Lesson Research Proposal for 5th Year Functions

Date of lesson: 7/02/2019
School name: Mercy Secondary School, Ballymahon, Co. Longford.
Teacher giving lesson: C. Halligan
Associate: Stacey Guinan
Lesson plan developed by: S. Costello, J. Dromey, C. Halligan, D. Mulvihill.

1. Title of the Lesson: In the air

2. Brief description of the lesson

To promote problem solving we have designed a problem that bridges the gap between the worded questions and graphing functions. This problem can also be simplified to use with an ordinary level class.

3. Research Theme

In Mercy Secondary School, as a focus for our school improvement plan and school self-evaluation, we want students to:

- Reflect on their progress as learners and develop a sense of ownership and responsibility for their learning.
- Enjoy their learning, are motivated to learn and expect to achieve as learners.

As a maths department we actively support the achievement of these goals in the following ways:

- Collectively develop and implement consistent and dependable formative and summative assessment practices – focusing on two stars and a wish as formative feedback for our students.
- As part of our numeracy strategy we ask students to reflect on their grades by using a grade tracker which allows them to establish areas of weakness.
- Select and use teaching approaches to the learning intentions and the students learning needs by using differentiated learning outcomes so that all students can achieve.

4. Background & Rationale

This lesson will be aimed at fifth year students. After a discussion with colleagues we decided to look at the area of word problems involving simultaneous equations as we found that our students had trouble with problem solving and not knowing where to start when faced with a word problem. We also found that students didn't read the problem properly and had difficulties with the maths language used in the problem. They were unable to dissect the problem independently and make the link between solving a simultaneous equation and what it represents.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
In Junior Cert, students should − explore patterns and formulate conjectures − explain findings − justify conclusions − communicate mathematics verbally and in written form − apply their knowledge and skills to solve problems in familiar and unfamiliar contexts − analyse information presented verbally and translate it into mathematical form − devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions. - solve first degree equations in one or two variables, with coefficients elements of **Z** and solutions also elements of **Z** **-** solve first degree equations in one or two variables with coefficients elements of **Q** and solutions also in **Q** - use tables, diagrams and graphs as tools for representing and analysing linear, quadratic and exponential patterns and relations (exponential relations limited to doubling and tripling)	During this unit, students should - select and use suitable strategies (graphic, numeric, algebraic and mental) for finding solutions to simultaneous linear equations with three unknowns and also one linear equation and one equation of order 2 with two unknown and interpret the results - select and use suitable strategies (graphic, numeric, algebraic, mental) for finding solutions to simultaneous linear equations with two unknowns and interpret the results and also one linear equation and one equation of order 2 with two unknowns and interpret the results - graph functions of the form • $ax + b \text{ where } a, b \in \boldsymbol{Q}, x \in \boldsymbol{R}$ • $ax^2 + bx + c$ $\text{where } a, b, c \in \boldsymbol{Z}, x \in \boldsymbol{R}$ • $ax^3 + bx^2 + cx + d$ $\text{where } a, b, c, d \in \boldsymbol{Z}, x \in \boldsymbol{R}$ • $ab^x \text{ where } a \in \boldsymbol{N}, b, x \in \boldsymbol{R}$	After this unit, students should - solve problems involving a line and a circle.

6. Goals of the Unit

- Students will be able to plot points.
- Students will be able to graph linear, quadratic, cubic, exponential functions.
- Students will be able to recognize the link between equations and graphs.
- In the context of a real-life problem, students will be able to form an expression and see graphing as a tool to solve.
- Students will be able to scale to the axes.

7. Unit Plan

Lesson	Brief overview of lessons in unit
1	What is a Function?
2	Graphing Linear Functions (Plotting points, scaling axes)
3	Graphing Quadratic Functions
4	Research Lesson
5	Cubic Functions
6	Exponential (Log Functions)
7	Transformation of all functions.

8. Goals of the Research Lesson:

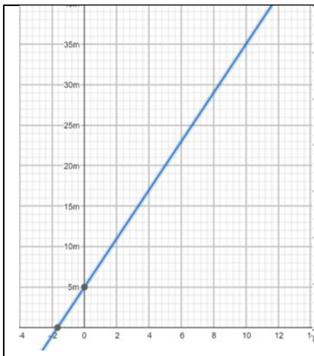
The mathematical goals of this lesson are:

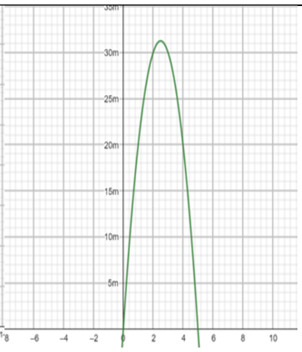
- Be able to read and comprehend the maths language in the context of a worded problem, and from that identify and assign variables to unknown quantities within the problem.
- Construct and solve a mathematical equation while exploring numerous methods and deciding on the most appropriate method.

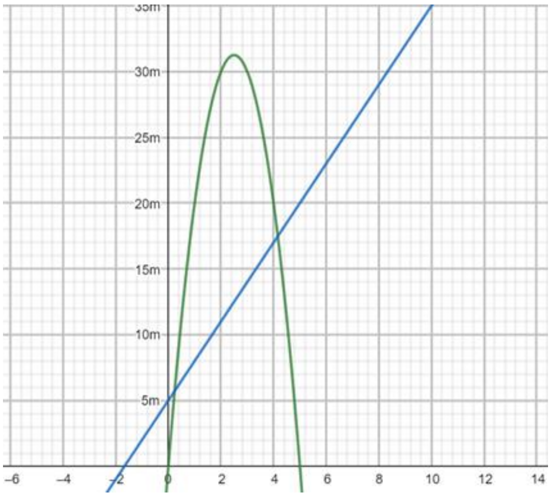
9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
This column shows the major events and flow of the lesson, including timings and what will go up on the board.	This column shows additional moves, questions, or statements that the teacher may need to make to help students.	This column identifies (a) what the teacher will look for (formative assessment) that indicates it makes sense to continue with the lesson, and (b) what observers should look for to determine whether each segment of the lesson is having the intended effect.
Introduction: **Two linear and two quadratic graphs along with their equations.**	What are the names of these graphs? What are the important features of this graph?	Recognise both types of graphs and match them

(5-10mins)	Where do they cross the x and y axis? What do the roots represent? Can you estimate the max/minimum value of each graph? Why is this estimate? Match the graphs to their equation. See resources at end of lesson plan	to their equations
Posing the Task A ball is thrown into the air (Presume this starts at ground level). After one and two seconds the height of the ball is twenty metres and thirty metres above the ground respectively. After five seconds the ball lands on the ground. The ball reaches a maximum height of 31.25 metres after two and a half seconds. Another ball is fired from a machine five metres above the ground. After one second this ball is eight metres above the ground. After five seconds the height of the ball is twenty metres above the ground. (Ignore the effects of gravity) 1) Estimate the time when the two balls are at the same height? 2) Find the height at this time? 3) This is only giving you an estimate; now find the accurate height of the ball to four decimal places? Can you answer the problem using different methods? (15 mins)	Handing out the problem. Teacher offers direction through questions like? Is there another way that you could approach the question? Teacher is recording responses for board work. Promote from teacher if needed: Clarify the problem (one solution) and for question 3 that we need to use algebra for the most accurate answer? Using any and all resources available to you?	Students working individually on the problem using various resources. iPad, GeoGebra, Graph paper.
Student Individual Work 1. Graphing both	Explain how you got the solution? Why did you choose this method? How did you know where to put the intercepts? How did you know which graph relates to which problems?	







2. Drawing two tables

Time	Height
0	0
1	20
2	30
2.5	31.25 2
3	30
4	20
5	0

Time	Height
0	5
1	8
2	11
3	14
4	17
5	20

3. Using iPad to graph equations to find the point of intersection.

Why did you intersect them?

Where did you look for the solution?

Is there anything you would like to change your method?

Have you answered the question?

How did you recognize the pattern?

How did you know it was quadratic or linear?

How did you estimate your answer from the table?

How can you present this better?

Can you clearly see your estimate from your table/pattern?

Is this answering all parts of the question?

How can you use the iPad (GeoGebra) as a tool for this problem?

Can you see any issues with this method?

Does this answer all the questions for this problem?

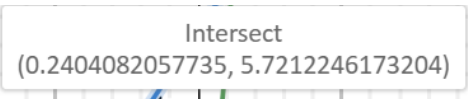
How did you construct the two equations?

How did you construct the equation of the line?

How did you form the quadratic equation?

What is a root?

See graph above and read off the points:



4. Constructing two equations:

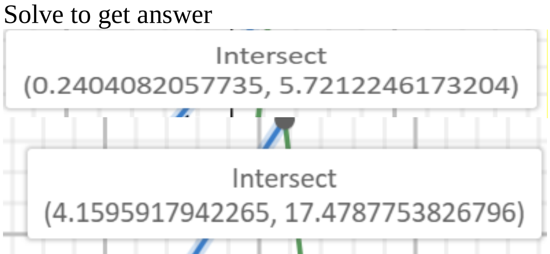
- Using 2a = second difference we find that the second difference is -10 and a= -5
- Using an^2+bn+c they will get $-5t^2+25t$ and using y=mx+c they will find the linear to be 5+3t.
- Or using roots of the quadratic and two points on the linear. i.e. x=0, x=5 etc.

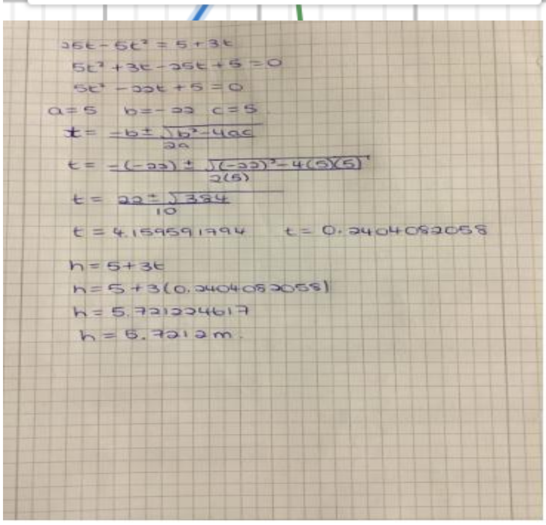
Using two equations to solve the heights.

$H_{1=}$ $-5t^2+25t$

$H_{2=}$ 5+3t.

Solve to get answer





(5-10 mins)

What is a factor?

Why did you equate them?

Why did you have to use the -b formula?

How did you know which value to use? Are both times relevant answers?

Do both solutions make sense in the contexts of the above question?

Can you think of another real-life scenario where this may happen?

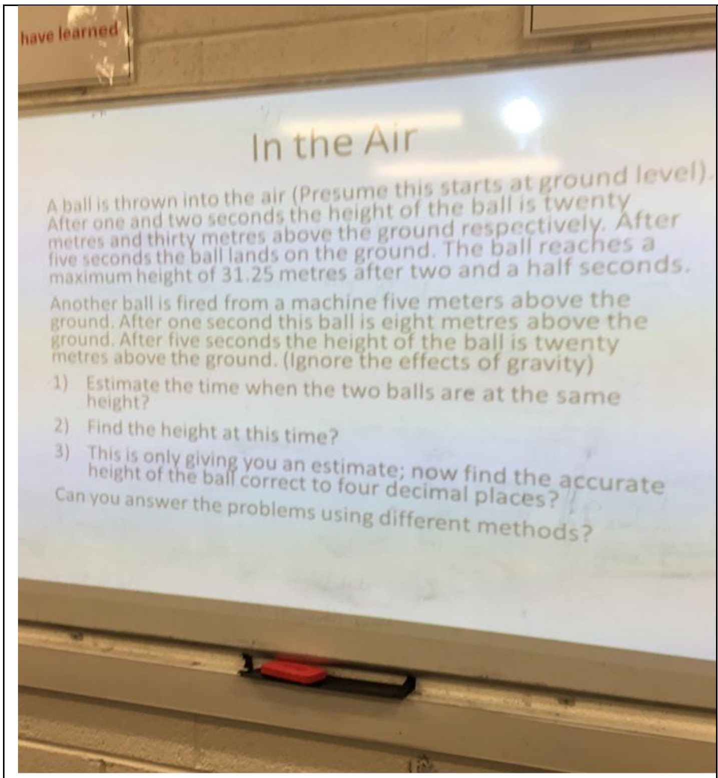
Does this answer all the questions for this problem?

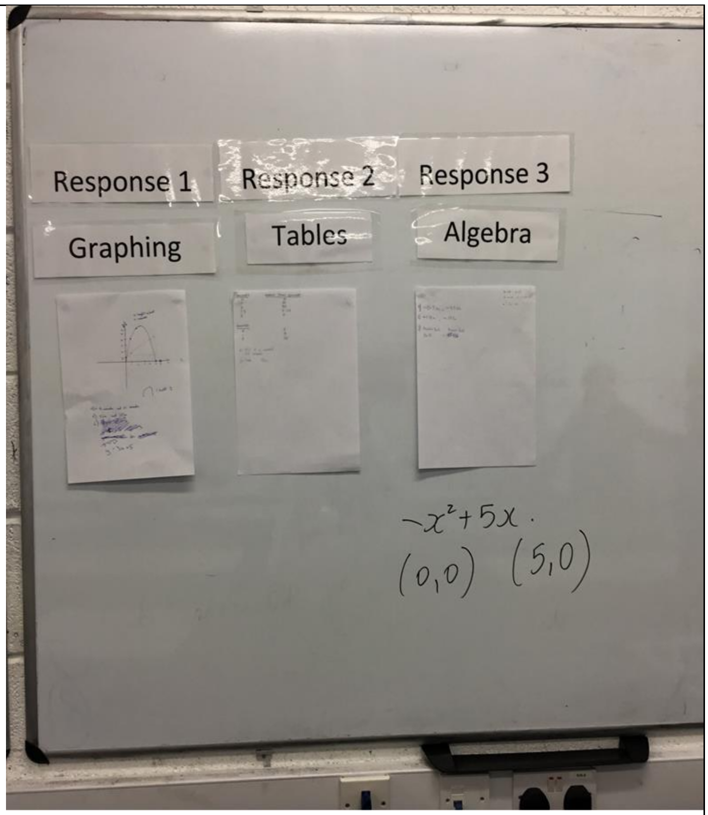
Using the table, they will be able to refine their answer to within two decimal places but not the four as requested.

Misconception: One student may come up with an equation just using the roots of the quadratic and not take into consideration the maximum point of the graph. If you see the board plan below this came up in the Ceardaiocht and was addressed by the teacher as a learning point e.g. $-x^2 + 5x$ equation if they only use the roots.		
Ceardaíocht /Comparing and Discussing Look at the solutions and compare and see if we have answered the questions. **Submarine v Missile** A submarine dives into the ocean to discover a new fish species, after 2 and 4 minutes the submarine is 16 and 24 metres below sea level respectively. After 10 minutes the submarine is back at sea level. The deepest the submarine dives for this exploration is 25 metres below sea level after 5 minutes. A missile is fired into the water 200 centimetres above sea level. After 2 minutes the missile is 2 metres below sea level, after 4 minutes the missile is 6 metres below sea level. At what time does the submarine need to change its path to avoid the missile? Complete this for homework question will be reviewed with students in the next class to clarify understanding.	How close is the estimate? What is the estimate answering? What method is answering all three questions? Which method is the best and most appropriate given the situation? Which method are you going to use to try to solve this problem? Can you use another method from today to help you to solve this problem?	Following on from this we hope the student will decide that the algebra is the best possible solution and will use this to solve this question.
Summing up & Reflection Two stars and a wish will be used as our formative feedback to assess the students' knowledge of this section. Following from this there will be an assessment at the end of this chapter	Teacher will review two stars and a wish and we will provide this feedback from the students in our evaluation and reflection at the end of the lesson proposal.	Students will complete Two stars and a wish on their post it.

10. Board Plan

Carefully plan the board work before the lesson takes place to decide on the order of the solutions and the links that will be made at the board. Put an image or a diagram of the pre-prepared board work here.





11. Evaluation

The lesson was evaluated using the following areas:

1. Any common misconceptions.
2. Any difficulties being encountered.
3. Each student has some level of engagement and success.
4. Students reflect on their progress.

To effectively evaluate the lesson the class was split into different groups which were then allocated to observers. Each observer has 6-7 students. There were 4 observers in the room. All observers had a copy of the seating plan and research lesson and recorded teacher student interaction and approaches taken to solve the problem. Photographs of the student's work were taken on iPads.

12. Reflection

__**Teacher Input**__

The lesson would have worked better if we had a longer class. It is more suited to a class that is 1 hour long instead of 40 minutes. This would allow for longer time for discussion of responses at the end of the lesson. Apart from this aspect, the lesson flowed well. The starter gave a good recap of linear and quadratic graphs and allowed the students to build up confidence before attempting the problem. Once the students started the problem, there was a period of individual work and then the students

started to work collaboratively, helping each other and assessing each other's work.
The iPad response was not included in the responses as even though the students were told they could use any and all responses available to them, none of the students used the iPad to solve the problem. Something that arose during the class was a question regarding the quadratic function. When using the roots to create the function, you get a function that does not satisfy all three points that are given in the question. The students would have to use the general form of a quadratic function to obtain the correct function and we did not have time to explore this in the lesson. This will be discussed in the next class.

Observing Teachers

At the beginning of the lesson, all students recognized the qualities and characteristics of linear and quadratic graphs. We discussed the possibility of doing the lesson again without the starter as we thought that it may have led the students straight to a graph solution. After discussion on this we found that some of the students started with a graph, but most students started with different version of the table. It was also observed that all students recognized the relationship between the point of intersection and graphs. The students clearly identified that when they were looking for the time when the balls were at the same height that they were looking for the point of intersection on the graph. All students had a good understanding of mathematical language used in the problem and as they were reading through the problem, they were translating it from English to maths. There was also a good use of mathematical language by the students when they were brought up to the board for the responses. All students were clearly engaged in the lesson and this was observed by the interaction between the teacher and students and also by the collaboration amongst the students when attempting the problem. It was noted by one of the observing teachers that the students were very confident to share their answers, that they took ownership of the responses that they had come up with and were willing to share their progress with their peers. One student approached the problem by obtaining the equation for the linear function and using trial and error to obtain heights by using values from his graph. We did not have this down as one of our responses.

John

All recognized the qualities and characteristics of each graph.
Every student recognized the relationship between the point of intersection and graphs.
Noticed different approaches used by students.
Good collaboration between students.

Sharon

Starter helped – gave a good understanding of linear and quadratic graph and important aspects of each graph.
Good understanding and use of maths language throughout lesson.
Highlighted and underlined keywords in the problem.
All students really engaged in the class. Students answering questions being asked. Students talking about scale of axis.
Good teacher-student interaction.

Dylan

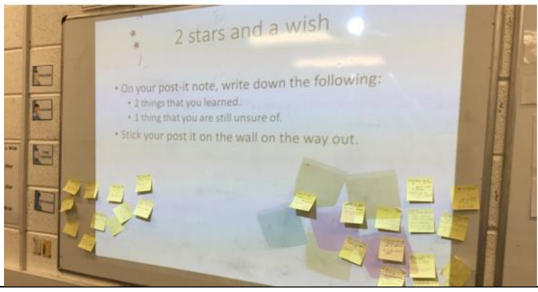
Possibility to do lesson again without using the starter.
Students confident to share their answers in class.

Stacey

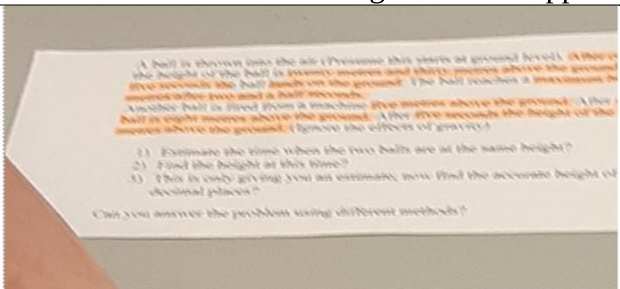
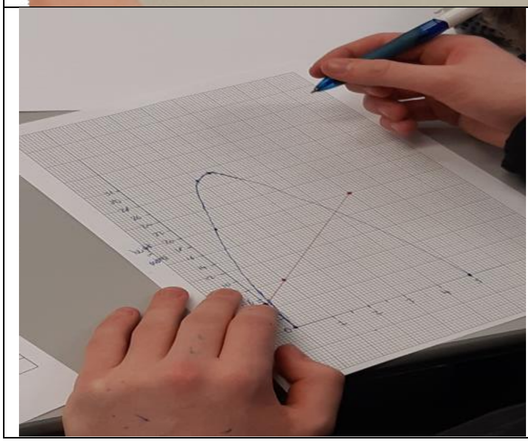
Great interaction between students.
Students underlining keywords and translating different aspects to maths.

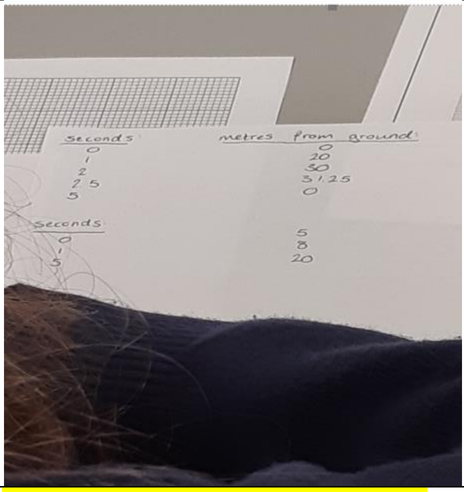
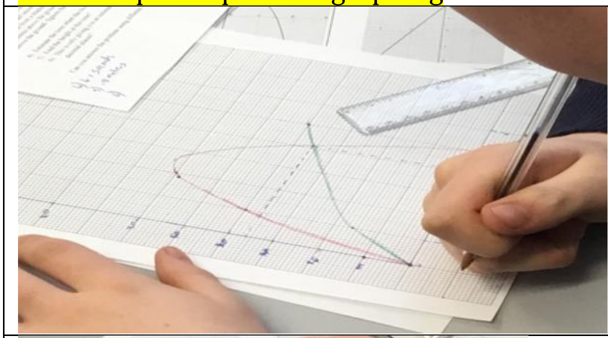
Students given loads of time to work independently and in groups.
Collaboration amongst students helped to clarify misconceptions.
Teacher gave students thinking time when clarifying the problem.
Flow of the lesson worked well.

<u>Students reflections:</u>

Two Stars	The wish
• My method would have worked had there been two linear graphs • Trying things even if they don't work • Drawing the diagram helps to visualize the problem • Finding the point of intersection using the graphs • There are three methods to solve this problem • Algebra is the best way and is more accurate. • Using tables to solve problems • Working under pressure • Graphs represent data really well but they cannot answer all parts of the questions all of the time	• How to get the quadratic equation from a graph? • How to get the slope from a quadratic graph? • Difficult to use the algebra to solve the question? • How to find the equation of a line? • How many points do you need in order to get a function? • How do you construct the quadratic equation when you have the graph? • Figuring out the right equation? 

We have highlighted the most popular answers from the students above. Some of the pics below reflect on the students activities during class and support the goals of our lesson.

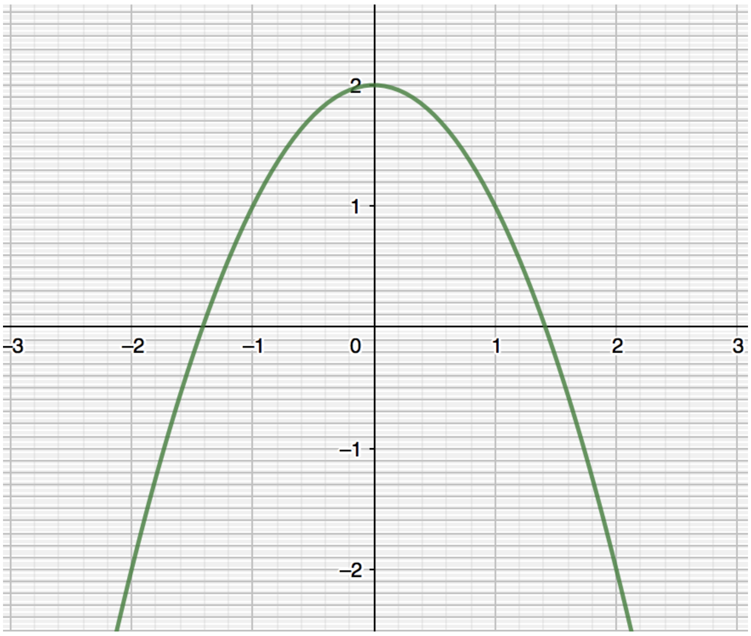
	Students underlining and highlighting the keywords to determine the variables and get the most useful information needed in order to answer the questions.
	Students attempting to graph the question the majority were correct and using this as the basis to sole the questions.

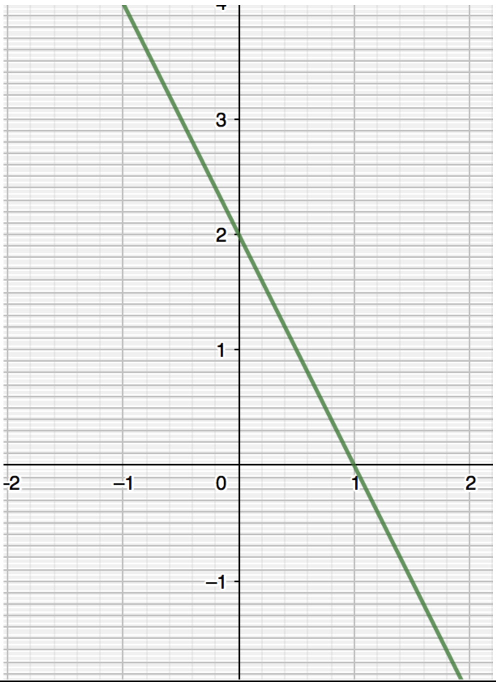
	One of the preferred methods was to construct the table first and then move on to the graphs from this.
Misconception spoted in graphing	
	The student graphed the quadratic correctly but missed one vital piece of information in his reading of the question.
	During a discussion with another student she pointed out the graph started at 5m above the ground and not at zero which the student then rectified.

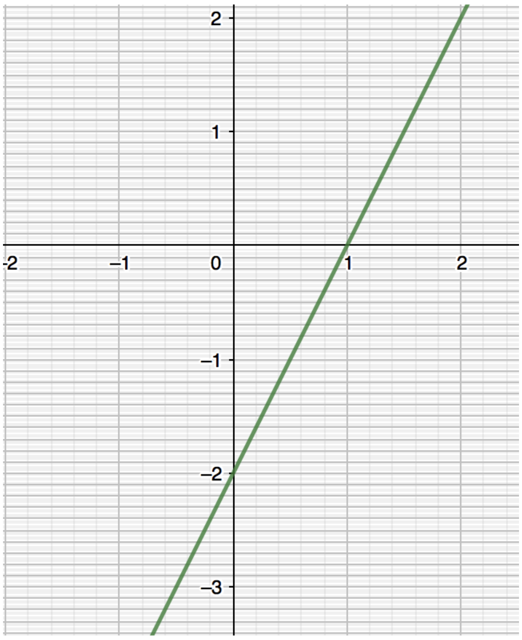
Resources: Introduction worksheet:

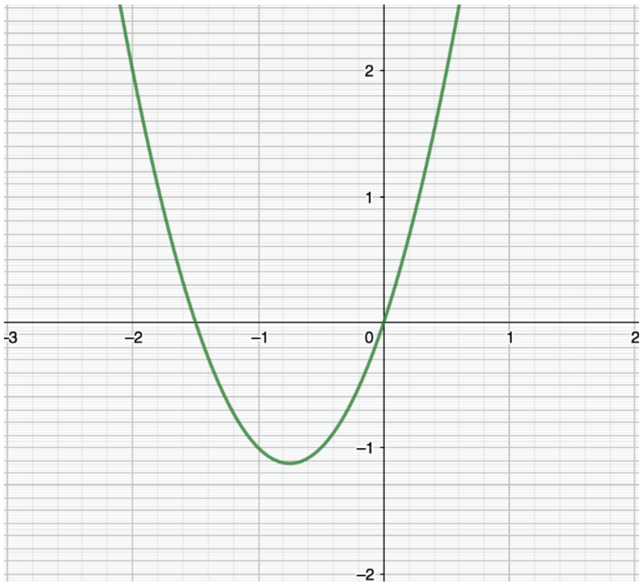
$$y = 2 - x^2 \qquad y = 2 - 2x$$

$$y = 2x - 2 \qquad y = 2x^2 + 3x$$









Submarine v Missile

A submarine dives into the ocean to discover a new fish species, after 2 and 4 minutes the submarine is 16 and 24 meters below sea level respectively. After 10 minutes the submarine is back at sea level. The deepest the submarine dives for this exploration is 25 meters below sea level after 5 minutes. A missile is fired into the water 200 centimeters above sea level. After 2 minutes the missile is 2 meters below sea level, after 4 minutes the missile is 6 meters below sea level.

At what time does the submarine need to change its path to avoid the missile?