Lesson Research Proposal for 2nd Year Algebra

Date of lesson: 28/02/2019
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1. **Title of the Lesson:** 'Place your Pix (Picture in X)'

2. **Brief description of the lesson**

In this lesson, we are focused on asking students to drawn upon their previous knowledge of area and algebra to work out how much space of a frame is wasted or uncovered when they place in it a picture of unknown length and area.

3. **Research Theme**

Our school's focus for School Self Evaluation (SSE) is embedding the key skills of Junior and Leaving Certificate into everyday teaching and learning as well as the teaching philosophy BLP (building learning power).
As part of this, we encourage our students to use thinking routines such as "What do I know, what do I need to know?" when faced with an unfamiliar problem. This scaffolds the learning for students to embrace the challenges of problem solving rather than looking for a direct answer, sticking with it to figure it out for themselves.

Background & Rationale

We discussed during our first meeting some of our observations from our recent House exams as well as previous house exams over the years. One of the areas that we highlighted that was still a common area of concern was of Algebra which is still posing problems. Coming from three different Maths departments, we all highlighted that algebraic manipulation in given contexts and in particular 'Difference of two squares' were questions that were very challenging for our second years. Analysis of our post-exam feedback from our students also identified it as an area they performed poorly in.

The issue of Algebra and algebraic manipulation in the problem-solving context has also been identified as an area that students find challenging by the Chief Examiners report. It was highlighted in the chief examiners report in 2015 that students struggled to complete longer, more involved problems both of a routine and non-routine kind.

'There were, however, a number of concerns raised over the course of the marking. At both Ordinary and Foundation level, concerns were expressed regarding candidates' lack of basic competency in algebra, and in particular in algebraic manipulation. A number of examiners observed that many of these candidates would struggle with aspects of the Leaving Certificate Ordinary level Mathematics syllabus as a result of this…. At Higher level, the standard demonstrated by candidates in basic algebraic manipulation shows some decline, with most candidates struggling to complete multi-step procedures accurately.'- Chief Examiner's Report, 2015.

After deciding upon our area of interest, we also decided to target our lesson towards 2nd Year students. Part of our rationale came from the practicality of 'Algebra' being a heavy component of the current second years scheme of works. Given the difficulty of Algebra as a strand, most students aren't introduced to concepts such as factorizing or the difference of two squares until Second Year. In First Year, they would have built up a basic introduction to dealing with unknown variables and constants, but not identified their applications to real-life situations or how they can be manipulated throughout other strands of the course.
The second reason for our choice of focusing on Second Years was due to the introduction of the new Junior Cycle Maths' specification for First Years this year. Since these First years will be the first cohort to learn Algebra within the scope of the new Junior Cycle learning strands, it made sense to look at targeting difficulties that were still being experienced by our current Second Years. This would allow us to not only reinforce understanding for the last of the old cycle students, but also to plan for and improve learning structures and opportunities for the new cycle students in years to come.

By focusing on this learning group, we are enabling our departments to plan effectively and implement changes that promote a deeper meaning and understanding within the math's classroom, both at the Junior Level and the Senior Level.
Furthermore, as Algebra can be argued to be fundamental for various areas of mathematics, it is important to overcome students' negative disposition towards the area of Algebra and encourage them to understand what is going on at a deeper level. This should help to alleviate issues and misconceptions as the students build on their problem-solving skills as they enter senior cycle and begin to apply their algebraic knowledge to areas such as patterns and functions.

Relationship of the Unit to the Syllabus
The beginning of the unit of learning will relate to the Junior Cycle curriculum and will build towards the Senior Cycle curriculum by the end of the unit.

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
The following learning outcomes have been adapted from the Junior Certificate mathematics syllabus • Examining algebraic expressions - show that relations have features that can be represented in a variety of ways • Be familiar with expressions of the form $x^2 - b^2$ and $a^2x^2 - b^2$, where a,b are natural numbers and x is a variable	The following learning outcomes have been adapted from the Junior Certificate mathematics syllabus Students should be able to: • Find an expression to represent the length of the side of a square, the area of a square and the difference in the area of two squares (using numbers and algebraically) • Manipulate and solve	The following learning outcomes have been adapted from the Leaving Certificate mathematics syllabus 3.4 Length, area and volume • solve problems involving the length of the perimeter and the area of plane figures: disc, triangle, rectangle, **square**, parallelogram, trapezium, sectors of discs, and figures made from combinations of these

• Factorise expressions of the form $a^2x^2 - b^2$ Select and use suitable strategies to find length of the perimeter and the area of the following plane figures: disc, triangle, rectangle, **square,** and figures made from combinations of these	an algebraic expression to find the difference in the area of two squares when given the side length of both squares	4.1 (a) Generating arithmetic expressions from repeating patterns. – generalise and explain patterns and relationships in words and numbers – write arithmetic expressions for particular terms in a sequence 4.1 (b) Representing situations with tables, diagrams and graphs. Relations derived from some kind of context – familiar, everyday situations, imaginary contexts or arrangements of tiles or blocks. – develop and use their own generalising strategies and ideas and consider those of others – present and interpret solutions, explaining and justifying methods, inferences and reasoning 4.1 (c) Finding formulae. Ways to express a general **relationship** arising from a pattern or context. 4.1 Expressions – - expand and re-group expressions – factorise expressions of order 2 – add and subtract expressions of the form • $(ax+by+c) \pm \ldots \pm (dx+ey+f)$ • $(ax^2+bx+c) \pm \ldots \pm (dx^2+ex+f)$ where $a,b,c,d,e,f \in Z$

4. Goals of the Unit

What we hope to achieve by the end of the unit is…

- To encourage our students to be more confident and competent in their approach to mathematical problems and that they will be more disposed to attempt a problem with a stronger sense of perseverance
- To improve students' procedural fluency in being able to carry out procedures flexibly, accurately, efficiently and appropriately to the given problem.
- To improve student's conceptual understanding and their comprehension of mathematical algorithms, operations and how to effectively use them
- To improve students' capacity to analyze the problem and try to interpret the problem – recognize which strategy to use when facing a problem (literacy in maths).
- Encourage students to develop a resilient attitude and improve their adaptive reasoning when faced with an unfamiliar problem in an unknown or known context
- Promote a more positive and productive attitude among students towards the learning strand of algebra and to view it in a more sensible and worthwhile manner.

5. Unit Plan

Sequence of lessons:

Lesson	Brief overview of lessons in unit
1	Revision of simplifying algebraic expressions by adding and subtracting like terms and expanding brackets
2	Revision of the 4 types of factorizing (HCF, grouping, guide number to solve quadratics and difference of two squares) and solving each of these types of expressions
3	Writing algebraic expressions from a word problem
4	Solving algebraic expressions formed from a word problem
5	Research Lesson – used to tie together the topics of writing algebraic expressions and simplifying and solving algebraic expressions

6. Goals of the Research Lesson:

Key skills:

Staying Well:

1. Being positive about learning
2. Being confident

Managing Information and Thinking:

1. Thinking creatively and critically

Being numerate:

1. Developing a positive disposition towards investigating, reasoning and problem-solving
2. Seeing patterns, trends and relationships

Being creative:

1. Exploring options and alternatives

Communicating:

2. Using language

3. Using number
4. Discussing and debating

Being literate:

1. Expressing ideas clearly and accurately

Statements of Learning:

1. communicates effectively using a variety of means in a range of contexts in L1,

15. recognizes the potential uses of mathematical knowledge, skills and understanding in all areas of learning.

16. describes, illustrates, interprets, predicts and explains patterns and relationships,

17. devises and evaluates strategies for investigating and solving problems using mathematical knowledge, reasoning and skills,

23. brings an idea from conception to realisation

By the end of the lesson students should be able to…

- Draw a diagram to illustrate a word problem
- Write a written expression for the area of a square using a variable
- Write a written algebraic expression to describe the difference in the area of two squares
- Solve an algebraic expression to find a value for the difference in the area of two squares
- Verbally explain how to find the difference of the area of two squares when the length of the side on one/both squares include variables/unknown quantities

7. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment
Introduction Before we start today's lesson, I would like to recap over some of the maths that we have learned in the past few weeks. What rules do we already know about adding and subtracting algebraic expressions? - Only constants can be added to constants - Only variables with the same power can be added to one another	Place a reminder of rules regarding adding/subtracting/ multiplying constants and variables on the board	Can students verbalise their learning so far?

What do we already know about multiplying variables? - $x . x = x^2$ What do we know about finding the area of a square? - Area of square = Length x Length What do we know about finding the area of a rectangle? - Area of a rectangle = Length x Width	Place the formula for finding the area of a square and the formula for finding the area of a rectangle on the board	
Posing the Task You place a square picture of side length x cm in a square frame of side length 18cm. In how many ways can you find the empty space in the frame after the picture has been placed in it?	Place a picture of the frame on the board indicating the side length in cm. Place a picture of the picture of Chris on the board indicating the side length in cm.	Do the students understand the task? Are they clear of what the question is asking?
Student Individual Work	Use your seating chart to record the model used by each student. Note the order in which you will call each student up during Ceardaiocht. If students are stuck, help them by asking appropriate questions eg. What is another word which we use in maths to describe 'space'? How do we find the area of a shape that we do not know? Can you draw a diagram to help you decide on/describe your approach?	Are students able to tackle the problem? Are students able to develop a mathematical expression for this problem? Do students understand that this is an area problem?

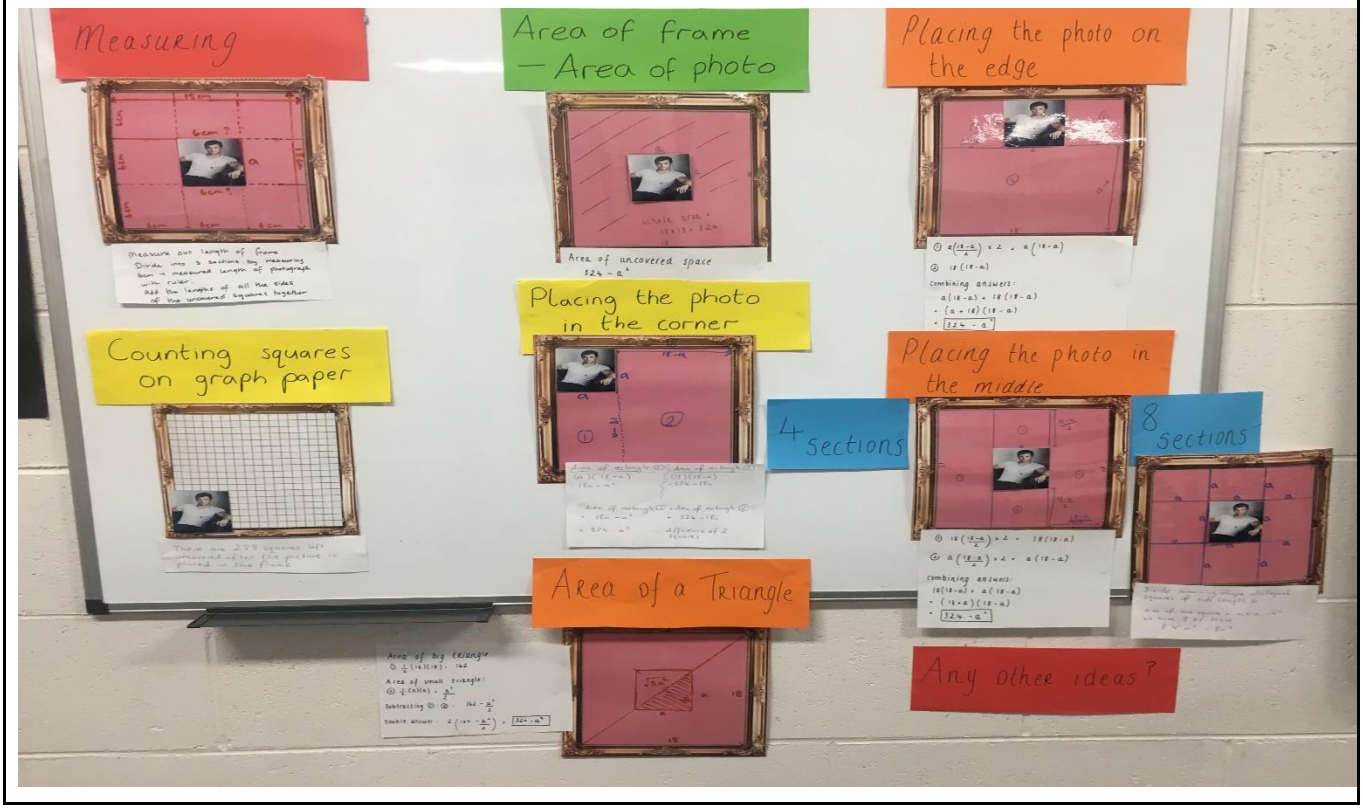
Ceardaíocht /Comparing and Discussing There are multiple models that students could use to find the space in the frame not occupied by the photo i.e. The difference in areas of the two squares Ask specific students to come to the board and explain how they calculated the space in the frame not occupied by the photo. **Student Response 1** – Measure the side lengths using a ruler to find a value for x, and calculate the two areas, subtracting the smaller area from the larger one. **Student Response 2** – Counting the squares **Student Response 3** – Using the area of a square 'formula', find area of frame, find area of photo and subtract the smaller from the larger. **Student Response 4** – Placing photo in the corner, divide the remaining shape into two rectangles, find the area of each and add them together. **Student Response 5** – Placing photo at the edge, divide the remaining shape into two squares and a rectangle, find the area of each and add them together. **Student Response 6** – Placing the photo in the middle. Option 1: Divide the remaining shape into two squares and two rectangles, find the area of each and add them together.	When students present work at the board, make sure to attach their name to it. Ask the students to raise their hand if they used this method. Did anyone use a different approach? Any other methods used?	Can students explain their approach? Do students recognise similarities/differences between their approach and that presented on the board? Do students offer alternative approaches to solving the problem?

Option 2: Divide the remaining shape into eight squares, find the area of each and add them together. **Student Response 7** – Divide both squares into two triangles and find the area of each. Subtract the smaller area from the larger area and double your answer.		
Summing up & Reflection We learned that: • That the area of a regular/an irregular shape can be represented algebraically in many different ways • That the area of a regular/an irregular shape can be calculated using several different methods • A difference of two squares problem can occur in real life		Do the student's reflections represent the teacher's view of the lesson?

8. Board Plan

Carefully plan the board work before the lesson takes place to decide on the order of the solutions and the links that will be made at the board. Put an image or a diagram of the pre-prepared board work here.

Board Plan



9. Evaluation

1. Do the students fully understand what they are being asked in this question and are they engaging with the problem?

They students appeared to find the question very accessible. They appeared to have no issue with identifying what the question was requiring of them. However, students tended to avoid using algebraic methods if at all possible, instead preferring to use numerical ones.

2. Did the lesson allow students to express the area of an irregular shape using an algebraic expression?

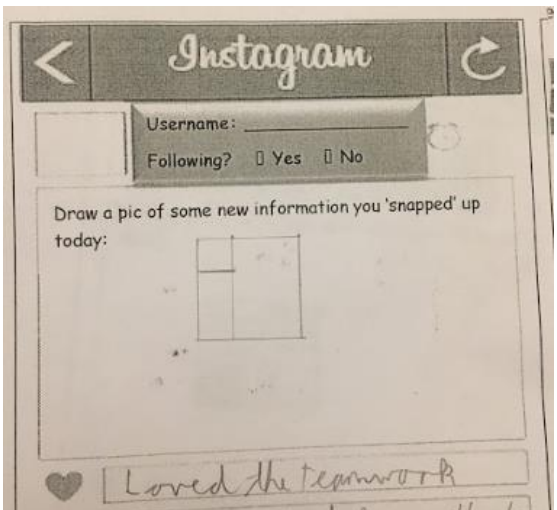
The lesson did allow the students to express the area of an irregular shape using an algebraic expression however, if we were to repeat the lesson again, we felt that we should have stated that students must provide at least one algebraic solution to the problem. This would help to avoid students ignoring algebraic approaches and favouring numerical ones.

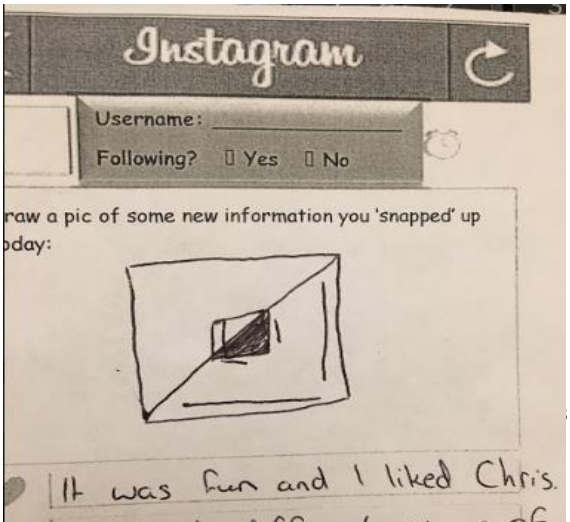
3. Did the students come up with all of the predicted responses? Which student response was the most/least common?

   The students did not come up with all of the proposed possible solutions. They came up with the following solutions
   - Measuring the side lengths of the picture and calculating the missing area using numerical values (This was the most common solution)
   - Counting the squares on the graph paper as a measurement tool
   - Found that three pictures fitted across the length of the pink section inside the frame and used this to establish that the side length of the photo was 6. Found that 36 units squared was one ninth of the total area and so the area of the uncovered space must represent eight ninths (This solution was not anticipated by the team).
   - Found the area of the small photo and subtracted it from the area of the large square (This was the second most common solution)
   - Students placed the photo in the middle and divided the remaining section into 8 equal squares. They found the area of each of these eight squares and added them together
   - Students placed the picture in the corner of the frame and divided the remaining shape into two rectangles. They then found the area of both of these rectangles and added them together
   - One group of students divided the large square and the small photo into two triangles, found the area of both triangles, subtracted the smaller area from the larger and then doubled their answer. (This approach was the one which students found most interesting according to the lesson evaluation slips)

4. Did the students enjoy the lesson?

   The students appeared to enjoy the lesson while they were being observed, and this was echoed in their exit tickets. The students enjoyed working together in groups to solve the problem (mentioned in 18 of 30 responses). The students also really liked the context in which the problem was posed (mentioned in 8 in the responses).





10. Reflection

a) Before the lesson, we had hoped that the lesson would engage the students and allow them to develop a more positive attitude towards algebra. We hoped that students would engage with the Key Skill of Working with Others (Junior Cycle), by working in groups of six and engaging in meaningful discussions about the problem through effective communication. We also hoped that students would come up with a variety of solutions to the problem of varying difficulty. We hoped that the students would enjoy the problem-solving process and that they would appreciate a use for algebra in everyday life.

b) During the lesson, students figured out a numerical value for x and used this to develop solutions instead of coming up with a general solution. Students also did not attempt to divide the unknown pink shape into known shapes until prompted by the teacher. Once prompted to do this, students were able to come up with a wider variety of solutions however, they did still tend to revert back to trying to find a numerical solution.

c) Each team member felt that overall, the lesson went well. The students particularly enjoyed the context in which the problem was presented and they enjoyed using the Instagram format to assess their own learning at the end of the lesson. It was felt that the students were 'fixated' on finding a numerical answer and were very reluctant to use algebra to form a solution. In turn, this limited the solutions that the students came up with. It was also felt that the Ceardaíocht section of the lesson was too short, possibly due to the lesson being carried out in a 40-minute period. All members of the team felt that the lesson would be more suited to an hour long class period.

d) The Lesson Study Associate felt that the lesson went very well, that students were engaged right throughout the lesson and that they enjoyed giving their post lesson feedback on the Instagram template. However, she also felt that the lesson was tough to carry out in a 40-minute lesson. As a result of this difficulty, the Ceardaíocht section of the lesson was cut short, which limited the student's ability to fully appreciate the purpose of the problem and all of its possible solutions.

   It was also felt that students worked very effectively in groups of six, with all students engaging in meaningful conversations about the problem and the many approaches which could be used to solve it.

e) If we were to repeat this lesson in the future, we would have been more explicit in stating that students must develop a general solution to the problem as opposed to a numerical one. This could be achieved by possibly restricting the students to only provide one numerical solution to the problem. We would also have reinforced that students are not being asked to solve for

x, and that a numerical answer is not required. We felt that if we had provided students with a whiteboard marker at the beginning of the lesson, they might be encouraged to divide the irregular shape themselves without having to be prompted. We would also have mentioned to the students that we were permitted to draw on the boards. We also felt that the lesson would have been better suited to an hour-long lesson to allow for a more in-depth and meaningful conversation during the Ceardaíocht section of the lesson to fully explore the solutions which the students developed. As a result, the teacher of the lesson did not have time to pose the questions 'Which solution provided the most accurate answer to the problem?' We felt that if this question was asked, it would have made the objectives of the lesson clearer to the students, and highlight the importance of algebra to the students. As a team we felt that we would like to further develop this lesson to encompass the idea that students would develop the 'formula' for factorizing the Difference of Two Squares expressions.