Module 4 – Activities & Problems

4.1 Mutually Exclusive Events

Which of the following are mutually exclusive events?

(a) A thumb tack falling head down and a thumb tack falling head up.
(b) A student studying Maths and Physics.
(c) Getting both a head and a tail when tossing a coin.
(d) Getting a six and a two when throwing a dice.
(e) Getting a king and a club when picking a card from a pack of playing cards.

4.2 Conditional Probability

The table below shows the percentages of boys and girls in a class and the percentages of each gender who wear glasses.

	Glasses	No Glasses	Totals
Boys	36	24	60
Girls	14	26	40
Totals	50	50	100

The teacher chooses a student at random from the class.

(a) What is the probability she chooses a boy?
(b) What is the probability that she chooses a boy who is not wearing glasses?
(c) What is the probability that she chooses a boy given that the person she chooses is not wearing glasses
(d) What is the probability that the person chosen is not wearing glasses given that it is a boy.
(e) Is P(NG|B) = P(B|NG)?
(f) What is the probability that she chooses a girl given that the person she chooses is wearing glasses?
(g) Is P(Girl|Glasses)= P(Glasses|Girl)?

Answer the above questions using the contingency table above and then using the formula for conditional probability.

4.3 A check of college students' accommodation amenities with a large group of students revealed that 38% had internet access, 52% had Games Consoles and 21% had both internet access and a Games Console. What is the probability that a randomly selected student accommodation has:

(a) a games console but no internet access?
(b) a games console or internet access, but not both?
(c) neither a games console nor a internet access?

4.4 Real estate ads suggest that 64% of homes for sale in the US have garages, 21% have swimming pools, and 17% have both features. What is the probability that a home for sale has:

(a) a pool or a garage?
(b) neither a pool nor a garage?
(c) a pool but no garage?
(d) if a home for sale has a garage, what's the probability that it has a pool, too?
(e) Are having a garage and a pool independent events? Explain.
(f) Are having a garage and a pool mutually exclusive? Explain.

4.5 A consumer organisation estimates that 29% of new cars have a cosmetic defect, such as a scratch or a dent, when they are delivered to dealers. This same organisation believes that 7% have a functional defect–something that doesn't work properly– and that 2% of new cars have both kinds of problems.

(a) If you buy a new car, what's the probability that it has some kind of defect?
(b) What's the probability it has a cosmetic effect but no functional defect?
(c) If you notice a dent on a new car, what's the probability it has a functional defect?
(d) Are the two events mutually exclusive? Explain.
(e) Do you think the two kinds of defects are independent events? Explain.

4.6 The events A and B are such that $P(A) = 0.7$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$.

(a) Find $P(A \cup B)$
(b) Find $P(A|B)$
(c) State whether A and B are independent events, justify your answer.

[2010 Sample Paper Higher Level]

4.7 A bag contains 4 red sweets and 5 blue sweets. Two sweets are taken out of the bag at random.
Draw a probability tree diagram, when:

(i) the sweets are taken with replacement.

(ii) the sweets are taken without replacement.

[N.B. Be sure to use the correct notation and figures on the branches of your tree]

4.8 Susan goes to work by one of two routes A or B. The probability of going by route A is 30%.
If she goes by route A the probability of being late for school is 5% and if she goes by route B, the probability of being late is 10%.
Draw a probability tree diagram, and then

(i) Find the probability that Susan is late for school

(ii) Given that Susan is late for school, find the probability that she went via route A.

4.9 Assume there is a test for cancer that is 98% accurate i.e. if someone has cancer, the test will be positive 98% of the time and if someone doesn't have cancer the test will be negative 98% of the time. Assume further that 0.5% (or 1 in 200 people) actually have cancer. Imagine someone has taken the test and the doctor informs them that they have cancer.

(a) What's the probability that a person tests positive for cancer?

(b) What's the probability that a person has cancer, given that they tested positive?

4.10 Expected Value
A roulette wheel has 38 numbers, 18 are black, 18 are red and 2 are green.

(a) If you bet €1 on a number and you are right you win €35 and you get your €1 back. If you are wrong you loose your €1. What is your outcome. Is this a fair game?

(b) If you bet €1 on a black number and you are right you win €35 and you get your €1 back. If you are wrong you loose your €1. What is your outcome. Is this a fair game?