WS08.02   Informal Introduction to Limits and Continuity of Functions

Student Activity: Concept of a Limit

Aim: To investigate the trends in the values of different functions as x approaches x_0

Given: $f(x) = x + 3$, $\quad g(x) = \dfrac{x^2 - 9}{x - 3}$ and $h(x) = \dfrac{1}{x - 3}$.

1.   Fill in the following table:

x	2.9	2.99	2.999	2.9999	3	3.0001	3.001	3.01	3.1
$f(x) = x + 3$									
$g(x) = \dfrac{x^2 - 9}{x - 3}$									
$h(x) = \dfrac{1}{x - 3}$									

2.   When x approaches 3 (but $x \neq 3$) does the value of each of the following functions approach a fixed value?
     If so, find that value.
     If not, can you describe what you think is happening?
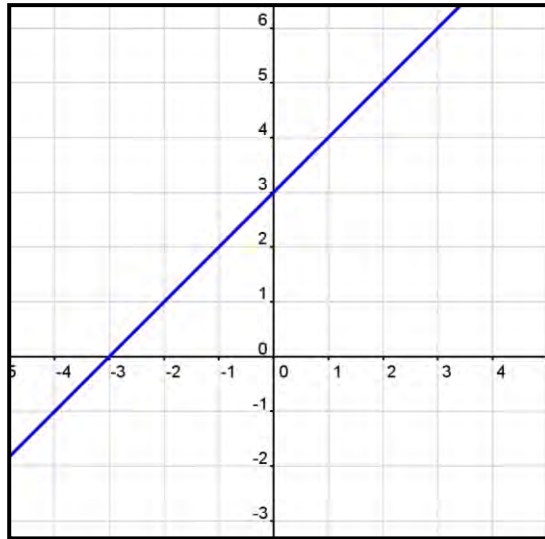
(i) $f(x) = x + 3$

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(ii) $g(x) = \dfrac{x^2 - 9}{x - 3}$

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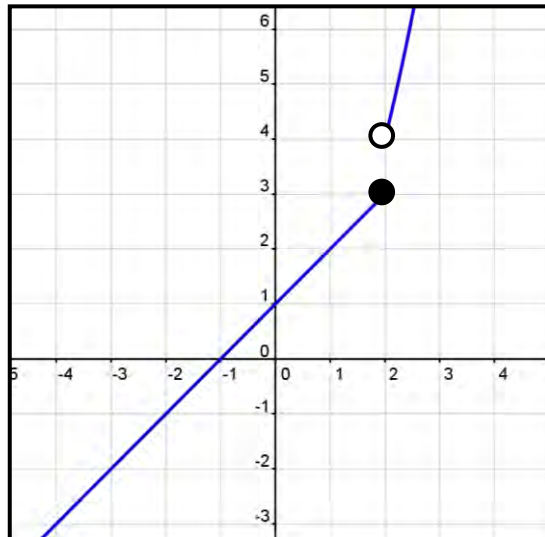**(iii)** $h(x) = \dfrac{1}{x - 3}$

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Analyse each graph below and determine whether the function it represents is continuous:
If not state why the function is discontinuous.

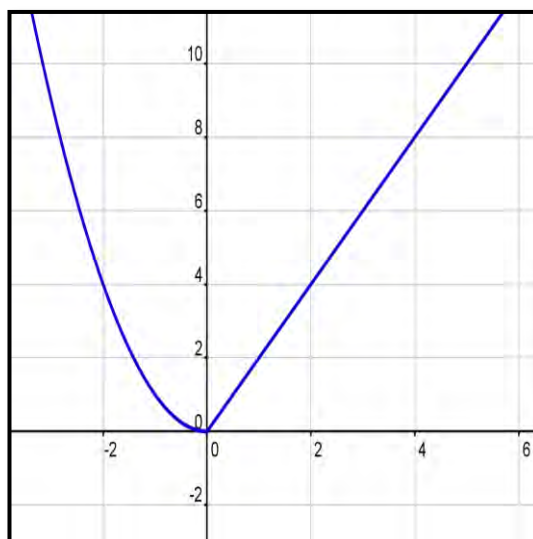**(i)** $f : x \mapsto x + 3$

(ii) $y = \begin{cases} x+1, & \text{if } x \leq 2 \\ x^2, & \text{if } x > 2 \end{cases}$

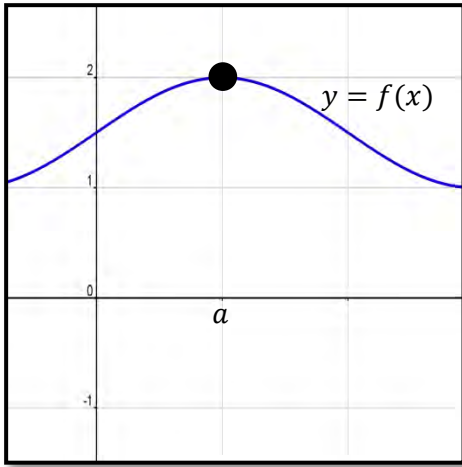**(iii)** $g(x) = \begin{cases} x^2, & \text{if } x < 0 \\ 2x, & \text{if } x \geq 0 \end{cases}$

Task 2

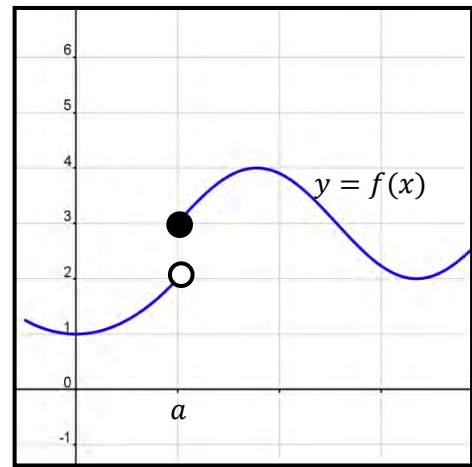According to each of the following graphs of functions $f(x)$:

(i)   Determine whether $f(x)$ is continuous at $x = a$.
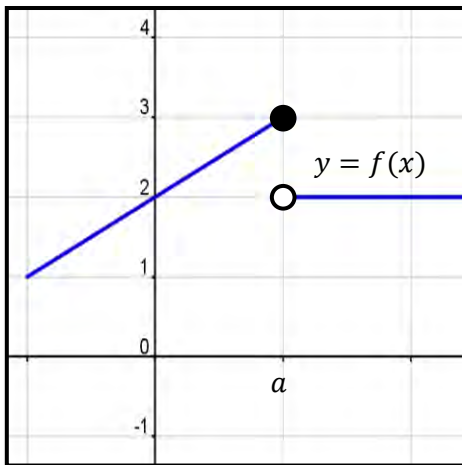
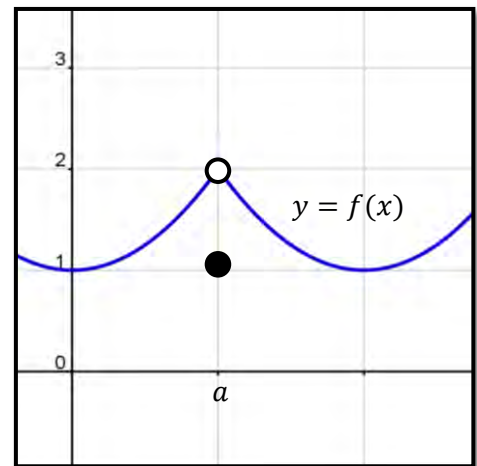**(ii)**   Find $\lim\limits_{x \to a} f(x)$.

(a)

(b)

(c)

(d)

(e)
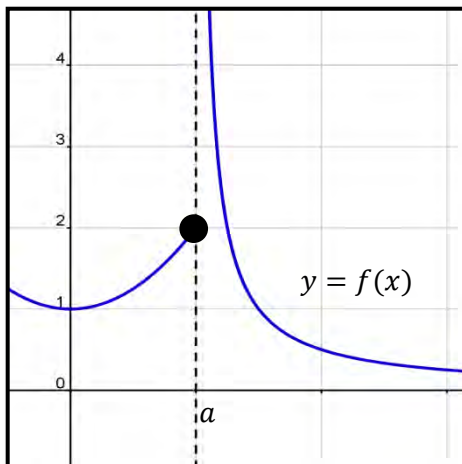
(f)
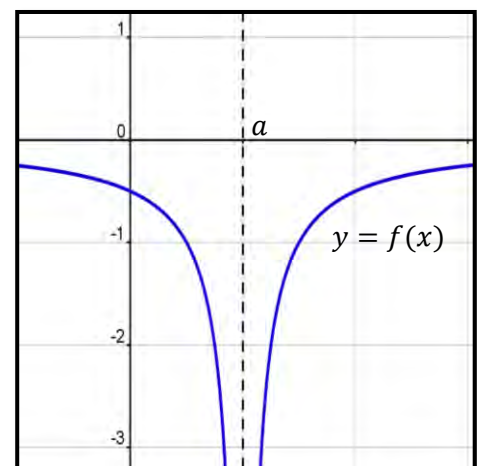
Solutions to **(a)** to **(f)** above:

(a)	**(i)**	Yes continuous at a	**(ii)**	Limit is 2
(b)	**(i)**	No not continuous at a	**(ii)**	Limit does not exist
(c)	**(i)**	No not continuous at a	**(ii)**	Limit does not exist
(d)	**(i)**	No not continuous at a	**(ii)**	Limit is 2
(e)	**(i)**	No not continuous at a	**(ii)**	Limit does not exist
(f)	**(i)**	No not continuous at a	**(ii)**	Limit does not exist