Lesson Research Proposal for 2nd Year Higher Level

Date of lesson: 4/6/2019
School name: Coláiste Mhuire Johnstown
Teacher giving lesson: Sean Brennan
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Lesson plan developed by: Brian Kennedy, Sean Brennan

1. **Title of the Lesson: Get to the Problem of the Root.**

2. **Brief description of the lesson:** Identifying relationships between quadratic equations, roots and graphs. From discovery of these connections a formula linking roots to quadratics will be derived.

3. Research Theme

At Coláiste Mhuire Johnstown, we want students to:
a) Enjoy their learning, are motivated to learn, and expect to achieve as learners.
b) Reflect on their progress as learners and develop a sense of ownership and responsibility for their learning

As Maths teachers, we will actively support the achievement of these goals by:
1) Providing a positive learning environment that allows for collaborative learning and values all ability levels.
2) Structuring our lesson to include positive feedback and promote personal reflections.

4. Background & Rationale

Why did we choose algebra as a research topic? We as a Maths department have identified Algebra as a problem area amongst our students. Common problems include trying to relate Algebra to their everyday life, remembering the skills and procedures associated when dealing with Algebra. Problems also arise when trying to relate their knowledge in Algebra to other topics present in the Mathematics curriculum. We find students struggle to identify the necessity to use Algebra to solve certain problems.

b) *Our Research findings:* We decided to look at our previous 3 years of Junior Cert results. We surveyed our TY/5^{th}/6^{th} year students on what questions they found most difficult from their Junior Cert paper. The majority of the students surveyed reported that they found Algebra the most difficult topic. On future discussion, we felt that the rote learning approach around algebra wasn't working effectively. Students weren't retaining the steps required to complete the algebraic questions correctly. When algebra is put with a secondary topic, for example- distance, speed and time, students feel better equipped to answer the question. We put this down to teaching approaches such as the use of problems, graphs and estimation techniques to help students grasp the concept.

5. Relationship of the Unit to the Syllabus

Describe how this unit relates to the syllabus/learning outcomes from prior years, for this year and for future learning.

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
In 6th class the child should be enabled to • explore the concept of a variable in the context of simple patterns, tables and simple formulae and substitute values for variables identify and discuss simple formulae from other strands	**AF.3** flexibly translate between the factorised and expanded forms of algebraic expressions of the form: I. axy, where a ∈ $\mathbb{Z}$ II. axy + byz, where a, b ∈ $\mathbb{Z}$ III. sx – ty + tx – sy, where s, t ∈ $\mathbb{Z}$ IV. $dx^2 + bx$; $x^2 + bx + c$; (and $ax^2 + bx + c$), where b, c, d ∈ $\mathbb{Z}$ and a ∈ $\mathbb{N}$ V. $x^2 – a^2$ (and $a^2 x^2 – b^2 y^2$), where a, b ∈ $\mathbb{N}$ **AF.5** form quadratic equations given integer root	**AF.4** select and use suitable strategies (graphic, numeric, algebraic, trial and improvement, working backwards) for finding solutions to: a) linear equations in one variable with coefficients in $\mathbb{Q}$ and solutions in $\mathbb{Z}$ or in $\mathbb{Q}$ b)quadratic equations in one variable with coefficients and solutions in $\mathbb{Z}$ (coefficients in $\mathbb{Q}$ and solutions in $\mathbb{R}$) c) simultaneous linear equations in two variables with coefficients and solutions in $\mathbb{Z}$ (or in $\mathbb{Q}$) d) linear inequalities in one variable of the form $g(x) < k$, and graph the solution sets on the number line for x ∈ $\mathbb{N}$, $\mathbb{Z}$, and $\mathbb{R}$

5. Goals of the Unit

- Allow students to consolidate their prior knowledge of manipulating and solving equations.
- Students understand algebraic equations can be used to solve real life situations
- Students will understand that not all problems result in the same types of equation.
- Use real life problems as motivation for the study and application of factors.
- Use appropriate graphing technologies.
- Enable students to be creative about solving different equations in their own particular way.

Key Skills and Statements of Learning:

In preparation for this lesson the Junior Cycle Key Skills and Statements of Learning have been taken into consideration. In this lesson we will address the Key Skills in the following ways:

I. Managing Information & Thinking: Students will be encouraged to think creatively and critically.

II. Being Numerate: Students will see patterns, trends and relationships.

III. Being Creative: By being creative, students will explore options and implement their ideas to solve problems.
IV. Working with Others: As the teacher takes solutions from the class and presents them on the board, students will learn by working with others.
V. Communicating: Students will be encouraged to present their thinking and explain the rationale behind it.
VI. Being Literate: Students will express their ideas during Ceardaíocht
VII. Managing Myself: Students will have an opportunity to reflect on their learning when the lesson is over
VIII. Staying Well: By applying their own prior knowledge and being engaged in avtive learning students will be positive about their learning.

This lesson also meets the following Junior Cycle Statements of Learning:
1. The student communicates effectively using a variety of means in a range of contexts.
15. The student recognizes the potential uses of mathematical knowledge, skills and understanding in all areas of learning.
16. The student describes, illustrates, interprets, predicts and explains patterns and relationships.
17. The student devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills.

6. Unit Plan

Lesson	Brief overview of lessons in unit
1	Recap: - Removing brackets - Evaluating expressions. - HCF - factorising with common terms. - factorising by grouping terms.
2	Explore the idea that x is a variable and that each input will have an output. Sub in values for x into same function. Acknowledge inputs and outputs form a couple.
3	Graphing functions. Recap on graphing linear functions; input and output. What does a linear graph look like? Use idea of couples covered in last lesson as coordinates to graph quadratic functions. Why is there a curve in a quadratic graph? Explore shapes certain functions take in graph form.
4	Solving quadratic equations through factorization. Sketching quadratic graphs using roots
5	Verifying roots of a quadratics Using the –b formula to solve quadratics and when/why it is needed
6	Research Lesson

7. Goals of the Research Lesson:

For students to recap on prior knowledge of factorizing and solving quadratics.
For students to create relationships between roots, graphs and equations using matching exercise problems.
For students to come up with as many ways as possible to complete the matching exercise
For students to derive the formula: $x^2 - x(sum\ of\ the\ roots) + (product\ of\ the\ roots) = 0$

8. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction 0-5 mins: Recap on quadratics - how to factorise and solve.	In groups, using mini whiteboards, factorise and solve the following equations.	Students work in groups factorising and solving the given equations. Teacher circulates looking at different answers. Students show whiteboards to the rest of the class.
Posing the Task 5-15 mins: **1st activity**: **Matching exercise 1 and 2** In pairs, try and match the following: Graph ↔ Roots ↔ Equation There are 2 matching exercises. Each pair gets one and tries to complete it. Once completed each pair then explains to the other pair in the group of 4 how they completed their sheet Let's go through the problem to see what connections were made through this activity. Teacher invites different groups up to the board to explain their answers. 15 – 25 mins: **2nd activity:** **Matching exercise 3** In pairs now try match the following: Roots ↔ Equation	Teacher hands out resources attached *(matching exercise 1 and 2)* Each pair gets four of each. Does everyone understand what is being asked of them? How do you know you are correct? Teacher hands out new resources minus the graphs. *(matching exercise 3)*	Can students complete the activity making connections between all three? What did you notice? What relationship does each correct match have?

Now let's discuss how we solved the problem without the graphs and see what connections we can make. Teacher invites different students up to the board to explain their reasoning on how they completed the matching exercise. Students are selected in order according to **anticipated student responses** below 25 -35 mins: **3rd activity:** Can you come up with another way of creating a relationship between roots and equations? Is there a quicker way of creating a quadratic from its roots apart from what we have just done? Can you derive your own formula to connect the roots and equations? 35- 40 mins: **Recap and Reflection** Teacher hands out reflection sheets for each student to complete individually. Teacher sets homework based on how 3rd activity went.	Each pair gets 4 sets. Teacher circulates classroom questioning the students reasoning. With a couple of examples of equations and their roots on the board, the class discusses any connections they can make between the two. Teacher prompts students to answer individually and honestly	Can students complete activity making connections between just roots and equations? How did you figure that out without the graph? What relationship can you see between roots and the quadratics? How do the roots relate to the quadratic? Can students come up with a formula that will represent the relationship between roots and their equations?
Student Individual Work **Student responses** **1st Activity:** All students should make the connection between the three, primarily using the graph. Further question negative and positive graphs. Response 1: Roots show where graph cuts the x-axis. **2nd Activity:** Response 1: Students work in reverse, put roots back into brackets, expand brackets and then simplify the quadratic Response 2: Students will verify the roots through substitution and matching roots and equations accordingly	Offer positive reinforcement on their pair work. Students work in pairs to come up with as many different ways as possible to complete the matching exercise.	Students come up to board to explain their reasoning on how to complete the matching exercise

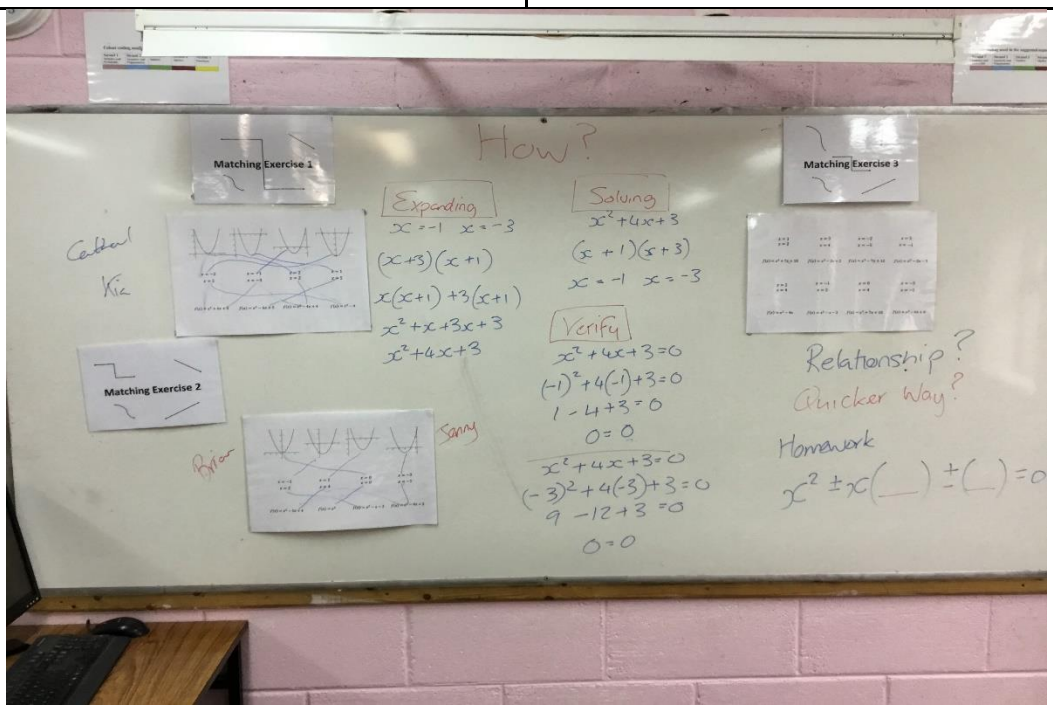
Response 3: Student solve the quadratic using factorization **3rd activity:** Some students will begin to see that the sum of the roots is the x coefficient and the product of the roots is the constant. Using this connect they can through trial and error derive the formula: $x^2 - x(sum\ of\ the\ roots) + (product\ of\ the\ roots) = 0$	Use higher order questions to promote problem solving. "What relationships exists between roots and their equations?"	Teacher prompts pairs where necessary with the following equation $x^2 \pm x(\ \ ) \pm (\ \ ) = 0$ Students use the above hint to figure out signs and fill in the blanks in the brackets
Ceardaíocht /Comparing and Discussing **Activity 1 and 2** Teacher selects appropriate answers in the order outlined above to bring the board to explain. Anyone not comfortable with this can use Think, Pair, Share and come to the board in a group. Students come to the board and present and discuss their findings from each activity. **Activity 3** Is there a quicker way to figure out which equation matches with which roots besides expanding and simplifying, verifying and solving? What relationships have you discovered from previous activities? Let's try us these discoveries to try derive a formula that will work for all roots. Use prompts where necessary	How did you match the graph with the roots with the equation? Why did that work? What relationship did you notice? What happened when there was a constant missing in the equation? $x^2 + 4x = 0$ What type of roots does this equation have? $x^2 + 4x + 4 = 0$ What does that equation look like on a graph?	
Summing up & Reflection Ask students to fill out individual reflection sheets Hand out homework sheets with different roots- students have to form quadratic equations in a variety of ways	Each reflection is individual. Be as honest as you want Recap with the students what connections and relationships they have discovered in today's lesson.	Do students understand the task? Students must write a sentence for each answer in the individual reflection.

9. Board Plan

Carefully plan the board work before the lesson takes place to decide on the order of the solutions and the links that will be made at the board. Put an image or a diagram of the pre-prepared board work here.

Board Plan

Matching Exercise 1: See resource below	Matching Exercise 3: See resource below
Matching Exercise 2: See resource below	Discussion: Creating the formula



10. Evaluation

The classroom will be divided up in 2 sections, approximately 14 students per each observer. Observers will take note of student interactions, engagement etc. keeping in mind the goals of the lesson. Student

worksheets will be collected and photographed to be reflected on later. The completed board will be photographed to be reflected on later. A post-lesson meeting will take place immediately after the lesson for reflection to take place. When observing the lesson, the following questions will be kept in mind:

a) Do students recognize that the problem can be solved in a variety of ways?
b) Did all students understand the problem?
c) Did students collaborate?
d) Do students recognize the link between equations, roots and graphs?
e) Were they engaged at all times?
f) Did students enjoy the lesson?
g) Did students have sufficient reflection time?

11. Reflection

Students demonstrated excellent prior knowledge at the start of class through teacher questioning. This we feel was essential as it prepared the class for the goals of the lesson and helped with the understanding of the matching exercise. The flow of the lesson worked well for the most part.
The students got involved very quickly and they seemed to have good discussions about the problem. The students worked well in groups and got involved in discussions about the different ways the matching exercises could be completed. Different problem solving strategies were demonstrated here as students used trial and improvement, working backwards and using an equation to solve the problem. The majority of the class had good reasoning for why each set of roots matched with the equations and the graphs.

During the lesson, students were fully engaged and were on task at all times. Giving students the opportunity to present at the board was powerful as other students were very engaged when this was happening. They were actively questioning their peers' board work which was great to see.

When the teacher was walking around, he did not need to spend too much time with one student. The teacher got to go around to everyone and was discussing higher order content with some students. We felt the structured format of the lesson allowed for this to happen and doesn't allow the focus of the lesson to become about what students do not know. We found the pair work essential as it in ensure any struggling student was helped by the other. The teacher was checking for what the students had discovered as opposed to what they had not understood. As a result, more able students had the time to explore higher order questions with the teacher because the teachers' time was not absorbed by one or two students who were struggling. The teacher got to discuss why one of the roots is zero, what equal roots are and what its associated graph looks like.
The team felt the matching exercises work great and the majority of the pairs were able to complete them. Some confusion occurred when the teacher further questioned a student on the difference between roots that are equal and roots of the difference of 2 squares. Several students helped with the answer and cleared up the confusion.
Despite the success of the matching exercises the final goal of the research lesson was not achieved. Students could write the quadratic equation of different associated roots but could not formularize the problem. They struggled with the concept of writing a generic equation that could be used for creating a quadratic from any roots. The majority of the class could see the connection between the sum of the roots and the x-coefficient and the product of the roots with the constant. Despite being able to

verbalize the relationship they could not form the equation that would work. The teacher stepped in with a prompt

$$x^2 \pm x(\ ) \pm (\ ) = 0$$

The teacher then decided that for homework the students had to decide on what signs the formula should have and also what words they should put into the brackets. They had to figure this out in as many ways as possible.

In addition the principal attended the lesson and remarked on the brilliant format and structure of the lesson. He really liked the idea of teaching for understanding and identifying the relationships between the roots, quadratics and the equations.

Overall, the lesson went well and we felt students enjoyed the lesson with many commenting that they loved the hands on approach of the matching exercise. Other feedback for the reflection sheets included 'loved the group work', 'everyone helped each other' 'the quick way is the best way', and 'I never realized that there were different ways to figure out the problem'. The lesson was well planned and carefully thought out and consequently the students achieved the majority of the lesson's learning outcomes

As the discussion came to a close, the chairperson asked the group their thoughts on the lesson study process. Everyone agreed unanimously that the benefits and potential of a lesson study approach to math education are huge. Although it can be time consuming the teachers felt that the benefits far outweigh this. Each member felt excited and all agreed to commit to future lesson study groups. We all felt that this structured problem solving approach through collective departmental meetings would be time well spent.