Lesson Research Proposal for First Year Geometry

For the lesson on 31st January 2018
At Ardee Community School, Ms. L Matthews class.
Lesson plan developed by: Lauren Matthews, Aishling Donohoe, Brigid Ring.

1.
Title of the Lesson: Untangle the Angles

2. Brief description of the lesson

Students will have prior knowledge on lines, line segments and rays, types of angles and types of triangles. Previously students measured angles using a protractor. During this lesson students will be introduced to a problem and asked to focus on the angles and find relationships between the angles. Students will present their workings and ideas on the board, listen and expand on each other's ideas. Students will categorize their ideas, introducing students to vertically opposite angles, alternate angles, corresponding angles and exterior angles of a triangle.

3. Research Theme

The teaching and learning goals identified by our Mathematics Department for the improvement of our students' learning and our teaching are as follows:

1. We want to select and use assessment practices that progress students' learning and
2. Value and engage in professional development and collaboration

On a regular basis to achieve the above as a Mathematics Department we are going to:
Focus on the family of techniques from Assessment for Learning by using more in-between desk work when students are at work to assess students learning in real time and as far as possible note them as the class progresses. We feel that by doing this we will gain a greater understanding of the general misconceptions, strengths and weaknesses of our students and address these in the following way:

- Encouraging our students to present and explain to their peers in class their various ideas/solutions. By making time for this, students will not only present but grow familiar with presenting at the board by saying "First I did this because……", "Next I did because…."
- Encourage students to actively listen to each other and make relationships, comparisons and differences in various methods presented leading to greater collaboration
- Encourage students to make explicit preferred ways to tackle a problem and why?
- Pay particular attention to our board-work so that the flow of the lesson can be seen
- Support students' reflections on the lesson
- Concentrate on effective questioning

For the coming academic year as a Mathematics Dept. we value our professional development and have decided to engage in Lesson Study as a means of greater collaboration. Developing a research lesson over 6 evening meetings is the key to unlocking collaboration. In addition, when the research lesson is conducted we will invite all Mathematics teachers into our classroom to engage in the classroom observation and post-lesson discussion. We hope to invite a knowledgeable other to the research lesson/

4. Background & Rationale

In primary school, students in fifth and sixth class learn to recognize, classify and describe angles and relate angles to shapes and the environment. They are also exposed to recognizing angles in terms of rotation. They should be able to estimate, measure and construct angles in degrees, using appropriate mathematical instruments. In addition, they explore the sum of angles in both triangles and quadrilateral. As a mathematics department, our collective experience has been that many of the students coming into first year lack the requisite skills to identify angles and their relationships to each other. Students are unable to discern specific angles or shapes within a complex problem or identify angles within shapes that are rotated. In addition, they are unfamiliar with the terminology relating to lines, angles and simple two-dimensional shapes. Finally, there is also concern that students lack competence using a ruler, protractor and compass. The principle reason for choosing Geometry as the focus of our research lesson is because the research lesson underpins so much of the content that will follow in the Junior Cycle course and the Senior Cycle course. Without these fundamental skills, students will lack the mindset to progress with more challenging units of the course.

From our research findings, we found that students found it difficult to recognize angles in different rotations, students were confused by naming angles. Students found using geometry equipment difficult, particularly measuring angles accurately in different rotations they were confused as to what their base line was or whether they were measuring the angles clockwise or anti-clockwise. Applying previously learned knowledge in unseen content, relating previously seen questions to questions they are solving now.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
➢ Identify and examine 3-D shapes and explore relationships, including tetrahedron, octahedron (faces, edges and vertices) 3-D shapes. ➢ Draw the nets of simple 3-D shapes and construct the shapes. ➢ Use angle and line properties to classify and describe triangles and quadrilaterals. ➢ Construct triangles from given sides or angles.	➢ Identify and distinguish the difference between a line, line segment and a ray ➢ Label a given angle. ➢ Identify, draw and describe the difference between right angle, straight angle, acute angle, obtuse angle and a reflex angle. ➢ Draw a given angle using a protractor.	➢ **Constructions.** ➢ Construct a bisector of any given angle, using only a compass and a straight edge. ➢ Construct a perpendicular bisector of a line segment, using only a compass and a straight edge. ➢ Construct a line perpendicular to a given line, passing through a given point on the line. ➢ Construct a line parallel to a given line

➢ Identify the properties of the circle. ➢ Construct a circle of given radius or diameter. ➢ Use 2-D shapes and properties to solve problems. ➢ Classify 2-D shapes according to their lines of symmetry. ➢ Tessellate combinations of 2-D shapes. ➢ Recognize, classify and describe angles and relate angles to shape and the environment. ➢ Recognize angles in terms of rotation. ➢ Estimate, measure and construct angles in degrees. ➢ Bisect an angle with a compass and straight edge*. ➢ Explore the sum of the angles Lines and angles in a triangle in a quadrilateral.	➢ Measure angles using a protractor. ➢ Calculate missing angles on a straight line. ➢ Calculate missing angles at a point. ➢ Convince themselves through investigation that vertically opposite angles are equal in measure. ➢ Convince themselves through investigation that corresponding angles are equal in measure. ➢ Convince themselves through investigation that alternate angles are equal in measure.	passing through a given point. ➢ Divide a line segment into two or three equal segments, without measuring it. ➢ Construct a line segment of a given length on a given ray. ➢ Construct an angle of a given number of degrees with a given ray as one arm. ➢ **Trigonometry.** ➢ Identify, draw and label the sides of a right angled triangle. ➢ Calculate the length of a missing side of a triangle, given the other two sides using the theorem of Pythagoras. ➢ Calculate an angle of a right angled triangle using trigonometric ratios. ➢ Calculate a missing side of a right angled triangle using trigonometric ratios. ➢ Recognize and identify angles of elevation and depression. ➢ **Area, Volume.** ➢ Calculate the area and perimeter of two-dimensional shapes. ➢ Solve problems involving area. ➢ Calculate the surface area and volume of three-dimensional shapes.

		- Solve problems involving surface area and volume. - Draw and interpret scaled diagrams. **Transformations – geometry.** - Recognize and identify axis of symmetry of various objects and shapes. - Reconstruct a shape using a given translation. - Reconstruct an object under a central symmetry in a point. - Identify the center of symmetry of various shapes. - Reconstruct a given shape through the axial symmetry.

6. Goals of the Unit

By the end of this unit, we want students to be able to confidently apply mathematical knowledge and theorem. We want to ensure that students will be able to use mathematical instruments effectively and use investigative methods to answer questions through thinking critically and creatively. Students should be able to see patterns, trends and relationships through exploring options and alternative methods to complete tasks. Students will work together in pairs and groups using appropriate language. They should develop the ability to express their ideas clearly and accurately and reflect on their personal goals and learning.

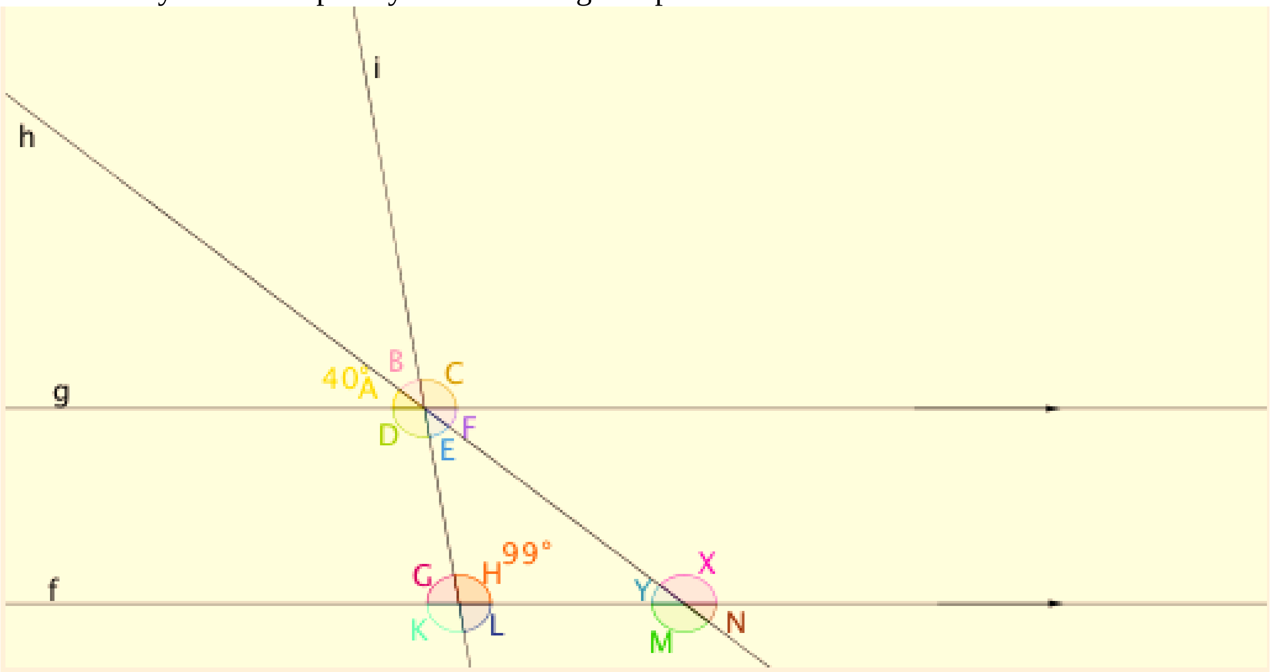
7. Unit Plan

Lesson	**Learning Goals**
1	Recognising basic geometrical concepts. Students will learn about the points on a plane, line, line segments and rays. Reintroducing properties of a line.
2	Different notations for line, line segment and rays. Measure the length of a line segment and also recognising they are unable to measure a ray or a line.
3	Identifying different types of angles. Reintroducing prior knowledge of types of angles. Recognise angles in terms of rotation.

4	Estimate and measure angles accurately.
5	Identify parallel and perpendicular lines, and find examples in room. *Axioms
6	Identify the properties of Isosceles, Scalene and Equilateral triangles.
7	Convince themselves through investigation that the three angles in a triangle equals 180°.
8	**RESEARCH LESSON**
9	Convince themselves through investigation that vertically opposite angles are equal in measure.
10	Convince themselves through investigation that alternate angles are equal in measure.
11	Convince themselves through investigation that corresponding angles are equal in measure.
12	Identify exterior angle of a triangle. Recognise that the sum of the two interior angles is equal to the exterior angle.

Proposed lesson question.

Find as many relationships as you can in the given problem.



Relationship	Reason

8. Goals of the Research Lesson:

Students need to understand what a relationship is, enabling students to discover new relationships relating to various angles. Students will need to know how to measure angles accurately in different rotations, this will enable students to measure angles accurately to discover these new relationships. Students will need to know how to apply their previous knowledge angles in a straight line, angles at a point and angles in a triangle, this will enable students to solve angles therefore discovering relationships between these angles.

The work carried out during the lesson will encourage students to think critically and creatively while exploring options and alternatives to solve a question. Students will be expressing, presenting and sharing their ideas clearly during the lesson, enabling students to work together as a group to build on each idea. Students will develop skills seeing patterns and trends, enabling students to discover each relationship.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction Discuss what students have worked on so far focusing on prior knowledge from previous lessons. Discuss what a relationship is.		
Posing the Task Present students a problem and explain to students what they are expected to do. 'Find relationships between angles..?' Ask students to estimate the amount of relationships they can find in the diagram. Explain to the students that they can measure the angles using a protractor or work out each of the angles using their prior knowledge of geometry. Ensure students are aware of the two given angles. Ask students to work on the problem individually for ten minutes and then we will come together as a class and use their work to discover some new relationships.	The problem will be handed out as an A3 worksheet.	
Student Individual Work **Student 1:** Students have identified that angles at a point add up to 360°. Students can do this by measuring angles using a protractor or using their prior knowledge to solve the angles. $40° + 41° + 99° + 99° + 41° + 40° = 360°$ $\lvert\angle A\rvert + \lvert\angle B\rvert + \lvert\angle C\rvert + \lvert\angle D\rvert + \lvert\angle E\rvert + \lvert\angle F\rvert = 360°$ $81° + 99° + 81° + 99° = 360°$ $\lvert\angle G\rvert + \lvert\angle H\rvert + \lvert\angle L\rvert + \lvert\angle K\rvert = 360°$ $\lvert\angle Y\rvert + \lvert\angle X\rvert + \lvert\angle M\rvert + \lvert\angle N\rvert = 360°$ **Student 2:** Students have identified that angles on a straight line add up to 180°. $\lvert\angle A\rvert + \lvert\angle B\rvert + \lvert\angle C\rvert = 180°$ $\lvert\angle B\rvert + \lvert\angle C\rvert + \lvert\angle F\rvert = 180°$ $\lvert\angle C\rvert + \lvert\angle F\rvert + \lvert\angle E\rvert = 180°$ $\lvert\angle F\rvert + \lvert\angle E\rvert + \lvert\angle D\rvert = 180°$ $\lvert\angle E\rvert + \lvert\angle D\rvert + \lvert\angle A\rvert = 180°$ $\lvert\angle D\rvert + \lvert\angle A\rvert + \lvert\angle B\rvert = 180°$ $\lvert\angle G\rvert + \lvert\angle H\rvert = 180°$ $\lvert\angle H\rvert + \lvert\angle L\rvert = 180°$	Observers will circulate the room and use a seating chart to record the approach each student used, while taking note of the order in which each student will be called up during the Ceardaiocht. Help students where necessary by asking appropriate questions. Encourage students to try and solve the problem a different way if they are finished.	Are students able to work out the problem? Are the students able to identify relationships by seeing patterns? Have any students stopped measuring angles because they can see these relationships?

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$$|\angle L| + |\angle K| = 180°$$
$$|\angle K| + |\angle G| = 180°$$

$$|\angle Y| + |\angle X| = 180°$$
$$|\angle X| + |\angle N| = 180°$$
$$|\angle N| + |\angle M| = 180°$$
$$|\angle M| + |\angle Y| = 180°$$

Student 3:
Students have identified that vertically opposite angles are equal in measure.

$$|\angle A| = |\angle F|$$
$$|\angle B| = |\angle E|$$
$$|\angle C| = |\angle D|$$

$$|\angle G| = |\angle L|$$
$$|\angle H| = |\angle K|$$

$$|\angle Y| = |\angle N|$$
$$|\angle X| = |\angle M|$$

Student 4:
Students have identified that corresponding angles are equal in measure.

$$|\angle K| = |\angle D|$$
$$|\angle H| = |\angle C|$$
$$|\angle N| = |\angle F|$$
$$|\angle Y| = |\angle A|$$
$$|\angle G| = |\angle A| + |\angle B|$$
$$|\angle L| = |\angle E| + |\angle F|$$
$$|\angle X| = |\angle B| + |\angle C|$$
$$|\angle M| = |\angle D| + |\angle E|$$

Student 5:
Students have identified that alternate angles are equal in measure.

$$|\angle F| = |\angle Y|$$
$$|\angle D| = |\angle H|$$
$$|\angle G| = |\angle F| + |\angle E|$$
$$|\angle X| = |\angle D| + |\angle E|$$

$$|\angle K| = |\angle C|$$
$$|\angle N| = |\angle A|$$
$$|\angle M| = |\angle B| + |\angle C|$$
$$|\angle L| = |\angle A| + |\angle B|$$

Student 6:
Students have identified that three angles in a triangle add up to 180°.

$$|\angle E| + |\angle H| + |\angle Y| = 180°$$

Student 7:
Students have identified that an exterior angle of a triangle is equal to the sum of the interior opposite angles.

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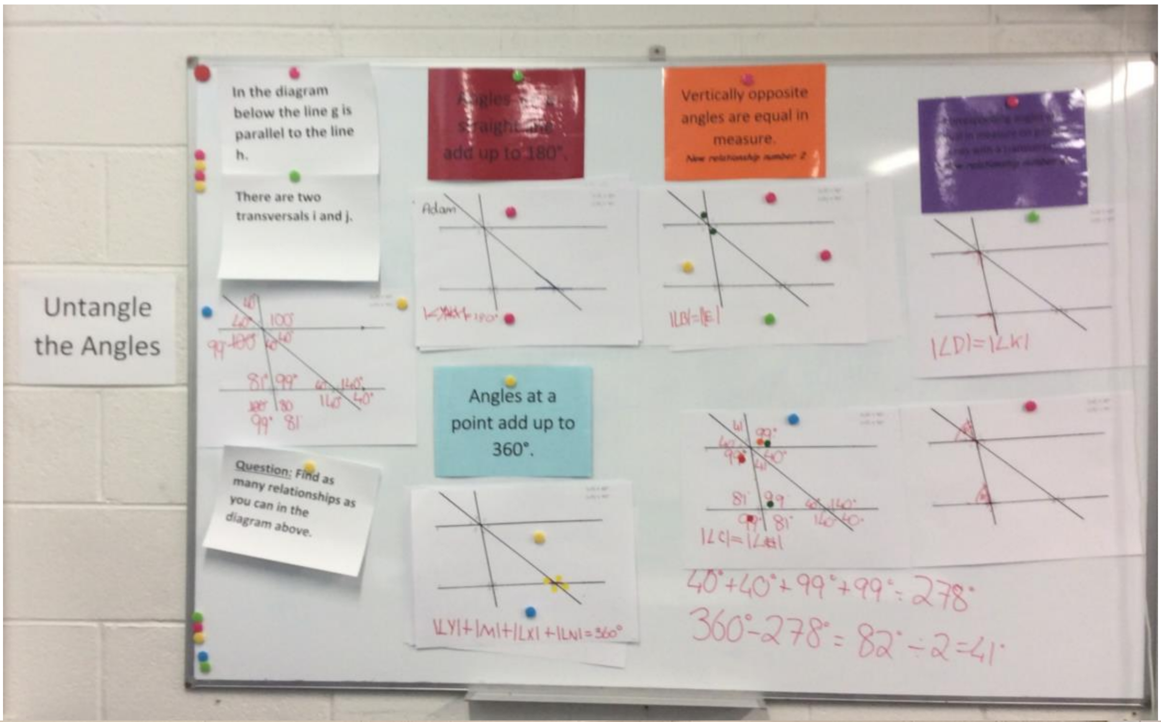
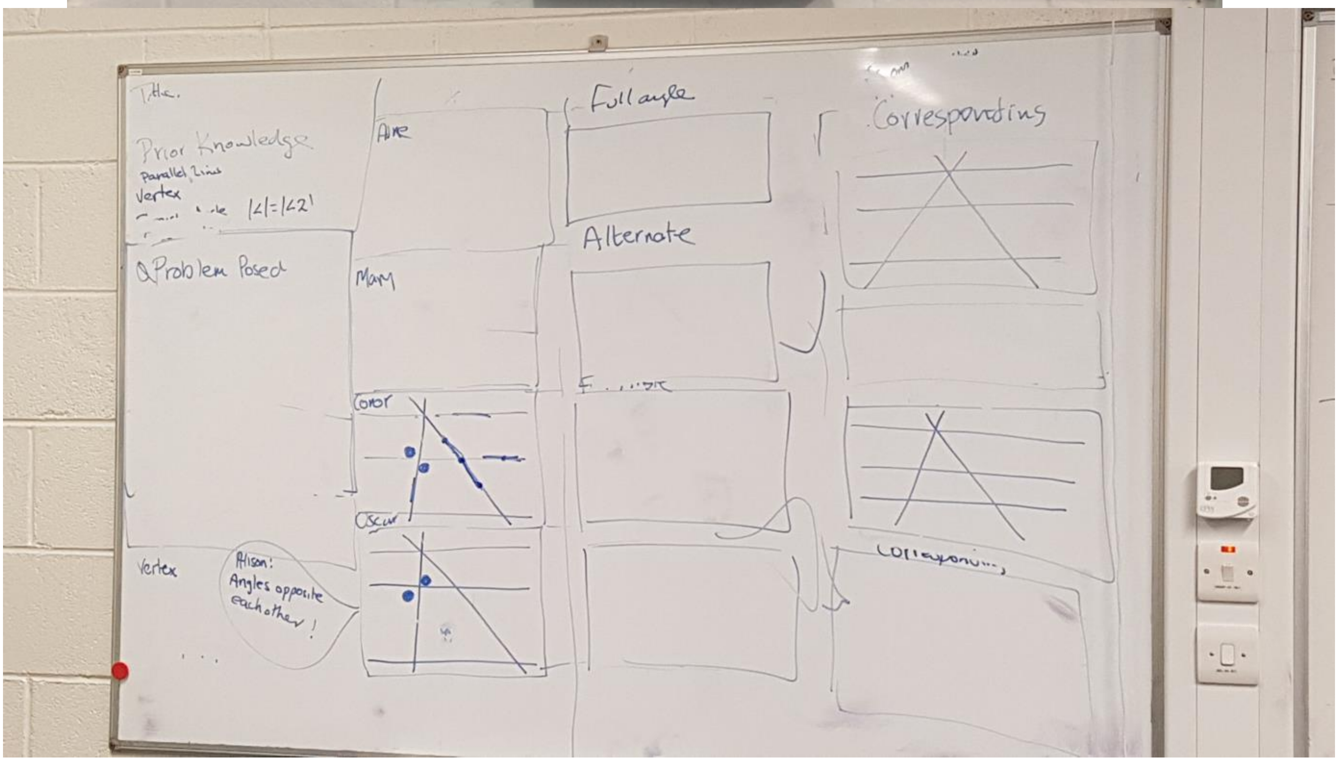
$$\lvert\angle X\rvert = \lvert\angle E\rvert + \lvert\angle H\rvert$$ $$\lvert\angle G\rvert = \lvert\angle E\rvert + \lvert\angle Y\rvert$$ $$\lvert\angle L\rvert = \lvert\angle E\rvert + \lvert\angle Y\rvert$$ $$\lvert\angle M\rvert = \lvert\angle H\rvert + \lvert\angle E\rvert$$ $$\lvert\angle C\rvert + \lvert\angle F\rvert = \lvert\angle Y\rvert + \lvert\angle H\rvert$$ $$\lvert\angle A\rvert + \lvert\angle D\rvert = \lvert\angle H\rvert + \lvert\angle Y\rvert$$		
Ceardaíocht /Comparing and Discussing Teacher: What relationships are visible in this problem? Is there any other patterns you can identify? How many strategies are there for completing the problem? Student: The most complex of problems can be broken down to find the solutions from knowing the angles of a triangle and the straight line rule.	As all the students solutions are posted on the board, students can engage in the task based on their level of ability. Students have their own copy of the problem and they post another copy on the board. Ask students if they see any new relationships / patterns arising?	Have students acknowledged that they are capable of completing this problem based on their prior knowledge? Can they identify and prove their solutions confidently?
(If needed, repeat 2, 3, & 4 above for additional tasks. Otherwise delete this row.)		
Summing up & Reflection Teacher: Using the board work summarize students ideas for the problem. Voice student's opinions on where difficulties arose. Students: Today I noticed that…. Questions I have after today are…. Today I learned from my friends ideas….. I understood from today's lesson…..	Using the board, reflect on what students did in class. Ask student to summarize the new relationships in a short reflection.	

10. Board Plan





11. Evaluation

Students initially were unclear about the word “relationship”. With the help and encouragement of the teacher they got to grips with the issue and soon began to see relationships between different angles. The teacher encouraged students to come up in turn to the board and explain thoroughly what they were doing. Students repeatedly demonstrated each relationship and the deep understanding was teased out of them.

The board work was clear and easy for the students to see and each new relationship clearly identified and then named.

12. Reflection

While great progress was made in teaching and learning, we may have been a bit ambitious in our goals.

Variations to the exact lesson could be to:

- Include the measures of each angle in the worksheet.
- Break up the lesson into two or three lessons.

The class group were slightly distracted by the presence of so many teachers in the room, however they did not try to illicit assistance from any observers.

Students were a little slow at realizing what the word relationship meant (even though this had been used extensively during the run up to the research lesson). However once they were sure about what to do they were eager and excited to contribute to the class.

The value of this question and approach to teaching many different theorems in geometry was recognized by everyone.