Exploring Trigonometric Graphs

Project Maths
Tionscadal Mata
Development Team

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Syllabus Extract: Relationship to syllabus. Leaving Certificate.

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
2.3 Trigonometry	– use the theorem of Pythagoras to solve problems (2D only)	– use trigonometry to calculate the area of a triangle – solve problems using the sine and cosine rules (2D) – define sin θ and cos θ for all values of θ – define tan θ – solve problems involving area of a sector of a circle and the length of an arc – work with trigonometric ratios in surd form	– use trigonometry to solve problems in 3D – graph the trigonometric functions sine, cosine, tangent – graph trigonometric functions of type $f(\theta) = a + b\text{Sin } c\theta$ $g(\theta) = a + b\text{Cos } c\theta$ for $a, b, c \in R$ – solve trigonometric equations such as Sin $n\theta = 0$ and Cos $n\theta = ½$ giving all solutions – use the radian measure of angles – derive the trigonometric formulae 1, 2, 3, 4, 5, 6, 7, 9 (see appendix) – apply the trigonometric formulae 1-24 (see appendix)

Students learn about	Students should be able to
2.5 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Unit 1

Upon completion of this Unit:

Students will:

Understand terms such as

- Maximum
- Minimum
- Period
- Range
- Amplitude
- Horizontal midway line
- Horizontal shape (stretch/shrink)
- Vertical shape (stretch/shrink)
- Transformations of the graphs of
  $f(x) = \sin x$ and $f(x) = \cos x$.

In this unit students will explore functions of the type

$f(x) = a \pm b\sin cx$

$f(x) = a \pm b\cos cx$ $\qquad a, b, c \in R$

and examine how the values of "a", "b" and "c" affect the curves.

Prior Knowledge

Students should be familiar with graphs of linear and quadratic functions.

Students should be familiar with the graphs of $f(x) = a \sin bx$ *and* $f(x) = a\cos bx$ from **Teaching and Learning Plan 10** and the **Student's CD**.

At the outset the students should be reminded of the features of the graphs of the functions $f(x) = \sin x$ and $f(x) = \cos x$. by completing **Student Activity Sheet 1** [Page 37].

Materials required

For each student you will need:

Mini white board

Student Activity Sheet 1 $f(x) = sin\ x$ and $f(x) = cos\ x$.

Student Activity Sheet2a (Mathematical Language / Properties of Trigonometric Graphs)

Student Activity Sheet 2b (Mathematical Language / Properties of Trigonometric Graphs)

Student Activity Sheet 2c (Mathematical Language / Properties of Trigonometric Graphs)

Student Activity Sheet 3 (Mathematical Language / Properties of Trigonometric Graph)

Student Activity Sheet 5 (Mathematical Language / Properties of Trigonometric Graphs)

Student Activity Sheet 6 (Transformations of Trigonometric Graphs)

Student Activity Sheet 7 (Mathematical Language / Properties of Trigonometric Graphs).

For each group of students you will need

Student's CD

Card Set A1 (Trigonometric Functions)

Card Set A2 (Trigonometric Functions)

Card Set B (Trigonometric Functions Important Features)

Card Set C

Student Activity 4 (Mathematical Language / Properties of Trigonometric Graphs).

Student Activities/Teacher's support and actions

Use the interactive *GeoGebra* files at: www.projectmaths.ie

Working in groups [1]

Ask students to work in pairs, distribute one or both of

Card Set A1 (Trigonometric Functions $g(x) = bsin\ x$) [Page 38]

Card Set A2 (Trigonometric Functions $g(x) = bcos\ x$) [Page 39]

to each pair of student.

Ask each group to sort the cards into three sets, selection criteria should be at the group's discretion.

For example, they may group the functions where the coefficient of $sin\ x$ or $cos\ x$ is 2.

Ask students to write a description of their selection criteria.

Ask students to write an equation of their own for each set (if working with cards from one set only).

Whole group activity using GeoGebra [1]

Share all the criteria students have come up with.

Hand out one of:

Student Activity Sheet 2a (Mathematical Language /Properties of Trigonometric Graphs) [Page 40]

and

Student Activity Sheet 2b (Mathematical Language /Properties of Trigonometric Graphs) [Page 41]

or

Student Activity Sheet 2c (Mathematical Language /Properties of Trigonometric Graphs) (getting the students to use a mixture of cards from Card Sets a1 and A2) [Page 42]

Now ask the students to use the interactive files

$f(x)=a+b\sin cx$ and $f(x)=a+b\cos cx$ at www.projectmaths.ie a to justify their descriptions of the functions contained on Card Sets A1/A2 and introduce students to the correct use of mathematical language using the previous student activity.

Instructions: In the interactive files
$f(x)=a+b\sin cx$ and $f(x)=a+b\cos cx$

Set Slider a to 0. Set Slider c to 1. Set Slider b to desired value.

Example:

A "b" value of 3 causes a stretch in the direction of the Y axis with scale factor of 3.

If student need more practice with transformations and trigonometric graphs use mini whiteboards to explore questions of the type:

Give an equation to represent the function which results from:

stretching $f(x)=\sin x$ in the direction of the Y axis with a scale factor of 4.

reflecting $f(x)=\cos x$ in the X axis.

Describe the transformation required to transform $f(x)=\sin x$ *to chlaochlú go dti* $f(x)=-2\sin x$

Define "amplitude", "range", "period" and link these to the idea of stretch/shrink

Check the students' understanding of how $f(x)=a+b\sin cx$ and $f(x)=a+b\cos cx$ Check the students' understanding of how give information about amplitude, range and period by asking them to find cards from **Card Set A1** and/or **Card Set A2** that fit certain criteria. For example, the students might be asked to find equation(s) which describe:

A graph having an amplitude of 2

Two graphs that have the same period

A graph that has been compressed along the X axis

A graph that has been stretched vertically

Two graphs that are images of each other by reflection.

Working in groups [2]

Distribute to each pair of students

Card Set B (Trigonometric Functions-Important Features) [Page 43]

Ask each pair to sort the cards into two sets. Selection criteria should be at their own discretion.

Ask students to write a description of the selection criteria.

Ask student to write an equation which would fit their selection criteria.

Whole group activity using GeoGebra [2]

Share all the criteria students have come up with.

Distribute to each pair of students.

Student Activity Sheet 3 (Mathematical Language/Properties of Trigonometric Graphs) [Page 44]

Now use the interactive files $f(x)=a+b\sin cx$ and $f(x)=a+b\cos cx$ at www.projectmaths.ie to justify the descriptions and introduce students to mathematical language.

Instructions: In the interactive files $f(x)=a+b\sin cx$ and $f(x)=a+b\cos cx$

Set Slider a to 0. Set Slider b to 1. Set Slider c to desired value.

Example

When "c" has a value of 3 this is a horizontal shrink with scale factor 3.

If students need more practice with transformations and trigonometric graphs use mini whiteboards to explore questions of the type:

Give an equation to represent the function which results from:

Stretch $f(x)=\sin x$ horizontally by a scale factor of 4

Shrink $f(x)=\cos x$ horizontally by a scale factor of 0.5

Decreasing the period of $f(x)=\sin x$ by a factor of 2

Increasing the period of $f(x)=\sin x$ by a factor of…

What transformation converts:

$f(x)=\sin x$ to $f(x)=\sin 2/5x$,

$f(x)=\cos x$ to $f(x)=\cos 5x$?

Check student understanding of how the functions

$f(x)=a+b\sin cx$ and $f(x)=a+b\cos cx$

give information about amplitude, range and period by asking them to find cards from **Card Set B** that fit certain criteria.

For example, the students might be asked to find equation(s) which describe:

A graph having amplitude of 2.

Two graphs having the same period.

A graph that has been shrunk along the X axis.

A graph that has been vertically stretched.

A graph that has been horizontally stretched.

Two graphs that are images of each other by reflection.

Two graphs that have the same amplitude.

Working in groups [3]

Distribute to each pair of students:

Card Set C [Pages 45 – 47]

Student Activity Sheet 4 (Mathematical Language/Properties of Trigonometric Graphs) [Page 48]

Student Activity Sheet 5 (Mathematical Language /Properties of Trigonometric Graphs) [Page 49]

Ask each pair of students to find cards from **Card Set C** to match the properties described on **Student Activity Sheet 4**.

When the students have found a suitable card they should place it in the correct box on **Student Activity Sheet 4** matching the property described therein. They should aim to match as many boxes and cards as possible.

Whole-group activity [3]

Choose one box from **Student Activity Sheet 4** and write down all the cards that were matched with it.

Discuss as a group if these cards are appropriate and agree reasons for any decisions reached.

Repeat the above for the remaining boxes.

Check students understanding of how the equations $f(x)=a+bsin\ cx$ and $f(x)=a+b\ coscx?$ give information about amplitude, range and period by asking them to fill in the second section of **Student Activity Sheet 5.**

Working in groups [4]

Distribute to each pair of students

Student Activity Sheet 6 (Transformation of Trigonometric Graphs)[Page 50]

Student Activity Sheet 7 (Mathematical Language / Properties of Trigonometric Graphs) [Page 51]

Discuss in groups how the graphs of $f(x)=a+bsin\ cx$ and $f(x)=a+b\ coscx$ are affected by changing the value of "a"

If required reinforce the students' prior learning by referring to the properties of the graphs of linear and quadratic functions.

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Whole group activity using GeoGebra [4]

Share all the criteria student have come up with in the previous activity Working in groups [4] and discuss the reasons for these criteria.

Now use the interactive files $f(x)=a+b\sin cx$ and $f(x)=a+b \cos cx$ at www.projectmaths.ie to justify the descriptions and introduce student to the appropriate use of mathematical language.

Instructions: In the interactive files $f(x)=a+b\sin cx$ and $f(x)=a+b \cos cx$

Set Slider b to 1. Set Slider c to 1. Set Slider a to desired value.

For example

When "a" has a value of 3 this causes a vertical translation upwards of length 3 units.

If student need more practice with transformations and trigonometric graphs use mini whiteboards to answer questions of the type:

> Give an equation to represent the function which results from translating: $f(x)=\sin x$ vertically upwards by 2 units.
>
> Give the amplitude of the function which results from translating $f(x)=\cos x$ vertically downwards by 3 units.
>
> Give the range of the function which results from translating $f(x)=\sin x$ vertically upwards by 4 units.
>
> Give the period of the function which results from translating $f(x)=\cos x$ vertically downwards by 2.5 units.
>
> Give the transformation which has been used in transforming $f(x)=\cos x$ to $f(x)=7 + \cos x$

Check student understanding of how the equation gives information about transformations amplitude, range and period by getting them to complete **Student Activity Sheet 7**.

Unit 2

Having completed this Unit:

Students will:

Understand how to identify the:

- Period
- Range
- Amplitude
- Horizontal midway line
- Horizontal shape in the direction of the X axis
- Vertical shape in the direction of the Y axis
- Transformations of trigonometric functions from their graphs.

Furthermore, the students will be able to sketch trigonometric graphs given their equations.

Prior Knowledge

Student should be familiar with the graphs $f(x)=a+b\sin cx$ and $f(x)=a+b\cos cx$ from Unit 1.

Materials required

For each student you will need:

Mini white board

Student Activity Sheet 8 (Trigonometric Graphs)

For each group of student you will need:

- Card Set D
- Card Set E
- Card Set F

Working in groups [5]

Distribute to each pair of students

Card Set D Trigonometric Graphs [Pages 52 – 53]

Student Activity Sheet 8 (Trigonometric Graphs) [Page 54]

Ask each group of students to fill in **Student Activity Sheet 8** using details from the graphs on **Card Set D** to do so.

Distribute to each pair of student **Card Set E** [Page 55]

Ask each group to match the graphs from **Card Set D** with the trigonometric functions from **Card Set E** using the information derived from the earlier group activity.

Check student understanding by asking them to explain why they matched the cards as they did.

Working in groups [6]

Distribute

Card Set D Trigonometric Graphs [Pages 52 – 53] to each pair of student if not already handed out.

Ask each group of students to name and identify a number of points which would allow them to draw a sketch of trigonometric graphs of the form: $f(x)=a+b\sin cx$ and $f(x)=a+b\cos cx$

Student can use mini whiteboards to draw sketches as part of their discussions.

Whole group activity [6]

Choose one group's set of points.

Discuss as a group the merits or otherwise of this set of points.

Ask questions such as: Will this set of points give a unique graph?

Will this set of points give the graph for all possible trigonometric functions of the form $f(x)=a+b\sin cx$ and $f(x)=a+b\cos cx$ is this the smallest set of points that will allow a sketch to be drawn?

Taking feedback from all the groups identify the five points required:

Maximum

Minimum

Three points of intersection with the horizontal midway line.

Working in groups [7]

Distribute to each pair of students

Card Set F [Page 56]

Check students understanding of how the equation gives the information required to sketch a trigonometric function of the form $f(x)=a+b\sin cx$ and $f(x)=a+b\cos cx$ by asking them to use mini whiteboards to sketch the graphs from **Card Set F**.

Each group of students can check their sketches by using the interactive files $f(x)=a+b\sin cx$ and $f(x)=a+b\cos cx$ at www.projectmaths.ie

Note 1: if students do not have **Card Set D** or **Student Activity Sheet 8** in their possession they could use **Card Set E** for this activity.

Note 2: Students could use the functions from **Student Activity Sheet 7** instead of using **Card Set F**, if this is considered desirable.

Unit 3

Having completed this Unit:

Student will:

be able to formulate conjectures form patterns

understand the need to explain their findings and justify their conclusions

have developed the skills to communicate mathematics verbally and in written form using appropriate mathematical language

be able to apply their knowledge and skills to solve problems in familiar and unfamiliar contexts

be able to analyse information presented verbally and translate it into mathematical form devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Prior Knowledge

Students should be familiar with the graphs of

$f(x)=a+b\sin cx$ and $f(x)=a+b\cos cx$

from Units 1 and 2 of Exploring Trigonometric Graphs.

Materials required

For each students you will need:

Mini white board

Question Sheets (Trigonometric Graphs in Context)

Working in groups [7]

Ask student to work in pairs.

Distribute to each student **Question Sheets (Trigonometric Graphs in Context)** [Page 15 - 17]

Ask each group to attempt Question 1.

Take feedback from groups using appropriate questioning to establish and develop the students' understanding.

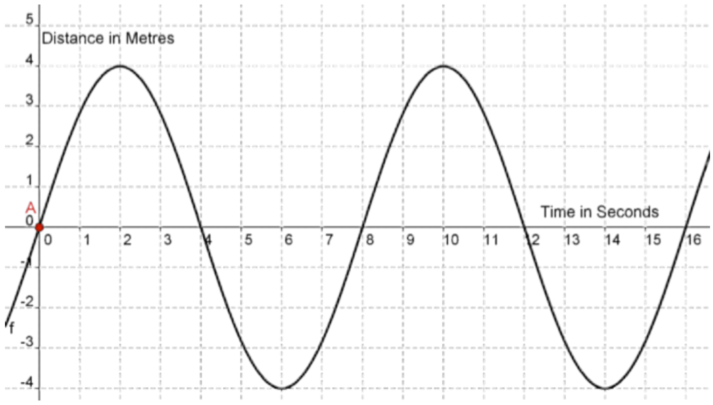
Repeat the above for two additional questions.

Now ask each student to work independently on the remainder of the questions on the **Question Sheets(Trigonometric Graphs in Context)**. Provide assistance to individuals as required and ensure that the students' learning is of a high quality and that their understanding of the material is being developed and deepened.

Questions Sheets - Trigonometric Graphs in Context

Question 1

A wrecking ball attached to a crane swings back and forth. The distance that the ball moves to the left and to the right of its resting position with respect to time is represented by the following graph.



a. What is the period of the crane's motion. Explain your answer?

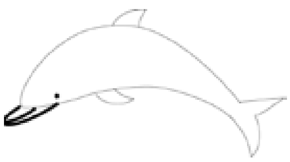
b. What is the equation of the horizontal midway line of the curve and what does it represent?

c. What is the amplitude of the crane's motion? Draw a diagram to represent what the amplitude represents in terms of the motion of the ball?

d. How many complete swings will the wrecking ball make in four minutes?

e. What does point A represent?

Question 2



As a dolphin swims along by the side of a cruise ship he jumps to a height of four metres above the water surface and dives to a depth of four meters below the surface. He does this in a regular motion (simple harmonic motion). A passenger using a stopwatch and starting it just as the dolphin is at the surface determines that the dolphin completes one cycle every 8 seconds.

a. Describe the displacement of the dolphin relative to the water surface using a sinusoid curve and sketch the graph of the dolphin's displacement relative to the water surface

b After three seconds how high above the surface is the dolphin?

c Is the dolphin above or below the surface of the water after 37 seconds?

Question 3

The inside rim of a bicycle wheel whose diameter is 25 inches, is 3 inches off the ground. An ant is sitting on the inside rim of the wheel at the point 3 inches off the ground. Sean starts riding the bicycle at a steady rate. The wheel makes one revolution every 1.6 seconds.

a. Find the equation of a sinusoid curve which describes the motion of the ant and draw a graph of the function.

b What height in centimetres from the ground will the ant be 25 seconds into the trip given that 1 inch = 2.54 cm.?

c Within the first 10 seconds how many times will the ant be at its starting height?

Question 4

The number of people in thousands employed in a resort town is represented by the function

$$f(x) = 3.8 - 1.7cos\frac{\pi}{6}t$$

Take t = 0 as last day of January

Take t = 1 as last day of February

Take t = 2 as last day of March

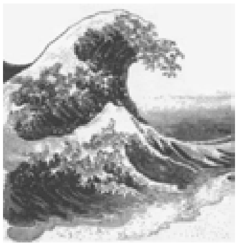
a. Draw a rough sketch showing the variation in the number of people employed in the town for one complete period.

b When is the maximum number of people employed and what is this maximum number?

c During which months of the year will the number of people employed be 4,650 or greater?

d Does this model have any drawbacks and if yes identify one?

Question 5



A tsunami (tidal wave) is a fast moving wave caused by an underwater earthquake. The water oscillates about its normal level, with equal amplitudes above and below this level The period is fifteen minutes. Suppose that a tsunami with an amplitude of ten metres approaches the pier at Honolulu, where the normal depth of water is nine metres. Assuming that the depth of water varies sinusoidally with time as the tsunami passes, predict the depth of the water at the following times after the tsunami first reaches the pier.

a Two minutes, four minutes and twelve minutes.

b According to your model what will be the minimum depth of the water?

c How do you interpret this answer in terms of what will happen in the real world?

Question 6

A buoy in the ocean is bobbing up and down in harmonic motion.

At $t = 0$ seconds, the buoy is at its high point and returns to that high point every 8 seconds. The buoy moves a distance of 1.44 meters from its highest point to its lowest point.

a. Using a sinusoidal function create a mathematical model to represent the depth of water under the buoy and sketch the curve.

b How high is the buoy at time = 29 seconds?

c Is it rising or falling at that time?

d How many times in 2 minutes will the buoy be at sea level?

Question 7

If the depth of water in a canal continually varies between a minimum 2m. below a specified buoy mark and a maximum of 2m above this mark over a 24-hour period.

a Construct a formula involving a trigonometric function to describe this situation.

The road to an island close to the shore is sometimes covered with water. When the water rises to the level of the road, the road is closed. *On a particular day, the water at high tide is 5 m above the mean sea level.*

b Construct a formula involving a trigonometric function to model this situation if high tide occurs every 12 hours and the tide behaves in simple harmonic motion. Draw a sketch of the model.

c Find the height of the road above sea level if the road is closed for 3 hours on the day in question.

d If the road were raised so that it is impassable for only 2 hours 20 minutes, by how much was it raised?

Question 8

At a certain latitude the number (d) of hours of daylight in each day is given by, $d = A + B \sin kt°$ where A and B are positive constants and t is the time in days after the spring equinox.

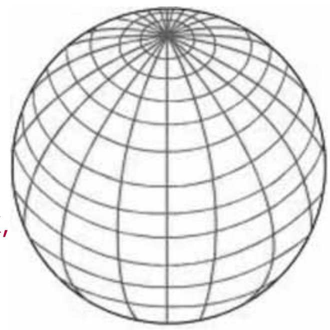
Assuming the number of hours of daylight follows an annual cycle of 365 days;

a Find the value of k correct to three decimal places.

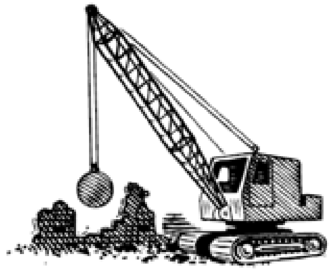
b If the shortest and longest days have 6 and 18 hours of daylight respectively state the values of A and B.

c Find in hours and minutes the amount of daylight on New Year's day which is 80 days before the spring equinox.

d A town at this latitude holds a fair twice a year on days that have exactly 10 hours of daylight. Find, in relation to the spring equinox, which two days these are.

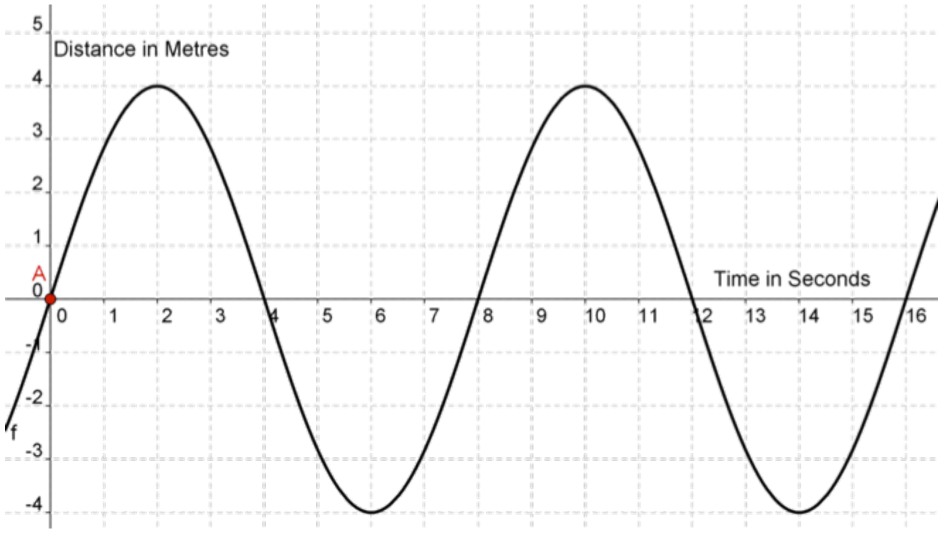


Solutions



Question 1

A wrecking ball attached to a crane swings back and forth. The distance that the ball moves to the left and to the right of its resting position with respect to time is represented by the following graph.



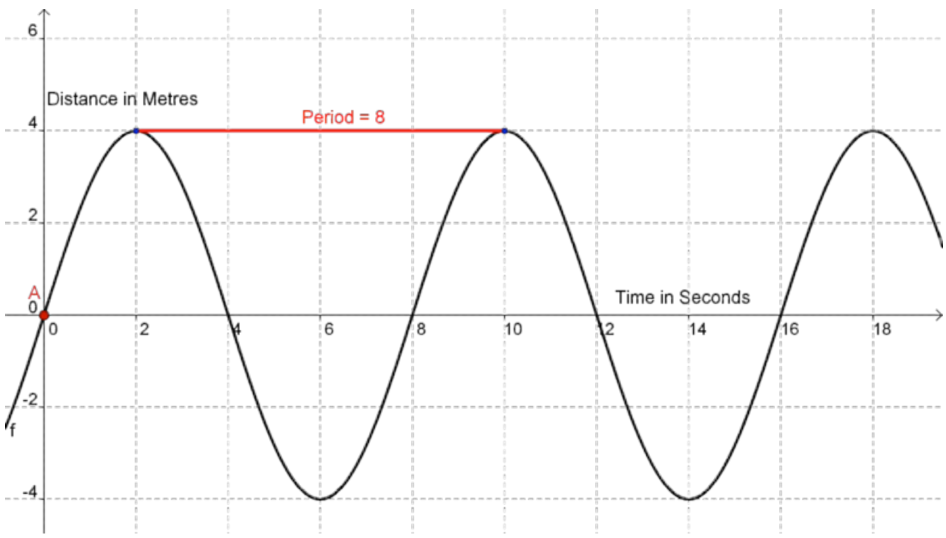
a. What is the period of the crane's motion. Explain your answer?

b. What is the equation of the horizontal midway line of the curve and what does it represent?

c. What is the amplitude of the crane's motion? Draw a diagram to represent what the amplitude represents in terms of the motion of the ball?

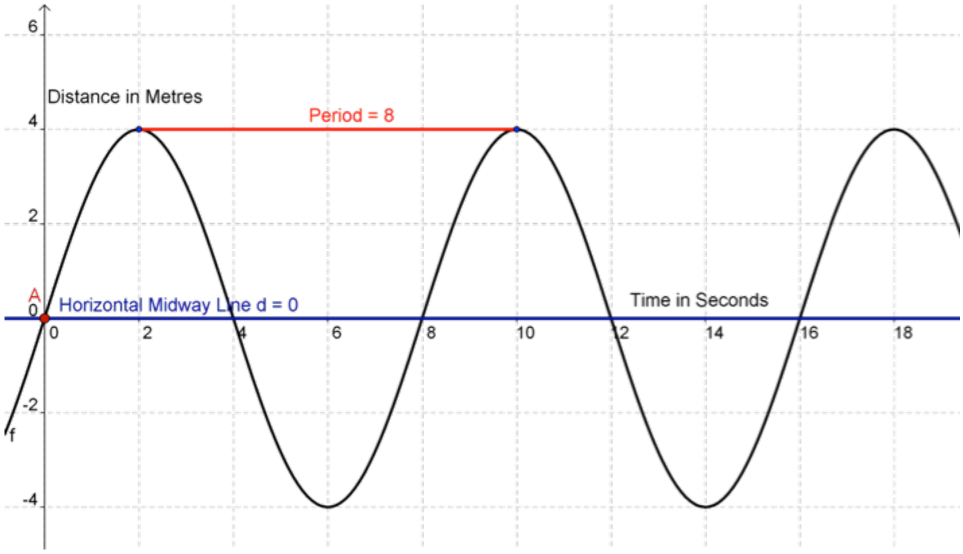
d. How many complete swings will the wrecking ball make in four minutes?

e. What does point A represent?

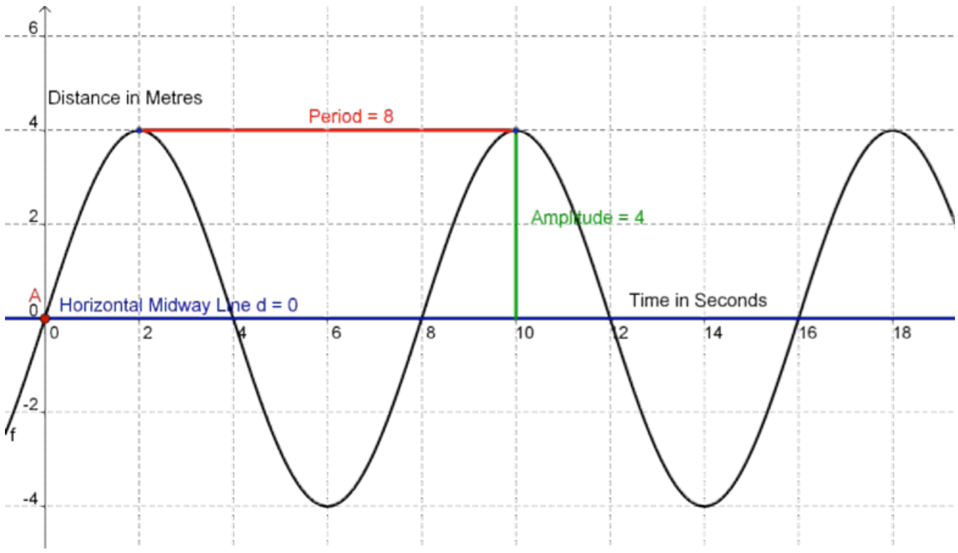


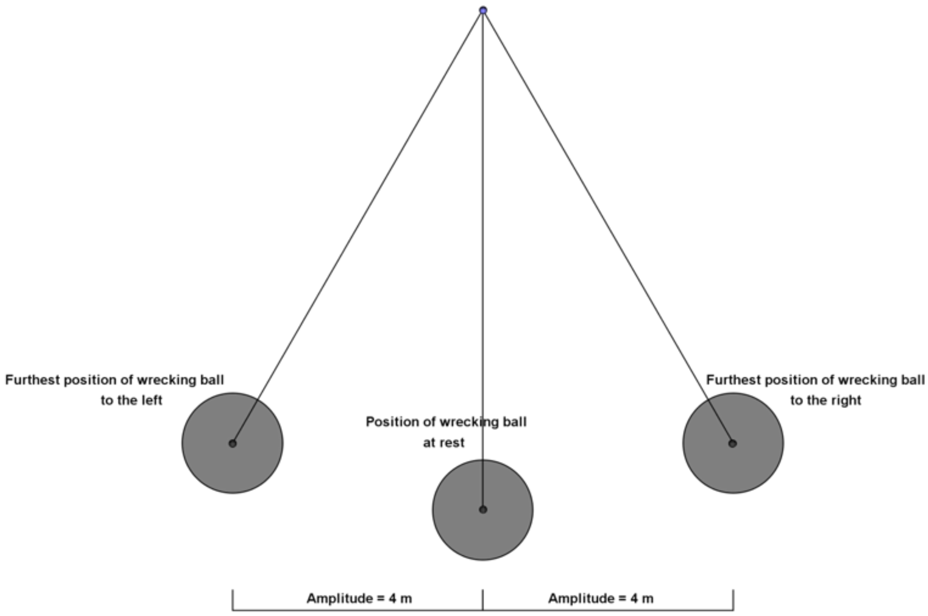
a. The period is 8 seconds. It represents the time taken for the wrecking ball to complete a full swing (i.e. from the extreme left to the extreme right and back to the extreme left again).

b. The equation of the horizontal midway line is $d = 0$. This represents the resting position of the ball.



c. The amplitude is 4 metres. This distance represents the maximum distance the wrecking ball swings to the left and right of its resting position.





d. 4 minutes = 240 seconds

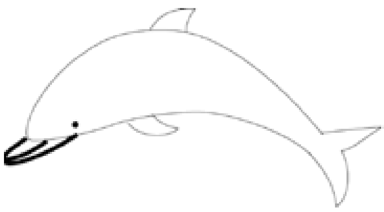
period = 8 seconds

number of swings = $\frac{240}{8} = 30$

e. Point A represents the position of the wrecking ball when collecting and recording of the data began

Question 2

As a dolphin swims along by the side of a cruise ship he jumps to a height of four metres above the water surface and dives to a depth of four meters below the surface. He does this in a regular motion (simple harmonic motion).



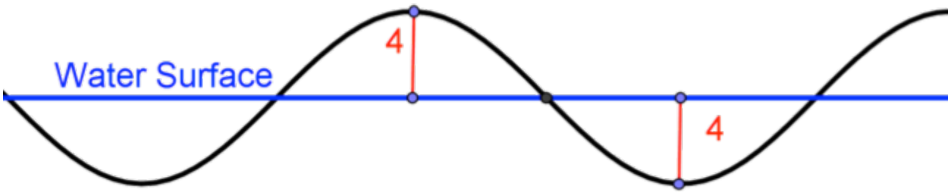
A passenger using a stopwatch and starting it just as the dolphin is at the surface determines that the dolphin completes one cycle every 8 seconds.

a Describe the displacement of the dolphin relative to the water surface using a sinusoid curve and sketch the graph of the dolphin's displacement relative to the water surface

b After three seconds how high above the surface is the dolphin?

c Is the dolphin above or below the surface of the water after 37 seconds?

a.



$a \pm b \sin cx$

$a \pm b \cos cx$

$a = \text{horizontal midway line}$

$a = 0$

$b = \text{aimplitude}$

$b = 4$

$period = \frac{2\pi}{c} = 8$

$c = \frac{\pi}{4}$

$shape = \text{function normal}$

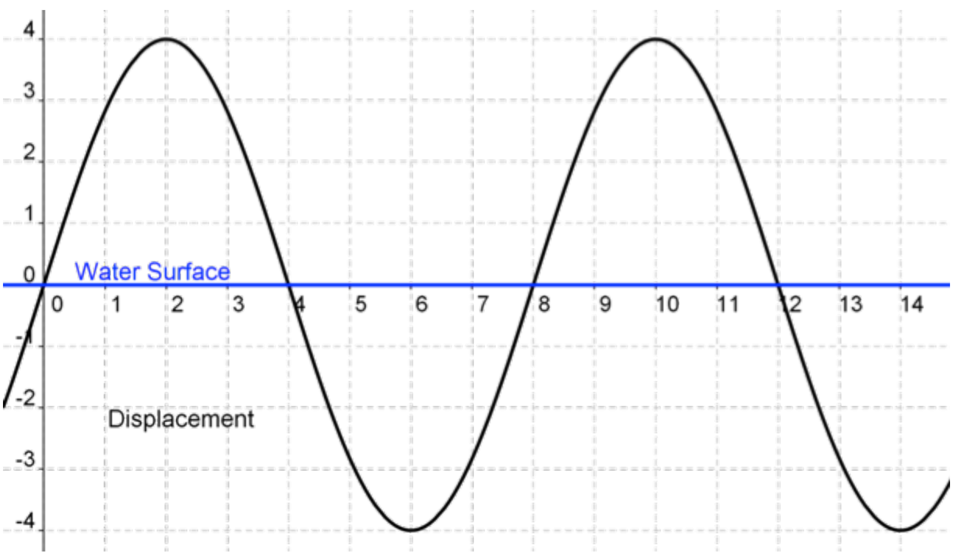
$h = 4\sin\frac{\pi}{4}t$

5 Point Sketch of a Sinusoid Curve

- **Maximum**
- **Minimum**
- **Three Intersections with Horizontal Midway Line**

These 5 points will be found at

$$0,\quad \frac{period}{4},\quad \frac{period}{2},\quad \frac{3period}{4},\quad period$$



b) After 3 seconds

$h = 4\sin\frac{\pi}{4}t$

h = 2.828

After 3 seconds the dolphin is 2.828m above the water

c) After 37 seconds

$h = 4sin\frac{\pi}{4}(37)$

h = -2.828

The dolphin is below the surface of the water

Question 3

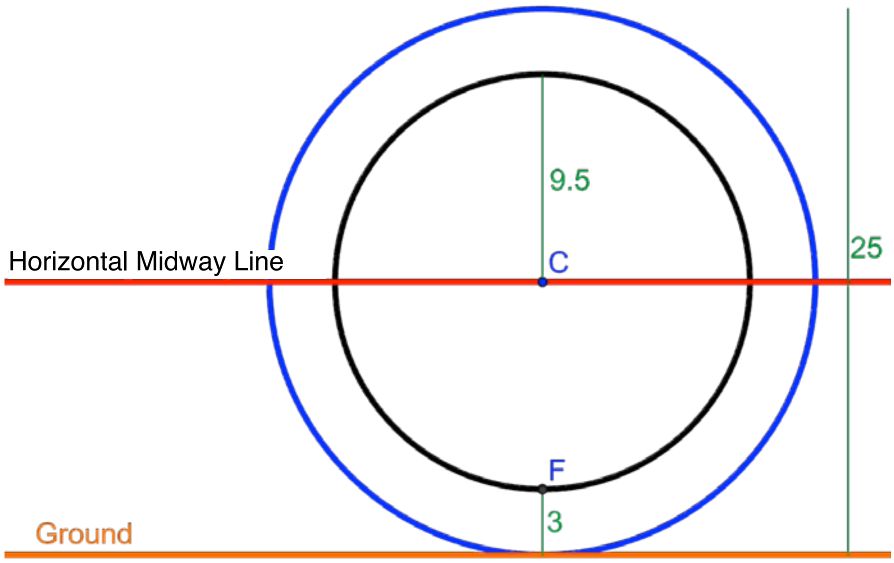
The inside rim of a bicycle wheel whose diameter is 25 inches, is 3 inches off the ground. An ant is sitting on the inside rim of the wheel at the point 3 inches off the ground. Sean starts riding the bicycle at a steady rate. The wheel makes one revolution every 1.6 seconds.

a Find the equation of a sinusoid curve which describes the motion of the ant and draw a graph of the function.

b What height in centimetres from the ground will the ant be 25 seconds into the trip given that 1 inch = 2.54 cm.?

c Within the first 10 seconds how many times will the ant be at its starting height?

a.



$a \pm b \sin cx$

$a \pm b \cos cx$

a = *horizontal midway line*

$a = 12.5$

b = *aimplitude*

$b = 9.5$

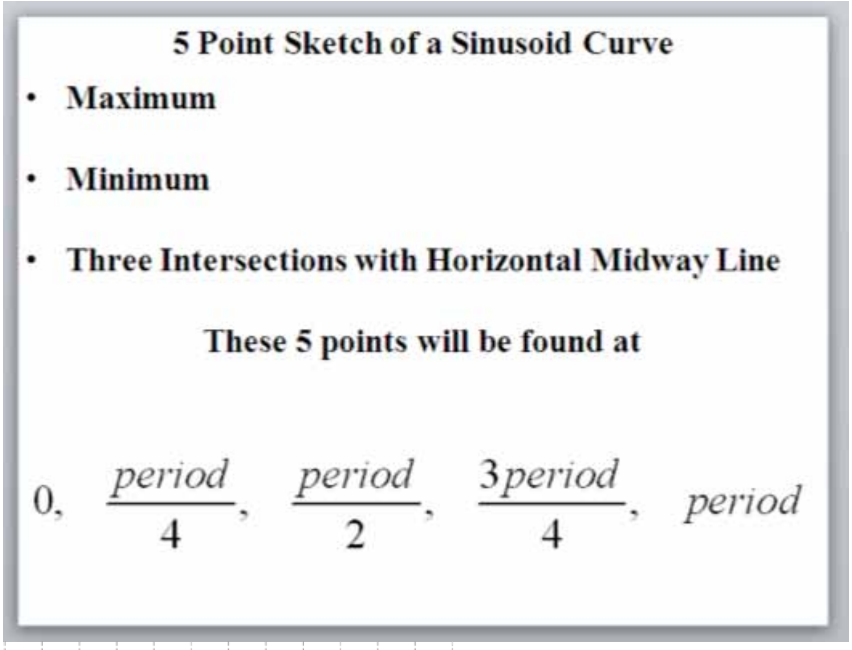
$period = \frac{2\pi}{c} = 1.6$

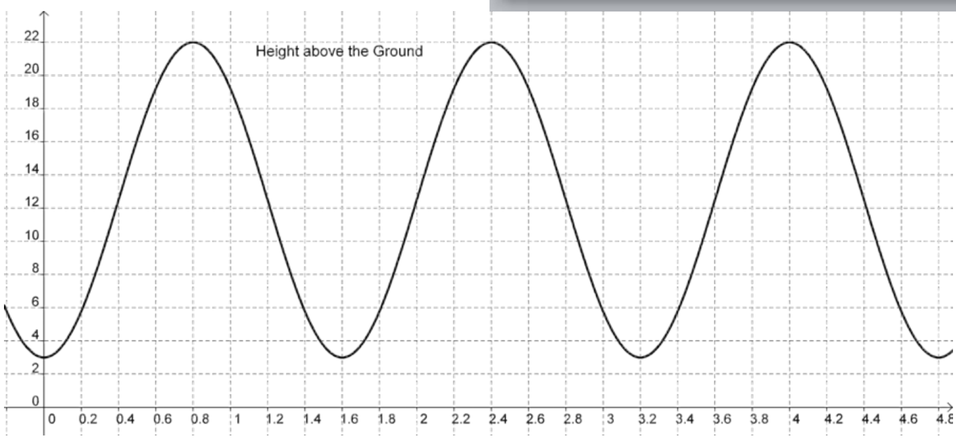
$c = 1.25\pi$

shape = cos function reflected

h = height off the ground

$h = 12.5\text{-}9.5cxos1.25\pi t$





b. 25 seconds into the trip

$t = 25$

$h = 12.5\text{-}9.5 \cos 1.2\pi (25)$

h = 19.218 inches = (19.218) (2.54) = 48.81372cm

Tar éis 25 soicind tá an seangán 48.81372cm ón talamh.

c. period = 1.6 seconds

in 10 seconds

$\frac{10}{1.6}$ cycle

6.25 cycle

at starting point: 7 times

Question 4

The number of people in thousands employed in a resort town is represented by the function

Take t = 0 as last day of January

Take t = 1 as last day of February

Take t = 2 as last day of March

a Draw a rough sketch showing the variation in the number of people employed in the town for one complete period.

b When is the maximum number of people employed and what is this maximum number?

c During which months of the year will the number of people employed be 4650 or greater?

d Does this model have any drawbacks and if yes identify one?

a.

$a - b \cos cx$

$3.8 - 1.7 \cos \frac{\pi}{6} x$

$a = \text{horizontal midway line} = 3.8$

$b = \text{aimplitude} = 1.7$

$$period = \frac{2\pi}{c} = \frac{2\pi}{\frac{\pi}{6}} = 12$$

shape = cos function reflected

5 Point Sketch of a Sinusoid Curve

- **Maximum**
- **Minimum**
- **Three Intersections with Horizontal Midway Line**

These 5 points will be found at

$$0, \quad \frac{period}{4}, \quad \frac{period}{2}, \quad \frac{3period}{4}, \quad period$$

a.

b. $3.8 - 1.7cos\frac{\pi}{6}x$

Range = [3.8 - 1.7, 3.8 + 1.7]

Range = [2.1, 5.5]

Maximum number employed = 5,500 people on the last day of July

c. $3.8 - 1.7cos\frac{\pi}{6}x = 4.65$

$-1.7cos\frac{\pi}{6}x = 4.65 - 3.8$

$-1.7cos\frac{\pi}{6}x = .85$

$1.7cos\frac{\pi}{6}x = -.85$

$\frac{\pi}{6}x = cos^{-1}\frac{-.85}{1.7}$

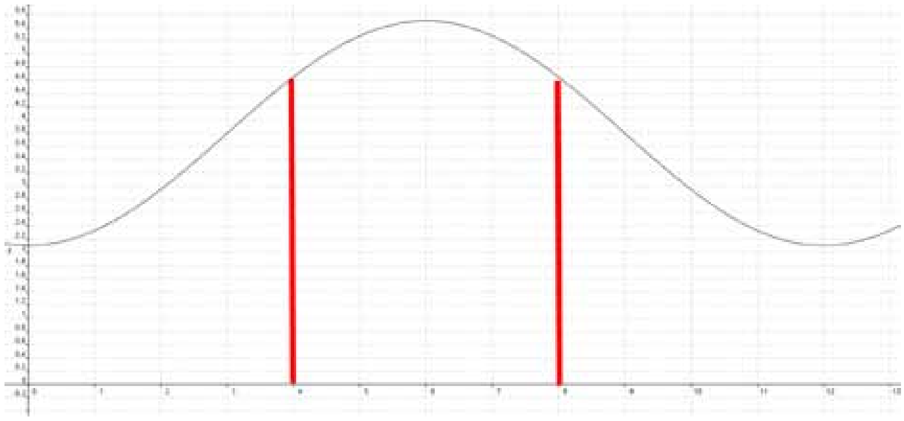
$\frac{\pi}{6}x = cos^{-1} - \frac{1}{2}$

$\frac{\pi}{6}x = \frac{2\pi}{3} \quad and \quad \frac{4\pi}{3}$

$x = \frac{\frac{2\pi}{3}}{\frac{\pi}{6}} \quad and \quad \frac{\frac{4\pi}{3}}{\frac{\pi}{6}}$

$x = 4 \quad and \quad 8$

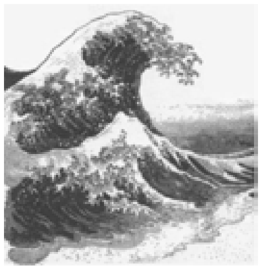
Numbers employed were 4650 or greater in the months of June, July, August, September.



d. A drawback of this model is that it assumes all months of the year have the same number of days.

Question 5

A tsunami (tidal wave) is a fast moving wave caused by an underwater earthquake. The water oscillates about its normal level, with equal amplitudes above and below this level The period is fifteen minutes. Suppose that a tsunami with an amplitude of ten metres approaches the pier at Honolulu, where the normal depth of water is nine metres. Assuming that the depth of water varies sinusoidally with time as the tsunami passes, predict the depth of the water at the following times after the tsunami first reaches the pier.

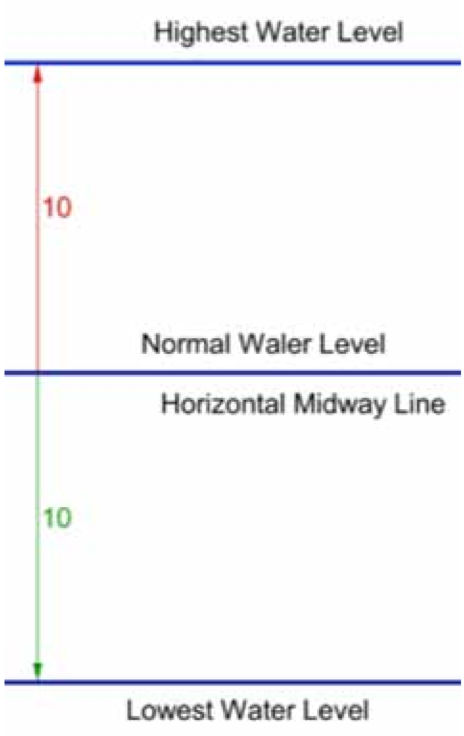


a Two minutes, four minutes and twelve minutes.

b According to your model what will be the minimum depth of the water?

c How do you interpret this answer in terms of what will happen in the real world?

a.



$a \pm b\sin cx$

$a \pm b\cos cx$

$a = \text{horizontal midway line}$

$a = 9$

$b = \text{amplitude}$

$b = 10$

$$period = \frac{2\pi}{c} = 15$$

$$c = \frac{2\pi}{15}$$

$shape = \text{sin function reflected}$

$h = \text{depth of water}$

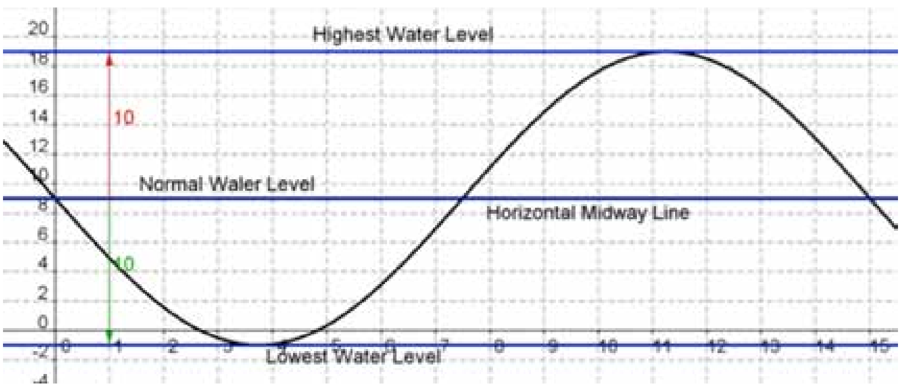
$$h = 9 - 10\sin\frac{2\pi}{15}t$$

5 Point Sketch of a Sinusoid Curve

- **Maximum**
- **Minimum**
- **Three Intersections with Horizontal Midway Line**

These 5 points will be found at

$$0,\ \frac{period}{4},\ \frac{period}{2},\ \frac{3period}{4},\ period$$



after 2 mins.

$$h = 9 - 10 sin\frac{2\pi}{15}(2)$$

$$h = 1.569\ m$$

after 4 mins.

$$h = 9 - 10 sin\frac{2\pi}{15}(4)$$

$$h = -0.945\ m$$

after 12 mins.

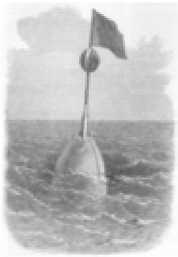
$$h = 9 - 10 sin\frac{2\pi}{15}(12)$$

$$h = 18.511\ m$$

b The minimum level of the water appears to be -1m.

c The water level can never be less than 0m. The water would drain away from the pier for a short period of time.

Question 6



A buoy in the ocean is bobbing up and down in harmonic motion.

At t = 0 seconds, the buoy is at its high point and returns to that high point every 8 seconds. The buoy moves a distance of 1.44 meters from its highest point to its lowest point.

a Using a sinusoidal function create a mathematical model to represent the depth of water under the buoy and sketch the curve.

b How high is the buoy at time = 29 seconds?

c Is it rising or falling at that time?

d How many times in 2 minutes will the buoy be at sea level?

a.

$a \pm bsincx$

$a \pm bcoscx$

$a =$ *horizontal midway line*

$a = n$

$b =$ *amplitude*

$b = .72$

$period \ = \dfrac{2\pi}{c} = 8$

$c = \dfrac{\pi}{4}$

shape = cos function normal

$h =$ *depth of water under bouy*

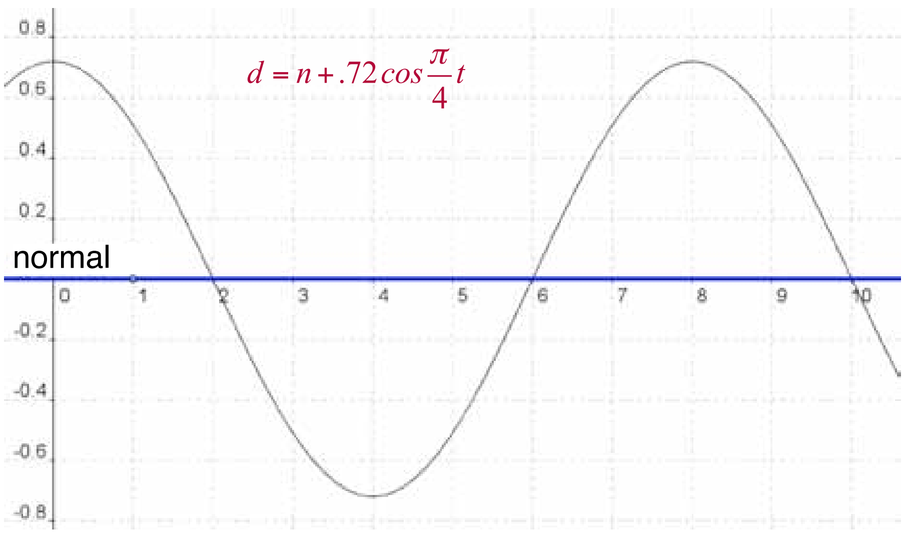
$d = n + .72\cos \ \dfrac{\pi}{4}t$

5 Point Sketch of a Sinusoid Curve

- **Maximum**
- **Minimum**
- **Three Intersections with Horizontal Midway Line**

These 5 points will be found at

$0, \ \dfrac{period}{4}, \ \dfrac{period}{2}, \ \dfrac{3period}{4}, \ period$



b. *after* 29 seconds

$d = n + .72\cos\frac{\pi}{4}(29)$

$d = (n - .509)m.$

i.e. after 29 seconds *the bouy is .509m. below normal depth*

c. *period* = 8

$\frac{29}{8} = 3.625$

this is more than half way into the fourth cycle

$\therefore$ *the bouy is rising after* 29 *seconds*

d. 2 minutes = 120 seconds

period = 8 seconds

number of cycles = $\frac{120}{8} = 15$

the bouy is at sea level 30 *times*

(2 *per cycle*)

Question 7

If the depth of water in a canal continually varies between a minimum 2m. below a specified buoy mark and a maximum of 2m above this mark over a 24-hour period.

a. Construct a formula involving a trigonometric function to describe this situation.

The road to an island close to the shore is sometimes covered with water. When the water rises to the level of the road, the road is closed. On a particular day, the water at high tide is 5 m above the mean sea level.

b Construct a formula involving a trigonometric function to model this situation if high tide occurs every 12 hours and the tide behaves in simple harmonic motion. Draw a sketch of the model.

c Find the height of the road above sea level if the road is closed for 3 hours on the day in question.

d If the road were raised so that it is impassable for only 2 hours 20 minutes, by how much was it raised?

a. $a \pm b\sin cx$

$a \pm b\cos cx$

$a = \text{horizontal midway line}$

$a = n$

$b = \text{amplitude}$

$b = 2$

$$period = \frac{2\pi}{c} = 24$$

$$c = \frac{\pi}{12}$$

if observations begin at high tide with tide begining to drop

$$h = n + 2\cos\frac{\pi}{12}t$$

if observations begin at low tide with tide rising

$$h = n - 2\cos\frac{\pi}{12}t$$

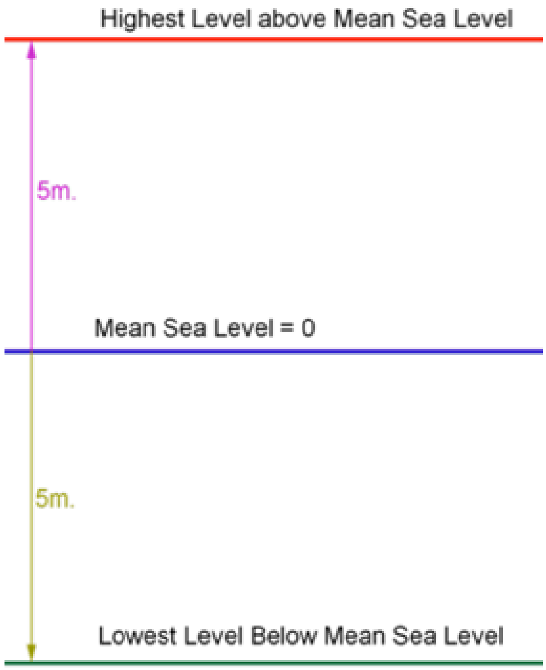
if observations begin at mean sea level with tide rising

$$h = n + \sin\frac{\pi}{12}t$$

if observations begin at mean sea level with tide dropping

$$h = n - \sin\frac{\pi}{12}t$$

b.



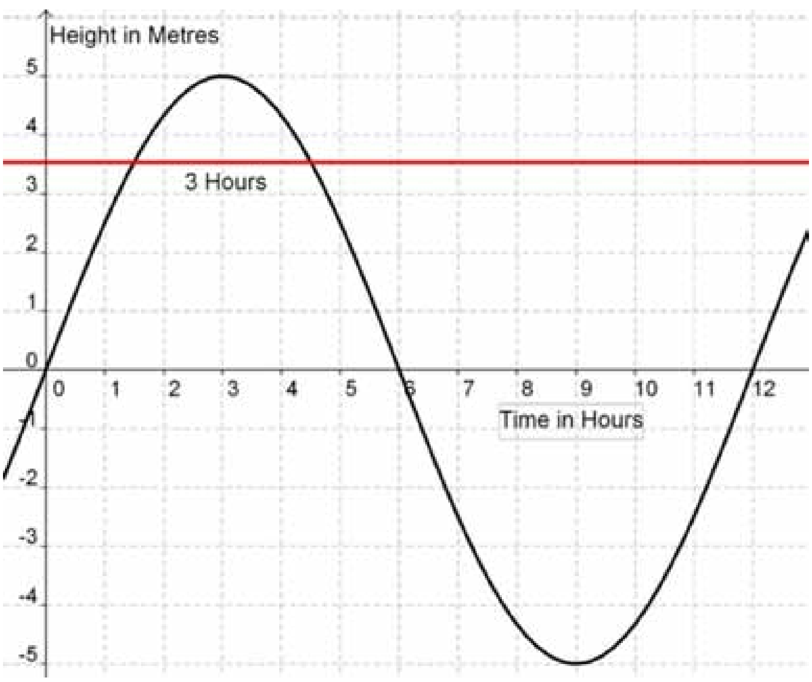
5 Point Sketch of a Sinusoid Curve

- **Maximum**
- **Minimum**
- **Three Intersections with Horizontal Midway Line**

These 5 points will be found at

$$0, \quad \frac{period}{4}, \quad \frac{period}{2}, \quad \frac{3period}{4}, \quad period$$

$$a \pm bsincx$$
$$a \pm bcoscx$$
$$a = horizontal\ midway\ line$$
$$a = 0$$
$$b = amplitude$$
$$b = 5$$
$$period = \frac{2\pi}{c} = 12$$
$$c = \frac{\pi}{6}$$
if observations begin at mean sea level with tide rising
$$h = 5sin\frac{\pi}{6}t$$



c. *Road closed for 3 hours*

a 1.5 *hour period before the* maximum *and*
a 1.5 *hour period after the* maximum

Max at 3 hours

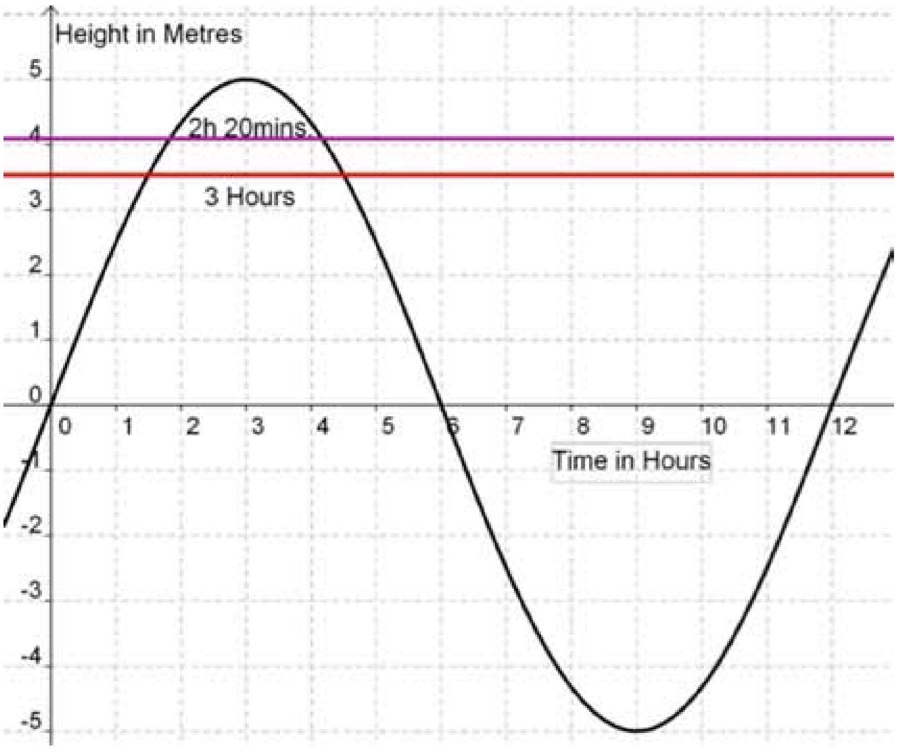
water reaches road after 1.5 hours

$$h = 5sin\frac{\pi}{6}(1.5)$$

$$h = 3.536\ m$$

The road is located 3.536m. above mean sea level

d.



Road closed for 2 hours 20 minutes.

a 1 hour 10 minute period before the maximum *and*
a 1 hour 10 minute period after the maximum

Max at 3 hours

water reaches road after 1 hour 50 minutes

$$1 \text{ hour } 50 \text{ mins.} = \frac{11}{6} \text{ hours}$$

$$h = 5 sin \frac{\pi}{6}\left(\frac{11}{6}\right)$$

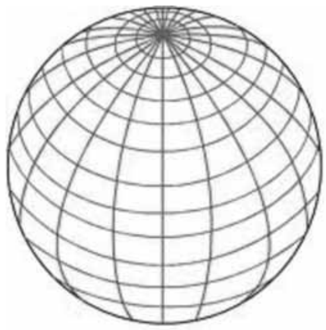
$$h = 4.096 \ m$$

Road raised by

$$4.096 - 3.536 = 0.56 \ m.$$

Question 8

At a certain latitude the number (d) of hours of daylight in each day is given by, $d = A + B \sin kt°$ where A and B are positive constants and t is the time in days after the spring equinox. Assuming the number of hours of daylight follows an annual cycle of 365 days;



a Find the value of k correct to three decimal places.

b If the shortest and longest days have 6 and 18 hours of daylight respectively state the values of A and B.

c Find in hours and minutes the amount of daylight on New Year's day which is 80 days before the spring equinox.

d A town at this latitude holds a fair twice a year on days that have exactly 10 hours of daylight. Find, in relation to the spring equinox, which two days these are.

a. $d = A + B\sin(kt)$

$$period = \frac{2\pi}{k} = 365 \ days$$

$$k = \frac{360}{365} = 0.986 \text{ (using degrees)}$$

b. $d = A + B\sin(kt)$

$y = A \text{ is horizontal midway line}$

$Max = 18$

$Min = 6$

$\Rightarrow A = 12$

$B = amplitude = 18 - 12 = 6$

$d = 12 + 6sin \cdot 986t$

c. $d = 12 + 6sin(\frac{360}{365}(-80))$

$d = 6.11 \text{ hours}$

amount of daylight on New Years Day = 6 hours 7 minutes

d. $12 + 6sin(\frac{360}{365})t = 10$

$6sin(\frac{360}{365})t = -2$

$sin(0.9863t) = -.333$

$0.9863t = sin^{-1} - .333$

$0.9863t = 360^\circ - 19.47^\circ = 340.53^\circ$

or

$0.9863t = 180 + 19.47 = 199.47^\circ$

$t = \frac{340.53}{0.9863} = 345.2\,6 \text{ days}$

i.e. approx. 21 days before Spring equinox

$\approx$ *March 1st /February 28th*

(Is the Spring equinox 21st or 22nd March?)

or

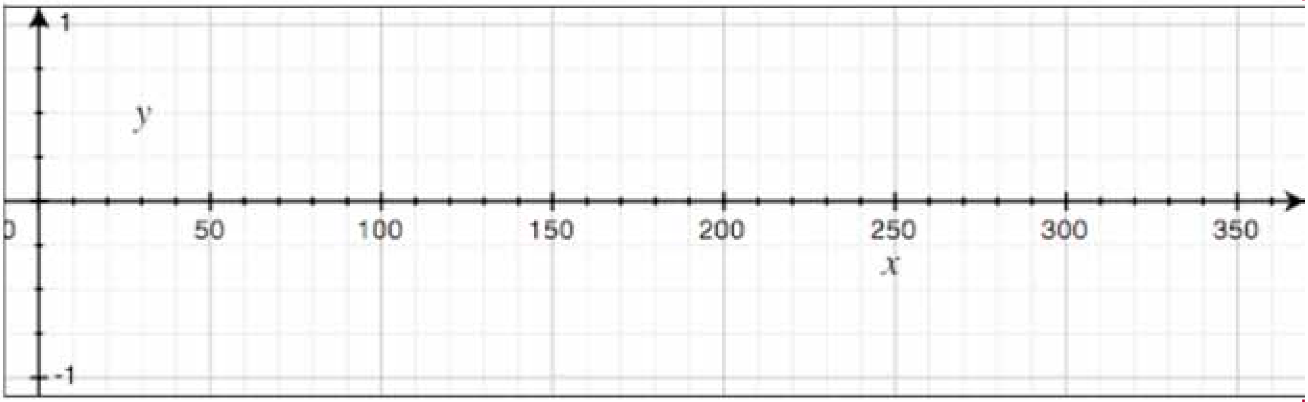
$t = \frac{199.47}{0.9863} = 202.24 \text{ days after the Spring equinox}$

$\approx$ *October 9th*

Student Activity Sheet 1 Graphs of $sinx$ and $cosx$

Complete the table below and then draw the graph of $sinx$ and $cosx$

x	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
$y = \sin x$																	
$y = \cos x$																	



x	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
$y = \sin x$																	
$y = \cos x$																	

What is the horizontal midway line of each graph?

$sin\ x$____________________ $cos\ x$____________________

What is the maximum of each graph?

$sin\ x$____________________ $cos\ x$____________________

What is the minimum of each graph?

$sin\ x$____________________ $cos\ x$____________________

What is the range of each graph?

$sin\ x$____________________ $cos\ x$____________________

The amplitude of a graph is the greatest height the graph is above the horizontal midway line. What is the amplitude of each graph?

$sin\ x$____________________ $cos\ x$____________________

When a graph repeats itself after a particular interval the interval is known as the period. What is the period of each graph?

$sin\ x$____________________ $cos\ x$____________________

What difference if any is there between the curves of $sin\ x$ and $cos\ x$?

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Card Set A1 Trigonometric Functions $g(x) = b\sin x$

A1 $g(x) = -3 \sin x$	A2 $g(x) = 2 \sin x$
A3 $g(x) = \frac{1}{2} \sin x$	A4 $g(x) = 5 \sin x$
A5 $g(x) = -4 \sin x$	A6 $g(x) = \frac{2}{3} \sin x$
A7 $g(x) = -\frac{1}{4} \sin x$	A8 $g(x) = \sin x$
A9 $g(x) = -\sin x$	A10 $g(x) = -\frac{1}{3} \sin x$

Card Set A2 Trigonometric Functions $g(x) = b\cos x$

A11 $g(x) = -2 \cos x$	**A12** $g(x) = \cos x$
A13 $g(x) = \frac{1}{4} \cos x$	**A14** $g(x) = \cos x$
A15 $g(x) = -5 \cos x$	**A16** $g(x) = \frac{1}{2} \cos x$
A17 $g(x) = -\frac{3}{2} \cos x$	**A18** $g(x) = 6\cos x$
A19 $g(x) = -\cos x$	**A20** $g(x) = -\frac{1}{5} \cos x$

Project Maths Tionscadal Mata Development Team

Student Activity Sheet 2a: Mathematical Language / Properties of Trigonometric Graphs

Card	Curve	Amplitude	Range	Period	Vertical Transformation (3)	Vertical Shape (3)	Horizontal Shape (3)
A1	$g(x) = -3 \sin x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A2	$g(x) = 2 \sin x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A3	$g(x) = \frac{1}{2} \sin x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A4	$g(x) = 5 \sin x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A5	$g(x) = -4 \sin x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A6	$g(x) = \frac{2}{3} \sin x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A7	$g(x) = -\frac{1}{4} \sin x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A8	$g(x) = \sin x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A9	$g(x) = -\sin x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A10	$g(x) = -\frac{1}{3} \sin x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink

From your observations above what is the effect of b on the curve?

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Student Activity Sheet 2b: Mathematical Language / Properties of Trigonometric Graphs

Card	Curve	Amplitude	Range	Period	Vertical Transforma-tion (3)	Vertical Shape (3)	Horizontal Shape (3)
A1	$g(x) = -2 \cos x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A2	$g(x) = \cos x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A3	$g(x) = \frac{1}{4} \cos x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A4	$g(x) = \cos x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A5	$g(x) = -5 \cos x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A6	$g(x) = \frac{1}{2} \cos x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A7	$g(x) = -\frac{3}{2} \cos x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A8	$g(x) = 6\cos x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A9	$g(x) = -\cos x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
A10	$g(x) = -\frac{1}{3} \cos x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink

From your observations above what is the effect of b on the curve?

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Student Activity Sheet 2c: Mathematical Language / Properties of Trigonometric Graphs

Card	Curve	Amplitude	Range	Period	Vertical Transformation (3)	Vertical Shape (3)	Horizontal Shape (3)
					Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
					Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
					Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
					Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
					Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
					Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
					Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
					Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
					Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
					Normal	Stretch	Stretch
					Reflected	Shrink	Shrink

From your observations above what is the effect of b on the curve?

Card Set B: Trigonometric Functions Important features

B1 $g(x) = \cos \tfrac{1}{2}x$	**B2** $g(x) = \sin 2x$
B3 $g(x) = \sin 3x$	**B4** $g(x) = \cos 4x$
B5 $g(x) = \cos 3x$	**B6** $g(x) = \sin \tfrac{4}{5}x$
B7 $g(x) = \sin 4x$	**B8** $g(x) = \cos \tfrac{2}{3}x$
B9 $g(x) = \cos 2x$	**B10** $g(x) = \sin \tfrac{1}{4}x$

Student Activity Sheet 3: Mathematical Language / Properties of Trigonometric Graphs

Card	Curve	Amplitude	Range	Period	Vertical Transformation (3)	Vertical Shape (3)	Horizontal Shape (3)
B1	$g(x) = \cos \frac{1}{2}x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
B2	$g(x) = \sin 2x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
B3	$g(x) = \sin 3x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
B4	$g(x) = \cos 4x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
B5	$g(x) = \cos 3x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
B6	$g(x) = \sin \frac{4}{5}x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
B7	$g(x) = \sin 4x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
B8	$g(x) = \cos \frac{2}{3}x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
B9	$g(x) = \cos 2x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink
B10	$g(x) = \sin \frac{1}{4}x$				Normal	Stretch	Stretch
					Reflected	Shrink	Shrink

From your observations above what is the effect of c on the curve?

Card Set C

$g\,(x) = 3\sin \tfrac{1}{2}x$	$g\,(x) = 3\sin \tfrac{1}{2}x$
$g\,(x) = 3\sin \tfrac{1}{2}x$	$g\,(x) = -5\cos 3x$
$g\,(x) = -5\cos 3x$	$g\,(x) = -5\cos 3x$
$g\,(x) = \tfrac{1}{2}\sin 4x$	$g\,(x) = \tfrac{1}{2}\sin 4x$
$g\,(x) = \tfrac{1}{2}\sin 4x$	$g\,(x) = -\tfrac{2}{3}\cos \tfrac{1}{4}x$
$g\,(x) = -\tfrac{2}{3}\cos \tfrac{1}{4}x$	$g\,(x) = -\tfrac{2}{3}\cos \tfrac{1}{4}x$

Card Set C (continued)

$g(x) = 5\cos 2x$	$g(x) = 5\cos 2x$
$g(x) = 5\cos 2x$	$g(x) = \frac{1}{2}\sin \frac{2}{3}x$
$g(x) = \frac{1}{2}\sin \frac{2}{3}x$	$g(x) = \frac{1}{2}\sin \frac{2}{3}x$
$g(x) = -4\cos \frac{1}{3}x$	$g(x) = -4\cos \frac{1}{3}x$
$g(x) = -4\cos \frac{1}{3}x$	$g(x) = -\frac{2}{5}\cos 6x$
$g(x) = -\frac{2}{5}\cos 6x$	$g(x) = -\frac{2}{5}\cos 6x$

Card Set C (continued)

$g\,(x) = 5\cos 2x$	$g\,(x) = \frac{1}{2}\sin \frac{2}{3}x$
$g\,(x) = \frac{1}{2}\sin \frac{2}{3}x$	$g\,(x) = \frac{1}{2}\sin \frac{2}{3}x$
$g\,(x) = -4\cos \frac{1}{3}x$	$g\,(x) = -4\cos \frac{1}{3}x$

Student Activity Sheet 4 Mathematical Language / Properties of Trigonometric Graphs

Vertical Transformation	
Normal	Reflected
Vertical Shape	
Stretch	Shrink
Horizontal Shape	
Shrink	Stretch

Student Activity Sheet 5 Mathematical Language / Properties of Trigonometric Graphs

	Curve	Amplitude	Range	Period
	$g(x) = 3sin\frac{1}{2}x$			
	$g(x) = -5cos3x$			
	$g(x) = \frac{1}{2}sin4x$			
	$g(x) = -\frac{2}{3}cos\frac{1}{4}x$			
	$g(x) = 5cos2x$			
	$g(x) = \frac{1}{2}sin\frac{2}{3}x$			
	$g(x) = -4cos\frac{1}{3}x$			
	$g(x) =$	6		$2\pi/3$
	$g(x) =$		[-4, 4]	3π
	$g(x) =$	2		$\pi/2$
	$g(x) =$		[-½, ½]	6π
	$g(x) =$	⅕		2π
	$g(x) =$		[-2, 2]	π
	$g(x) =$	3	[-3, 3]	
	$g(x) =$			4π
	$g(x) =$	¼		

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Student Activity Sheet 6: Transformation of Trigonometric Graphs

	Curve	Effect of the value a in $f(x) = a + \sin x$ and $f(x) = a + \cos x$		
		Horizontal Midway Line	Amplitude	Range
	$h(x) = 1 + \sin x$			
	$h(x) = 2 + \sin x$			
	$h(x) = 3 + \sin x$			
	$h(x) = -1 + \sin x$			
	$h(x) = -2 + \sin x$			
	$h(x) = -3 + \sin x$			
	$h(x) = 1 + \cos x$			
	$h(x) = 2 + \cos x$			
	$h(x) = 3 + \cos x$			
	$h(x) = -1 + \cos x$			
	$h(x) = -2 + \cos x$			
	$h(x) = -3 + \cos x$			

From your observations above what is the effect of a on the curve?

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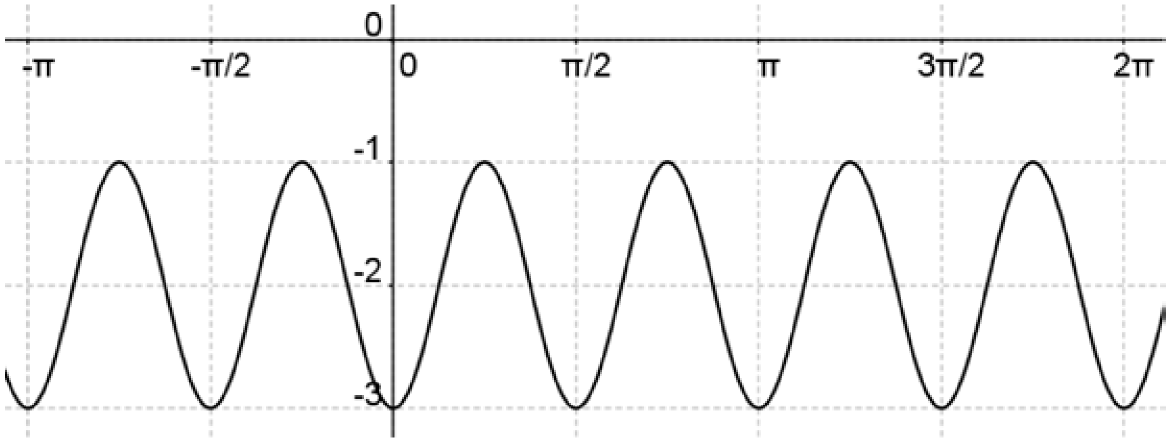
Student Activity Sheet 7: Mathematical Language / Properties of Trigonometric Graphs

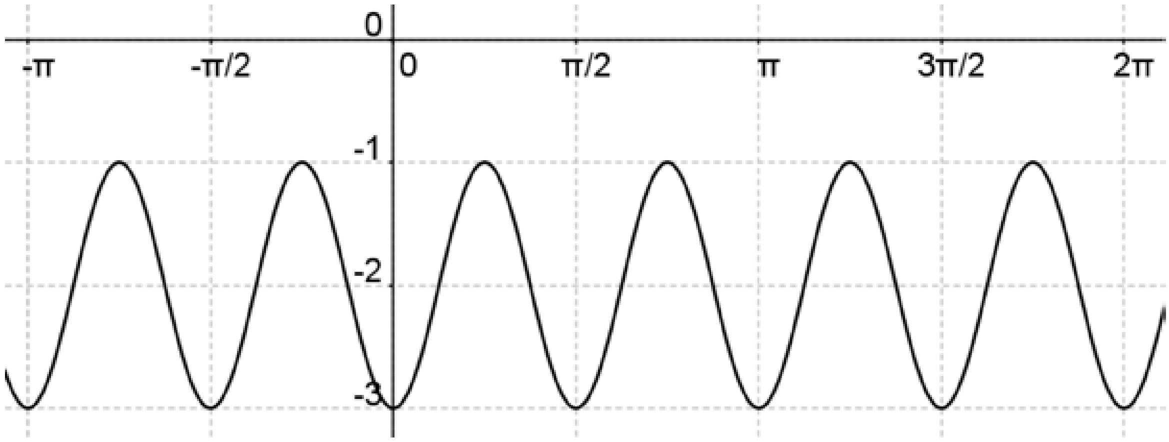
	Curve	Vertical Translation	Vertical Shape	Amplitude	Range	Period
a	$g(x) = 2 + 3\ sin\ 4$					
b	$g(x) = 3 - 4\ cos\ 2x$					
c	$g(x) = -4 - 2\ sin\ 3$					
d	$g(x) = -1 + 5\ sin\ 2$					
e	$g(x) = 5 - cos\ {}^{x}\!/_{2}$					
f	$g(x) = 3\ sin\ 3x$					
g	$g(x) = 1 + cos\ 4x$					
h	$g(x) = \frac{1}{2} + 5\ cos\ x$					
i	$g(x) = 3 - 3\ sin3x$					
j	$g(x) = \frac{1}{4} - \frac{1}{2}\ sin\ \frac{1}{3}x$					
k	$g(x) = 2 + 3\ sin\ 4x$					
l	$g(x) = 1 + 3\ cosx$					
m	$g(x) = 4 + 3\ cos\ \frac{1}{2}x$					
n	$g(x) = 5\ sinx + 1$					
o	$g(x) = -1\ -sinx\ \frac{2}{3}x$					

Card Set D: Trigonometric Graphs

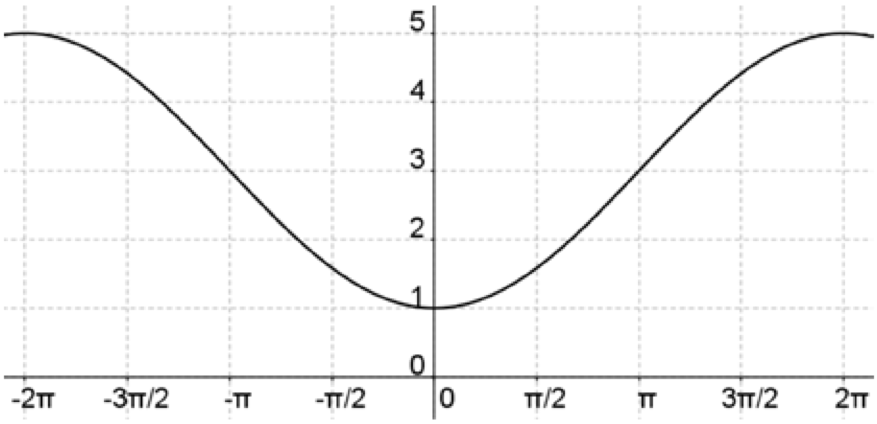
D1



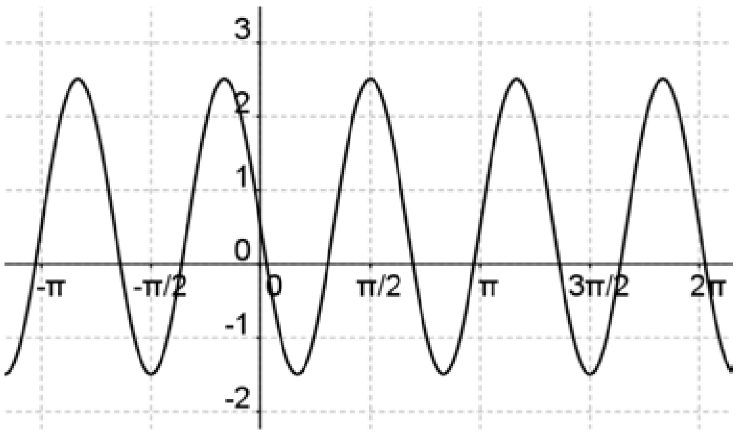
D2



D3

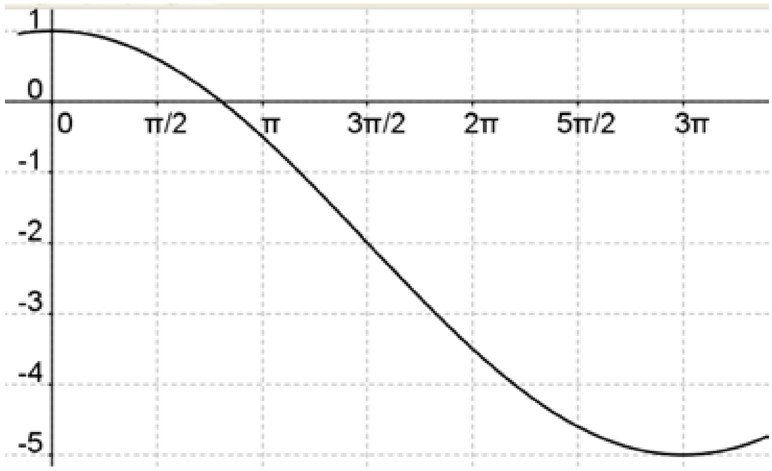


D4

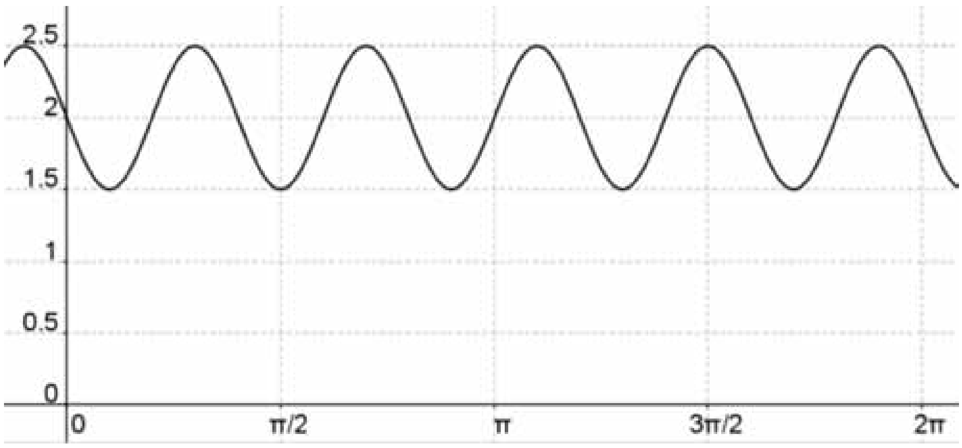


Card Set D: Trigonometric Graphs (continued)

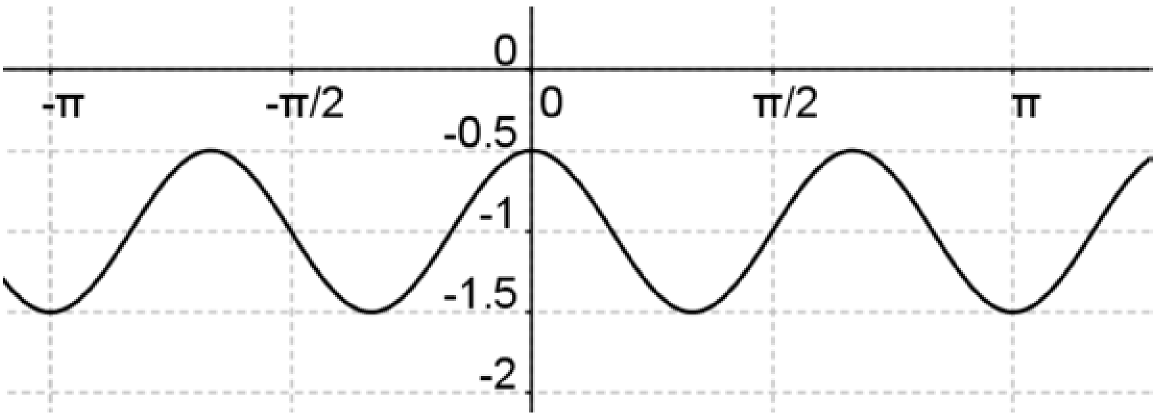
D5



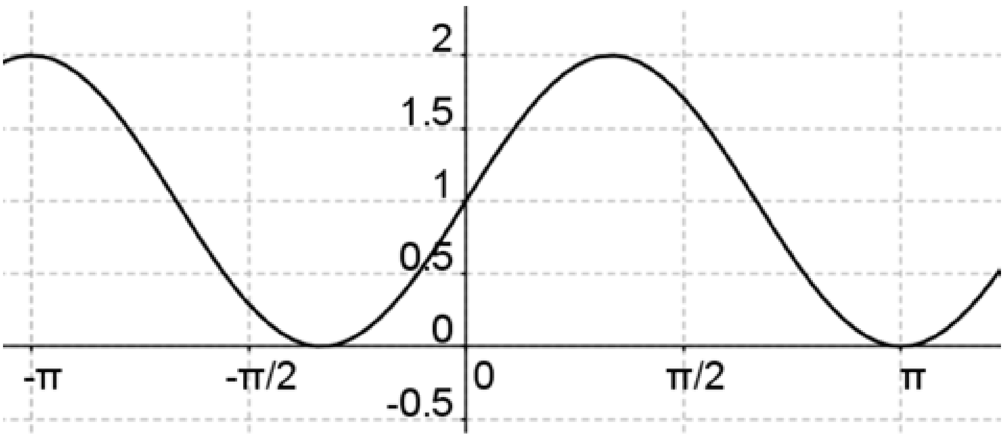
D6



D7



D8



Student Activity Sheet 8: Trigonometric Graphs

Graph	Equation Horizontal Midway line	Function Type And Vertical Shape	Amplitude	Range	Number of cycles in 2π	Period
D1						
D2						
D3						
D4						
D5						
D6						
D7						
D8						

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Card Set E: Trigonometric Graphs

E1 $f(x) = \frac{1}{2} - 2\sin 3x$	E2 $f(x) = 1 + \sin \frac{3}{2}x$
E3 $f(x) = -2 + 3\cos \frac{1}{3}x$	E4 $f(x) = 1 + 2\sin 3x$
E5 $f(x) = 2 - \frac{1}{2}\sin 5x$	E6 $f(x) = -1 + \frac{1}{2}\cos 3x$
E7 $f(x) = 3 - 2\cos \frac{1}{2}x$	E8 $f(x) = -2 - \cos 4x$

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Card Set F

F1 $k(x) = 2 + 3\sin 4x$	F2 $k(x) = 5 - 4\cos 2x$
F3 $k(x) = 1 + 3\cos \tfrac{1}{2} x$	F4 $k(x) = \tfrac{1}{2} - \sin 3x$
F5 $k(x) = 3 + 2\cos \tfrac{1}{4} x$	F6 $k(x) = 1 - \tfrac{1}{2}\sin x$
F7 $k(x) = 2 + \tfrac{1}{4}\cos 3x$	F8 $k(x) = 4\sin 2x$
F9 $k(x) = -1 - 2\sin 3x$	F10 $k(x) = -2 + 3\cos 4x$