Lesson Research Proposal for 2nd year mixed/higher level students on Trigonometry

Date of lesson:	29/1/19
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1. **Title of the Lesson:** Run Johnny Run...Maths to the rescue

2. Brief description of the lesson

In this lesson, students (Second year mixed or higher level), will solve a problem in trigonometry which allows for various paths to a solution. By presenting the problem with the challenge to the students of finding as many approaches as possible, it will give feedback on how well students have integrated, and can access, recently learned trigonometry into their usable mathematical skill set.

3. Research Theme

In reference to our schools' SSE priorities (from *Looking at our Schools 2016) - A Quality Framework for Post-Primary Schools*)

We, as maths teachers, through this lesson study collaboration, aim to:

- Provide, and look for, constructive, developmental oral feedback in a whole-class problem solving context.
- Work with students on clear strategies for seeing connections and improvements in how they approach, explain and justify their solutions in open maths problems.

In pursuit of these aims, we wish to use a range of questioning and suggestive techniques effectively for a variety of purposes including:

- stimulating substantial, rather than low detail, student responses, which make use of recently acquired mathematical learning.
- promoting connections to other areas of the syllabus by encouraging active listening to other students as they explain alternative approaches.
- facilitating deeper student engagement with lesson content and reflection on how they are progressing, especially from exploring as a whole-class, the outcomes of a research lesson.

4. Background & Rationale

We hope, through this research lesson, to engender a good foundation of reflection by students on feedback that they can carry forward in their mathematical studies. This involves metacognition or "thinking about thinking" so that students consider their own approaches to problems in light of their learning style and stretch themselves to incorporate the ways that others have thought and presented, and how these ways may relate to their own.

In our experience of teaching maths, we notice that students often rush into a problem to get a solution without reflecting on what they are trying to find in the context of a real world problem. As a result of this, they struggle, beyond the blind procedure, when problem solving, to apply techniques as a means of modelling the real situation.

Relating this to teaching introductory trigonometry, students often don't have a clear understanding of what sine, cosine and tan mean and so struggle to transfer their use to the many applications presented in exam questions and beyond. Sometimes they will even write out something like sin = opp / hyp, without recognising the connection to an angle.

Because of this lack of relating ratios to angles, they become confused when naming the sides of the triangle, particularly that the adjacent and opposite is dependent on the angle given.

They easily use their calculator, for example, to find sin 30 = ½ but find it hard to make use of this in finding an unknown length through use of the ratio opposite / hypotenuse.

Along with these concerns, we also wished to encourage the students to compare approaches to solving a problem and develop some sense of one approach being more efficient, faster, easier etc. than another. They are often met with solving an equation in trigonometry where the unknown is a denominator and, while this is something they must become competent at carrying out, we hope they will explore alternatives that make the task a little easier (in this case, by creating an equation where the unknown is a numerator). This reflection on use of procedure and its suitability/efficiency for a given task, is a form of self-assessment we would like students to develop.

The following quotes add weight to these points, as well as highlighting the need for students to, not only have procedures (such as drawing, or adapting, appropriate diagrams) in their repertoire, but to also recognise when they could be used to solve real-life based problems:

> "In many topics, including coordinate geometry and trigonometry, drawing sketches or diagrams may aid candidate in understanding how to solve the problem." (Chief Examiners Report, Leaving Cert Maths, 2015)
> The chief examiner's report for Junior Cert maths 2015, states "Candidates continue to find geometry and trigonometry challenging…with trigonometry causing particular difficulty (p.13).

They also point out that in paper 2, Q.13, candidates "struggled" in part (b), "which required them to connect the photograph to the diagram given and to draw their own diagram (or adapt substantially the one given) before using trigonometry to find the height of the water tank itself. It is worth mentioning that, in examination papers under the previous syllabus, candidates would generally have been aware what questions on the examination paper required, say, trigonometry. This was not the case here, so candidates first had to identify that trigonometry was required to complete the question before they were able to start it at all". (p.27)

Our problem was designed to address some of these concerns, as well as giving us feedback on how well students could use what they would have recently learned (as well as allowing for use of less efficient, established methods) in a new situation i.e., would they now add the new methods to the ones that were easier but not as efficient(accurate)?

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later learning outcomes
Be able to convert fractions to decimals Know what an angle is Have a basic understanding of different types of triangles - scalene, isosceles, equilateral with an emphasis	Find the size of an angle in a right-angled triangle given the length of two sides Students recognise that the naming of the adjacent and opposite sides changes depending on the angle	Consider the truth of the statement cos 75 = cos 45 + cos 30 That students can relate sine, cosine and tan to angles greater than 90 degrees.

of right-angle Be able to use a protractor to measure an angle Need to know what a ratio is The sum of the internal angles in a triangle is 180 degrees. Students understand an inverse operation Students understand similar triangles have equal angles Pythagoras Theorem Students understand slope and can find the slope Naming the sides - hypotenuse, opposite, adjacent	chosen Students recognise there is a relationship between the sine of an angle and the cosine of the complementary angle Students use the inverse trigonometric functions to find the angle	

6. Goals of the Unit

Students will be able to:

- Recognise and investigate similar right-angled triangles, especially how the ratio of one side length to another is constant for a common angle
- Recognise that adjacent and opposite as descriptions of the two shorter sides in a right angled triangle depend on the acute angle being used at the time
- Explain what complementary means in relation to acute angles in right angled triangles

- Understand that sine, cosine and tan correspond to ratios of side lengths of right-angled triangles.
- Identify the hypotenuse through its (1) position as opposite the right angle, and (2) being the longest side.
- Develop greater confidence in their mathematical abilities and feel a sense of achievement through:
  - being led to such discoveries as the sine of an angle (for example) is the same as the cosine of its complementary angle.
- Engaging in justifying methods they have chosen to use for themselves.
- Reflecting on their achievements and using feedback as positive guidance for mathematical growth.

7. Unit Plan

In the Draft *Specifications for Junior Cycle Mathematics* it is stated that "Students use geometry and trigonometry to model and solve problems involving area, length, volume, and angle measure". The most directly relevant learning outcomes for this unit would include:

GT.4 evaluate and use trigonometric ratios (sin, cos, and tan, defined in terms of right-angled triangles) …… involving angles between 0° and 90° at integer values and in decimal form

U.4 represent a mathematical situation in a variety of different ways, including: numerically, algebraically, graphically, physically, in words; and to interpret, analyse, and compare such representations.

Lesson	**Brief overview of lessons in unit**
1	Investigate Pythagoras Theorem
2	Students look at a variety of Right-Angled Triangles and identity the Hypotenuse, Opposite and adjacent sides. Building on this they are given a number of similar triangles (with angle 30 degree) and asked to firstly measure the length of the opposite side, hypotenuse and given angle, and then suggest a relationship between the three. Homework: Investigate the relationship between the adjacent side,

	hypotenuse and given angle.
3	Students use similar Right-Angled Triangles with angles 45 degrees and 60 degrees (if time also 20 degrees and 70 degrees) to compare sin and cos of angles.
4	Students investigate right angled triangles with angles of different degrees and compare them to Tan - the idea of slope should be introduced here and integrated into questions. Each student makes a poster with a given trig ratio and angles (30, 60 and 45). They should start this in class.
5	Research Lesson

8. Goals of the Research Lesson:

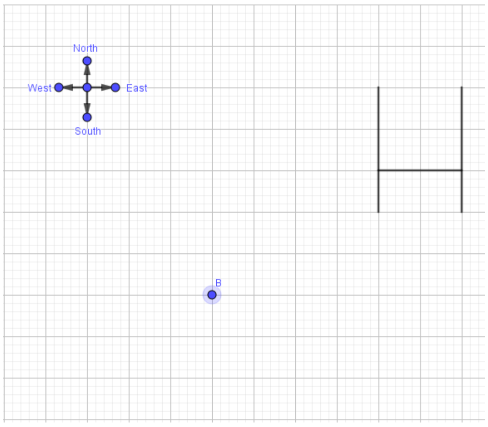
Looking at the goals of the research lesson itself from two perspectives:

1. Provide an opportunity for students to:
   - integrate their learning in Trigonometry
   - show their competency in using it fluently and creatively, without prompting of memorised rules.
2. Provide teachers the opportunity to:
   - receive feedback on how well students have grasped the trigonometry taught to them in recent weeks.
   - strengthen students' communication skills in a whole class context, by providing opportunities for them to justify choices as well as listen to those of other students and the teacher(s)
   - promote reflection in students on how they access maths in new situations.

9. Flow of the Research Lesson:

Steps, Learning Activities Teacher's Questions and Expected Student Reactions	Teacher Support	Assessment

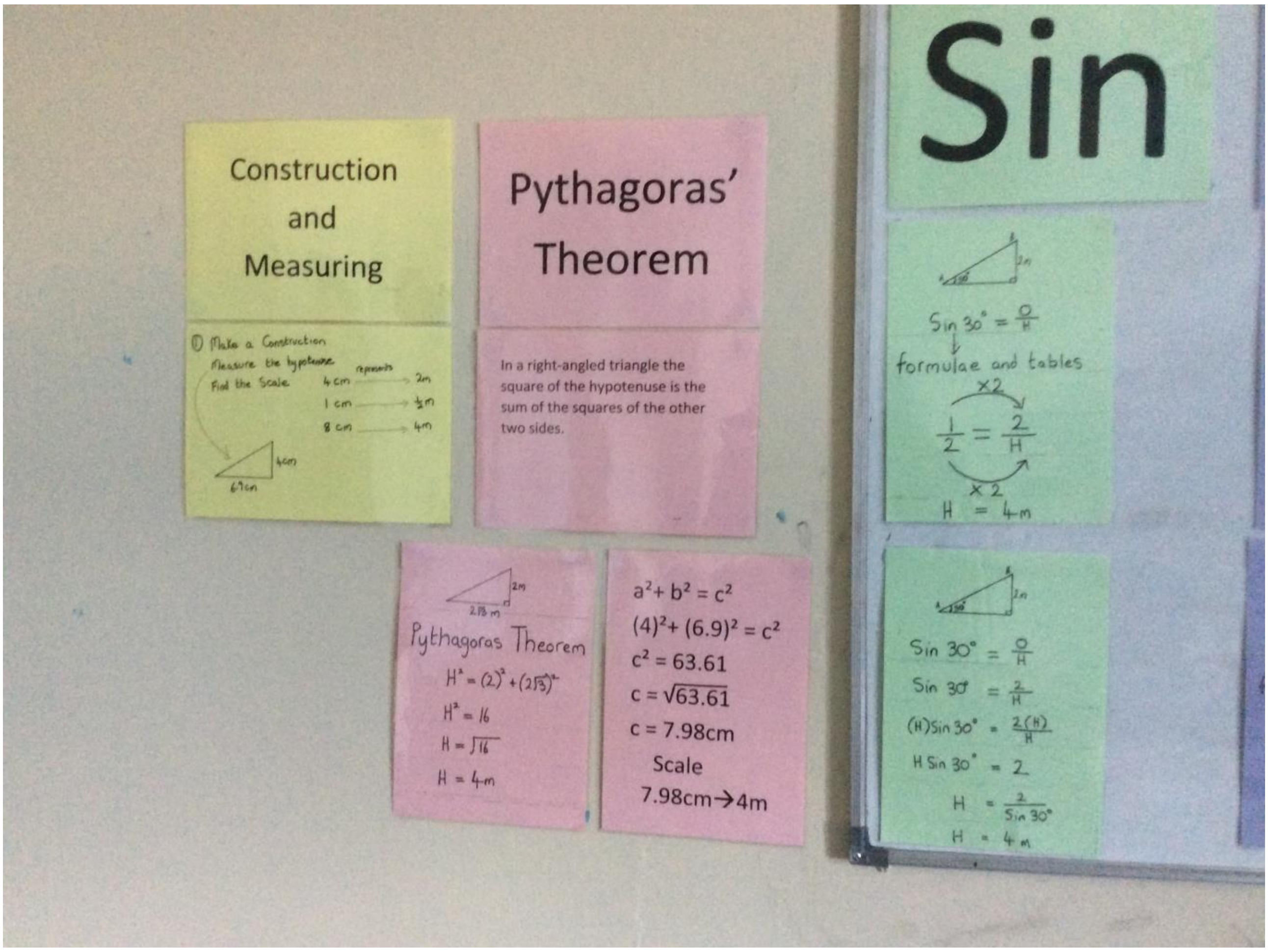
Introduction (5 minutes) Recap: Teacher tells class they will today engage in a task for which the last few weeks of maths classes will be useful.	Ensure that students have their formula & tables to hand.	Teacher gets a sense of level of students' confidence by watching how they respond to this open ended introduction to the lesson. Are students adopting a readiness by checking/taking out equipment (f & t etc.) or asking questions?
Posing the Task **Show students clip (or something similar in relation to taking a place kick).** https://www.youtube.com/watch?v=9FDHprs9Jzw 2min 40 sec	Pass around worksheet. Show video of rugby player on the run up to kick the penalty (1min 40 seconds). Display the question on the board to the class. Read it together. Is there anything unclear about the question?	Are students attentive to the video? Do they give reasonable time to reading the question?

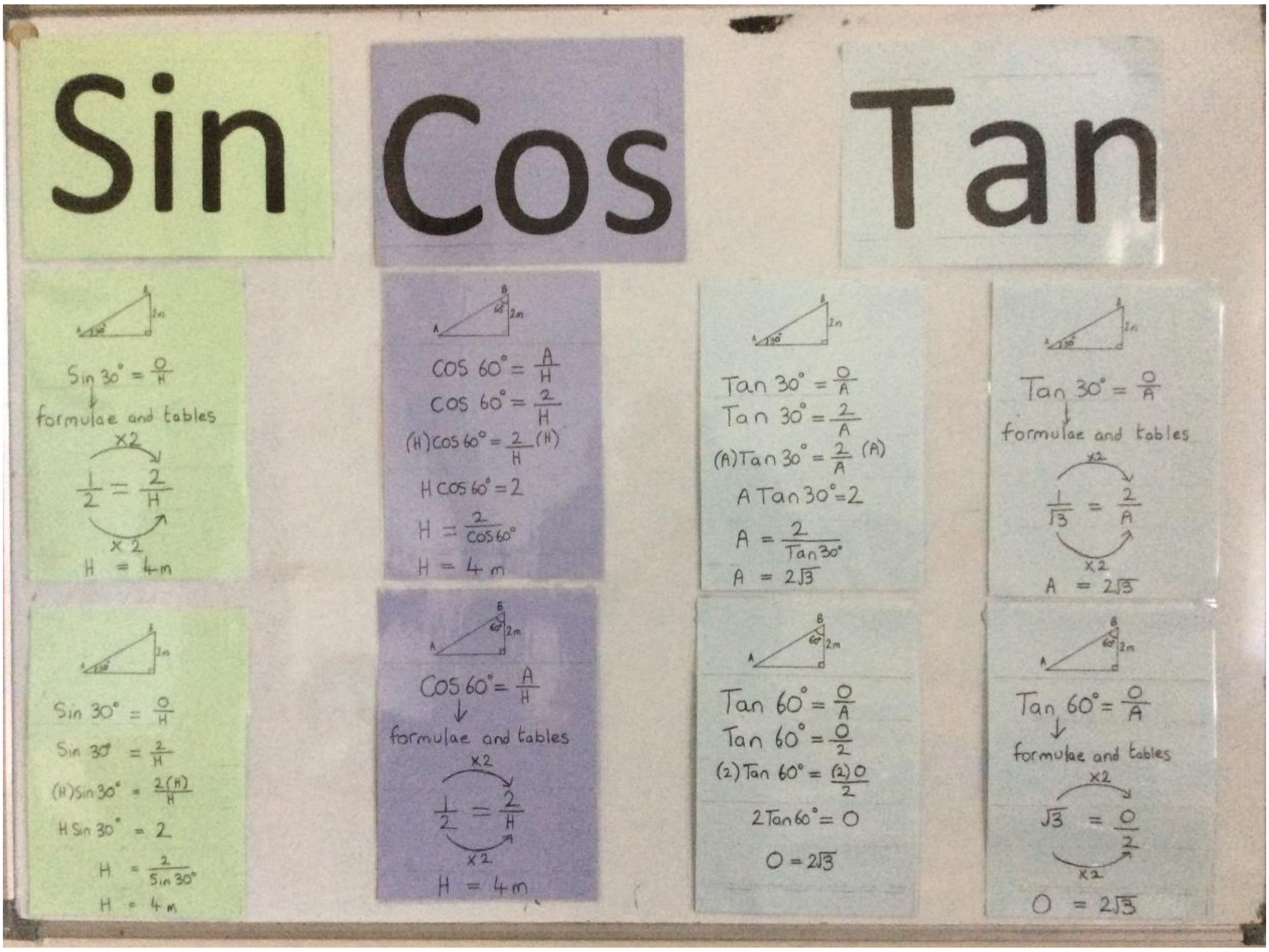
 Johnny Sexton places a ball on the pitch at B to take a place kick. He needs to calculate the perfect run up. He paces a distance of 2 m south. from the ball. He then turns due west and walks a certain distance. He then turns 150 degrees clockwise (to his right) to face the ball. 1. How long is his run up to the nearest metre? 2. What is the total distance he walks for the place kick?	Are there any further questions? You may request extra worksheets if you need them. You now all have 15 minutes to solve the question **in as many different ways** as possible. During that time, we will monitor your work but we will not ask you any questions or give you any tips. You can check any proposed answer by attempting the question using a different method. At the end of the 15 minutes I will call upon different students to explain their method to the class. I will give them prior warning of this.	How do they approach finding their first solution? Teacher(s) walk around taking notes on methods being used
Student Individual Work (15 minutes)	Teachers walk around the room studying and noting different approaches, filling, in their seating plan, which of the students will	Pictures and notes can be taken for later use in post lesson evaluation.

	present each method and in what order.	
Ceardaíocht /Comparing and Discussing (15 minutes) ***Student solutions that were used and the order in which they were discussed is shown on board plan below*** ****Student solutions and student thinking is discussed in the evaluation and reflection below****	Teacher puts up posters one by one, outlining each approach used, from simplest to most elegant, above students' writing, as the selected students explain their approach. Which solution do you think is the easiest? the quickest? Which of the trigonometry ratios could not be used for question 1?	Students come to the front of the class to explain their different methodologies and teacher watches for use of mathematical language. If not presenting, a student watches, listens or asks questions. They are encouraged to take notes and teacher can observe these at the end.
Summing up & Reflection (5 Minutes)	Write homework on the board: a. What did you recap about trig in today's	Students jot down the homework and ask questions where necessary.

	lesson – 3 points. b. Please design a question and solution on a right-angled triangle using special angles and we will look at them in our next class.	Evaluation sheet is given out and students asked to fill it out and then return to teacher as they leave.

10. Board Plan





11. & 12. Evaluation and Reflection

We decided not to do the recap at the start. We hoped they would know to use what was covered in trigonometry as well as other methods. Construction was the first choice in most cases - moving the protractor over until 150 degrees in line with B and then using ruler to measure hypotenuse. Then mixture of construction/measuring and use of Pythagoras.

It was impressive to see all students engaged fully in the task and getting into it right away. There was also active recording of the methods being described by the students presenting. All students recognised that a right-angled triangle was involved, some form of the theorem of Pythagoras was used by each.

Using other recently taught material for right angled triangles (the trigonometric ratios) to solve a real world problem was also in evidence.

21 of the students used sine, cosine or tan in one of their methods. They also came up with a correct solution. This was helpful feedback as it indicated that the trigonometric ratios were now usable approaches for them.

The other 8 students led us to consider what prevented them accessing the ratios. We also feel that it would be beneficial to provide opportunities for assessment during the unit, rather than only at the end, so that this disconnect from use of the ratios can be addressed with immediacy and allow change during the lessons.

More real world tasks in groups where students set up plans for solution, without having to go all the way to a solution could emphasize method and allow greater variety of contexts.

Two students out of the class of 29 used the tan ratio with both angles (60 and 30 degree) and this allowed them in their presentation to highlight one being the reciprocal of the other.

Encouraging students to reflect on their efforts by prompting them to try for more solutions was helpful. We could see first-hand the value of asking for many solutions rather than just the first one that comes to mind, because it was only in the second or third attempts that over half the girls called upon Sin, Cos or Tan.

"Constructive, developmental oral feedback" was met by the teacher commenting on students' solutions during their presentations. Having time in our meetings to prepare together for this Ceardaíocht and how we would interact when students were presenting, heightened our awareness of student thinking in their solutions. It also increased our genuine curiosity and listening skills which the students seemed to appreciate as they made the effort to be more clear in how they had undertaken the solution they were presenting to the rest of the class.
Doing the construction/measuring approach as first solutions, helped students understand the problem and primed them for the use of the ratios and thus understanding more fully their use, and how they provide greater accuracy than a ruler and protractor.

This is one example of how lesson study covers much of what is required in the new JC courses - students learning to communicate through presentation in this case.

Teachers felt that, particularly through the Ceardaíocht, students were engaging in metacognition as they listened to each other's approaches with the help of the teacher's weaving the approaches together through questioning and connecting verbally. There were signs of students taking ownership of their work as they justified an approach to the listening class, as well as when they integrated a different approach by accepting the presentation of a fellow student.

An unexpected approach was to create a rectangle and get the diagonal length - a version of Pythagoras. Another was using similar triangles.

Students are familiar with going up to the board now with doing CBAs. The class was only 45 minutes but it would have been better to have a double class in order to get into the depth required.

One teacher: "I was impressed with how students handled sin 60 where the unknown is in the denominator and they use scale to find it: ½=2/x."

Students used the calculator a lot but it was useful that the teacher asked if the sin 30 could be found elsewhere and the student presenting at that stage was able to respond "in the tables". The class was at the end of a unit so it was good to see that they used what they learned. Since some students were permitted to work with others (tables being tight together) it would be interesting to know how well they would do on their own. Also, as this was a higher level class, it would be useful to try the lesson with a mixed or ordinary class for comparison. Would they come up with as many solutions?

References

Looking at our Schools 2016

https://www.education.ie/en/Publications/Inspection-Reports-Publications/Evaluation-Reports-Guidelines/Looking-at-Our-School-2016-A-Quality-Framework-for-Post-Primary-schools.pdf

Chief Examiners Report, Junior Cert Cert Maths, 2015

https://www.examinations.ie/misc-doc/EN-EN-25073660.pdf

Chief Examiners Report, Leaving Cert Maths, 2015

https://docs.google.com/document/d/1wPSXevJhSMKWPH3lKq_pHNFjOvW9SEn-HYuBaChsHn8/edit#

Draft Specifications for Junior Cycle Mathematics

https://www.ncca.ie/media/3163/jcmathematics_draft_specification.pdf

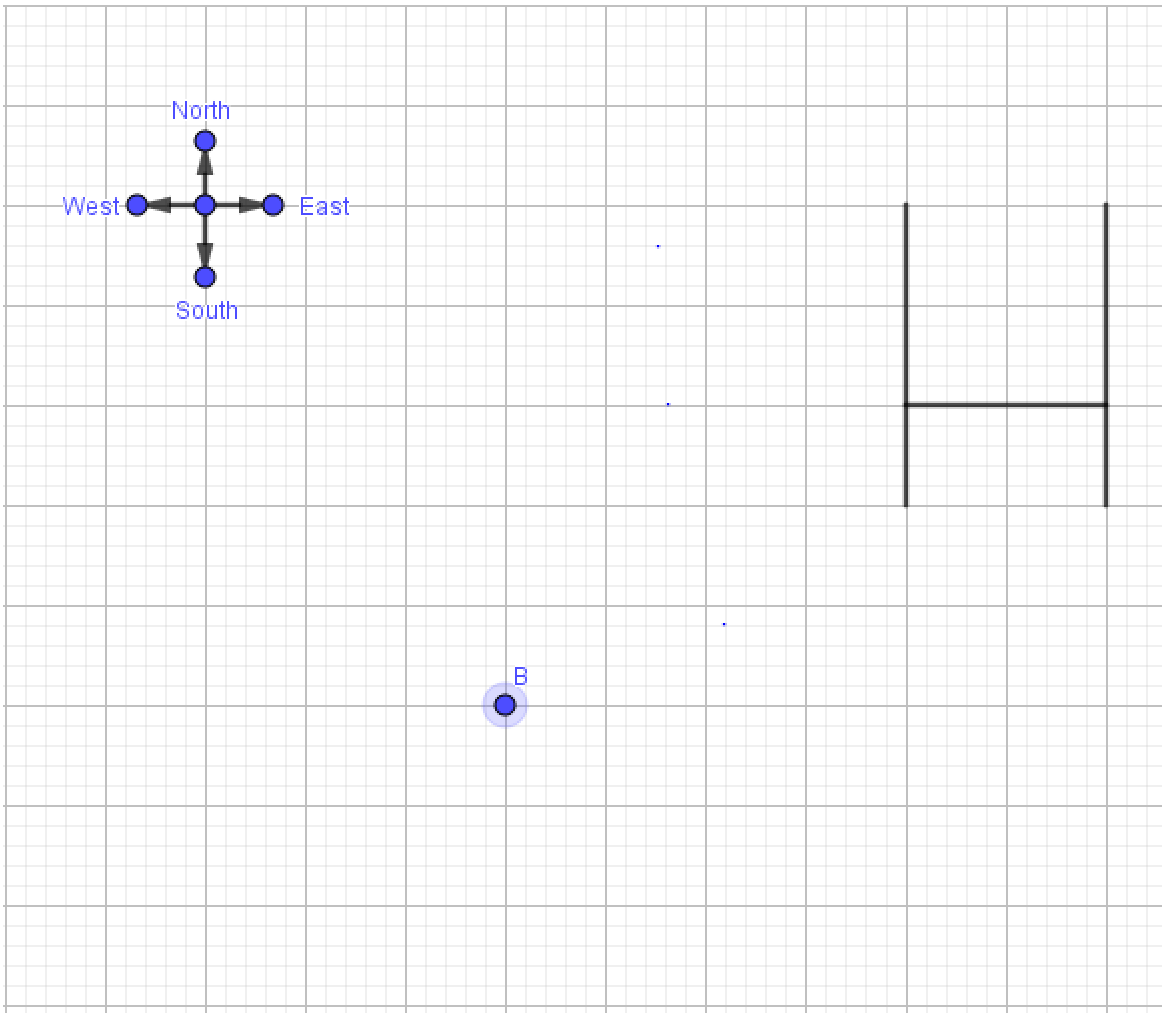
APPENDIX 1

THE PROBLEM

Johnny Sexton places a ball on the pitch at B to take a place kick. He needs to calculate the perfect run up. He takes 2 paces south from the ball. He then turns due west and walks a certain distance. He then turns 150 degrees clockwise (to his left) to face the ball.

1. How long is his run up to the nearest metre?

2. What is the total distance he walks for the place kick?



APPENDIX 2

STUDENT EVALUATION OF LESSON STUDY

Name _______________________

1. Circle the number that best indicates how clear the question was to you at the beginning.

Very clear **Not clear at all**

1 2 3 4 5

2. How many different ways did you find for representing the rectangles?

3. State one thing that you learned in the class:

a) From your own individual work:

b) From your peers/class discussion.

4. How challenging did you find the task, once you were clear what you were being asked to do?

(Circle the appropriate number)

Very easy **Very hard**

1 2 3 4 5

5. Name something that you found helpful today for your learning of mathematics.

6. Name one thing that could be improved, in this lesson, for your learning of mathematics.

7. Which method did you think was the best?

8. Which method was the easiest?

9. Which method was the quickest?

Thank you!