

Teaching & Learning Plans

Using Pythagoras' Theorem to establish the Distance Formula

(Draft)

Junior Certificate Syllabus



The Teaching & Learning Plans are structured as follows:

Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

- i. **Student Learning Tasks – Teacher Input:** This section focuses on teacher input and gives details of the key student tasks and teacher questions which move the lesson forward.
- ii. **Student Activities – Possible and Expected Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.
- iii. **Teacher’s Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.
- iv. **Checking Understanding:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plan:

Using Pythagoras' Theorem to establish the distance formula

Aims

- To familiarise students with the concept of distance and how distance can be measured
- To engage students in appreciating the power of the co-ordinate plane
- To allow students to discover the distance formula using Pythagoras' Theorem by calculating the length of the hypotenuse

Prior Knowledge

- *The operations $+$, $-$, \times , \div of integers*
- *The meaning and calculation of perfect squares and the values of square roots including surds*
- *Decimal places and significant figures*
- *Employing letters as symbols in order to represent numerical quantities that can take on different values*
- *The concept of measure and units of measure*
- *Pythagoras' Theorem*
- *The co-ordinate plane and plotting points on the co-ordinate plane*

Note: For a diagnostic Activity Sheet see Appendix A

Learning Outcomes

As a result of studying this topic, students will be enabled to discover the distance formula using Pythagoras' Theorem by:

- Recalling and discovering the various units of measure and measuring instruments available to measure distance
- Using the co-ordinate plane to plot points
- Understanding the terms vertical and horizontal and finding the distance between two points on horizontal and vertical lines
- Accepting the need for discovering a formula without having to physically measure the distance
- Applying knowledge of Pythagoras' Theorem in the discovery process
- Being aware of the use of letters rather than numbers in representing mathematical formulas
- Using the formula to calculate the distance between two points on the co-ordinate plane

Relationship to Junior Certificate Syllabus

Topic: Geometry and Trigonometry	Description of topic <i>Students learn about</i>	Learning outcomes <i>Students should be able to</i>
2.1 Synthetic Geometry	<p>Theorems</p> <p>14. [Theorem of Pythagoras] In a right angled triangle the square of the hypotenuse is equal to the sum of the squares on the other two sides</p> <p>15. If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle</p>	
2.3 Co-ordinate Geometry	<p>Co-ordinating the plane</p> <p>Properties of lines and line segments</p>	<p>-Read and plot points in the four quadrants</p> <p>-Calculate the distance between two points</p>

Relationship to Leaving Certificate Syllabus

Students learn about	Students working at FL should be able to	In addition students working at OL should be able to	In addition students working at HL should be able to
2.2 Co-ordinate Geometry	<p>-Co-ordinate the plane</p> <p>-Calculate the distance between two points</p>		

Resources Required

Graph paper (See Appendix B), a ruler (straight edge)

Introducing the Topic

Rene Descartes was a French mathematician who was the first person to apply Algebra to Geometry. In the 17th century, Descartes and Pierre de Fermat, a French lawyer with a passion for mathematics showed how a point could be represented by a pair of numbers, giving its distance from two axes.

Real Life Context

- From very early in human history, there was a need to represent the positions of places and to calculate the distance between them. Some of the earliest known maps are preserved on ancient clay tablets from Babylonian times
- Maps are used to represent the world and specifically Ordnance Survey maps include a two dimensional co-ordinate system for specifying position
- Mobile phone networks use GPS or global positioning systems for specifying location
- Sat Navs also use GPS technology to plot routes and calculate distances

Lesson Interaction

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<p>» Today we are going to extend our knowledge of the co-ordinate plane. Once you have a reference system you can find out many things and in this lesson we are going to discover a formula to find the distance between two points on the coordinate plane or the length of a line segment joining two points.</p>		<p>» Prepare board:</p> <ul style="list-style-type: none"> • title of lesson: 'Distance on the Coordinate Plane' • word bank • co-ordinate plane • reference column 	<p>» Are students comfortable with the mathematical terms such as co-ordinate plane, reference system, formula and line segment used in the introduction?</p>
<p>» How do we usually measure distance between two points?</p>	<ul style="list-style-type: none"> •Using a ruler •Sat nav •Pacing •Measuring tape etc. 		

Teaching & Learning Plan: Using Pythagoras' Theorem to establish the distance formula

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<p>» We are now going to do an activity that looks at how distances are measured using measuring instruments and units of measure.</p> <p>» Working in pairs, complete student activity 1.</p>	<p>• Students are to report to the class on the activity.</p>	<p>» Distribute Student Activity 1. Assign roles and a time limit.</p> <p>» Explain the word odometer.</p> <p>» Add 'Odometer' to the word bank.</p> <p>» Walk around the classroom to see what students are writing down and provide help where required.</p> <p>» Get a selection of the groups to report on Student Activity 1.</p>	<p>» Can students match distance to be measured with instrument and units?</p> <p>» Are students interacting with each other in a cooperative learning way?</p> <p>» Are students comfortable with the language used?</p>
<p>» In Student Activity 1 we have seen that we already know a lot about distances and how they are measured. Today we are going to discover a formula to calculate distances without having to physically measure the distance or use a scale or physically travel the route.</p>			

Teaching & Learning Plan: Using Pythagoras' Theorem to establish the distance formula

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
Section A	Section A	Section A	Section A
<p>» In your groups, using your knowledge of the co-ordinate plane, plot the points (10,3) and (2,3) and join the points with a straight edge.</p>	<ul style="list-style-type: none"> • Students draw x and y axes on the graph paper and plot the points. 	<p>» Distribute graph paper</p> <p>» Write the points, (10,3) and (2,3) into the reference column on the board.</p> <p>» Plot the points on the board.</p>	<p>» Walk around the classroom to assess whether the students can apply their knowledge of the co-ordinate plane to plot the points.</p>
<p>» Could we measure this horizontal distance between these points without using a ruler?</p> <p>» Yes, by counting the blocks between the two points.</p> <p>In this case a block is a unit of measure like mm, cm, km or light years which we used to describe distances in Student Activity 1.</p>	<ul style="list-style-type: none"> • Counting the blocks. • Counting the numbers between the points. 	<p>» Add 'horizontal' to word bank.</p>	<p>» Are students making valuable contributions to solving this problem?</p>

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Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<p>» What do you notice about the answer 8 units?</p> <p>» Record your answer on your graph.</p>	<ul style="list-style-type: none"> • It is 8 blocks. • It is the difference between the x co-ordinates. • y co-ordinates are the same. 	<p>Write the word 'x co-ordinate' in the word bank.</p> <p>» Highlight the distance from the origin to each point on the board.</p> <p>» Get students to record their answer on their graph using the word 'units'.</p> <p>» Add the measurements to the diagram on the board.</p>	<p>» Are students beginning to associate horizontal distance as the difference between the x co-ordinates?</p>

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Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<p>» Now plot the point (2,9) on your graph and with a straight edge join it to the point (2,3) already on your graph.</p> <p>» In units, what is the vertical distance between these two points?</p> <p>» What do you notice about the answer 6 units?</p> <p>» Record your answer on your graph.</p>	<ul style="list-style-type: none"> • Students plot the point and draw the line. • 6 units • It is the difference between the y co-ordinates 	<p>» Add (2,9) to the reference column on the board.</p> <p>» Add the word 'vertical' to the word bank.</p> <p>» Add 'y coordinate' to the word bank.</p> <p>» Highlight the distance from the origin for each point on the board.</p>	<p>» Are students using the word 'unit'?</p> <p>» Are students associating vertical distance as the difference between the y co-ordinates?</p>
Section B	Section B	Section B	Section B
<p>» Join the points (2,9) and (10,3).</p> <p>» Can we measure the distance between these two points without using a measuring instrument?</p> <p>» Why not?</p>	<ul style="list-style-type: none"> • We can't do it. • We can't count the blocks because this line isn't horizontal or vertical. 	<p>» Join the points on the graph on the board</p>	<p>» Students awareness of the fact that counting the blocks isn't possible in this context and there is a need to explore other ways</p>

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Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<p>» Looking at this shape, in your groups can you identify some features of the shape and come up with a way to solve this problem?</p>	<ul style="list-style-type: none"> • It's a triangle. • It's a right angled triangle. • We are looking for the length of the hypotenuse. • We can measure the length of the side using the Pythagorean Theorem. • $a^2 + b^2 = c^2$ 	<p>» Pointing at the diagram on the board.</p> <p>» Walk around, ask probing questions where help is required.</p> <p>» Ask students for feedback.</p>	<p>» Listen to students' responses and assess participation in answering the questions. Are students making a link between existing knowledge of the Pythagorean Theorem and getting the length of the side on the co-ordinate plane?</p>
<p>» That's right, using Pythagoras' Theorem.</p>		<p>» Write in word bank: Pythagoras' theorem $a^2 + b^2 = c^2$</p>	

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Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<p>» Now using Pythagoras' Theorem find the length of the hypotenuse.</p> <p>» So by applying the Theorem of Pythagoras, you have found that the distance between (2,9) and (10,3) is 10 units.</p> <p>» We are now going to practise this using Student Activity 2.</p>	<ul style="list-style-type: none"> • Students calculate the distance using Pythagoras' Theorem. • 10 units. 	<p>» Walk around the classroom observing what student's are writing. Assist them as required.</p> <p>» Distribute Student Activity 2</p> <p>» Walk around the classroom observing what student's are writing. Assist them as required.</p>	<p>» Can students apply Pythagoras' Theorem to this problem?</p>

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Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
Section C	Section C	Section C	Section C
<p>» Now let's move to a general co-ordinate system. On a new sheet of graph paper draw an x and y axis with no scales – with no numbers on the axes. Plot any two points that do not form a vertical or horizontal line.</p> <p>» Let's look at one of these points. I don't know the number coordinates of this point. What coordinates could I give it?</p> <p>» Why can't we call every point (x,y) on the co-ordinate plane?</p>	<ul style="list-style-type: none"> • (x,y) • (a,b) • Infinite number of points on the co-ordinate plane. 	<p>» Draw a second coordinate plane on the board. Mark two points on it.</p>	<p>» Are students applying their prior knowledge that every co-ordinate has a 'x' and a 'y' component?</p>
<p>» The international convention is to label the points we don't know in this way (x_1,y_1) and (x_2,y_2) – note the use of subscripts.</p> <p>» Now label your two points.</p>	<ul style="list-style-type: none"> • Students are labelling their points (x_1, y_1) and (x_2, y_2) 	<p>» Label the two points on the board as (x_1,y_1) and (x_2,y_2).</p> <p>Write 'subscripts' into the word bank.</p> <p>» Walk around class. Make sure that all groups have these two general points labelled correctly. Clarify where necessary.</p>	<p>» Are students labelling the points correctly?</p>

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Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<p>» As we did earlier, we want to find the distance between these two points, or the length of the line segment joining them. How did we do that before?</p> <p>» That's right. So now construct the right angled triangle where your two points (x_1, y_1) and (x_2, y_2) are the end points of the hypotenuse. In your groups, find the third vertex of the triangle using the coordinates we have.</p>	<ul style="list-style-type: none"> • constructing a right angle triangle. • by Pythagoras' Theorem. • by getting the horizontal and vertical distances first. <p>• (x_2, y_1)</p>	<p>» Write 'line segment' into the word bank.</p> <p>» Construct the right angled triangle on the board.</p> <p>» Write the word 'vertex' in the word bank.</p> <p>» Walk around and check students' work giving guidance where required.</p> <p>» Probe with questions about the next step.</p>	<p>» Are students relating earlier work to this general case?</p> <p>» Can students transfer their knowledge of the coordinate plane to this task?</p>

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Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
<p>» In reviewing your work what steps do we need to take to find the length of the line joining (x_1, y_1) and (x_2, y_2), the hypotenuse, which we'll now call 'd'?</p> <p>» That's right. We need the horizontal and vertical distances.</p> <p>» In your groups. Work out the vertical distance, the horizontal distance and use Pythagoras' Theorem work out the length of the hypotenuse in this general case.</p>	<ul style="list-style-type: none"> • get the horizontal and vertical distances first. • get the lengths of the sides of the triangle. • get the length of the hypotenuse. • $x_2 - x_1$ • $y_2 - y_1$ • $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ 	<p>» Highlight the line joining (x_1, y_1) and (x_2, y_2) in a different colour and label it 'd'</p> <p>» Walk around helping students where required.</p> <p>» Ask groups for feedback</p> <p>» Write the general vertical, horizontal, and hypotenuse distances on the board.</p>	<p>» Can students apply the work they did earlier in the lesson to this general task?</p> <p>» Are students using the words from the word bank in their interactions and questioning?</p>
<p>» You have now discovered the distance formula. It enables us to get the distance between any two points on the co-ordinate plane without using a measuring instrument. This is how modern technologies like Sat Navs and Mobile phone networks find positions and calculate distances between them.</p>		<p>» Write the distance formula on the board.</p>	

Student Activity 1

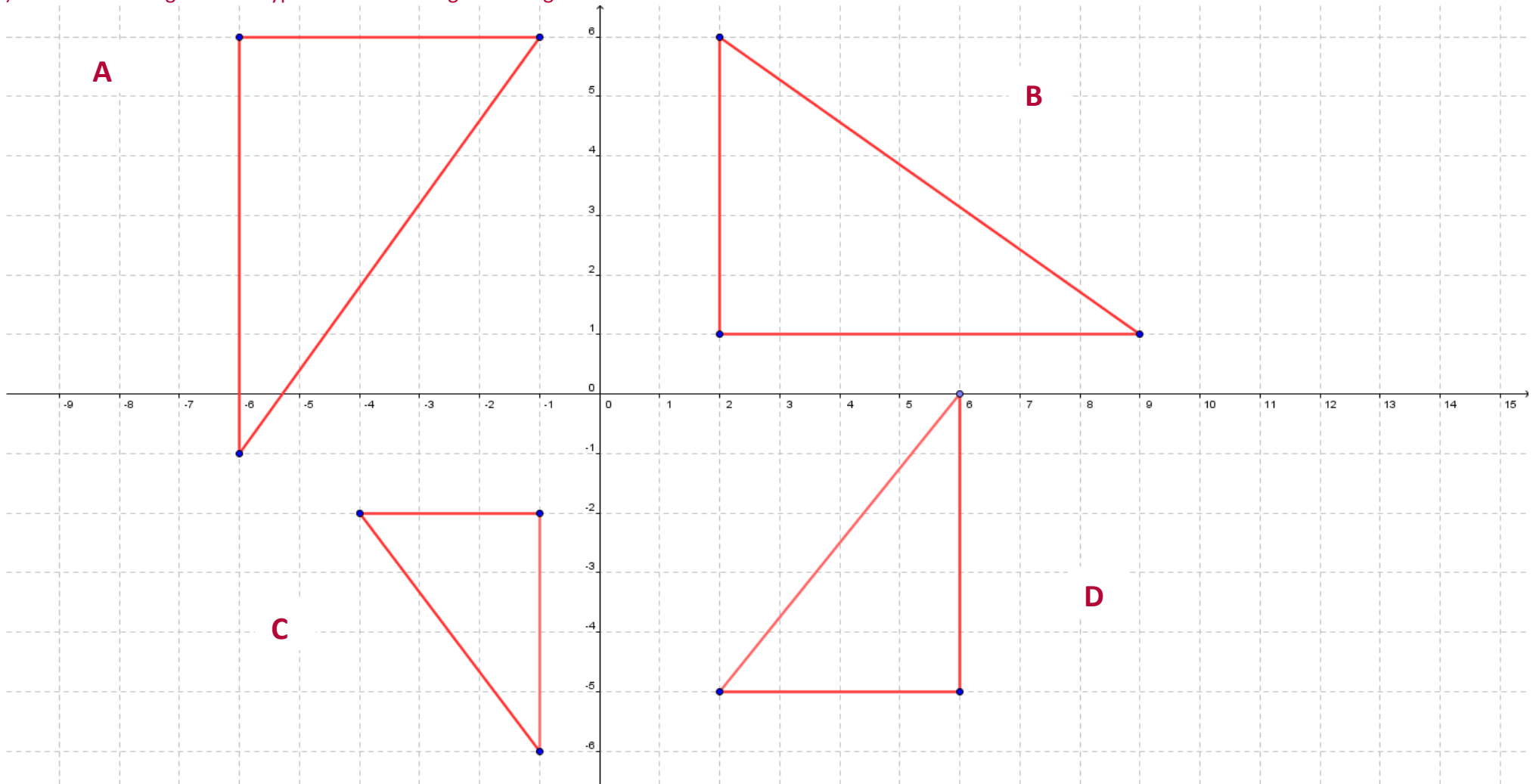
Measuring Distances

Distance	Instrument	unit
1. Length of a pen		
2. Length of a side of a desk		
3. The length of the classroom		
4. Length of school pitch		
5. School to your house		
6. Your house to nearest airport		
7. Your nearest Airport to Heathrow Airport, London		
8. Heathrow Airport to NASA Station, Houston, Texas		
9. Cape Canaveral to International space station		
10. Earth to Neptune		

Student Activity 2

For each of the triangles A, B, C and D complete the following:

- (i) using the scale, write the coordinates of each of the three vertices
- (ii) calculate the vertical distance
- (iii) calculate the horizontal distance
- (iv) calculate the length of the hypotenuse to two significant figures



Appendix A

1. Operations and Integers

Simplify:

(a) $-3+5$ (b) $-3-5$ (c) $-3+5$ (d) $-3-(-5)$

(e) $\frac{4}{2}$ (f) $\frac{-4}{2}$ (g) $\frac{4}{-2}$ (h) $\frac{-4}{-2}$

2. Squares

Simplify:

(a) $(5)^2$ (b) $(2)^2$ (c) $(-3)^2$ (d) $(-10)^2$

(e) $(-3+5)^2$ (f) $(8-(-2))^2$ (g) $(-3-4)^2$ (h) $(-12+4)^2$

3. Square roots

Simplify:

(a) $\sqrt{25}$ (b) $\sqrt{36}$ (c) $\sqrt{81}$ (d) $\sqrt{100}$

(e) $\sqrt{(4)^2}$ (f) $\sqrt{(-3)^2}$ (g) $\sqrt{(-4+6)^2}$ (h) $\sqrt{(2-(-2))^2}$

4. Substitution

If $a = 2$ and $b = -3$, find the value of each of the following:

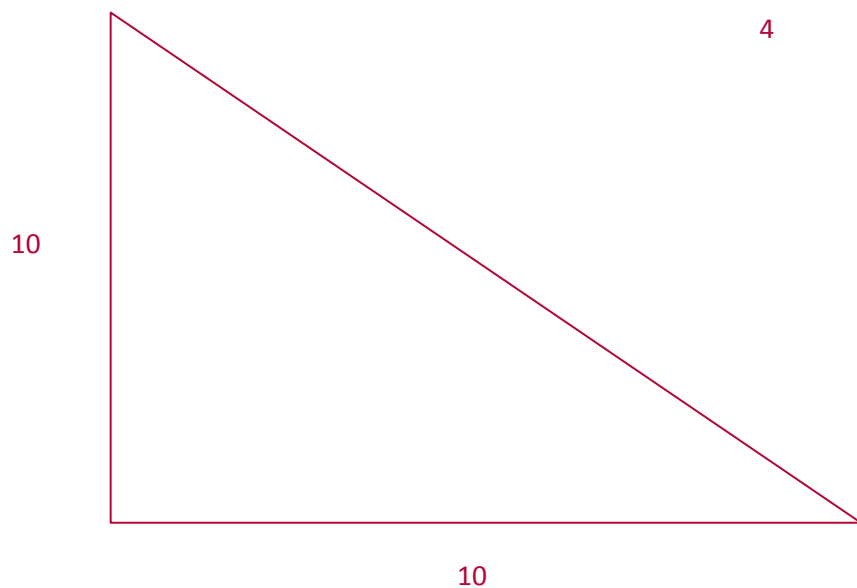
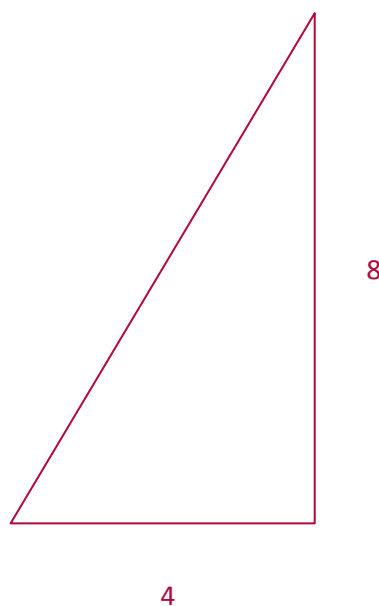
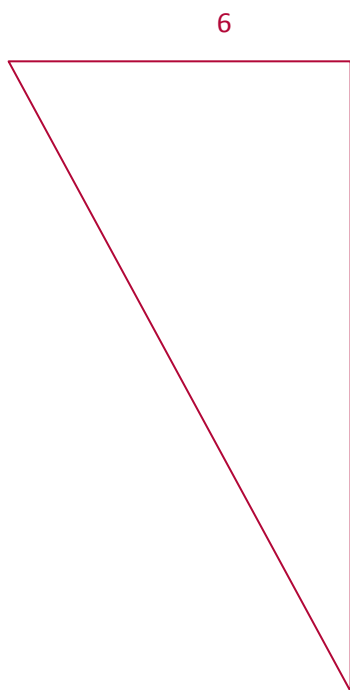
(a) $a + 2$ (b) $b - 2$ (c) $a + b$ (d) $b - a$

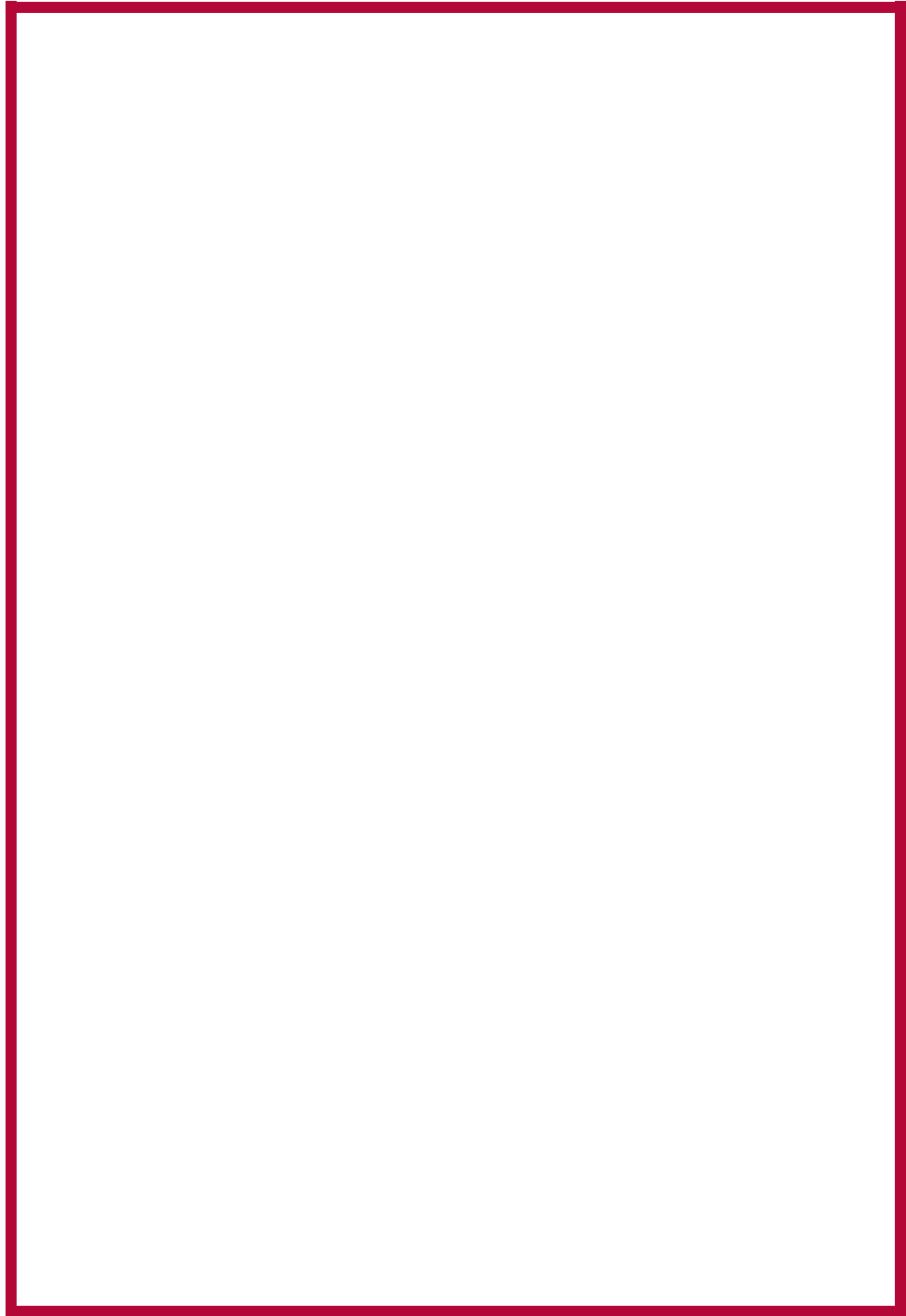
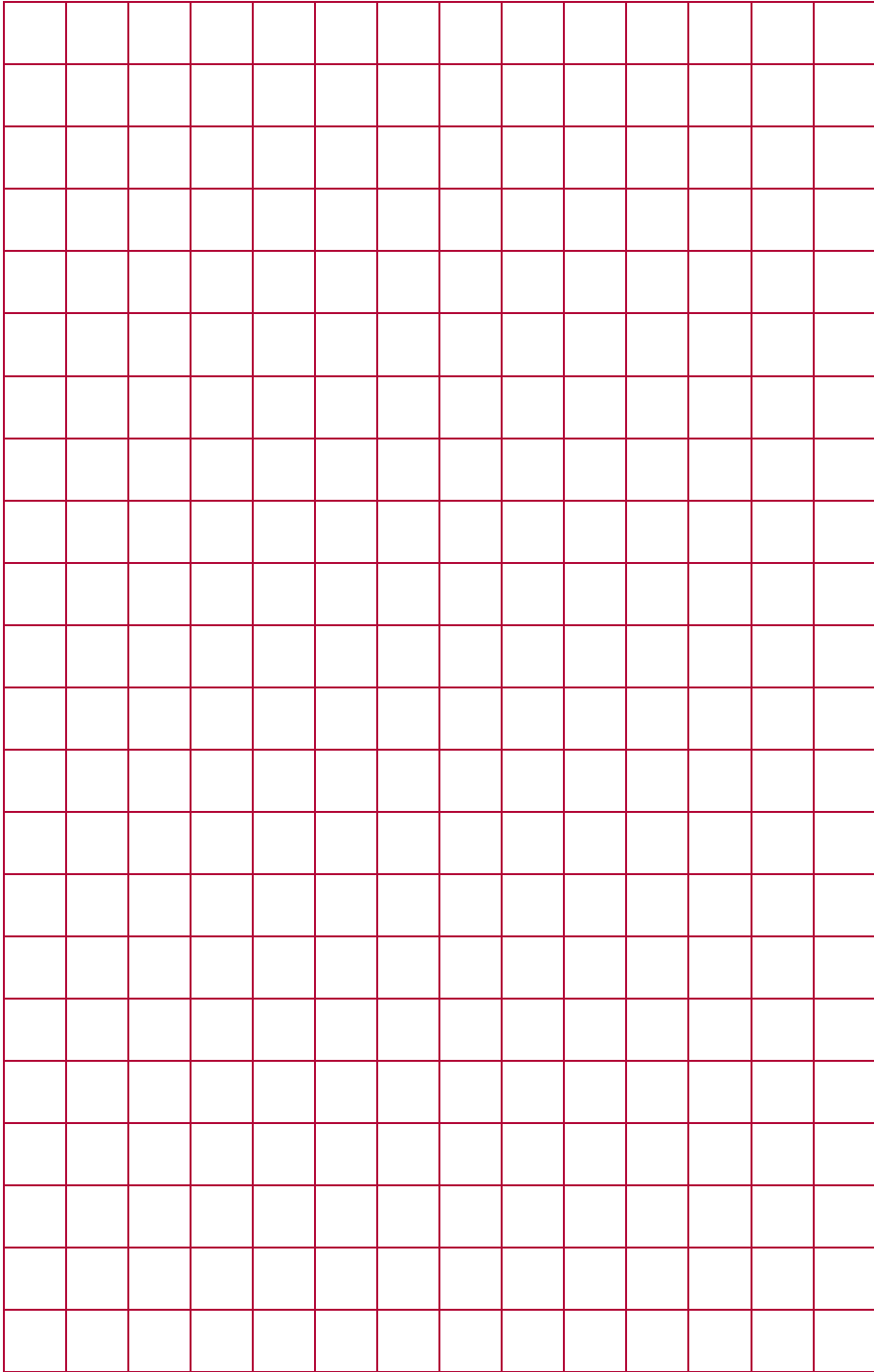
(e) $a - b$ (f) $(b-a)^2$ (g) $(a - b)^2$ (h) $5 + (b-(-b))^2$

Appendix A

5. Pythagoras' Theorem

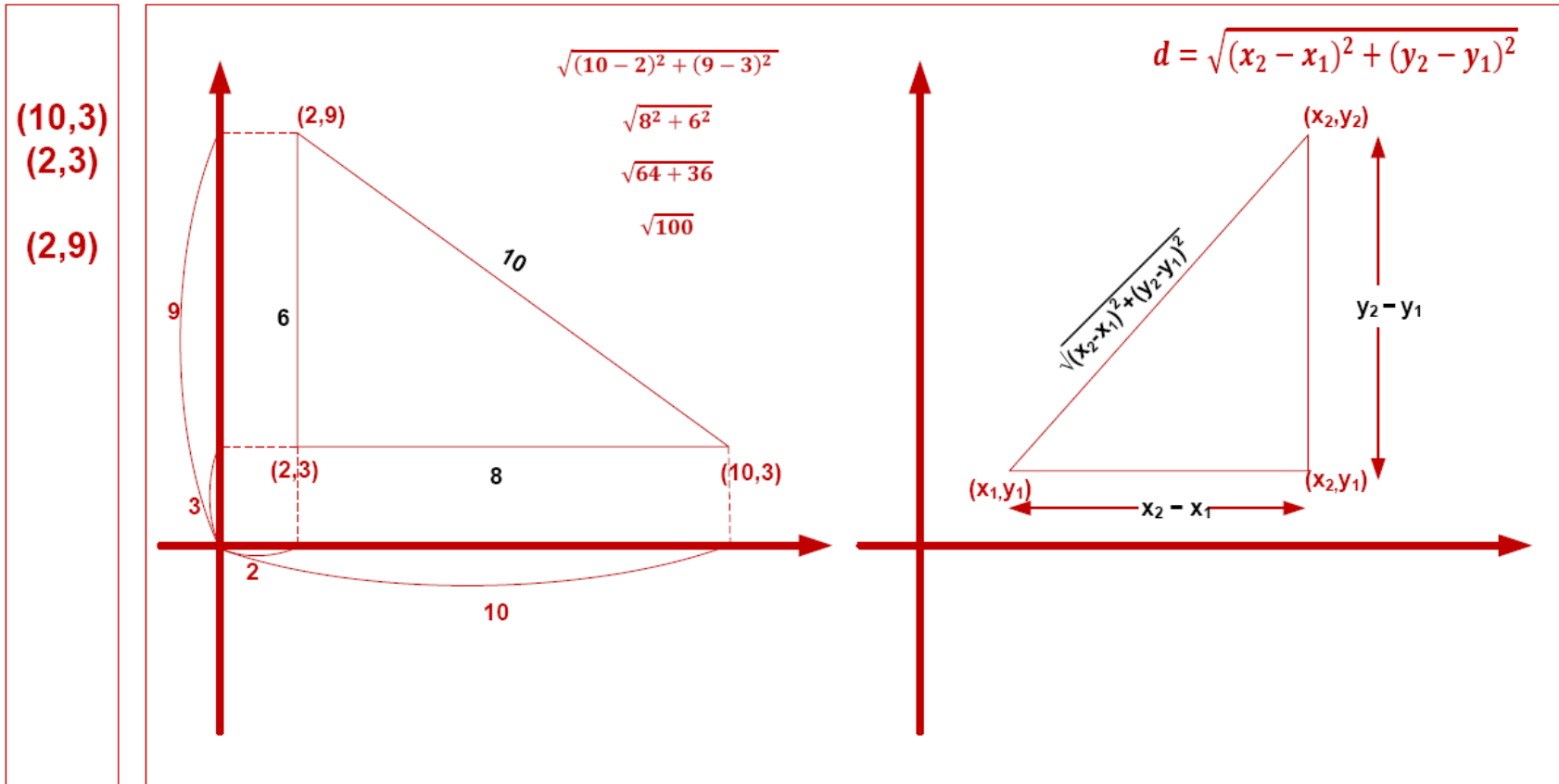
By using Pythagoras' Theorem calculate the length of the unknown side in each of the following triangles





Appendix C Board Plan

DISTANCE ON THE CO-ORDINATE PLANE



WORD BANK

- horizontal
- x co-ordinate
- hypotenuse
- Pythagoras' Theorem $c^2 = a^2 + b^2$
- vertical
- y co-ordinate
- line segment
- vertex
- subscripts