Teaching & Learning Plans

Plan 10: Trigonometric Functions

Leaving Certificate Syllabus

Project Maths
Tionscadal Mata
Development Team

The Teaching & Learning Plans are structured as follows:

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Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input:** This section focuses on teacher input and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities – Possible and Expected Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.

iv. **Checking Understanding:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plan 10: Trigonometric Functions

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Aims

- To help students to graph trigonometric functions sin x, cos x and tan x
- To recognise the period and range of each function
- To enable students to relate sin x and cos x to the unit circle
- To reinforce understanding of the terms function, domain, range, period, inverse function, asymptote

Prior Knowledge

Students should be familiar with the introduction to trigonometry (T&L Plan 8) and the unit circle (T&L Plan 9)

Learning Outcomes

As a result of studying this topic, students will be able to

- plot the graph of y= sin x, where x is an angle in standard position in the unit circle by projecting the y coordinates (sin x) of points on the unit circle onto an x-y Cartesian plane [This will be done for more than one revolution of the circle to emphasise the periodic nature of the functions. As small intervals are used the non linearity will be emphasised, and they will see that these graphs form smooth curves, which are "rounded" at the top and bottom.]
- recognise the periodicity of y = sin x, and be able to give the value of the period and the range
- explain why y = sin x is a function for all values of x and why its inverse is not a function for all values of x but could be if we restricted the x-values [more detail on this later in inverse functions]
- solve equations of the form $\sin x = \pm 0.5$, from the graph, for the domain plotted
- plot the graph of y= cos x by plotting the x coordinates of points on the unit circle against the corresponding angle in standard position, on an x-y Cartesian plane
- plot graphs of sin x, cos x and tan x using tables
- plot graphs of the form a sin x, a cos x and state their period and range, and explain the effect of changing the value of a
- plot graphs of the form sin bx, cos bx, state their period and range and explain the effect of changing the value of b

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- state the period and range of graphs of the form asin bx, acos bx for a, b $\in$ N
- solve equations of the type y = asin bx, y =acos bx for the domain used in the graph
- sketch functions of the form asin bx or a cos bx for a, b $\in$ N from a knowledge of the period and range and the shape of the function

Relationship to Leaving Certificate Syllabus

Sub-topics	Higher Level
2.3 Trigonometry	Graph the trigonometric functions sine, cosine, tangent. Graph trigonometric functions of type asin nθ, acos nθ for a, n $\in$ N.

Resources Required

Compasses, protractors, clear rulers, pencils, formulae and tables booklet, Geogebra, Autograph, Perspex Model of Unit Circle (last 3 desirable but not essential)

Note

The CD icon is used throughout this document to indicate that there is a resource on the Student Disc relating to this topic.

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Lesson Interaction

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» We have seen on the unit circle that sin, cos and tan vary as the angle varies from 0° to 360°. The sin function is represented by the y coordinate on the unit circle.		» Show http://www.projectmaths.ie/system/files/Presentation%20London%20eye%20sine.pptx and Perspex Model of unit circle.	» Does this presentation help students in the task of plotting the graph $y=\sin x$?
» Using **Student Activity 1A**, project the values of the y coordinate, which is sin x onto the Cartesian plane to get a graph of $y=\sin x$ $0 \leq x \leq 570°$	» Students plot the points and join them in a smooth curve.	» Distribute **Student Activity 1**. » Tell students they will need a clear ruler and pencil.	
» Describe the graph **Student Activity 1B** Q 1.	• It consists of repeating patterns. • It has a max of 1 and a min of -1. • The graph is "rounded" – no sharp points.	» Ask individual students to describe the graph in their own words.	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
		» Show an animation of this exercise from Autograph. (File, New Extras Page, Trigonometry, sin) » See also (scroll to end of page) piston animation: http://www.intmath.com/Trigonometric-graphs/2_Graphs-sine-cosine-period.php » GeoGebra interactive webpage f(x) =asin$x$. Set slider with a=1	» Does the animation summarise the exercise for students and give them increased understanding of the relationship between sin x and the y-coordinate on the unit circle?
» Any function which is repeating such that $f(x) = f(x+c)$, where c is a constant is called a periodic function. » We refer to the length of the repeating pattern on the x- axis as the period and the max and min y values are the range written as $[\text{min } value, \text{max } value]$ » Write down the period and range of $y=\sin x$ **Student Activity 1B** Q 2 and 3.	• Period = 360° • Range = $[-1,1]$		

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Is $y=\sin x$ a function? **Student Activity 1B** Q4	• Students explain in **Student Activity 1B** Q4 that for each value of x there is only one value of y, hence $y=\sin x$ is a function.		
» Is the inverse of $y=\sin x$ a function? **Student Activity 1B** Q5. » Explain your answer.	» Students explain that for any y value there is an infinite number of x values, so the inverse of $y=\sin x$ is not a function. » They may also draw a horizontal line through any value of y in the range, showing that it has multiple solutions.	» Remind students that a function is a set of ordered pairs, where each x value has one unique y value.	» Do students remember what a function is? » Can they explain why the inverse of $y=\sin x$ is not a function?
» Complete **Student Activity 1B** Q6 and 7.			» Can students solve the equation $\sin x=0.5$?
» Having seen the connection between $y = \sin x$ and motion in a circle, we will now plot the graph of $y=\sin x$ using a table of values, then $y= 2\sin x$ and $y= 3\sin x$, using the same axes and scale.	» Students plot the graphs on graph paper accompanying **Student Activity 2** using different colours for each graph.	» Distribute **Student Activity 2**.	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» What is the effect of varying a? **Student Activity 2.**	• The range changes. The range is [-a, a]	» If required give students some help in making this conclusion.	» Do all students understand that in the equation $y=a \sin x$ the period remains unchanged as a changes and that only the range changes?
» If this was a sound wave what effect do you think it would have on the sound to change the value of a?	• The volume would increase as a increased.		
» In the function $y=a \sin bx$ we have seen the effect of varying a, and now we will keep a constant at a=1, and vary b.	» Students fill in the table and plot the graphs in 3 different colours (**Student Activity 3**).	» Distribute **Student Activity 3**. » Remind students that they are plotting the functions $f : x \to \sin x$ $g : x \to \sin 2x$ $h : x \to \sin 3x$	» Are students keeping the correct shape on the curves even when they have fewer points to plot for each cycle?
» Fill in **Student Activity 3.** » What is the effect of varying b?	» Students fill in **Student Activity 3**. • As b varies the period varies.		
		» Students should be able to move from the specific to the general here and should be allowed to try it out on their own first. » Using GeoGebra interactive webpage show graph of $y=a \sin bx$, varying a and b using the sliders.	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» How can you calculate the period given b?	• When b is 1, the period is 2π rad = 360°. When b=2 the period is π rad = 180°. $\frac{2\pi}{2} = \pi \text{ rad} = 180^\circ$ • In general the period is $\frac{2\pi}{b} \text{ rad} = \frac{360^\circ}{b}$		» Can students now calculate the period and range of any function of the type y=a sin bx, from the values of a and b? » Use examples of functions of this type to check.
» If this was a sound wave what would be the effect of varying b? **Note:** Students do not need to know this for the syllabus, but it is good for real life context.	• The frequency and hence the pitch of the note would change, increasing as b increased.		
» We saw how sin x is the y-coordinate of a point on the circumference of the unit circle and that cos x is the x-coordinate. Hence by plotting the x coordinates of points on the unit circle onto a Cartesian plane we should get the graph of y=cos x.		» Show an animation of this exercise from Autograph. (File, New Extras Page, Trigonometry, cos) and Perspex model of unit circle. » Show, using a GeoGebra interactive webpage, where sin is dragged 90° to the left giving the cosine function.	

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Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Describe the graph of $y = \cos x$	• Its shape is the same as the sine graph. It is the graph of $\sin x$ shifted 90° to the left. • It is periodic with a period of 360° and a range of $[-1,1]$		» Are students now able to describe the graph of $y = \cos x$ in terms of periodicity and range, having worked with $y = \sin x$?
» Having seen the connection between $y = \cos x$ and motion in a circle, we will now plot the graph of $y = \cos x$ using a table of values, then $y = 2\cos x$ and $y = 3\cos x$, using the same axes and scale (**Student Activity 4**).	» Students plot the graphs using different colours for each graph.	» Distribute **Student Activity 4**.	
» What is the period and range of $y = \cos x$, $y = 2\cos x$ and $y = 3\cos x$? **Student Activity 4**.	» Students fill in **Student Activity 4**.		» Do all students understand that in the equation $y = a\cos x$, the period remains unchanged as a changes and that only the range changes?
» What is the effect of varying a? **Student Activity 4**.	• The range changes. The range is $[-a,a]$		
» In the function $y = a \cos bx$ we have seen the effect of varying a, and now we will keep a constant and vary b.	• Students fill in the table and plot the graphs in 3 different colours. **Student Activity 5.**	» Distribute **Student Activity 5.**	» Are students keeping the correct shape on the curves even when they have fewer points to plot for each cycle?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» What is the period and range of $y = \cos x$ $y = \cos 2x$ and $y = \cos 3x$? **Student Activity 5**. » What is the effect of varying b?	» Students fill in **Student Activity 5**. • As b varies the period varies.		
» How can you calculate the period given b?	• When b is 1, the period is 2π rad = 360°. When b=2 the period is π rad = 180°. $\frac{2\pi}{2} = \pi \text{ rad} = 180^\circ$ • In general the period is $\frac{2\pi}{b} \text{ rad} = \frac{360^\circ}{b}$	» Students may need some help in moving from the specific to the general here; allow them to try it out on their own first.	» Can students now calculate the period and range of any function of the type y=a cos bx, from the values of a and b? » Use more examples of functions of this type to check understanding.
» Sketch the graphs of the functions on **Student Activity 6**, without tables of values, by first calculating the period and range, and knowing the shape of functions of the type y=a sin bx and y=a cos bx.	» Students make 4 evenly spaced marks on the x axis and mark the range on the y axis, and plot the curves.	» Distribute **Student Activity 6**. » Walk around, looking at the graphs, referring students back to the previous activity sheets when they are in doubt. » Show GeoGebra interactive webpage y=a sin bx and y=a cos bx	» Are students able to sketch graphs of the form y=a sin bx and y=a cos bx from the values of period and range?
» We will now look at the graph of the function y=a tan x.		» Distribute **Student Activity 7**.	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» What is the relationship between sin, cos and tan of an angle x?	$\tan x = \frac{\sin x}{\cos x}$		
» What measurement in the diagram represents for instance the tan <AOA"? Explain.	• Tan <AOA" = intercept the radius from O to A" cuts off the line x=1. $\tan AOA'' = \frac{opp}{adj} = \frac{OA''}{1} = OA''$		
» Fill in the table on **Student Activity 7A**.	» Students fill in **Student Activity 7A**.		
	» Students plot the points. (**Student Activity 7B**)		
» Using a calculator, find tan 89°, then the tan 89.999°, then the tan 89,99999°. » What is the tan 91°, 90.001°, and 90.00001°?	• 57.29, 57295.78, 5729577.951 • -57.29, -57295.78,-5729577.95	» Students who may have difficulty understanding that tan x tends to infinity can be helped in this by using the calculator to evaluate tan of angles very close to 90°.	» Do students see that as x gets closer to 90°, tan x increases very rapidly?
» Describe the graph of $y = \tan x$?	• Its shape is unlike the graphs of $y = \sin x$ and $y = \cos x$. • There are gaps in it. • It is periodic but it repeats more often than $\sin x$ or $\cos x$ » Students complete **Student Activity 7C**.	» Encourage students to describe the graph in their own words, but because of familiarity with sin and cos graphs they should be using the words period and range.	
» What is the period $y = \tan x$? (**Student Activity 7C**)	• Its period is 180°.		

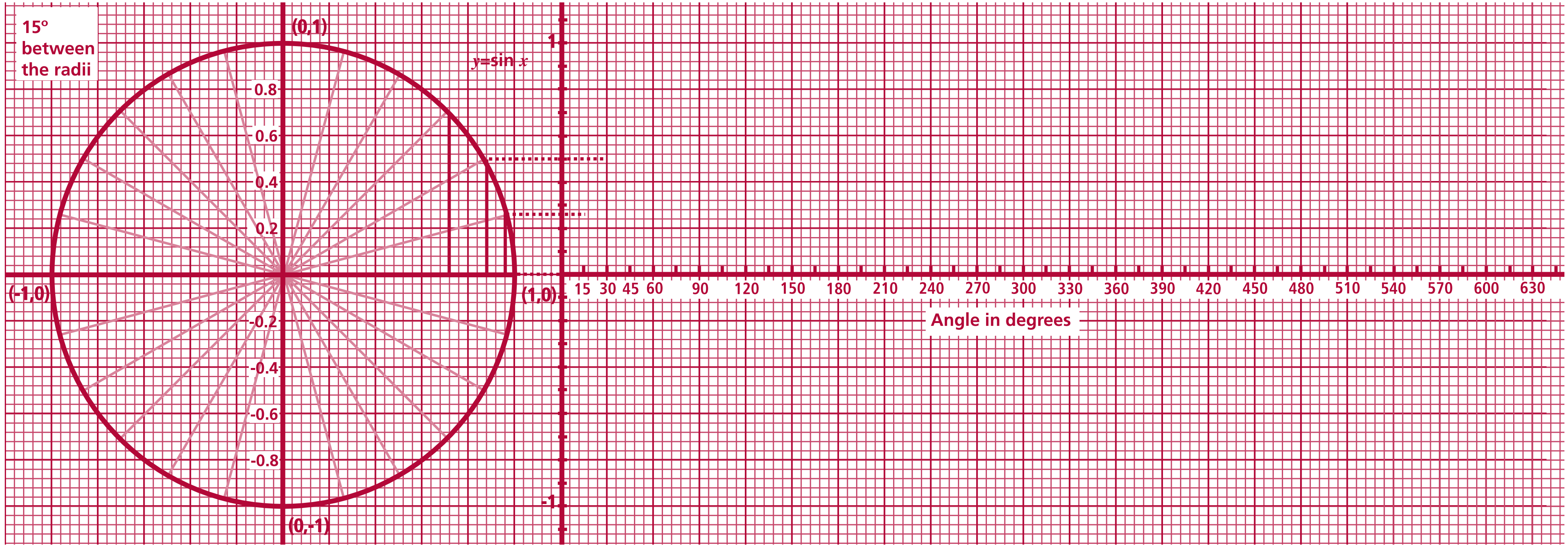
Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Describe how tan x varies as x approaches 90°?	• As the angle approaches 90°, tan x increases very rapidly and tends to + ∞ as it approaches 90° from the left, and to - ∞ as it approaches 90° from the right.	» Show the GeoGebra interactive webpage on y=tan x	» Do students appreciate the phrase "tends to infinity" as x tends to 90°?
» What is the range of y = tan x (**Student Activity 7C**).	• $[-\infty,+\infty]$		Are students able to plot the graph of y = tan x and list the period and range?
» What is the tan 90°?	• Undefined		
» The graph of y = tan x approaches a line as x approaches for example, 90° and 270°. These lines are called ASYMPTOTES – a straight line to which the graph becomes closer and closer but doesn't touch. » Draw in the asymptotes on the graph.	» Students draw in lines $y = -90°$ $y = 90°$ $y = 270°$		

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
Reflection » Write down 3 things you learned about today trigonometric functions. » Write down anything you found difficult. » Write down any questions you may have.	1. Draw the graph of sin x, a sin x, sin bx and a sin bx. 2. Find the period and range of sin x, a sin x, sin bx and a sin bx. 3. Concept of varying a and b in y = a sin bx 4. Draw the graph of cos x, a cos x, cos bx and a cos bx. 5. Find the period and range of cos x, a cos x, cos bx and a cos bx. 6. Concept of varying a and b in y = a cos bx 7. Draw the graph of y = tan x. 8. Find the period and range of tan x. 9. Concept of asymptotes 10. Draw asymptotes on the graph y = tan x.	» Circulate and take note particularly of any questions students have and help them to answer them.	» Have all students learned and understood these items? » Are they using the terminology with understanding and communicating with each other using these terms?

Student Activity 1

Student Activity 1A

Graph of y= sin x Complete the projection of sin values from the unit circle onto the Cartesian plane on the right and then join the points with a smooth curve.



Student Activity 1B

1. Describe the graph of y = sin x. ______________________
2. What is the period of y = sin x? ______________________
3. What is the range of y = sin x? ______________________
4. Is y = sin x a function? Explain. ______________________
5. Is the inverse of y = sin x a function? Explain. ______________________
6. Using the graph solve for x the equation sin x = 0.5 ______________________
7. How many solutions has the equation in Q6? ______________________

Student Activity 2

Using a calculator, or the unit circle, fill in the table for the following graphs and plot all of them using the same axes.
$y = \sin x$, $y = 2\sin x$, $y = 3\sin x$ Use different colours for each graph.

$x/^o$	-90	-60	-30	0	30	60	90	120	150	180	210	240	270	300	330	360
$\sin x$																
$2\sin x$																
$3\sin x$																

	Period	Range
$y = \sin x$		
$y = 2\sin x$		
$y = 3\sin x$		
$y = a\sin x$		

In the function, what is the effect on the graph of varying a in asin x? ______________________

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____

Student Activity 3

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Fill in the table first, and using the same axes but different colours for each graph, draw the graphs of:
$y = \sin x$, $y = \sin 2x = \sin (2 \times x)$, $y = \sin 3x = \sin (3 \times x)$ Graphs of the form $y = \sin bx$

$x/^o$	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360
$\sin x$																									
$2x$																									
$\sin 2x$																									
$3x$																									
$\sin 3x$																									

	Period	Range
$y = \sin x$		
$y = \sin 2x$		
$y = \sin 3x$		
$y = \sin bx$		

In the graph of $y = \sin bx$, what is the effect on the graph of varying b in $\sin bx$?

Student Activity 4

Using a table, find the coordinates for the following graphs and plot all of them using the same axes: $y = \cos x$, $y = 2\cos x$, $y = 3\cos x$

$x/^o$	-90	-60	-30	0	30	60	90	120	150	180	210	240	270	300	330	360
$\cos x$																
$2\cos x$																
$3\cos x$																

	Period	Range
$y = \cos x$		
$y = 2\cos x$		
$y = 3\cos x$		

In the function $y = a\cos x$, what is the effect on the graph of varying a in $a\cos x$? ____________________

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____

____

Student Activity 5

By filling in a table first, and using the same axes but different colours for each graph, draw the graphs of

$x/^o$	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360
$\cos x$																									
$2x$																									
$\cos 2x$																									
$3x$																									
$\cos 3x$																									

	Period	Range
$y = \cos x$		
$y = \cos 2x$		
$y = \cos 3x$		
$y = \cos bx$		

In the graph of $y = a\cos bx$, what is the effect on the graph of varying b in $\cos bx$? ______________________

____

____

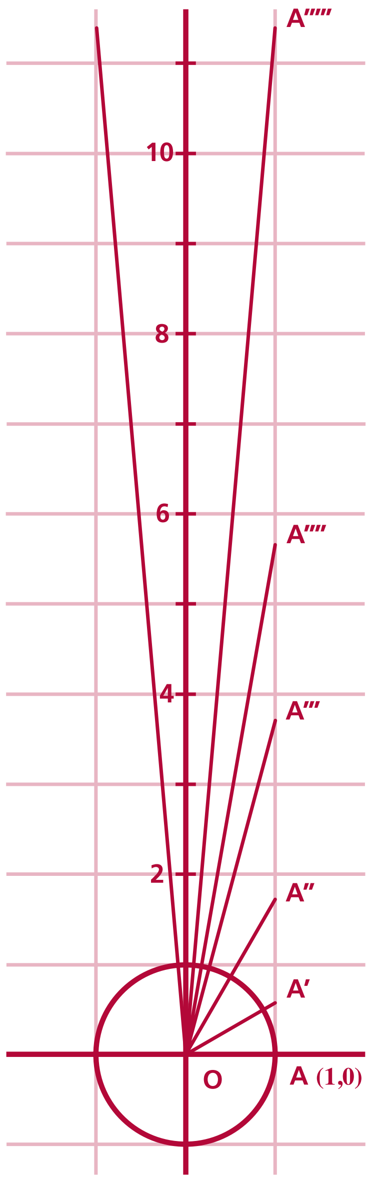
Student Activity 6

Sketch each of the following graphs: $(0° \le x \le 360°)$ $y = 4\sin x$ Period =________________ Range =________________

Sketch each of the following graphs: $(0° \le x \le 360°)$ $y = \cos 4x$ Period =________________ Range =________________

Sketch each of the following graphs: $(0° \le x \le 360°)$ $y = 2\sin 3x$ Period =________________ Range =________________

Student Activity 7



Student Activity 7A

$\lvert\angle AOA'\rvert$	$\lvert\angle AOA''\rvert$	$\lvert\angle AOA'''\rvert$	$\lvert\angle AOA''''\rvert$	$\lvert\angle AOA'''''\rvert$
30.00°	60.00°	75.00°	80.00°	85.00°

$y = \tan x$

Using the diagram of the unit circle, read the approximate value of the tan of the angles in the table using the trigonometric ratios.

Angle θ/°	0	30	60	75	80	85
tan θ						

Student Activity 7

Student Activity 7B

By filling in a table first, and using the same axes but different colours for each graph, draw the graphs of

$x/^o$	-90	-75	-60	-45	-30	0	30	45	60	75	90	105	120	135	150	180	210	225	240	255	270	285	300	330	360
$\tan x$																									

Student Activity 7C

$y = \tan x$	Period	Range