Teaching & Learning Plans

The Correlation Coefficient

Leaving Certificate Syllabus

Project Maths
Tionscadal Mata
Development Team
The Teaching & Learning Plans are structured as follows:

**Aims** outline what the lesson, or series of lessons, hopes to achieve.

**Prior Knowledge** points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

**Learning Outcomes** outline what a student will be able to do, know and understand having completed the topic.

**Relationship to Syllabus** refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

**Resources Required** lists the resources which will be needed in the teaching and learning of a particular topic.

**Introducing the topic** (in some plans only) outlines an approach to introducing the topic.

**Lesson Interaction** is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input:** This section focuses on possible lines of inquiry and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities – Possible Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher’s Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.

iv. **Assessing the Learning:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

**Student Activities** linked to the lesson(s) are provided at the end of each plan.
Teaching & Learning Plan: The Correlation Coefficient

Aims
• To familiarise students with scatter plots and the concept of correlation

Prior Knowledge
• Plotting points on the x and y axis
• Finding the slope between two points
• Finding the equation of a line
• Linear Relationships

Learning Outcomes
As a result of studying this topic, students will be able to:
• use scatter plots to determine the relationship between variables (OL)
• recognise that correlation is a value from -1 to +1 (OL)
• match correlation coefficients to appropriate scatter plots (OL)
• understand that correlation does not imply causality (OL)
• draw the line of best fit (HL)
• use the line of best fit to make predictions (HL)
• calculate the correlation coefficient by calculator (HL)

Catering for Learner Diversity
In class, the needs of all students whatever their ability level are equally important. In daily classroom teaching, teachers can cater for different abilities by providing students with different activities and assignments graded according to levels of difficulty so that students can work on exercises that match their progress in learning. Some students may only be able to engage in activities which are relatively straightforward, while others may be able to engage in more open-ended and challenging activities. Selecting and assigning activities appropriate to a student’s ability will cultivate and sustain his/her interest in learning.

In interacting with the whole class, teachers can employ effective and inclusive questioning. Questions can be pitched at different levels and can move from basic questioning to ones which are of a higher order nature. In this T & L Plan, some students may be required to answer a question such as: Estimate the correlation coefficient from a particular scatter plot? A more challenging question can be reserved for others: Interpret the correlation coefficient in the context of the variables? Besides whole-class teaching, teachers can consider different grouping strategies – such as group and pair work – to encourage student interaction, help students to verbalise
their mathematical understanding and help to build student self-confidence and mathematical understanding. During the activities in this T&L students are encouraged to work in pairs and discuss the context.

**Relationship to Leaving Certificate Syllabus**

<table>
<thead>
<tr>
<th>Students learn about</th>
<th>Students working at OL should be able to</th>
<th>In addition, students working at HL should be able to</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6 Representing data graphically and numerically</td>
<td><strong>Graphical</strong>&lt;br&gt;• determine the relationship between variables using scatterplots&lt;br&gt;• recognise that correlation is a value from -1 to +1 and that it measures the extent of the linear relationship between two variables&lt;br&gt;• match correlation coefficient values to appropriate scatter plots&lt;br&gt;• understand that correlation does not imply causality</td>
<td><strong>Graphical</strong>&lt;br&gt;• draw the line of best fit by eye&lt;br&gt;• make predictions based on the line of best fit&lt;br&gt;• calculate the correlation coefficient by calculator <strong>Numerical</strong>&lt;br&gt;• recognise the effect of outliers&lt;br&gt;• use percentiles to assign relative standing</td>
</tr>
</tbody>
</table>

**Teacher Notes**

There are many situations where we may want to investigate the relationship between variables e.g. the capacity of an engine and the fuel efficiency of an engine, the sales of ice cream and the temperature and the number of hours studied by a student and their subsequent performance in an exam etc.

The correlation coefficient is a mathematical way of measuring the linear relationship between variables.

**Some terms:**

**Bivariate data:** Bivariate data is a fancy way to say, ‘two-variable data.’ The easiest way to visualize bivariate data is through a scatter plot

**Scatter plots:** Scatter plots show the relationships between two variables measured on the same cases

**Correlation:** The correlation coefficient is a measure of the direction and strength of a linear relationship

**Outlier(s):** One or more points that do not fit the overall pattern as seen in the scatter plot
**Line of best fit:** A line on a scatter plot which can be drawn near the points to more clearly show the trend between two sets of data.

**Causation:** Correlation does not imply causation i.e. a high correlation does not automatically imply that changes in one variable cause the changes in the other variable.

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**Note**

Information and Communication Technologies (ICT) are used whenever and wherever appropriate to help support student learning. In this Teaching & Learning Plan the CD icon appears at the corresponding position of the content to indicate that an interactive ICT module is available on the Project Maths Student's CD.
### Lesson Interaction

<table>
<thead>
<tr>
<th>Student Learning Tasks: Teacher Input</th>
<th>Student Activities: Possible Responses</th>
<th>Teacher’s Support and Actions</th>
<th>Assessing the Learning</th>
</tr>
</thead>
</table>
| » Do you think there is a link between the size of an engine and the fuel efficiency of a particular car? | • Engine size influences fuel efficiency.  
• Larger engines use more fuel.  
• The bigger the engine size the more fuel is needed.  
• The engine size does not have a big influence on the amount of fuel a car uses. | » Get students, in pairs, to write down a sentence about fuel efficiency and engine size.  
» Write students’ statements on the board. | » Are students interacting with each other?  
» Can students verbalise their reasoning? |
| » Why is fuel efficiency important? | | | |
| » What does your sentence mean? | | | |
| » Would fuel efficiency influence your choice of car? | | | |
| » Can you think of other pairs of variables that may be linked? | • Ice cream sales and temperature.  
• Hours spent studying and marks in exams.  
• The amount of hours you work and the amount of money you earn. | » You may have to lead students into answering this question e.g. Is there a link between ice cream sales and temperature? | » Are students able to provide examples of pairs of variable?  
» Are their pairs of variables reasonable to try and link together?  
» Are students able to make clear statements about the potential links between variables? |
<p>| » Why do you think there is a link between the variables you have chosen? | | | |</p>
<table>
<thead>
<tr>
<th>Student Learning Tasks: Teacher Input</th>
<th>Student Activities: Possible Responses</th>
<th>Teacher’s Support and Actions</th>
<th>Assessing the Learning</th>
</tr>
</thead>
</table>
| » We are now going to look at some recent information about engine sizes and their fuel efficiencies. | • Students report back to the rest of the class on **Student Activity 1**.  
• Bigger engines are less efficient.  
• Some larger cars use lots of fuel.  
• Smaller engines use less fuel. | » Distribute **Student Activity 1** and ask students to display this data on a pair of axes. | » Are students interacting and cooperating with each other to find the solutions to **Student Activity 1**?  
» Are students able to accurately plot the points?  
» Can students explain their answers to parts (ii) and (iii) of **Student Activity 1**? |
| » Working in pairs, complete **Student Activity 1**. | | » Circulate around the room to see what students are doing and provide help if necessary. | |
| » How strong do you think the relationship is? | | Note: State that the correlation coefficient has to be calculated at Higher Level only. | |

**Note:** Discuss the above before introducing the idea of Correlation Coefficient.

» It seems that there is a relationship between engine size and fuel efficiency. (How sure are students that there is a relationship?)
### Teaching & Learning Plan: The Correlation Coefficient

<table>
<thead>
<tr>
<th>Student Learning Tasks: Teacher Input</th>
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<th>Teacher’s Support and Actions</th>
<th>Assessing the Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>» How could you measure the strength of the relationship?</td>
<td></td>
<td>Note: There is no universally accepted criterion for applying the adjectives “strong”, “moderate” and “weak” to correlation coefficients. State Examinations Commission (January 2010). Report on the Trialling of Leaving Certificate Sample Papers for Phase 1 Project Maths, 33. <a href="http://www.examinations.ie/schools/Report_on_Trial_final.pdf">http://www.examinations.ie/schools/Report_on_Trial_final.pdf</a> [accessed September 2011]. The following is a guide:</td>
<td></td>
</tr>
<tr>
<td>» To check mathematically if there is a relationship we calculate or are given the correlation coefficient.</td>
<td></td>
<td>» Range of values of correlation coefficient $-1 \leq r \leq 1$</td>
<td></td>
</tr>
<tr>
<td>» The correlation coefficient ($r$) measures the linear relationship between variables. The coefficient lies between 1 and -1 and if the coefficient is greater 0.6 than we say there is a strong positive correlation and if the coefficient is smaller -0.6 than we say there is a strong negative correlation.</td>
<td></td>
<td>» Strong positive correlation $0.6 \leq r \leq 1$</td>
<td></td>
</tr>
<tr>
<td>» Suggestion: Ask students to summarise what they have learned.</td>
<td>• Students write a summary in their own words.</td>
<td>» Weak positive correlation $0 &lt; r &lt; 0.6$</td>
<td></td>
</tr>
</tbody>
</table>

### Assessing the Learning

- **Note**: Support this work graphically showing scatter plots having the relevant correlation coefficient.
### Student Learning Tasks: Teacher Input

<table>
<thead>
<tr>
<th>Question</th>
<th>Student Responses</th>
<th>Teacher’s Support and Actions</th>
<th>Assessing the Learning</th>
</tr>
</thead>
<tbody>
<tr>
<td>What type of correlation do we have if the correlation coefficient is -0.43?</td>
<td>• Weak negative.</td>
<td>Get the students to draw scatter plots that might have these values for $r$.</td>
<td>Are students able to relate the decimals to the scale?</td>
</tr>
<tr>
<td>What type of correlation do we have if the correlation coefficient is 0.74?</td>
<td>• Strong positive.</td>
<td></td>
<td>Do students understand decimals in the range $-1 \leq r \leq 1$?</td>
</tr>
<tr>
<td>What type of correlation do we have if the correlation coefficient is 0.03?</td>
<td>• Weak positive.</td>
<td></td>
<td>Are students able to link the $r$ values to the correct mathematical statement e.g. strong positive etc.?</td>
</tr>
<tr>
<td>What type of correlation do we have if the correlation coefficient is -0.81?</td>
<td>• Strong negative.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What type of correlation do we have if the correlation coefficient is -0.61?</td>
<td>• Strong negative.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What do you think the correlation coefficient would be for the scatter plot in <strong>Student Activity 1</strong>?</td>
<td>• Students here may give a variety of responses.</td>
<td><strong>Note</strong>: The correlation coefficient for <strong>Student Activity 1</strong> is -0.91.</td>
<td>Are students able to make a reasonable and meaningful estimate of the correlation coefficient?</td>
</tr>
<tr>
<td>Why is the coefficient here negative?</td>
<td></td>
<td></td>
<td>Are students able to make a reasonable and meaningful estimate of the correlation coefficient?</td>
</tr>
<tr>
<td>Could it be -0.4?</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Teaching & Learning Plan: The Correlation Coefficient

<table>
<thead>
<tr>
<th>Student Learning Tasks: Teacher Input</th>
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<th>Teacher’s Support and Actions</th>
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</tr>
</thead>
<tbody>
<tr>
<td>» Draw a scatter plot that would have a strong positive correlation?</td>
<td>▶ Students may give a variety of answers here.</td>
<td>» Get students in pairs to draw their own plots.</td>
<td>» Are students drawing plots that make sense?</td>
</tr>
<tr>
<td>» Draw a scatter plot that would have a strong negative correlation?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>» Draw a scatter plot that would have a correlation close to 0?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>» Here are some number lines. Can you shade on the lines the values of the correlation coefficient which indicate different types of correlation?</td>
<td>▶ Student responses here will be dependent on their prior knowledge of inequalities.</td>
<td>» Distribute Student Activity 2 A.</td>
<td>» Are students shading the lines correctly?</td>
</tr>
<tr>
<td>» Complete Student Activity 2A.</td>
<td></td>
<td>» Circulate the room.</td>
<td>» Are students able to interpret the sets from the now shaded number lines?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>» Get students in groups to put their answers on the board.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>» Some extra work may be required here on inequalities e.g. ages in the classroom today are $15 \leq A \leq 17$</td>
<td></td>
</tr>
</tbody>
</table>
### Student Learning Tasks: Teacher Input

- Here are some scatter plots with their corresponding correlation coefficients. Can you match each graph with its correlation coefficient?

### Student Activities: Possible Responses

- Students may ask about slope and attempt to calculate the slope.

### Teacher’s Support and Actions

- Distribute **Student Activity 2B** and **Student Activity 2C**.
- Circulate the room and ask students to justify their selections.
- Use the slope as an aid to understanding positive and negative correlation.

### Assessing the Learning

- Are students matching the coefficients correctly to the graphs?
- Do students understand that the value of slope is not the value of the correlation coefficient?
- For extra work use [http://statistics.net/stat/Correlations/](http://statistics.net/stat/Correlations/)

### Teacher Reflections

- What do we mean by linear relationships?

- In **Student Activities 2B** and **Student Activity 2C** we were matching correlation coefficients to graphs now we are going to interpret the correlation so we can make statements about the linear links between variables.

- What is a linear relationship?

- What is a variable?

- A line.
- A formula.

- Something that can vary in value.
# Teaching & Learning Plan: The Correlation Coefficient

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<tr>
<td>» Now we are going to look at three activities that test your understanding of the correlation coefficient scale.</td>
<td>» Students may choose random answers. Compare answers around the room.</td>
<td>» Distribute Student Activity 3A.</td>
<td>» Are students labelling their axes correctly?</td>
</tr>
<tr>
<td>» Complete the scatter plot and answer the questions on the correlation coefficient.</td>
<td></td>
<td>» Circulate and take note of students’ difficulties. Encourage students to discuss these difficulties and talk them through.</td>
<td>» Can students explain their choices and give clear reasons to support those choices?</td>
</tr>
<tr>
<td>» Match each correlation coefficient with the correct statement in Student Activity 3B.</td>
<td></td>
<td>» Distribute Student Activity 3B and Student Activity 3C.</td>
<td>» Are students using correlation terminology in a suitable context?</td>
</tr>
<tr>
<td>» In Student Activity 3B write a sentence based on the value of the correlation coefficient.</td>
<td></td>
<td>» Give students time to discuss their choices.</td>
<td>» Are students able to interpret the correlations correctly?</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>» Can students justify each selection?</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>» Are students able to identify what is wrong with the other answers?</td>
</tr>
<tr>
<td>Student Learning Tasks: Teacher Input</td>
<td>Student Activities: Possible Responses</td>
<td>Teacher’s Support and Actions</td>
<td>Assessing the Learning</td>
</tr>
<tr>
<td>---------------------------------------------------------------------------------------------------</td>
<td>----------------------------------------</td>
<td>-----------------------------------------------------------------------------------------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>» The next activity is about how to calculate the correlation coefficient using a calculator and causation.</td>
<td>• Schools may be closed and students are buying more ice cream.</td>
<td>» For information on calculator use go to <a href="http://www.projectmaths.ie">www.projectmaths.ie</a></td>
<td>» For an applet that calculates the correlation coefficient go to: <a href="http://www.easycalculation.com/statistics/correlation.php">http://www.easycalculation.com/statistics/correlation.php</a></td>
</tr>
<tr>
<td>» Look back at Student 3A. In this activity there was a strong correlation between temperature and ice cream sales. Are there any other variables that can influence ice cream sales?</td>
<td>• There may be more sales at a weekend.</td>
<td>aturity and causation.</td>
<td></td>
</tr>
<tr>
<td>» It does however seem reasonable that the rise in temperature does have an effect on ice cream sales. We can say here that the strong positive correlation does indicate that temperature rises causes increased sales of ice cream.</td>
<td>• There may be more sales during a holiday period.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Teaching & Learning Plan: The Correlation Coefficient

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</tr>
</thead>
</table>
| • Students report back to the rest of the class.  
• Discuss with class their answers to and their interpretation of the answer to question (ii). | » Distribute **Student Activity 4**.  
» Read the introduction with the class and ask students to fill in the activity sheet.  
» Calculator use: Teacher hands out instructions or writes instructions on the board. An emulator can be used here if it is available.  
» For Ordinary Level students the correlation coefficients for **Student Activity 4** are  
  i 0.91  
  ii -0.93  
  iii 0.97  
  iv -0.73 | » Are student's calculating the coefficient correctly?  
» Are students interpreting the correlation coefficients correctly?  
» Are students discussing causation along with the correlation coefficient? |

» What pattern is emerging from the scatter plots you have drawn? See **Student Activity 1A** and **Student Activity 3A**.  
» If you were to draw a line to show the direction in these patterns where would you draw them?  
» The lines we have been trying to find are called the line of best fit. This is the line that is the closest fit to the data.  
**Note:** We draw this line only when we have a strong positive or strong negative correlation.  
» Compare students' work and let them discuss where they drew their lines.  
» Arrive at a decision for which line gives the best indication of the pattern.
### Teaching & Learning Plan: The Correlation Coefficient

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<tr>
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</thead>
<tbody>
<tr>
<td>» We are now going to try and draw the line of best fit. We are also going to use the line and interpret its slope.</td>
<td>» Compare student answers and discuss with the class the answers to questions (viii), (ix), (x) and (xi).</td>
<td>» Distribute <strong>Student Activity 5</strong>.</td>
<td>» Are students able to draw the line of best fit?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>» Encourage discussion amongst the students.</td>
<td>» For an interesting applet on the line of best fit go to: <a href="http://staff.argyll.epsb.ca/jreed/math9/strand4/scatterPlot.htm">http://staff.argyll.epsb.ca/jreed/math9/strand4/scatterPlot.htm</a></td>
</tr>
<tr>
<td>» Ask students to draw the scatter plot and follow the steps to draw the line of best fit.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>» When is it appropriate to draw this line?</td>
<td>» When there is a high positive or negative correlation and when the relationship looks linear.</td>
<td></td>
<td>» Do students understand the usefulness of the line of best fit?</td>
</tr>
<tr>
<td>» Why would it be useful?</td>
<td>» It can help to estimate outcomes.</td>
<td></td>
<td>» Do students understand the limitations of the line of best fit?</td>
</tr>
<tr>
<td>» Is this line always a correct measure of the relationship between the variables?</td>
<td>» It is not exact but it is the best “fit”.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Student Learning Tasks: Teacher Input

- Sometimes when we do a scatter plot we can come across a result that is out of step with the rest of the data. Just because the piece of data does not “fit” this does not mean it is wrong. Data like this are called outliers and can have a major effect on the correlation coefficient and the line of best fit.

- We are now going to look at **Student Activity 6**.

- Outliers can distort the correlation dramatically. An outlier(s) can make a weak correlation look big or hide a strong correlation.

- In this activity Identify the outlier(s)?

- What effect does it have?

### Student Activities: Possible Responses

### Teacher’s Support and Actions

- Distribute **Student Activity 6**.

- Write on the board the following notes about what to do with outliers:
  - Do the calculation with or without the outlier.
  - Check for error in data.
  - Investigate the data point.

**Note:** This piece of data was investigated and found to be correct. Can students explain how there can be such a piece of data?
Student Activity 1

Drawing a Scatter Plot

Below is some recent research about engine sizes. This data shows the engine size and the fuel economy of a range of petrol car.

<table>
<thead>
<tr>
<th>Engine Size (cc)</th>
<th>1.0</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.6</th>
<th>1.7</th>
<th>2.0</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuel Economy (k.p.l.)</td>
<td>18</td>
<td>16</td>
<td>14</td>
<td>16</td>
<td>13</td>
<td>12</td>
<td>12.5</td>
<td>12</td>
<td>11</td>
<td>8.8</td>
<td>6.5</td>
</tr>
</tbody>
</table>

i  Show this information on a scatter diagram.

ii  Describe, in your own words the relationship between the engine size and fuel economy of these cars.

iii A car manufacturer produces a new car with a 1.8 litre engine and a fuel efficiency of 17 kilometres per litre. Plot this car’s performance on your scatter plot. If you were interested in buying a new car that was fuel efficient with this size engine would you buy this car? Write a short comment.
Student Activity 2A

Correlation Coefficient: The scale

On the number line below shade in the possible values of the correlation coefficient.

-2 -1 0 1 2

On the number line below shade in the possible values of the correlation coefficient that indicate a strong positive correlation.

-2 -1 0 1 2

On the number line below shade in the possible values of the correlation coefficient that indicate a weak positive correlation.

-2 -1 0 1 2

On the number line below shade in the possible values of the correlation coefficient that indicate a strong negative correlation.

-2 -1 0 1 2

On the number line below shade in the possible values of the correlation coefficient that indicate a weak negative correlation.

-2 -1 0 1 2
Matching Correlation Coefficients to Scatter Plots

The table shows the correlations for the four graphs below. Match each graph to the correlation coefficient.

<table>
<thead>
<tr>
<th>Correlation</th>
<th>-0.72</th>
<th>1</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Graph A

Graph B

Graph C

Graph D
Matching Correlation Coefficients to Scatter Plots

The table shows the correlations for the four graphs below. Match each graph to its correlation coefficient.

<table>
<thead>
<tr>
<th>Correlation</th>
<th>0.72</th>
<th>-0.90</th>
<th>0.96</th>
<th>-0.42</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For extra work use http://istics.net/stat/Correlations/
# Solutions

## Activity 2B

<table>
<thead>
<tr>
<th>Correlation</th>
<th>-0.72</th>
<th>1</th>
<th>-1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph</td>
<td>D</td>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
</tbody>
</table>

## Activity 2C

<table>
<thead>
<tr>
<th>Correlation</th>
<th>0.72</th>
<th>-0.90</th>
<th>0.96</th>
<th>-0.42</th>
</tr>
</thead>
<tbody>
<tr>
<td>Graph</td>
<td>C</td>
<td>B</td>
<td>A</td>
<td>D</td>
</tr>
</tbody>
</table>
Student Activity 3A

Interpreting the data and the Correlation Coefficient

An ice cream seller records the maximum daily temperature and the number of ice creams she sells each day. Her results for a period of ten days are shown in the table.

<table>
<thead>
<tr>
<th>Maximum temperature (°C)</th>
<th>22</th>
<th>26</th>
<th>25</th>
<th>27</th>
<th>25</th>
<th>20</th>
<th>24</th>
<th>26</th>
<th>28</th>
<th>27</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of ice creams sold</td>
<td>85</td>
<td>102</td>
<td>94</td>
<td>103</td>
<td>92</td>
<td>72</td>
<td>89</td>
<td>100</td>
<td>107</td>
<td>105</td>
</tr>
</tbody>
</table>

(i) Display the data in a way that allows you to examine the relationship between the two variables.
(ii) The correlation coefficient here is 0.98. With this in mind and looking at the scatter plot which of the following statements is correct. Give a reason for your answer.

A. As the temperature increases ice cream sales increase.
B. As the temperature increases ice cream sales tend to increase.
C. There is no evidence of a linear relationship between temperature and ice cream sales.
D. As the temperature increases ice cream sales decrease.
E. As the temperature increases ice cream sales tend to decrease.

Choice: ______________________________________________________

Reason for your answer:________________________________________________________
____________________________________________________________________________
____________________________________________________________________________
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If the correlation coefficient had been 0.4 which of the above statements would be correct.

Choice: ______________________________________________________

If the correlation coefficient had been -0.7 which of the above statements would be correct?

Choice: ______________________________________________________

Identify two statements that are always incorrect no matter what the value of the correlation coefficient is. Give a reason for your answer.

Choices: ______________________________________________________

Reason for your answer:________________________________________________________
____________________________________________________________________________
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Interpreting the data and the Correlation Coefficient

Here are the results of some other research with their correlation coefficients:

Pick the appropriate statement for each survey and give a reason for your answer.

1. In a survey the correlation coefficient between engine size and fuel economy was found to be -0.9.
   A. As the engine size increases fuel efficiency increases.
   B. As the engine size increases fuel efficiency tends to increase.
   C. There is no evidence of a linear relationship between engine size and fuel efficiency.
   D. As the engine size increases fuel efficiency decreases.
   E. As the engine size increases fuel efficiency tends to decrease.

   Choices: _________________________________________________________

   Reason for your answer: ____________________________________________
   ________________________________________________________________
   ________________________________________________________________
   ________________________________________________________________

2. In a survey the correlation between the numbers of hours per week students spent studying and their performance in an exam was 0.7.
   A. As the number of hours spent studying increases student exam performance increases.
   B. As the number of hours spent studying increases student exam performance tends to increase.
   C. There is no evidence of a linear relationship between the number of hours spent studying and student exam performance.
   D. As the number of hours spent studying increases student exam performance decreases.
   E. As the number of hours spent studying increases student exam performance tends to decrease.

   Choices: _________________________________________________________

   Reason for your answer: ____________________________________________
   ________________________________________________________________
   ________________________________________________________________
   ________________________________________________________________
   ________________________________________________________________
Student Activity 3C

Interpreting the data and the Correlation Coefficient

Here are the results of some other research with their correlation coefficients:

(i) In a survey the correlation between the heights of male students and their shoe sizes was -0.01.

(ii) In the 1960’s the U.S. army undertook research into the relationship between a man’s height and the chances of him receiving a promotion. The results showed that the correlation between a man’s height and his chance of promotion was 0.8.

Pretend you are a journalist, write a sentence based on the value of the correlation coefficients in (i) and (ii) above.

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Student Activity 4

Calculating the Correlation Coefficient (HL) and Causation (OL)

For each of the following sets of data calculate the correlation coefficient and write a conclusion.

Note: In some of the surveys below there are strong correlations but this does not always imply causation. In some cases there may be a lurking variable that can explain the strong correlation. For example in the last 10 years there is a strong positive correlation between the sales of cars and the sales of electrical items. Can we say a rise in the sale of cars tends to lead to a rise in the sale of electrical items? No we cannot. The lurking variable here is the performance of the economy as a whole.

Correlation does not always imply causation.

Using your calculator calculate the correlation coefficient and write a conclusion.

i  Roller coasters get their speed as a result of dropping down a steep incline. The table below gives height of a drop and the speed achieved for different roller coasters around the world.

<table>
<thead>
<tr>
<th>Drop in Metres (m)</th>
<th>25</th>
<th>30</th>
<th>28</th>
<th>58</th>
<th>55</th>
<th>40</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed of the coaster (k.p.h)</td>
<td>80</td>
<td>90</td>
<td>78</td>
<td>93</td>
<td>90</td>
<td>93</td>
<td>105</td>
<td>128</td>
</tr>
</tbody>
</table>

Correlation Coefficient: ____________________________________________________________

Conclusion: ____________________________________________________________________
_______________________________________________________________________________
_______________________________________________________________________________
_______________________________________________________________________________

ii The table shows the number of units of electricity used in heating a house on ten different days and the average temperature for each day.

<table>
<thead>
<tr>
<th>Average temperature</th>
<th>6</th>
<th>2</th>
<th>0</th>
<th>6</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>8</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Units of electricity used</td>
<td>28</td>
<td>38</td>
<td>41</td>
<td>34</td>
<td>31</td>
<td>31</td>
<td>22</td>
<td>25</td>
<td>23</td>
<td>22</td>
</tr>
</tbody>
</table>

Correlation Coefficient: ____________________________________________________________

Conclusion: ____________________________________________________________________
_______________________________________________________________________________
_______________________________________________________________________________
_______________________________________________________________________________

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Student Activity 4 (continued)

iii The table shows the number washing machines sold on 8 different months in an electrical supply shop and the number of dishwashers sold for each of those months.

<table>
<thead>
<tr>
<th>Number of Washing Machines sold</th>
<th>42</th>
<th>65</th>
<th>80</th>
<th>110</th>
<th>28</th>
<th>40</th>
<th>120</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Dishwashers sold</td>
<td>28</td>
<td>38</td>
<td>41</td>
<td>58</td>
<td>10</td>
<td>26</td>
<td>72</td>
<td>22</td>
</tr>
</tbody>
</table>

Correlation Coefficient: ____________________________________________________________

Conclusion: _____________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________

iv The table shows the average number of hours spent daily by students watching television and the average mark they achieved in their summer exams.

<table>
<thead>
<tr>
<th>Hours spent watching T.V. daily</th>
<th>6</th>
<th>2</th>
<th>0</th>
<th>6</th>
<th>3</th>
<th>5</th>
<th>10</th>
<th>8</th>
<th>9</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average scores in their exams</td>
<td>28</td>
<td>38</td>
<td>41</td>
<td>34</td>
<td>31</td>
<td>31</td>
<td>22</td>
<td>25</td>
<td>23</td>
<td>22</td>
</tr>
</tbody>
</table>

Correlation Coefficient: ____________________________________________________________

Conclusion: _____________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
______________________________________________________________________________
Student Activity 5

Line of Best Fit (HL only)

The following table shows the weekly rainfall ($x$ cm) and the number of tourists ($y$ thousand) visiting a certain beauty spot, for 9 successive weeks.

<table>
<thead>
<tr>
<th>Rainfall ($x$ cm)</th>
<th>4.5</th>
<th>3.0</th>
<th>5.2</th>
<th>5.0</th>
<th>2.1</th>
<th>0</th>
<th>0</th>
<th>1.2</th>
<th>3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of tourists ($y$ thousand)</td>
<td>5.0</td>
<td>8.0</td>
<td>0.8</td>
<td>4.2</td>
<td>4.8</td>
<td>7.4</td>
<td>9.4</td>
<td>8.6</td>
<td>2.6</td>
</tr>
</tbody>
</table>

i  Draw a scatter plot for this data.

ii  Find the mean rainfall ($\bar{x}$).

iii  Find the mean number of tourists ($\bar{y}$).

iv  Plot the point ($\bar{x}, \bar{y}$). Draw lines parallel to the $x$-axis and $y$-axis through this point.

v  This splits the scatter plot into 4 quadrants. In which quadrants do you find the most points?
**Line of Best Fit (HL only)**

vi Draw a line of best fit. Draw an oval around the data. The line must go through \((\bar{x}, \bar{y})\). The line of best fit should go through the two quadrants that contain the most data points.

vii On the 10th week there was 4cm of rainfall. Use your line of best fit to estimate the number of tourists that had visited the beauty spot in the 10th week. 

viii By picking appropriate points find the slope of the line of best fit.

ix Interpret the slope in the context of rainfall and number of tourists.

x Find the equation of the line of best fit and use it to check your answer to part vii.

xi The manager of the café at this beauty spot has to plan staffing levels. A mix of full time and part time staff are employed. In the light of the information above and the fact that the correlation coefficient is -0.78 what advice would you give the manager?
Outliers

A Company surveyed 12 of its employees. Below is a table of their year’s experience with the company and their income.

<table>
<thead>
<tr>
<th>Full Years experience</th>
<th>10</th>
<th>8</th>
<th>4</th>
<th>12</th>
<th>7</th>
<th>6</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>12</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Salary (€ 000's)</td>
<td>41</td>
<td>33</td>
<td>27</td>
<td>46</td>
<td>29</td>
<td>33</td>
<td>20</td>
<td>27</td>
<td>44</td>
<td>32</td>
<td>38</td>
<td>32</td>
</tr>
</tbody>
</table>

i  Draw a scatter plot of the data.

ii Circle one or more outliers.

iii Calculate the correlation coefficient for all the data.

iv Remove the outlier and calculate the correlation coefficient.

   What do you notice?

v Draw your line of best fit for all the data and draw the line of best fit when the outlier is removed.

vi What effect, if any, does removing the outlier have on the line of best fit?