Algebra

	Pre-Algebra →	Understanding Variables →	Algebra →	Extension
"For effective learning, algebraic thinking must be nurtured in parallel with arithmetic understanding" *Lynn Arthur Steen*	**Number Theory** Solid understanding of Number Theory from Strand 3 Useful Methodology: Array Models, T&L on Integers, Fractions & Ratio 1		**Algebra skills seen as "generalised arithmetic"**. Make an explicit association between symbols and numbers. Use array models and algebra tiles (drawings) to help misconceptions. 3	"Most of the major principles of algebra and geometry emerge as generalisations of patterns in number and shape"
	2 **Patterns** **Fostering 'Algebraic Thinking' through exposure to patterns, relationships, generalising and problem solving.** Develop pattern-based thinking - recognise, construct and extend patterns (T&L on Patterns) - use tables to represent a pattern (patterns with unifix cubes) - use patterns to represent real-world situations - develop language to describe patterns precisely, both orally and in writing, as a prelude to using symbols. - use patterns to solve problems (Locker Problem) Deliberate focus on relationships involving two variables - develop an understanding of how one quantity changes as a result of the change in another quantity: $y = mx + c$ - Methodologies: Money Box Problem/ Sunflowers Problem - Students use tables and graphs to represent a relationship - Students introduced to linear relationships, constant rate of change, variables, increasing/decreasing change, slope = rise/run Generalising using symbols - Simplification: Letters employed to reduce the language used to describe patterns. (Doesn't matter what letter/symbol is used) - Students generalise the pattern, using symbols, and make their first formula. The Power of Pattern-Based Thinking: Problem Solving - Patterns and relationships are used to model maths and real-world situations, particularly for solving problems. - Symbols are used to generalise the rule of a pattern observed in a situation. Then that rule can be used to solve the problem. *By doing Patterns first:* *Algebra is seen as the language we use to describe patterns and relationships for the ultimate goal of problem solving.* *Students also get a very good introduction to a variable as a changing quantity.*	2 "Algebra provides finite ways of managing the infinite." **Variables can be used in 4 different ways:** - A formula like $A = l \times b$ (infinite amount of possibilities) - A Law/Identity like the Commutative Law, $x + y = y + x$ (for all cases) - A Relationship/Rule like $\{(x,y)\vert\, y = 2x + 3,\ x \epsilon R\}$ (infinite amount of points that fit a rule) - An unknown like $2x = 6$ (one number from an infinite set of possibilities) All of the above can be explored using patterns. **Problem Solving:** Using a variable as an unknown can be introduced and explored through problem solving. Example: For how many days did John need to save in order to accumulate €45 for a new computer game?	3 **Money Box Problem extended:** We can show adding like terms as part of a real-world problem solving question. For example: 2 family members combining their savings to buy a computer console costing €249 **Skills for Solving Equations:** After Money Box / Sunflowers Problem is used to explain an unknown in context of a real-world problem, extend this to teach the skills for solving equations. Methodology: T&L on Equations, stabilisers **Solving Word Problems using Algebra**: Show that algebra allows choice and flexibility in solving problems. Let students discover that algebra is often the most efficient way to solve a problem, especially word problems. ***Overview of the learning outcome for teaching algebra:*** *The relationship based approach to learning algebra should culminate in students having a deep understanding of algebra which allows easy movement between story, table, graph and equation.* *Learners should also have an appreciation that the power of algebra lies in its capacity to describe relationships for the purpose of problem solving.*	- **Factorising** - Construct some **Perimeter and Area Formulae** using patterns and variables - Discover **theorems** through patterns - Extend rise over run triangle into the formula for slope, then the distance between 2 points. **Co-ordinate Geometry** understood as the marriage of geometry and algebra. - Discover **quadratic, cubic and exponential relationships** through patterns - Look at patterns in **Statistics** - Discover **Trigonometric Ratios** through patterns - Investigate patterns of change in **Periodic and Trigonometric functions** - Rates of change observed in patterns can be extended to change at an instantaneous point in **Calculus.** - Extend patterns and symbols into **Sequences and Series**
	Functions Introduce the terms inputs, outputs, a mapping, domain and range. Money Box Problem N → N, Sunflowers Question N → R	Play "Guess the Rule" game.		- Formalise **Functions**

Draw the following arrays:

$$x, \ y, \ 2x, \ x^2, \ 4x^2, \ 2(x+y), \ 2x+2y$$

where $x \neq y$.

Question: Is $2(x+y) = 2x+2y$? Discuss.

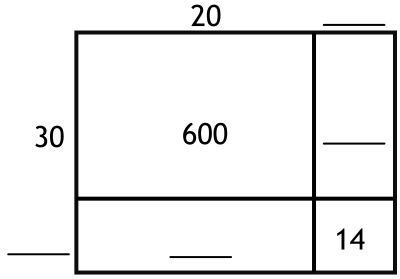
Question: Is $2x \neq x^2$ always, sometimes or never?

WS5.17 Array Model with Numbers

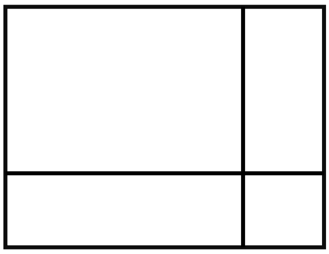
(a) $27 \times 32 = (20 + \_\_\_\_\_) \times (30 + \_\_\_\_\_)$

$= \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_ + \_\_\_\_\_\_$

$= \_\_\_\_\_\_$



(b) $35 \times 41 =$



WS5.18 Array Model with Algebra

Example

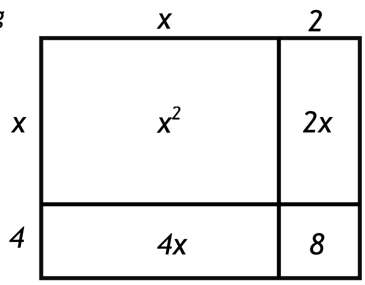
$(x + 2)(x + 4)$

$= x(x + 4) + 2(x + 4)$

$= x^2 + 4x + 2x + 8$

$= x^2 + 6x + 8$

Check work using an array model

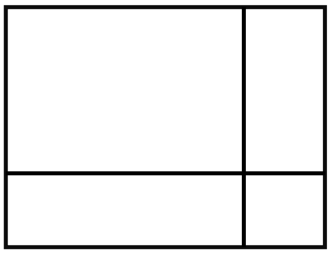


(c) $(x + 5)(x + 3)$

$=$

$=$

$=$



Worksheets available on ***http://www.projectmaths.ie***

- *Examples* **(a)** *and* **(b)** *above are taken from worksheets found under Teachers/Strand 3/Junior Cycle/supplementary material*
- *Example* **(c)** *above is taken from worksheets found under Teachers/Strand 4/Junior Cycle/supplementary material*

1. Evaluate $2+3\times4$.

Answer: ☐

Class discussion on everyone's answers

Mathematicians made an agreement that:

multiplication takes precedence over addition.

2. Considering the agreement, which word problem below describes the arithmetic sentence $2+3\times4$

A. You work for 3 hours babysitting and you normally get €4 per hour. But this time the people tip you an extra €2. How much did you earn?

B. A gardener decides to plant trees around the edges of a square park. He decides to plant 3 willow trees and 2 cherry blossom trees on each edge of the park. How many trees does he plant?

A or B? ☐

3. If we want to have addition done before multiplication we use brackets: $(2+3)\times4$

we always simplify inside the brackets first

Put brackets on the following statements to make them true.

(i) $7\times8+2=70$ **(iii)** $6+3\times2+5=23$

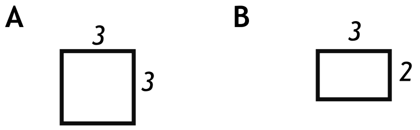
(ii) $2+3\times4+5=45$ **(iv)** $3\times7+1+1=25$

4. Another operation to consider is powers. Match the numerical expressions with their corresponding array models by placing A, B, C or D into the box.

(i) 3×2 ☐

3^2 ☐

Place A or B in the boxes



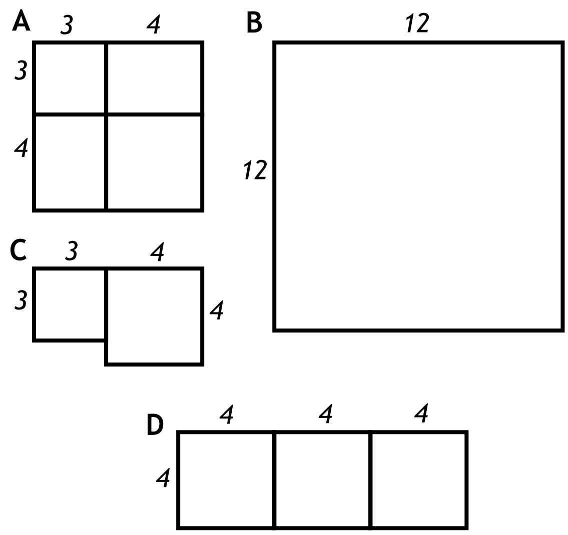
(ii) 3^2+4^2 ☐

3×4^2 ☐

$(3\times4)^2$ ☐

$(3+4)^2$ ☐

Place one of the letters A, B, C or D in each box



Class discussion on where the powers come in the order of operations and formalise:

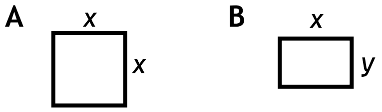
B I
M D
A S

WS5.20 *Algebraic Expressions with Corresponding Array Models*

Match the algebraic expressions with their corresponding array models by placing A, B, C or D into the box

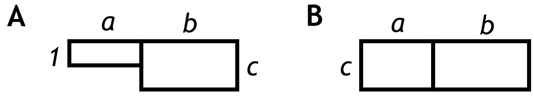
(i) $x \times y$ ☐

x^2 ☐



(ii) $a + b \times c$ ☐

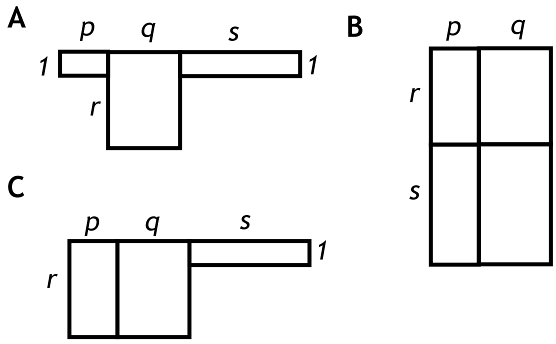
$(a+b) \times c$ ☐



(iii) $(p+q) \times (r+s)$ ☐

$p + q \times r + s$ ☐

$(p+q) \times r + s$ ☐

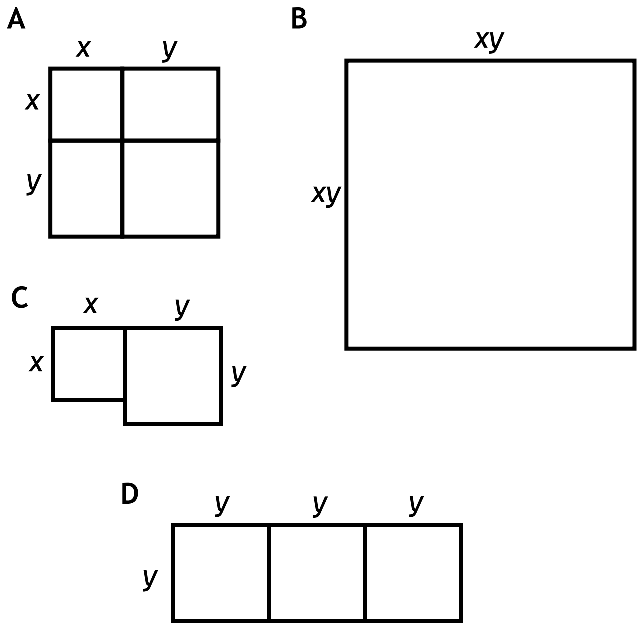


(iv) $x^2 + y^2$ ☐

$3 \times y^2$ ☐

$(x \times y)^2$ ☐

$(x+y)^2$ ☐



(v) Is $x^2 + y^2 = (x+y)^2$?
Justify your answer:

1. Parking Bays

You work for a campsite owner. He wants to sell bays in his campsite and wants to include parking for the campers' car beside their tent. The owner wants the parking bay to be suitable for different sized cars and so wants the bays to be as follows:

The length of a bay is 5 m longer than the width of the camper's car.
The width of the bay is 2 m longer than the width of the camper's car.

Draw a diagram to show the area of the car parking space for any width of car.

If the width of John's car is 1.5 m, what area will his parking space be when he buys a campsite bay.

2. Sums of Pairs

Caroline has three numbers. She adds them in pairs and records the answer in each case. When she does this she has three different totals: 11, 17 and 22.

What are the three numbers Caroline had to start with?

Can you describe a method that would enable you to work out the three numbers given any three totals?

3. A Walk Around the Earth

Suppose you are six feet tall and walk around the Earth's equator.
How much farther does your head travel than your feet?

4. Burning Candles

Two different candles are lit. They burn at different rates and one is 3 cm longer than the other.

The longer one was lit at 5.30 p.m. and the shorter one at 7 p.m.
At 9.30 p.m. they were both the same length.

The longer one, burned out at 11.30 p.m. and the shorter one burned out at 11 p.m.

How long was each candle originally?

5. Bernie's Field

Bernie has been given a field in the shape of a triangle. Two sides of the triangle are exactly 10 metres long.

What is the largest possible area, in square metres, of Bernie's triangular field?

1. Taking out a common factor

Factorise $3x + 6$

	x	2
3	$3x$	6

The factors are $3(x + 2)$

Over to you:
Factorise $5x^2 + 20x$, using the table model.

2. Grouping

Factorise $ab - bc + da - dc$

	a	$-c$
b	ab	$-bc$
d	da	$-dc$

The factors are $(b + d)(a - c)$

Over to you:
Factorise $2ax - 6ay - 3x + 9y$ using a table model.

3. Factorising Quadratics: Reuse the Grouping Method

Example A

Guide Number

+6

6×1

-6×-1

3×2

-3×-2

$x^2 - 5x + 6$

$x^2 - 3x - 2x + 6$

$x(x-3) - 2(x-3)$

$(x-3)(x-2)$

	x	-3
x	x^2	$-3x$
-2	$-2x$	$+6$

Example B

Guide Number

- 42

$\pm \begin{cases} 1 \times 42 \\ 2 \times 21 \\ 3 \times 14 \\ 6 \times 7 \end{cases}$

$2x^2 - 11x - 21$

$2x^2 - 14x + 3x - 21$

$2x(x-7) + 3(x-7)$

$(2x+3)(x-7)$

	x	-7
$2x$	$2x^2$	$-14x$
$+3$	$+3x$	-21

Over to you:

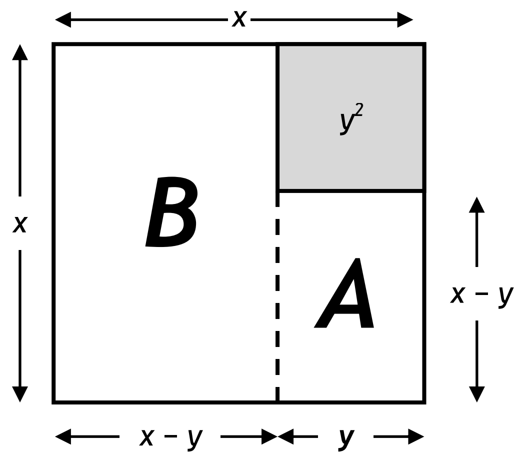
Factorise the following quadratic using grouping: $3x^2 - 17x + 20$.

4. *Difference of Two Squares*

Factorise: $x^2 - y^2$

Area of $A = y(x - y)$
Area of $B = x(x - y)$

Area of $A + B = y(x - y) + x(x - y)$
$\qquad\qquad\quad = (x - y)(x + y)$



Over to you:
Factorise $9a^2 - 4b^2$ using an area model.