Lesson Research Proposal for 3rd Year Higher Level

Date of lesson: 4th March 2019
School names: St. Louis Secondary School and St. Mary's College
Teacher giving lesson: Cathal Halpenny
Associate: Cathal Halpenny
Lesson plan developed by: Anne Marie Kirk, Denyse Hughes and Cathal Halpenny

1. Title of the Lesson: All aboard QUADRATIC Airlines

2. Brief description of the lesson

This lessons aims to provoke students understanding of how quadratic graphs relate to their equation and how they can relate to other similar equations.

3. Research Theme

The teaching and learning goals identified by the group are in the domains of (a) Learner experiences and (b) Teachers' individual practice. These two areas are linked to the SSE priorities that have been established in each school.

- In the domain of learner experiences the standard which the lesson is targeted is for students to reflect on their progress as learners and develop a sense of ownership of and responsibility for their learning.
- In the domain of teachers' individual practice, we are driven to achieve the goal of the teacher responding to individual learning needs and differentiates teaching and learning activities as necessary.

4. Background & Rationale

Should be based on discussion within the group
The topic chosen to focus on is functions to 3rd year higher level students.
The main reasons for this choice are as follows:

- The group felt the topic of functions is received with a negative attitude by students.
- A significant number of students have difficulty with understanding of functions.
- Teachers felt functions would be a good topic to incorporate skills in digital technology which spans many of the Junior Cycle key skills.
- It was also discussed the lack of students understanding of the links that functions has with many other topics on the junior cycle course e.g. algebra, patterns and co-ordinate geometry.

The rationale of why the group chose the topic of functions was mainly from teachers' collective experience. Evidence was also found from recent Chief examiner reports. One teacher in the group has corrected state exams for many years and has seen first-hand the common misconceptions and lack of conceptual understanding from students.

5. Relationship of the Unit to the Syllabus

Describe how this unit relates to the syllabus/learning outcomes from prior years, for this year and for future learning.

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
1st Year Common introductory course **Strand 4: 4.2 Representing situations with tables diagrams and graphs** − use tables, diagrams and graphs as a tool for analysing relations − develop and use their own mathematical strategies and ideas and consider those of others − present and interpret solutions, explaining and justifying methods, inferences and reasoning **All strands** − explore patterns and formulate conjectures − explain findings − justify conclusions − communicate mathematics verbally and in written form − apply their knowledge and skills to solve problems in familiar and unfamiliar contexts − analyse information presented verbally and translate it into mathematical form - devise, select and use	Functions - Interpreting and representing quadratic functions in graphical form. draw graphs of the following functions and interpret equations of the form • $f(x) = ax^2 + bx + c$, where $a, b, c \in Z, x \in R$ Find maximum and minimum values of quadratic functions from a graph	Interpret inequalities of the form $f(x) \leq g(x)$ as a comparison of functions of the above form; use graphical methods to find approximate solution sets of these inequalities and interpret the results

appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions		

6. Goals of the Unit

● Be capable of factorising quadratic equations and will be able to form these equations from their roots.
- Plot points on an x y axis.
● Use different methods to find the roots of a quadratic equation.
● Understand the relationship between roots, factors and quadratic functions.
● Proficiently sketch a graph of a quadratic function.
● Be confident to solve problems using graphs. e.g. finding values for x and y.
● Understand the shifting vertically and horizontally of the function x^2 and the resulting equations that are formed.
● Find the minimum or maximum point on a graph and understand how it relates to the quadratic equation in perfect square form.

7. Unit Plan

Lesson	Brief overview of lessons in unit
1 – 3	Graphing quadratic Functions
4	Solving problems using graphs e.g. finding outputs from given inputs and vice versa
5 – 6	Using Roots to form a quadratic equation
7	RESEARCH LESSON
8 . .	Solving problems on the max and min points of quadratic graphs. • .

8. Goals of the Research Lesson:

Looking at the goals of the research lesson itself from two perspectives:

a. Mathematical goals (what students will know/understand by the end of the lesson)
b. Key Skills and Statements of Learning

- We want students to form quadratic equations from a given graph.
- We want students to link quadratic equations and where their corresponding graphs cross the x and y axis.
- Recognise the roots of a quadratic and their importance when solving quadratic equations.
- Recognise the graph undergoes axial symmetry through the y axis.
- Relate the difference of two squares to their graph.
- Recognise the vertical shifting of the graph x^2 and its corresponding equations.
- Students to take ownership of their learning

Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction (7 mins) Students will be given a quadratic graph and asked to determine its equation. This is used to revise students' knowledge from previous lessons in the unit.	Students are asked to find the equation of the quadratic graph given using methods learned from previous lessons.	The Teacher will ask students to explain their method(s). This will be shared with students on the board.
Posing the Task (3 mins) The problem is distributed to each student. Students are asked to read the problem carefully and ask any questions.	Teacher may need to clarify terms and what is being asked.	
Student Individual Work (15 mins) ***Part (i) Solutions*** **Student 1** Using the roots to find the factors of the quadratic (i) Multiplying out or (ii) Realising it's the difference of two squares $x = 1 \quad x = -1$ $(x - 1) \quad (x + 1)$ $x^2 - 1$ **Student 2** Using the roots and the equation $x^2 - sumx + product$ $x = 1 \quad x = -1$ $x^2 - (1 + (-1))x + (1)(-1)$ $x^2 - 0x - 1$ $x^2 - 1$ **Student 3** The loop is the shifting of the function x^2 vertically by -1. This is also where the graph crosses the y axis. Therefore the equation is $x^2 - 1$ **Student 4** Writing out the pattern and finding a general term.	The teacher circles the room to question students on their ideas and methods and prompt/clarify if necessary.	The teacher may ask students. "How can you be sure the equation you have found is correct?" "Can you verify your answer?" This will hopefully prompt students to pick other points on the graph and use substitution.

T0	T1	T2	T3	T4	Tn
-1	0	3	8	15	n^2-1

By inspection of using the equation
Tn = an^2+bn+c
where a=2nd diff/2 => a = 1, b =0 and c = -1

Part (ii) Solutions

Student 1

Using similar roots to find the factors of the quadratic

(ii) Multiplying out or (ii) Realising it's the difference of two squares
e.g.

$$x = 2 \quad x = -2$$
$$(x - 2) \quad (x + 2)$$
$$x^2 - 4$$

Student 2

Using the roots and the equation $x^2 - sumx + product$
e.g.

$$x = 3 \quad x = -3$$
$$x^2 - (3 + (-3))x + (3)(-3)$$
$$x^2 - 0x - 9$$
$$x^2 - 9$$

Student 3

The original quadratic is shifted by drawing and guesswork but no equation found.

Student 4

A student draws any graph that clearly crosses the y axis at a point that corresponds to the constant in their equation but is an incorrect graph. e.g. a student draws a graph that cuts the y axis at -3 and writes down the equation $x^2 - 3$ but the roots and other points are incorrect.

Student 5

A translation is correctly drawn by shifting and the new equation is found.

Student 6

A student who realises the roots are the square root of where the graph cuts the y axis. e.g if the graph cuts the y-axis at 5 the roots are $\pm\sqrt{5}$

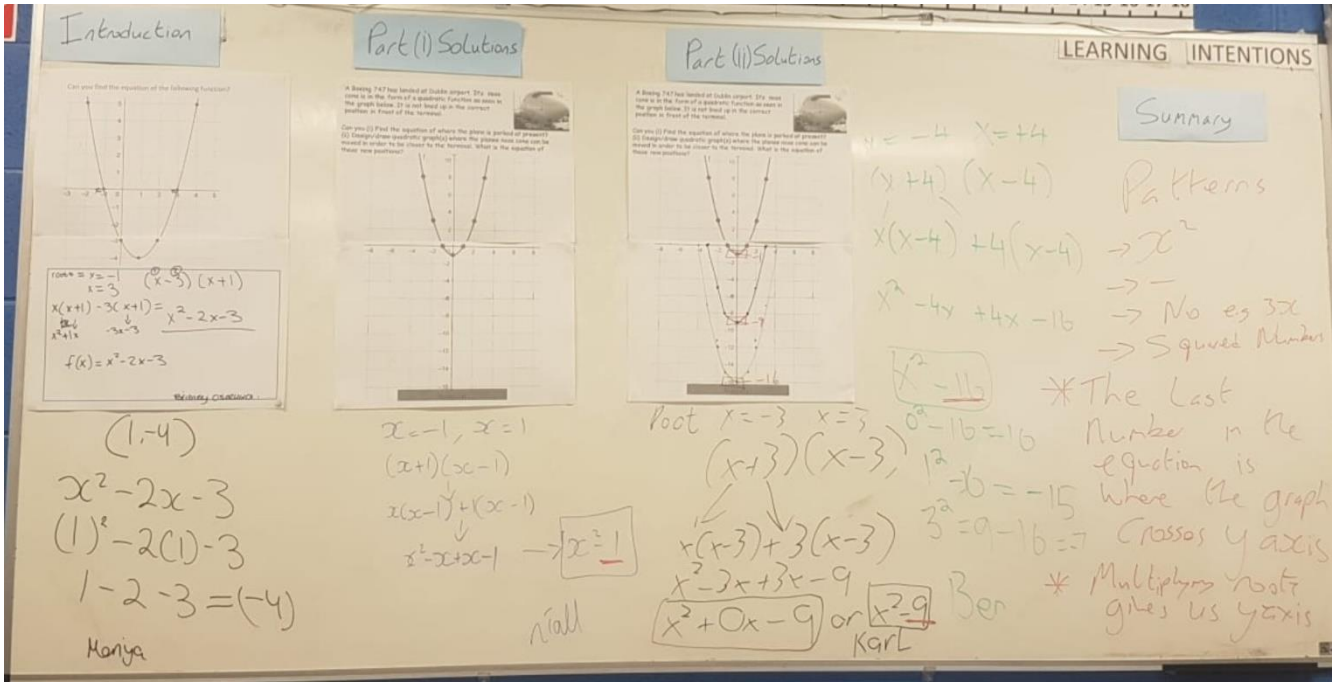
Student 7

A student uses the method of the minimum point being in the form $(x-p)^2+q$, where the shifting of the x^2 graph horizontally and

vertically to give a minimum point of (p, q)		
Ceardaíocht /Comparing and Discussing (15 mins) The first students who will be brought to the board will be those who used the same method as in the introduction. e.g. roots and factors. Students who have designed similar equations using the difference of two squares will be brought to the board to show their findings.	The teacher will The teacher will ask students what patterns do they see between the graphs, e.g. x^2-4, x^2-9, x^2-16 Do students see any patterns with points on the graph and the following functions? When a student has written the equation of a graph, how do they know it is correct for the graph they have drawn?	Students should see patterns such as; • Symmetry through the y axis • The value of the product is the value where the graph cuts the y axis.
Summing up & Reflection (10 mins) Students should be reminded of how roots of a quadratic relate to its equation. Students should be confident of shifting a quadratic graph vertically.	Teacher summaries student's ideas on the board.	

9. Board Plan

Board Plan



10. Evaluation

- The majority of students were able to identify the roots of a quadratic graph and establish its equation using them.
- Many students were capable of verifying points on a graph with its equation. Students have a good understanding of this.

11. Reflection

- Students' showing their solutions on the board was very beneficial to the rest of the class. Students develop ideas and thinking, as well as gaining motivation to persevere with their attempts at solving the problem.

- Making links and having an exercise to promote students prior learning really helps and motivates weaker students to tackle the problem.

A Boeing 747 has landed at Dublin airport. It's nose cone is in the form of a quadratic function as seen in the graph below. It is not lined up in the correct position in front of the terminal.

Can you (i) Find the equation of where the plane is parked at present? (ii) Design/draw quadratic graph(s) where the planes nose cone can be moved in order to be closer to the terminal. What is the equation of these new positions?