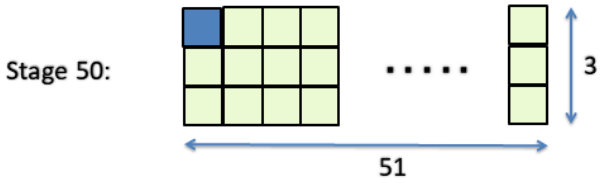
Stage 50: 2 + 50(3) = 152. There are 2 at the start and then 50 groups of 3.

Any Stage: 2 + n(3). There are 2 at the start and then the stage numbers times 3.

Anticipated Response 3:



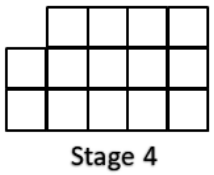
If we had 1 more square, we would have a Rectangle.



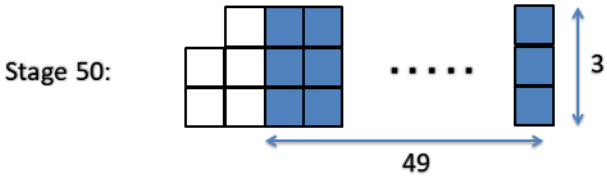
Stage 50: 51(3) -1 = 152. If there was 1 more square we would have 51 groups of 3.

Any Stage: (n+1)(3) – 1. Subtract 1 from 3times 1 more than the stage number.

Anticipated Response 4:



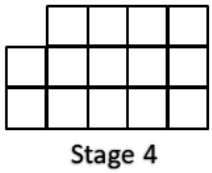
We start with 5and add 3 each time.



Stage 50: 5 + 49(3) = 152. We had 5 at the start and 3 was added on 49 times.

Any Stage: 5 + (n-1)(3). Add 5 to 3 times 1 less than the stage number.

Anticipated Response 5:



Stage 4

Using a table, Stage 1 plus 3 extra each time.

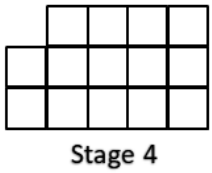
Stage Number (n)	Number of Squares (s)	s	s
1	5	5	5 + 0(3)
2	8	5+3	5 + 1(3)
3	11	5+3+3	5 + 2(3)
4	14	5+3+3+3	5 + 3(3)
50			5 + 49(3)

Stage 50: 5 + 49(3) = 152

Any Stage: 5 + (n-1)3

{**NB: Do not use a + (n-1)d formula as it is not recommended for JC course.**}

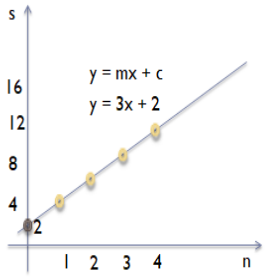
Anticipated Response 6 (May have to be developed by the teacher)



Stage 4

Using a graph.

Stage Number (n)	Number of Squares (s)
1	5
2	8
3	11
4	14
50	?



Stage 50: 3(50) + 2 =152

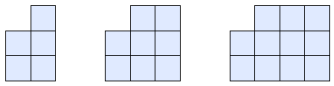
Any Stage (Linear Expression): 3n + 2

{NB:The line is not continuous as shown here. This is because it is discrete data. There is no stage 3.5 for example! Nor is there a zero stage as the graphic may indicate. But rather the linear expression is identical to the shown line but exists only in the domain n=N}

Ceardaíocht /Comparing and Discussing Ask specific students to come to the board with their solution in the following order: Anticipated Response 5 Anticipated Response 2 Anticipated Response 3 Anticipated Response 6 Are these different expressions equivalent?? How many squares in the 100th pattern? Which expression did you choose?	Make sure to attach a student's name to their work when it is presented on the board. Ask students to raise their hand if they got a certain method. Ask different students to provide rationale behind different ideas and how formulae were derived. Put in hints for questions.	Can students explain their approach? Are there similarities between what others did and what is presented on the board? Is there any particular approach that is the best?
Summing up & Reflection Get students to fill out a Reflection on their Learning sheet (appendix 1)	Use the layout of the boardwork to help provide students with a summary of the progression in their learning.	

10. Board Plan

The Task



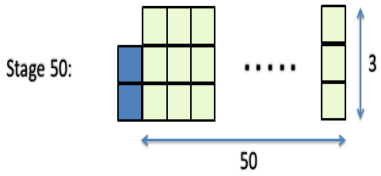
Can you see a trend?
What would the next stage look like?
Explain your thinking.
What would the 50th stage be like?
Can you write a general term for a stage?

Solution 1
Using a Table - Stage 1 plus 3

Stage Number (n)	Number of Squares (s)	s	s
1	5	5	5 + 0(3)
2	8	5+3	5 + 1(3)
3	11	5+3+3	5 + 2(3)
4	14	5+3+3+3	5 + 3(3)
50			5 + 49(3)

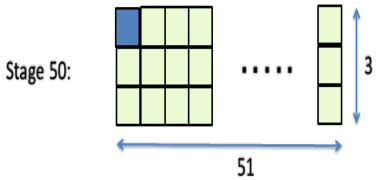
Any Stage: 5 + (n - 1)(3)

Solution 2
There are 2 at the start plus a Rectangle that is 3 squares high.



Any Stage: 2 + n(3)

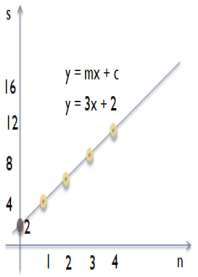
Solution 3
If we had one more Square, we would have a Rectangle



Any Stage: (n+1)(3) – 1

Solution 4
Using a Graph

Stage Number (n)	Number of Squares (s)
1	5
2	8
3	11
4	14
50	?



Stage 50: 3(50) + 2 =152

Any Stage (Linear Expression): 3n + 2

Are these different expressions equivalent??

How many squares in the 100th pattern?

Which expression did you choose?

11. Evaluation

a) Do students recognise that real life problems can be solved in a variety of ways.
b) Did all students understand the problem?
c) Did students collaborate?
d) Do students recognise the link between Algebra, Patterns and Co-ordinate Geometry?

12. Reflection

As a team we hoped that students would use a variety of ways to solve the Great Wall of Nenagh problem. We found that students used at least 4 methods to find solutions. They then furthered their learning by graphing this linear relationship and linking it back to their prior learning in Co-ordinate Geometry.

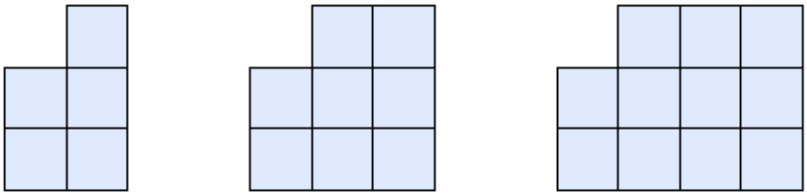
We observed many fantastic problem solving skills amongst the group. While students were in groups of 4, we should have perhaps stated at the start for them to find as many solutions as they could together. This would have prompted them to collaborate more.

At the end of the lesson students were asked to write a reflection on the lesson. The overall feedback was very positive. As a group we were also very impressed with the lesson study process. While we found it hard not to jump in and help the students straight away, we saw that by stepping back and being more facilitator than teacher, the students achieved more. We are currently thinking up ideas to do next year already!!

The teacher was very organised which meant that the lesson ran smoothly. The problem being posed first and using prior student knowledge was found to lead to deeper understanding.

Appendix 1 – Student Reflection

Reflecting on My Learning



Stage 1 Stage 2 Stage 3

What area of mathematics do you think this lesson was about?

What did you learn today?

What did you not understand?

Did you enjoy the lesson? Why or why not?

Student: