Partitioning the whole/unit into equal parts

Multiplying and dividing fractions

Adding and Subtracting fractions

Fraction Equivalence

Ordering fractions

Partitioning
the whole / unit into equal parts

Diagnostic Test

Overview of Fractions

**Partitioning:** Partitioning is defined as the “act of dividing a quantity into a given number of parts which are themselves qualitatively equal” (Kieran & Nelson 1981 p. 39)

**Aims**

* To use partitioning in the development of students’ understanding of fractions (It forms the basis for equivalence which forms the foundation for fraction operations.)
* To help students to acquire the idea of equal shares and of combining and recombining fractions
* To provide students with an opportunity to discuss their strategies

**Prior knowledge**

Fractions in primary school

Similar problems given as homework after the diagnostic test on fractions -students would have had an opportunity to work on these problems on their own. They can now work with their peers and thus share ideas and strategies. Students would make their own fraction strips for reference.

**Learning Outcomes**

**As a result of studying this topic, students will be able to**

* use word problems, diagrams and arithmetic sentences to represent the addition of rational numbers
* see division as “fair sharing”
* express word problems as arithmetic sentences and vice versa.
* understand the importance of identifying the whole to which a fraction belong
* have an understanding for the algorithm for the sum of rational numbers
* consolidate an understanding of the equivalence of top heavy fractions and mixed numbers and understand the algorithm to convert from one to the other.

**Resources required**

Students draw pictures and partition the shapes. They could also model the situation using uni -fix cubes or fraction strips.

**Relationship to the Junior Cert syllabus**

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| **Strand/Topic Title** | **Learning Outcomes***Students will be able to*  |
| **Strand 3 :** 3.1 Number systems Students will devise strategies for computation that can be applied to any number. Implicit in such computational methods are generalisations about numerical relationships with the operations being used. Students will articulate the generalisation that underlies their strategy, firstly in common language and then in symbolic language. |  * investigate models to help think about the operations of addition, subtraction, multiplication and division of rational numbers
* consolidate the idea that equality is a relationship in which two mathematical expressions have the same value
* analyse solution strategies to problems
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| **Lesson Interaction** |
| **Student Learning Tasks:****Teacher Input** | **Student Activities:** **Possible and Expected Responses** | **Teacher’s support and actions**  | **Checking Understanding** |
| **Student Activity 1** **Time: 10 mins**Opening question: Would you rather have ${1}/{2}$ of a bar or ${1}/{4}$ of a bar?* I have 7 bars to divide in fair shares between myself and my 3 friends.

Possibly split class into groups of 4 as opposed to working in pairs as the activity involves dividing the bars among 4 people. Establish whether the students understand the meaning of “fair share”. Questions – Can anyone explain what fair share means? If 8 bags of Tayto have to be divided between 4 people how many bags would each person have to get to ensure that each person has a fair share? Working in pairs, show how you would divide out the7 bars on the Activity sheet among 4 people so that everyone would get a fair share. | * Give each person a bar – this uses up 4 bars with 3 left.Divide 2 of these in ½ and give each person ½.Divide the remaining bar in 4 parts and give each person ¼.Each gets 1 + ½ + ¼ = 1 ¾ bars.
* Divide each of the 7 bars into fourths. Each person gets a fourth from each bar, so each person gets 7/4.
* Give everyone 1 bar. Take ¼ of a bar from each of the 3 remaining bars. Give these 3 one quarter pieces to one person and then each of the ¾ bars to each of the remaining 3 people.

Each person gets 1 ¾. | * Distribute **Student Activity 1**
* Circulate to see what strategies students are coming up with.
* When they have finished, ask groups with different strategies to present their strategies to the class so they all get to see the different viewpoints which are all valid.
* Some students may say for the last 3 bars that it is 3 bars divided by 4, hence everyone gets ¾ of a bar – they should also explain this as in the third strategy.
 | * Are there many different strategies among the students?
* Are students relating fractions to division in the context of fair shares?

Are there many different ways to divide the 7 bars in fair shares between 4 people? We’ve shared this bar into parts, what do we call these parts? How many quarters are there in 1 bar? How would you split 9 bars among 4 people? |
| * What does 7/4 mean?

Shade in ${7}/{4}$. | * 7 equal parts, each of which is ¼ of a whole
 | * Ask different students from the class these 4 questions and ask them to use what they have just done in partitioning in their explanations.
 | * By discussing the different strategies are students seeing that 7/4 = 1 ¾ (Recombining fractions) – the equivalence between mixed numbers and improper or top heavy fractions?
* Do students use the terms top heavy or improper fraction?
 |
| * Why is 7/4 the same as 1 ¾ ?
 | * 4 fourths = 1 unit + 3 fourths more
 |
| * Some people convert 1 ¾ to an improper or top heavy fraction by multiplying 1 by 4 and adding 3. Can you explain this?

If I have 7 bars and I split them into 4 equal parts how many ${1}/{4}$s will I have? What is ${9}/{4}$ equal to if it is written as a whole number and a fraction? What do top heavy fractions look like? Can you give me an example of a mixed number? Is there another name for top heavy fractions/Is there another name for improper fractions? | * Multiply 1 by 4 to convert it to quarters giving 4 quarters and add 3 quarters to 4 quarters to give 7/4.
 |
| * Some people convert 7/4 to a mixed number by saying 4 goes into 7 once with 3 left over so 7/4 is 1 ¾. Can you explain this?
 | * How many 4 quarters in 7 quarters is got by dividing 4 into 7. This gives us the number of whole units in 7/4 as every 4 makes a unit and then we have 3 quarters left over.
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|  **Student Activity 2a Time: 5 mins*** Workings in pairs solve the following problem:

I invited 8 people (including myself) to my party but I only had three chocolate bars to share. How much did each person get if they all had fair shares.Was this activity more or less difficult than the first? Why? Which is bigger, ${1}/{4}$ or ${1}/{8}$? By how much is it bigger? Can we write ${1}/{4}$ as another fraction? If I have ${1}/{4}$ and ${1}/{8}$ how much in total do I have? | * Divide the 3 bars in eights and give each person an eight from each of the 3 bars

Using symbols: * Divide 2 bars into quarters giving each person ¼. Then divide the last bar into eights giving each person 1/8 of this bar. Altogether each person receives ¼ + 1/8 = 3/8
 | * Distribute **Student Activity 1**
* Teacher listens to students talking through the activity and is alert for any misconceptions.
* Different groups present their strategies.
 | * Has this been more difficult than the first task or are students comfortable with both tasks?
* Do students understand repeated addition represented as multiplication?
* This will be an indication of whether they need other examples of this type or not.
* Check if students understand that ¼ and 1/8 represent quantities which have a relation to each other.
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| **Student Activity 2b Time: 5 mins*** After we had eaten I found 7 more chocolate bars. This time how much did each person get?

 How much chocolate had each person eaten altogether?Draw a picture to show how you would add ${1}/{2+{1}/{4+{1}/{8}}}$. | * Split 4 bars into halves and give each person ½. Split 2 bars into quarters and give each person ¼ and split last bar into eights and give each person 1/8 .

 Each person gets ½ + ¼ +1/8 = 7/8* Split each bar into eights and give each person 7/8.
* **Total chocolate eaten from 2a and 2b:** 3/8 +7/8=10/8 bar = 1 1/4
 | * Use paper folding with students who have difficulty with adding ½ +1/4 +1/8 of fraction strips – see sheet at the end of the lesson.
 | * Are students able to add ½ + ¼ +1/8 to get 7/8 without having to use calculators?
* Have they mental pictures of converting them to a common denominator without actually having to write them down?
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| **What is the unit?*** Discuss and answer the following problem in pairs.

Jack has 2 pizzas – one Hawaiian and one cheese. Each pizza is the same size and each is cut into 8 equal slices. Jack eats 2 slices of the Hawaiian and 1 slice of the cheese pizza. Use this fact to write 3 different word problems about Jack’s pizza eating, one for each of the following answers.a) 3b) 3/8c) 3/16. | * a) How many slices of pizza did Jack eat? A slice is the whole
* b) What fraction of a pizza did Jack eat? A pizza is the whole
* c) What fraction of the 2 pizzas did Jack eat? 2 pizzas represent the whole.
 | * Draw the 2 pizzas on the board and divide each into 8 equal parts. Write the 3 answers underneath.
* Circulate and listen to students’ discussions.
* Ask a couple of groups to report back and justify their answers.
 | * Do students see the that the 3 questions refers to 3 different wholes a) a slice b) a pizza c) 2 pizzas ?
 |
| **Homework :** Student Activity 3 – Do in your copies* Divide 14 bars of chocolate among 8 people.
 | * Give each person a full bar.There are 6 left over. Divide 4 of these in half and give each person ½ .Divide the last 2 into quarters and give each person ¼. Each now gets 1+1/2 + ¼ = 1 ¾
* Divide every bar into 8 parts and give each person 1/8 from each of 14 bars. Every 8 eights is 1. Each person gets 1 and 6/8 = 1 ¾ .
 | * This will be corrected next day or students who have moved ahead can do this in class.
 | * Do students see that when they get 1 ¾ they get 7/4?
* Do students see that in the second method they have 14 lots of 1/8 reinforcing the idea of multiplication as repeated addition?
* Can students recombine 14/8 to give 1 ¾ ?
* Are students using ideas of equivalent fractions 6/8 = ¾ ?
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| * Write down at least 3 things you learned about fractions today.
 | 1. When we partition a unit each part must be the same size – unit fractions involve fair sharing
2. Mixed numbers are formed by recombining equal parts into whole and counting how many parts are left over.
3. The whole can change in different parts of a question as in the pizza problem.
4. The denominator of a fraction is the number of equal parts the whole is divided into and the numerator counts how many of those equal parts we have in a particular case.
 | * Ask different students to input what they have learned and perhaps use what you heard in circulating during activities to remind students of what they discovered themselves.
 | * Are students clear on the role of the numerator and denominator? Do they understand how the algorithm for converting mixed numbers to top heavy fractions and vice versa works? Do they understand the vital role of identifying the “whole” in any given question on fractions?

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**Homework : 14 bars divided equally among 8 people - using pictures, words and arithmetic sentences**

**Student Activity 1: Divide 7 bars equally /fairly among 4 people**

Describe how you did this.



**Student Activity 2a – Divide 3 bars equally among 8 people**

Describe how you did this:



**Student Activity 2b – Divide 7 bars equally among 8 people – Describe how to do this?**

How many bars altogether have the 8 people got from 2a and 2b?