Lesson Research Proposal for Second Year Trigonometry

Date of lesson: 27th February 2019
School name: Presentation Secondary School, Castleisland, Co. Kerry.
Teacher giving lesson: Liz Cosgrave
Associate: Conleth Dillon
Lesson plan developed by: Annette Leen & Liz Cosgrave

1. Title of the Lesson: A step in the right direction

2. Brief description of the lesson

This lesson was developed to introduce the idea of trigonometrical ratios, specifically

$\sin A = \frac{opposite}{hypothenuse}$

The lesson involves students discovering the idea of ratio using triangles formed by high heeled shoes. Students will calculate the heel height and instep length to discover if the heel height changes in relation to the size of the shoe. Through this process it is envisaged that students will work towards the discovery and understanding of trigonometrical ratios.

3. Research Theme

Although all of the domains in the 'Looking at our schools' (LAOS) document are relevant to ensuring excellence in teaching and learning we have decided to focus on two domains; Learner experiences and Teachers' collaborative practice. Our research theme emerged after reflecting and clarifying our goals as a Maths department while cognisant of the school's SSE and SIP documents. Our main priority for this research project is to improve teaching and learning in our school and to ensure that our practices are at a highly effective level. The Schools SSE priorities for this year are:

We have chosen to concentrate on the following domains from the LAOS document:

Learner Experiences:

Standards	Statements of highly effective practice
Students engage purposefully in meaningful learning activities	Students demonstrate **very high** levels of interest and participation in learning.
Students reflect on their progress as learners and develop a sense of ownership of and responsibility for their learning.	They have a **sense of ownership of their work**, take pride in it, and **take responsibility for improving it.**

Teachers' Collaborative Practice:

Standards	Statements of highly effective practice
Teachers value and engage in professional development and professional collaboration	Teachers view collaboration as a means to improve student learning and to enhance their own professional development. They engage in constructive collaborative practice, and **in collaborative review of practice.**
Teachers work together to devise learning opportunities for students across the curriculum	Teachers collaborate with relevant and appropriate outside personnel to provide meaningful learning experiences for students, **and work together to ensure that the learning is integrated**.

In order to achieve these goals we have engaged in research involving analysis of exam results, engaging in peer observation, reflection and professional dialogue. Lesson Study affords us the opportunity to collaborate with colleagues and outside personnel to improve the learning experience of students in our school.

4. Background & Rationale

Year: Second Year
Topic: Trigonometry
Level: Higher level, mixed ability class

Research findings:
Following a group discussion Trigonometry was identified as an area where students understanding could be improved. Currently students find this topic challenging, they have trouble with mathematical language and often struggle to link the basic trigonometric ratios to more difficult problems. Students also have great difficulty identifying where the skills learned in this topic can be useful in life outside the classroom. The topic brings together learning from several different strands which students can find difficult. We observed from our own teaching experience and previous exam results that students don't tend to recall the skills learned in this topic after a short time. The basic skills often need to be re-taught in senior cycle, a situation we would like to improve as the senior cycle curriculum can be challenging time wise. Our experience as examiners for the SEC also lead us to identify Trigonometry as an area of significant weakness in both Junior and Leaving certificates. This was reflected in the Chief Examiners report for Junior Certificate Mathematics in 2015 where trigonometry was identified as "challenging". Identifying the hypotenuse caused difficulty that year as the triangle was orientated with the hypotenuse facing downwards. This is an example of the issues that concern us as it seemed that students could not adapt their knowledge to deal with slight differences in how the problem was presented to them. Another question on the examination that year required the use of trigonometry and many students didn't recognise that this was the case.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
From the Primary School curriculum, they should be able to: identify types of angles in the environment identify, describe and classify 2-D shapes: equilateral, isosceles and scalene triangle explore the sum of the	Students should be able to: use trigonometric ratios to solve problems involving angles (integer values) between 0˚ and 90˚ solve problems involving right-angled triangles	From the Leaving Certificate Foundation Level curriculum: apply the result of the theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving heights and distances use trigonometric ratios to solve real world problems

angles in a triangle use angle and line properties to classify and describe triangles and quadrilaterals estimate, measure and construct angles in degrees From the Common Introductory course students should be able to: Synthesis and problem-solving skills: explore patterns and formulate conjectures − explain findings − justify conclusions − communicate mathematics verbally and in written form − apply their knowledge and skills to solve problems in familiar and unfamiliar contexts − analyse information presented verbally and translate it into mathematical form − devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant Conclusions. From the junior Certificate Mathematics syllabus students should be able to: -Prove that the angles in any triangle add to 180˚ -Use equipment to measure angles	consolidate their understanding of the concept of equality to be able to use calculators appropriately and accurately, as well as carrying out calculations by hand and mentally. solve problems involving surds manipulate measure of angles in both decimal and DMS forms From the Junior Cycle Specifications GT. 2 Investigate 2D shapes so that they can interpret scaled diagrams GT.4 evaluate and use trigonometric ratios (sin, cos, and tan, defined in terms of right-angled triangles) and their inverses, involving angles between 0 and 90 in decimal form N1 Present numerical answers to the degree of accuracy specified N2 (a)(b) Investigate equivalent representatives of rational numbers so that they can flexibly convert between fractions and decimals, use and understand ratio and proportion N3 (a) Investigate situations involving proportionality so that they can use absolute and relative comparison where	involving angles From the Leaving Certificate Ordinary Level curriculum: use of the theorem of Pythagoras to solve problems (2D only) use trigonometry to calculate the area of a triangle solve problems using the sine and cosine rules (2D) define sin θ and cos θ for all values of θ define tan θ solve problems involving the area of a sector of a circle and the length of an arc work with trigonometric ratios in surd form From the Leaving Certificate Higher Level curriculum: use trigonometry to solve problems in 3D – graph the trigonometric functions sine, cosine, tangent – graph trigonometric functions of type • f(θ)= a+bSin cθ • g(θ) = a+bCos cθ for a,b,c ∈ R – solve trigonometric equations such as Sin nθ=0 and Cos nθ= ½ giving all solutions – use the radian measure of angles – derive the trigonometric formulae 1, 2, 3, 4, 5, 6, 7, 9 (see appendix) – apply the

− apply the theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving heights and distances solve first degree equations in one or two variables, with coefficients elements of Z and solutions also elements of Z	appropriate. N4 Analyse numerical patterns in different ways U.4 represent a mathematical situation in a variety of different ways, including: numerically, algebraically, graphically, physically, in words; and to interpret, analyse, and compare such representations U.6 make connections between mathematics and the real world U.7 make sense of a given problem, and if necessary mathematise a situation U.8 apply their knowledge and skills to solve a problem, including decomposing it into manageable parts and/or simplifying it using appropriate assumptions U.9 interpret their solution to a problem in terms of the original question U.13 communicate mathematics effectively: justify their reasoning, interpret their results, explain their conclusions, and use the language and notation of mathematics to express mathematical ideas precisely	trigonometric formulae 1-24

6. Goals of the Unit

Students should be able to:

- understand the terms and definitions associated with Trigonometry

- calculate trigonometrical ratios
- recognise the relationship between the Tan of an angle and the slope
- apply their prior knowledge to scaffold their learning
- interpret information given in graphical form
- select appropriate techniques to solve problems

7. Unit Plan

Lesson	Brief overview of lessons in unit
1	Introduction to Pythagoras
2	Using the theorem of Pythagoras to find the sides of right angled triangles
3	Using the theorem of Pythagoras to find the sides of right angled triangles in more complicated problems
4 **Research Lesson**	Discovering sine-using a real life example to help understand what is meant by the sine of an angle.
5	Cosine and tangent-building on the learning from yesterday to find Cos and Tan ratios
6	Finding the length of a side in a right-angled triangle
7	Finding the length of a side in a right-angled triangle
8	Finding an angle given Sin, Cos, Tan
9	Finding an angle given two sides
10	Solving more difficult problems
11	Solving more difficult problems

8. Goals of the Research Lesson:

Looking at the goals of the research lesson itself from two perspectives:

a. Mathematical goals (what students will know/understand by the end of the lesson)
Students should:
   - have an increased understanding of the relationship between angles and distance
   - realise that there are practical applications to what they learn in Maths class
   - understand that trigonometric ratios can be used to solve this problem
   - enjoy the lesson and be motivated to learn
   - understand that ratio is another word for fraction
   - realise the sine is a trigonometric ratio and that sin A $=\frac{opposite}{hypothenuse}$
   - realise that with respect to an angle of 30˚ the ratio of the opposite to the hypotenuse in a right-angled triangle is $\frac{1}{2}$.

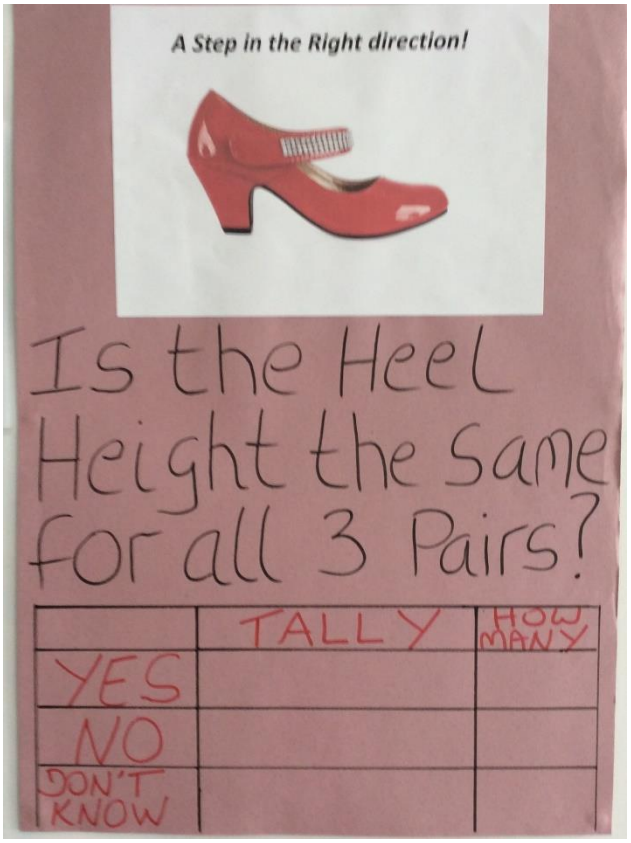
b. Key Skills and Statements of Learning
Key skills
   - Being Literate: Students will show an understanding of key words and will be encouraged to express their answers verbally.
   - Being numerate: students will use their numeracy skills to reason out the problem.
   - Communication: Students will present their solution to the problem.
   - Managing information and thinking: Students will be encouraged to think about the problem individually and to critically analyse the information provided.
   - Managing myself: Students should reflect on how they did and see where they could

improve.
- Staying well: Students should grow in confidence through the discovery of possible solutions.
- Being creative: Students will be required to think creatively to solve the problem.
- Working with others: Students will work in teams and engage in peer led learning.

Statements of learning
The student should be able to:
- communicate effectively using a variety of means in a range of contexts
- recognise the potential uses of mathematical knowledge, skills and understanding in all areas of learning
- describe, illustrate, interpret, predict and explain patterns and relationships
- devise and evaluate strategies for investigating and solving problems using mathematical knowledge, reasoning and skills

9. Flow of the Research Lesson:

Steps, Learning Activities **Teacher's Questions and Expected Student Reactions**	**Teacher Support**	**Assessment**
This column shows the major events and flow of the lesson, including timings and what will go up on the board.	This column shows additional moves, questions, or statements that the teacher may need to make to help students.	This column identifies (a) what the teacher will look for (formative assessment) that indicates it makes sense to continue with the lesson, and (b) what observers should look for to determine whether each segment of the lesson is having the intended effect.
Introduction **Does heel-height vary according to shoe size?**	Teacher introduces the lesson and prompts students to display prior knowledge	Do students agree or disagree? Why? Can the students adequately communicate their arguments? (Key skill)
Posing the Task Examination of the heel height in relation to the length of the instep	Teacher displays some shoes and, on the overhead, poses the question in the form of a story.	Can students use their numeracy skills to reason out the problem (Key skill)

 	Teacher takes a tally of the students' opinions prior to introducing the individual task	
Student Individual Work Students are given three pictures of shoes of various sizes with a triangle superimposed on each picture and a table to fill in. Students measure the heel and instep using a ruler and front angle using a protractor	Teacher prompts the class to suggest the correct unit of measurement for the task.	Are the students able to measure lines and angles accurately? Can students critically analyse their data? (Key skill) Are the students able to notice patterns in their data and draw conclusions from them?

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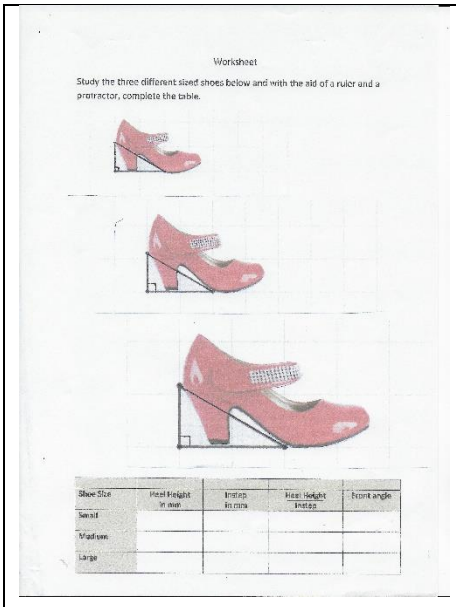
<td> 
</td>
<td>Students get a larger picture of the triangles the same size as the ones that will be on the board to measure and compare findings.</td>
<td>Can students work in teams and engage in peer led learning to arrive at conclusions?</td>
</tr>
<tr>
<td>Ceardaíocht /Comparing and Discussing

Students are called to the board and fill in the master copy of the worksheet

<table>
<tr><th>SIZE</th><th>HEEL HEIGHT</th><th>INSTEP</th><th>HEEL HEIGHT / INSTEP</th><th>FRONT ANGLE</th></tr>
<tr><td>SMALL</td><td></td><td></td><td></td><td></td></tr>
<tr><td>MEDIUM</td><td></td><td></td><td></td><td></td></tr>
<tr><td>LARGE</td><td></td><td></td><td></td><td></td></tr>
</table>

The length of the heel height varies

The length of the instep varies

The length of the heel height is half the length of the hypotenuse

Angles measured are all 30˚</td>
<td>Large version of the shoe pictures on the board and the Master copy of table on the board

What do you notice about the measurements of the heel heights?

What do you notice about the three insteps?

Can you explain the relationship between the heel height and the instep?

What do you notice about the measurement of the three angles?

Would this be the case for any right-angled triangles with an angle of 30˚?</td>
<td>Can students explain the relationship/ratio between the two sides in question?

Are students more confident and enthusiastic due to this discovery?

Can students explain their discovery using mathematical keywords? (Keyskill)

Are students curious to see if this is the case for other angles?</td>
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<table>
<tr><td></td><td></td><td></td></tr>
<tr><td>Student Individual Work
Students are then given various right angled triangles with an angle of 30°. They measure the opposite and hypotenuse using a ruler and front angle using a protractor.</td><td>Teacher prompts the students to suggest the correct unit of measurement for the task.</td><td>Are the students able to measure lines and angles accurately?

Can students critically analyse their data? (Key skill)

Are the students able to notice patterns in their data and draw conclusions from them?

Can students work in teams and engage in peer led learning to arrive at conclusions?</td></tr>
<tr><td>In right-angled triangles containing an angle of 30°, the length of the opposite side is half the length of the hypotenuse</td><td></td><td></td></tr>
<tr><td>Ceardaíocht /Comparing and Discussing

Students are called to the board and fill in the master copy of the worksheet
<table><tr><th>SIZE</th><th>OPPOSITE</th><th>HYPOTENUSE</th><th>OPPOSITE / HYPOTENUSE</th><th>FRONT ANGLE</th></tr><tr><td>SMALL</td><td></td><td></td><td></td><td></td></tr><tr><td>MEDIUM</td><td></td><td></td><td></td><td></td></tr><tr><td>LARGE</td><td></td><td></td><td></td><td></td></tr></table>
Students replace “shoe terms” with trigonometrical terms i.e. opposite, hypotenuse

The length of the opposite side is half the length of the hypotenuse</td><td>Large version of the shoe pictures on the board
Master copy of table on the board

Teacher asks students to replace shoe terms with pre-prepared trigonometrical terms.

What do you notice about the three sides opposite the angle (heel height)?

What do you notice about the three hypotenuses?

Can you explain the relationship between the opposite and the hypotenuse?</td><td>Are students curious to see if this is the case for other angles?

Can the students replace the terms correctly?

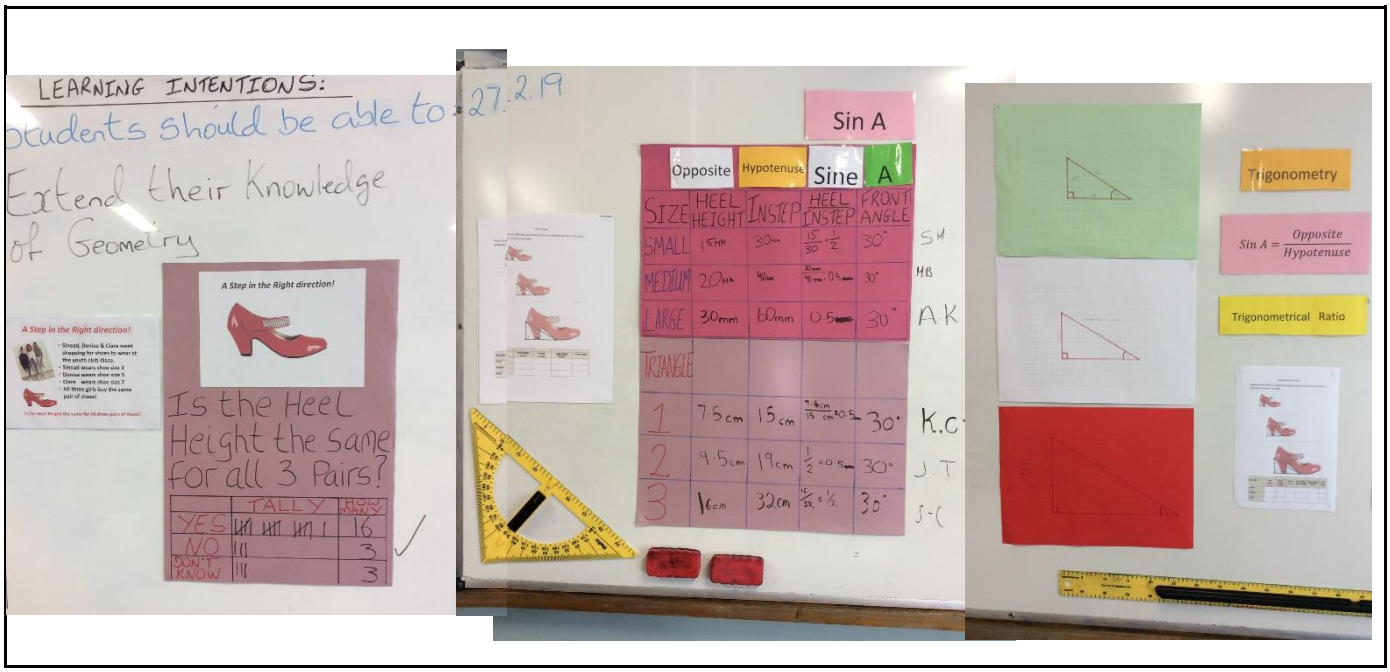
Can students explain that the length of the opposite is different for each triangle?

Can students explain that the length of the hypotenuse varies

Can students explain the relationship/ratio between the two sides in question?</td></tr>
</table>

Angles measured are all 30˚ In right-angled triangles containing an angle of 30°, the length of the opposite side is half the length of the hypotenuse	What do you notice about the measurement of the three angles? Would this be the case for any right-angled triangles with an angle of 30˚? Lets shorten the statement to $\sin 30° = \frac{1}{2}$	Are students more confident and enthusiastic due to this discovery? Can students explain their discovery using mathematical keywords? (Keyskill) Can students shorten the statement to $\sin 30° = \frac{1}{2}$
Summing up & Reflection Teacher revisits original question and the tally of the students' opinions. Students will be provided with a 2 stars and a wish reflection sheet	Teacher uses the board work to help students summarise. Students will be asked to reflect on the lesson and their learning and asked to note two positives and something they wish to find out to extend their learning	Have students changed their opinions? If so, why? Can they explain the terms opposite, hypotenuse, sine, sin A?

10. Board Plan



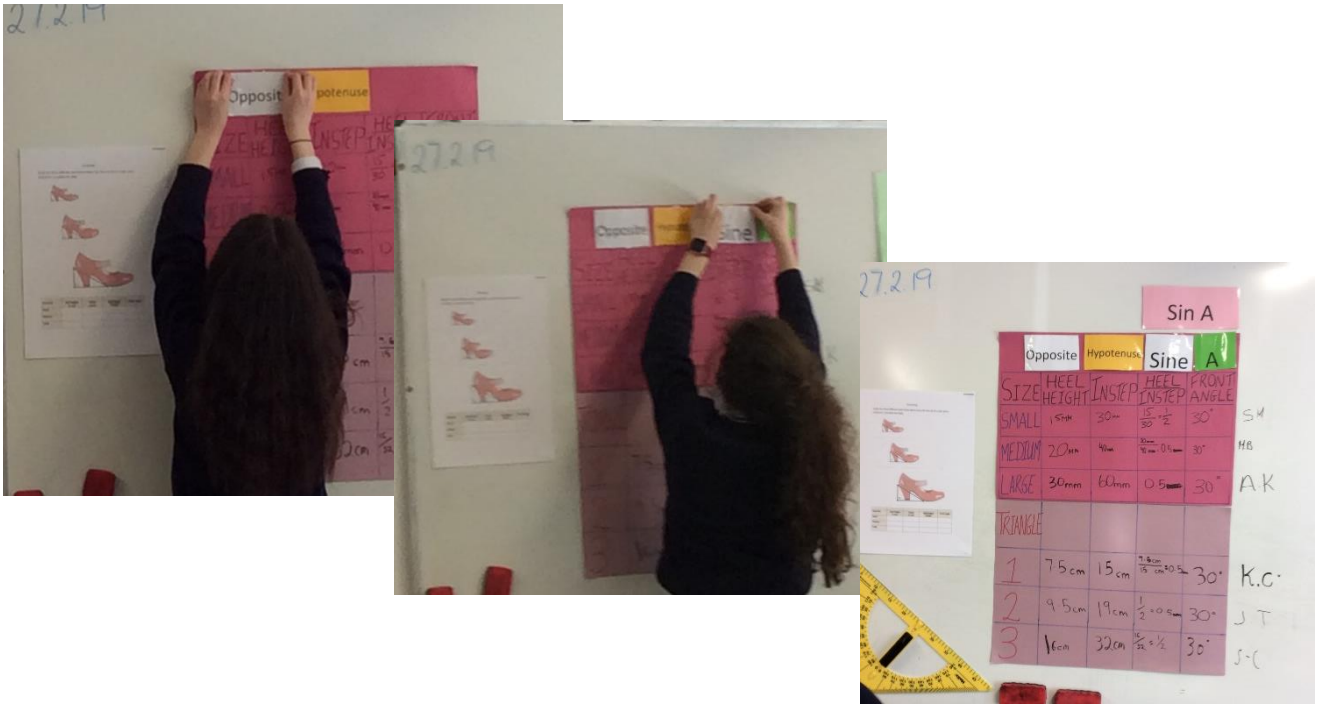
11. Evaluation

Through this lesson we hope to improve students understanding of trigonometrical ratios. Therefore, to evaluate the lesson we will analyse the answers to the following set of questions:

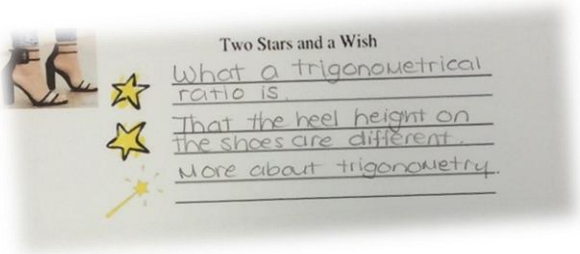
- Do students have a basic understanding of what a trigonometric ratio is and that there is a connection between angles and distance?
- Can students see the mathematical application of mathematics in real life problems?
- Do the students want to know more about this topic?
- Did the students enjoy the lesson?
- Were all students engaged in the lesson regardless of mathematical ability?
- What changes need to be made to the lesson if it were to be used again?

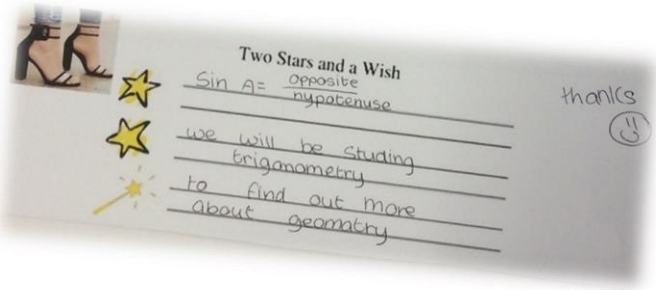
12. Reflection

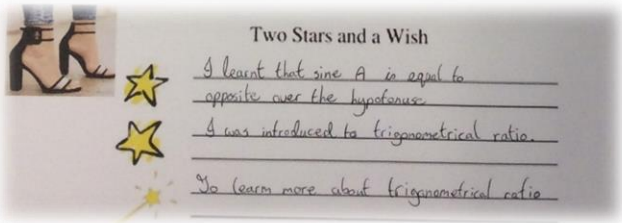
The lesson went very well. Students were on task and interested with a positive attitude throughout. They were not distracted by the observers and didn't seem to notice them as they moved around the room. During the lesson the team were amazed by the amount of maths the lesson contained from across many topics. The team were impressed by the amount of mathematical language the students were using in their discussion with each other.



They replaced the language of shoes with mathematical keywords very easily. Students displayed pride in their work on the board and were happy to initial their piece of work. The team were delighted to hear the students still discussing the problem and its practical applications to their own lives when they were leaving the room. The students' reflection sheets showed that they enjoyed the lesson, were more knowledgeable and that they are eager to learn more about the topic.







The teacher displayed various questioning techniques and a great ability to guide students towards solutions during the lesson. The learning outcome was displayed as well as the research question. These were referred to again at the end of the lesson to ensure the students realized what they had discovered over the course of the lesson. She emphasised keywords throughout and students repeated the words back to her when they were giving their input. Differentiation for students of different abilities was apparent throughout with weaker students given more support and stronger students encouraged to go further. Students were encouraged to come to the board and displayed confidence and pride in their work. The teacher's excellent relationship with the group was apparent as they clearly felt their opinions were valued and they were confident to discuss their findings. The lesson was slightly rushed

at the end due in part to the amount of debate and discussion the students engaged in. The time issue would need to be addressed if the lesson were to be reproduced.

The lesson study process itself has been very beneficial to us as teachers. It gave us time out to think and discuss various difficulties we encounter daily. It promoted a culture of collegial collaboration within the maths department. The importance of collaborating with colleagues who understand the context of the school cannot be underestimated. Both teachers met outside of the lesson study workshops and felt that the time spent on lesson study was time well spent. It encouraged us to think about our teaching in a different way and to use a more facilitation approach to problem solving. The in depth study of the topic meant that we developed our understanding of the primary syllabus as well as the new Junior Cycle specifications which will be of benefit to our teaching.

The main challenges we encountered during the lesson study process were problems using ICT to create the worksheets that we wanted to produce and time constraints. The benefits far outweigh any challenges we faced.

Overall it is very clear to us that we reached the goals we set for the research lesson, enriched the learning and teaching experience in the classroom through collaboration.