4 Week Modular Course in Geometry and Trigonometry
Strand 1

Project Maths
Tionscadal Mata
Development Team

Module 3

Junior Certificate Trigonometry

Ordinary Level

Right-angles triangles
Theorem of Pythagoras

Learning Outcomes:
Students should be able to apply the result of the theorem of Pythagoras to solve right-angled triangle problems of a simple nature involving heights and distances.

Trigonometric Ratios

Learning Outcomes:
Students should be able to use trigonometric ratios to solve problems involving angles (integer values) between 0^o and 90^o.

Extra on Higher Level

Trigonometric ratios in surd form for angles of 30^o, 45^o and 60^o

Learning Outcomes:
Students should be able to solve problems involving surds.

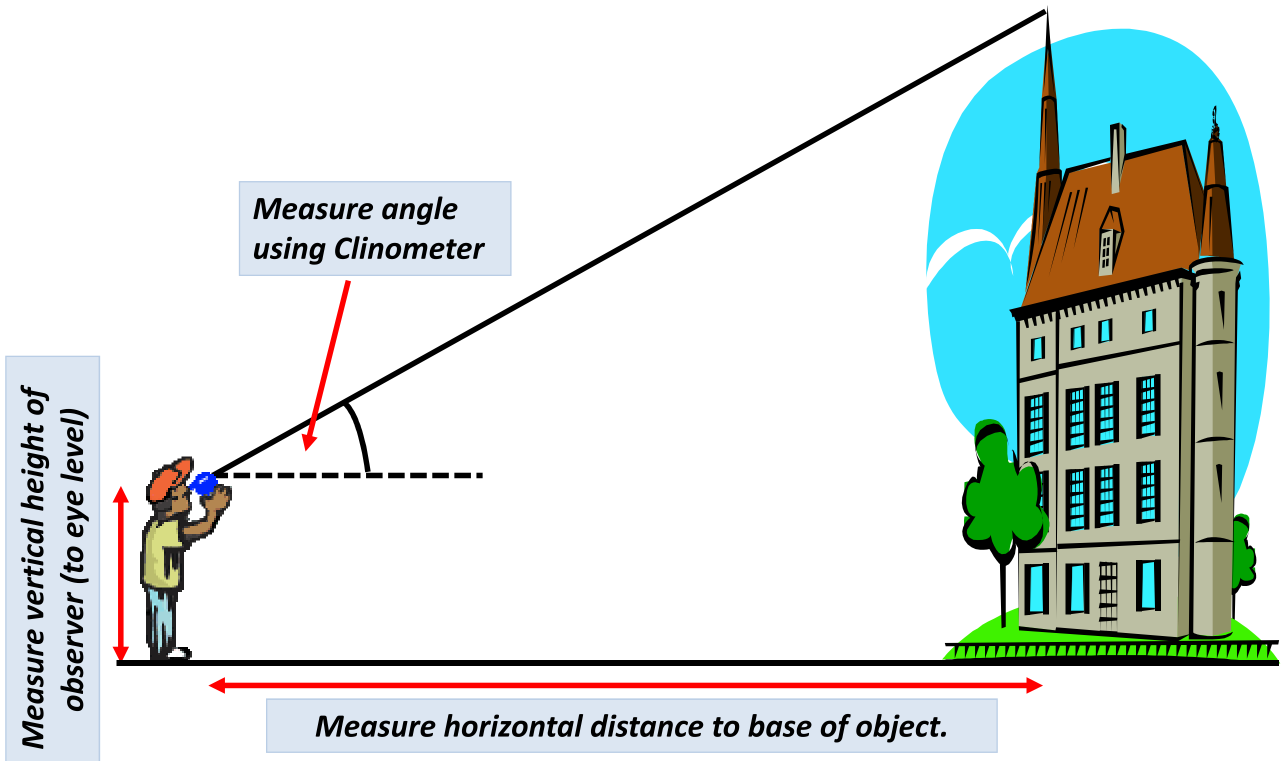
Right-angled triangles

Learning Outcomes:
Students should be able to solve problems involving right-angled triangles.

Decimal and DMS values of angles

Learning Outcomes:
Students should be able to manipulate measure of angles in both decimal and DMS forms.

Using a Clinometer



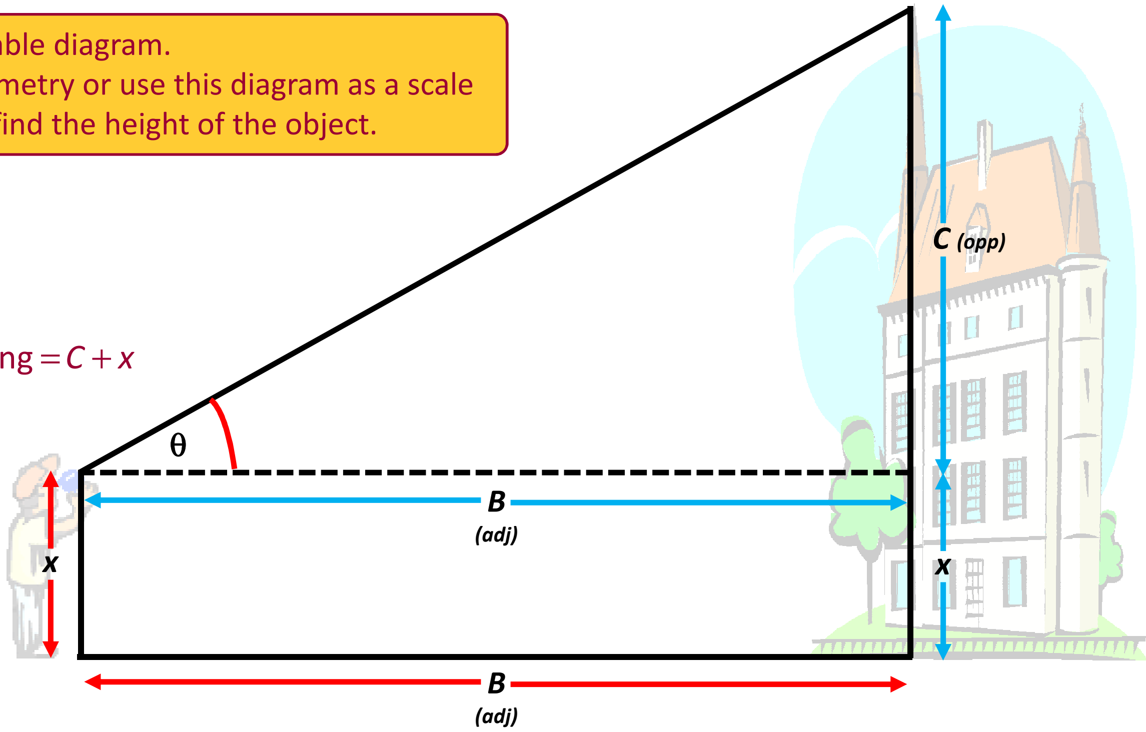
Using a Clinometer

1. Draw a suitable diagram.
2. Use trigonometry or use this diagram as a scale diagram to find the height of the object.

$$\tan\theta = \frac{C}{B}$$

$$B\tan\theta = C$$

$$\text{height of building} = C + x$$



Leaving Certificate Trigonometry

Foundation Level

Learning Outcomes:

1. *Students should be able to solve problems that involve finding heights and distances from right-angled triangles (2D only)*
2. *Students should be able to use the theorem of Pythagoras to solve problems (2D only).*
3. *Students should be able to solve problems that involve calculating the cosine, sine and tangent of angles between 0^o and 90^o.*

Ordinary Level

Learning Outcomes:

1. *Students should be able to use trigonometry to calculate the area of a triangle.*
2. *Students should be able to use the sine and cosine rules to solve problems (2D).*
3. *Students should be able to define sin θ and cos θ for all values of θ.*
4. *Students should be able to define tan θ.*
5. *Students should be able to calculate the area of a sector of a circle and the length of an arc and solve problems involving these calculations.*

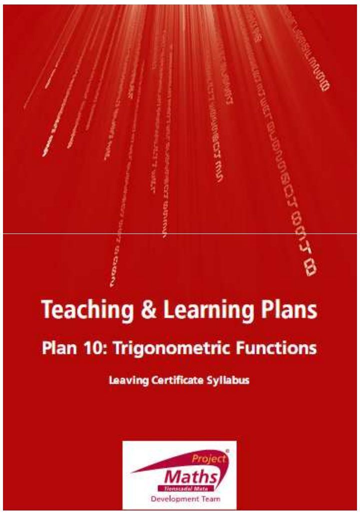
Higher Level

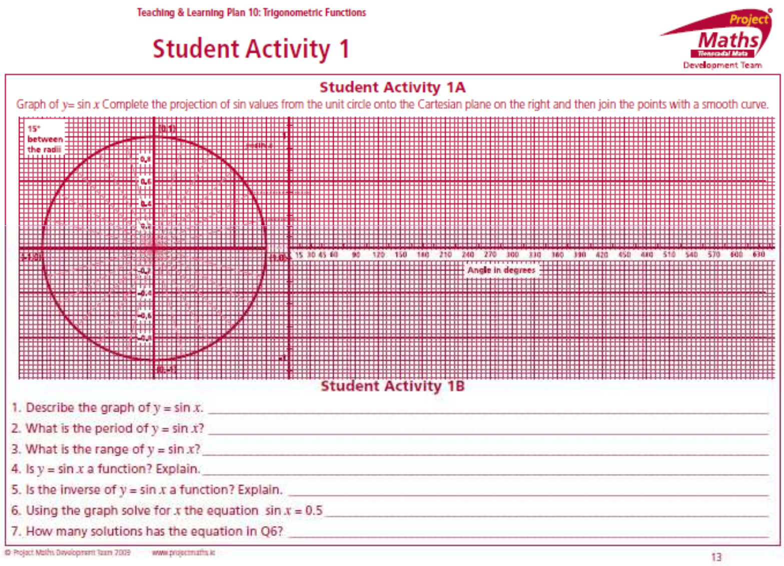
Learning Outcomes:

1. Students should be able to solve problems in 3D.
2. *Students should be able to graph the trig functions sin, cos and tan.*
3. *Students should be able to graph trig functions of the type asin nθ and asin nθ for a, n ∈ N.*
4. *Students should be able to solve trig equations such as sin nθ = 0 and cos nθ = ½ giving all solutions.*
5. *Students should be able to use the radian measure of angles.*
6. *Students should be able to derive the formulae 1, 2, 3, 4, 5, 6, 7, & 9 (see appendix).*
7. *Students should be able to apply the formulae 1 – 24.*

New Content – Periodic Trigonometric Graphs

Students should be able to graph trig functions of the type $a\sin n\theta$ and $a\cos n\theta$ for $a, n \in N$.




Teaching & Learning Plan 10: Trigonometric Functions

Student Activity 1

Student Activity 1A

Graph of $y = \sin x$ Complete the projection of sin values from the unit circle onto the Cartesian plane on the right and then join the points with a smooth curve.

Student Activity 1B

1. Describe the graph of $y = \sin x$. ______
2. What is the period of $y = \sin x$? ______
3. What is the range of $y = \sin x$? ______
4. Is $y = \sin x$ a function? Explain. ______
5. Is the inverse of $y = \sin x$ a function? Explain. ______
6. Using the graph solve for x the equation $\sin x = 0.5$ ______
7. How many solutions has the equation in Q6? ______

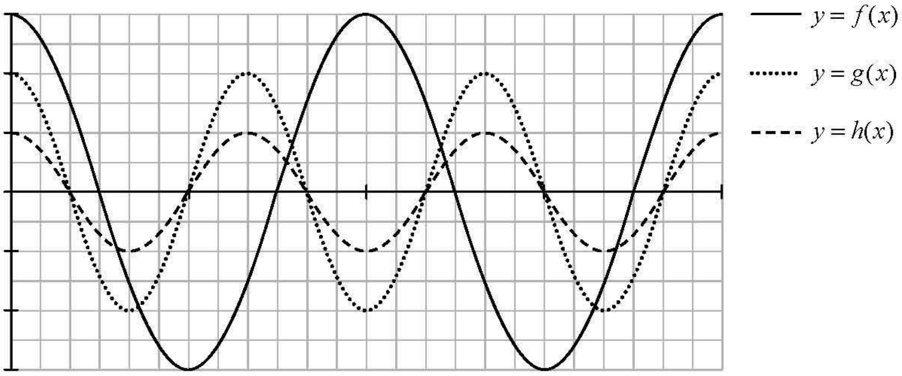
© Project Maths Development Team 2009 www.projectmaths.ie 13

LCHL 2010 Q5 (b)

(b) The graphs of three functions are shown on the diagram below. The scales on the axes are not labelled. The three functions are:

$$x \to \cos 3x$$
$$x \to 2\cos 3x$$
$$x \to 3\cos 2x$$

Identify which function is which, and write your answers in the spaces below the diagram.

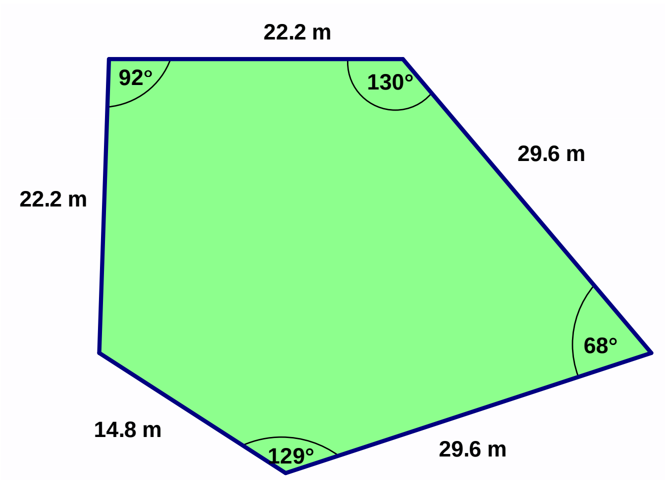


$f : x \to$ ______ $g : x \to$ ______ $h : x \to$ ______

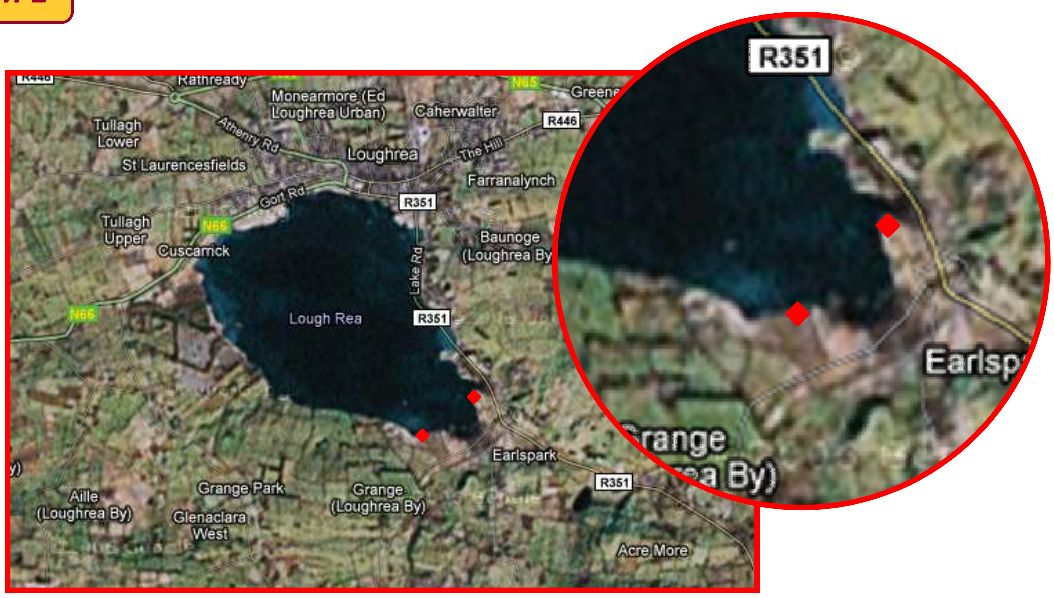
(c) Label the scales on the axes in the diagram in part **(b)**.

Practical Problem 1

A developer asks a surveyor to calculate the area of the following site which can be approximated to a pentagon as shown. The surveyor uses a theodolite to measure all the given angles. The surveyor does not need to measure the 5th angle in the diagram.
What is the measure of the 5th angle?
Find the area of the site.
A hectare is 10,000 m^2. What fraction of a hectare is the site?

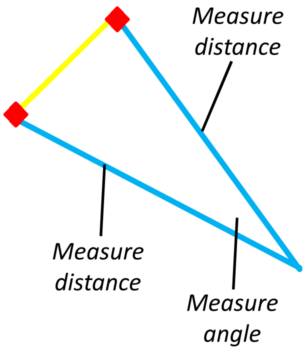


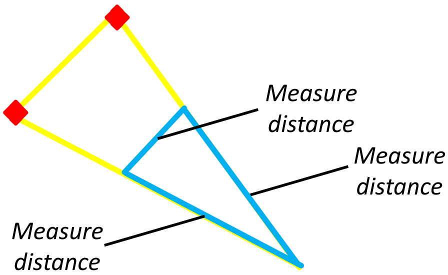
Practical Problem 1



A new bridge is to be built across Lough Rea from one red point on the map to the other red point. The surveyor wants to make an initial measurement between the two points on either side of the lake. Suggest two methods ***(1)*** *using trigonometry* ***(2)*** *using synthetic geometry by which the measurement between the two points can be taken without crossing the lake.*

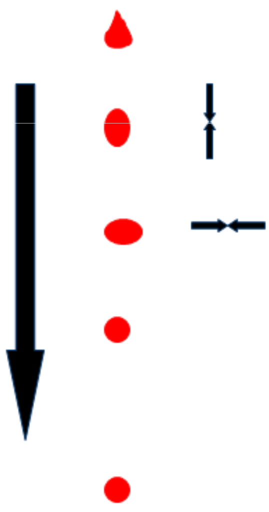
© Project Maths Development Team – Draft





Interactive

Practical Problem 3



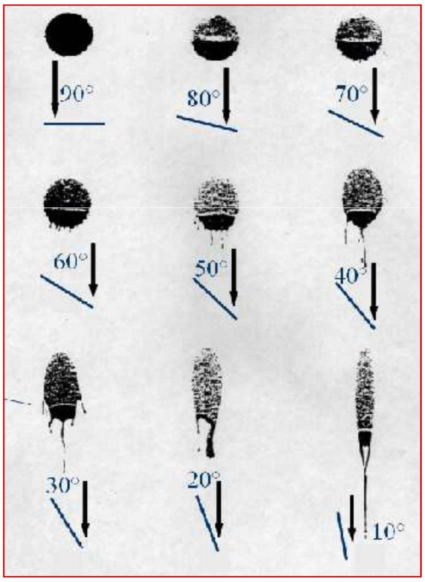
Single drop breaks free (teardrop shape)

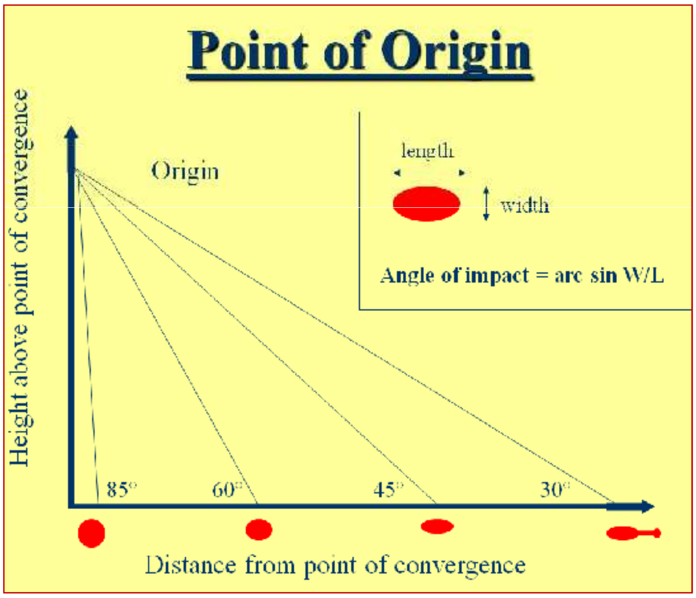
Surface tension pulls in vertically

And horizontally

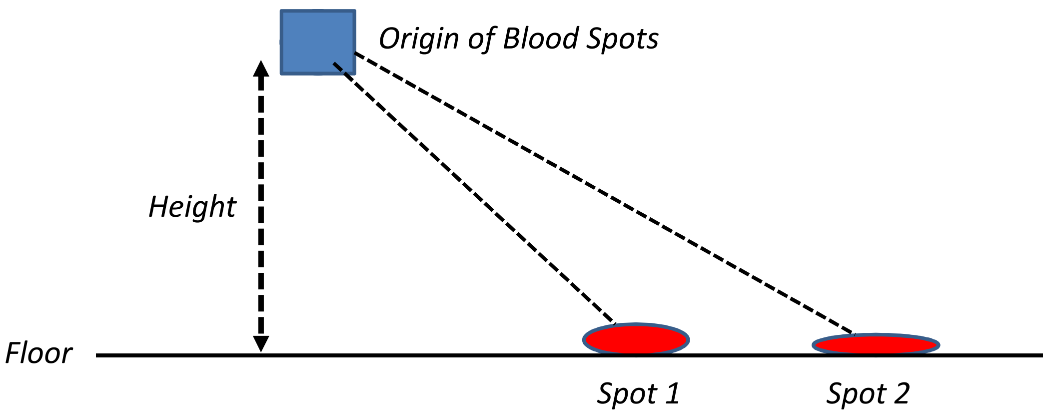
Shape settles into sphere (0.05 ml)

Does not break up until impact





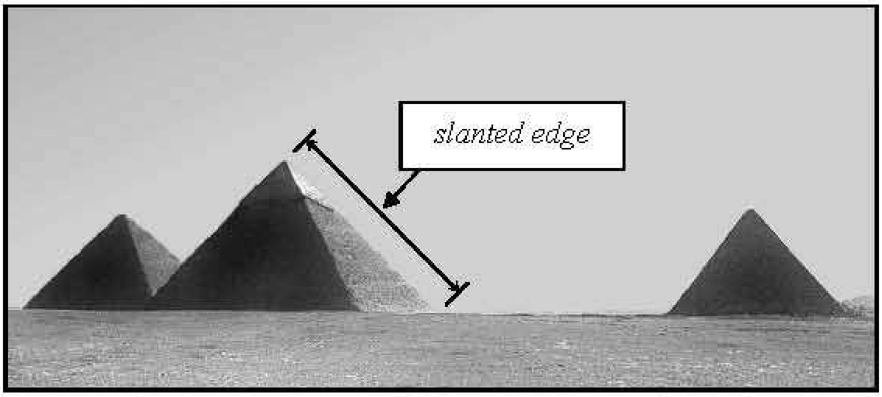
Interactive



Blood spot 1 has a width of 2.14 cm and a length of 5.36 cm. Blood spot 2 has a width of 2.28 cm and a length of 3.91 cm. if the distance between the two bloodspots is 1.5 cm calculate the perpendicular height from the origin of the blood spots to the floor.

Practical Problem 4, [LCHL 2005 Q5 (b)]

(b) The great pyramid at Giza in Egypt has a square base and four triangular faces. The base of the pyramid is of side 230 metres and the pyramid is 146 metres high. The top of the pyramid is directly above the centre of the base.



(i) Calculate the length of one of the slanted edges, correct to the nearest metre.

(ii) Calculate, correct to two significant figures, the total area of the four triangular faces of the pyramid (assuming they are smooth flat surfaces).

Interactive

Student activity on graphs of $y = a \sin bx$

Use in connection with the following file** f(x) = a sin bx **(angle measure in radians) on the Student's CD.

1. Drag the sliders so that a=1 and b=1.
Write down the period and range of $f(x) = \sin x$

(i) Period =

(ii) Range =

2. Drag slider a to vary the value of a. What is the effect of changing variable a on the function $f(x) = a \sin bx$?

3. Drag the **slider a** to vary the value of a, keeping b =1 and fill in the following table.

a	**1**	**2**	**3**	**4**
Range of f(x)				

4. Drag the **slider a** to vary the value of a, keeping b =1 and fill in the following table.

a	**-1**	**-2**	**-3**	**-4**
Range of f(x)				

You may wish to check your answer to Q2 having answered Q3 and Q4.

5. Drag the **slider b** to vary the value of b, keeping a constant. What is the effect of varying b on the function $f(x) = a \sin bx$?

6. Drag the **slider b** to vary the value of b, keeping a constant at e.g. a =2 and fill in the following table.

b	**1**	**2**	**3**	**4**
Period of f(x)				

7. Drag the **slider b** to vary the value of b, keeping a constant at e.g. a =2 and fill in the following table.

b	**1**	**2**	**3**	**4**
Period of f(x)				

8. Fill in the table below:

Function	**Range**	**Period**
$y = 3 \sin x$		
$y = \sin 4x$		
$y = 5 \sin 3x$		
$y = 2 \sin 2x$		

9. Given that $y = a \sin bx$, write down the range and period of this function in terms of a and b.

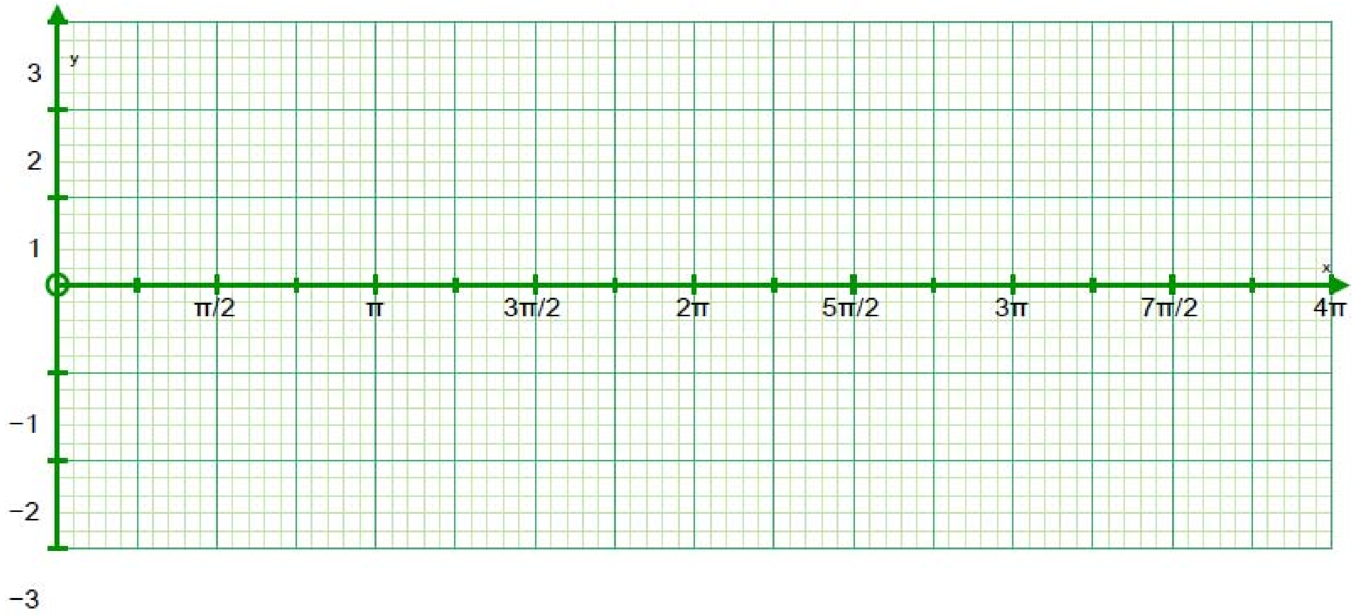
Range =

Period =

10. Fill in the last column in the table below, in the form *y* = *a* sin *bx*, for *a* and *b* ,given the range and period of each function:

Range	Period	$y = a \sin bx$
$[-1,1]$	π	
$[-3,3]$	$\frac{2\pi}{3}$	
$[-5,5]$	$\frac{\pi}{2}$	
$[-4,4]$	$\frac{\pi}{4}$	

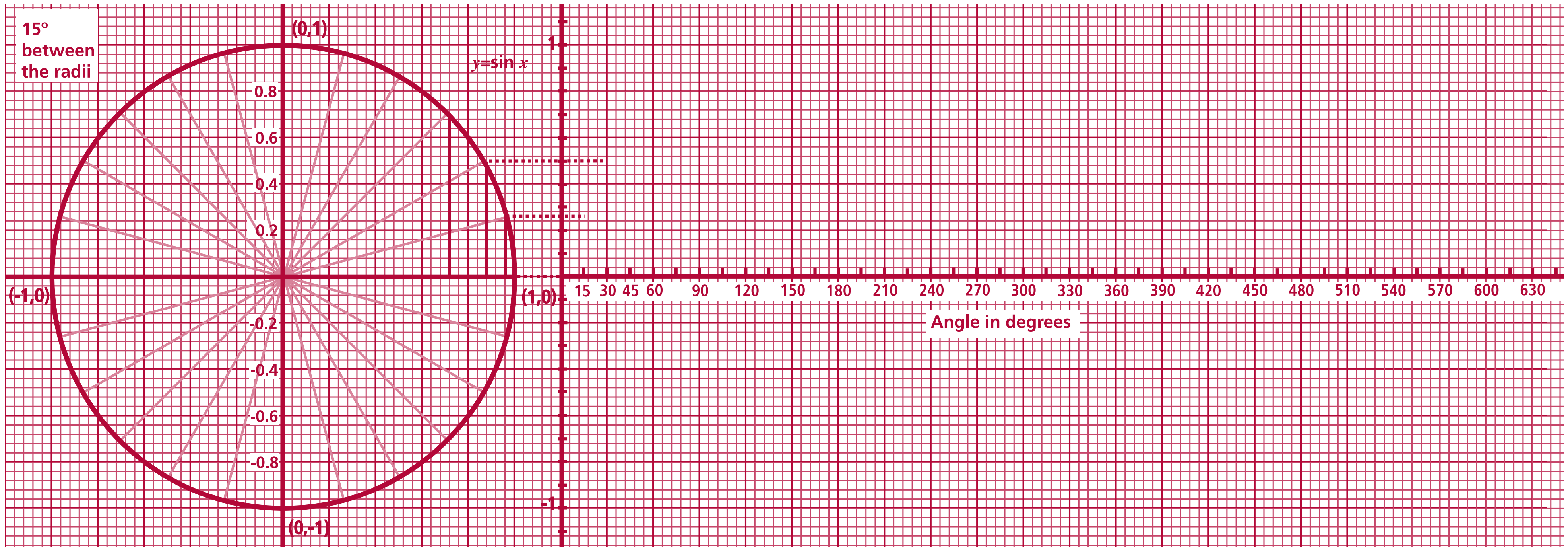
11. Given that the period of *f(x)* = *a* sin *bx* is pradians and the range is $[-2,2]$ sketch a graph of the function on the graph paper provided below for the domain 0 to 4.



Student Activity 1

Student Activity 1A

Graph of y= sin x Complete the projection of sin values from the unit circle onto the Cartesian plane on the right and then join the points with a smooth curve.



Student Activity 1B

1. Describe the graph of y = sin x. ______
2. What is the period of y = sin x? ______
3. What is the range of y = sin x? ______
4. Is y = sin x a function? Explain. ______
5. Is the inverse of y = sin x a function? Explain. ______
6. Using the graph solve for x the equation sin x = 0.5 ______
7. How many solutions has the equation in Q6? ______

Student Activity 2

Project Maths Tionscadal Mata Development Team

Using a calculator, or the unit circle, fill in the table for the following graphs and plot all of them using the same axes.
$y = \sin x$, $y = 2\sin x$, $y = 3\sin x$ Use different colours for each graph.

$x/^o$	-90	-60	-30	0	30	60	90	120	150	180	210	240	270	300	330	360
$\sin x$																
$2\sin x$																
$3\sin x$																

	Period	Range
$y = \sin x$		
$y = 2\sin x$		
$y = 3\sin x$		
$y = a\sin x$		

In the function, what is the effect on the graph of varying a in asin x? ______________________

____

____

____

____

Student Activity 3

Fill in the table first, and using the same axes but different colours for each graph, draw the graphs of:
$y = \sin x$, $y = \sin 2x = \sin (2 \times x)$, $y = \sin 3x = \sin (3 \times x)$ Graphs of the form $y = \sin bx$

$x/^o$	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360
$\sin x$																									
$2x$																									
$\sin 2x$																									
$3x$																									
$\sin 3x$																									

	Period	Range
$y = \sin x$		
$y = \sin 2x$		
$y = \sin 3x$		
$y = \sin bx$		

In the graph of $y = \sin bx$, what is the effect on the graph of varying b in $\sin bx$?____________________

Student Activity 4

Using a table, find the coordinates for the following graphs and plot all of them using the same axes: $y = \cos x$, $y = 2\cos x$, $y = 3\cos x$

$x/^o$	-90	-60	-30	0	30	60	90	120	150	180	210	240	270	300	330	360
$\cos x$																
$2\cos x$																
$3\cos x$																

	Period	Range
$y = \cos x$		
$y = 2\cos x$		
$y = 3\cos x$		

In the function $y = a\cos x$, what is the effect on the graph of varying a in $a\cos x$? ______________________________

____

____

____

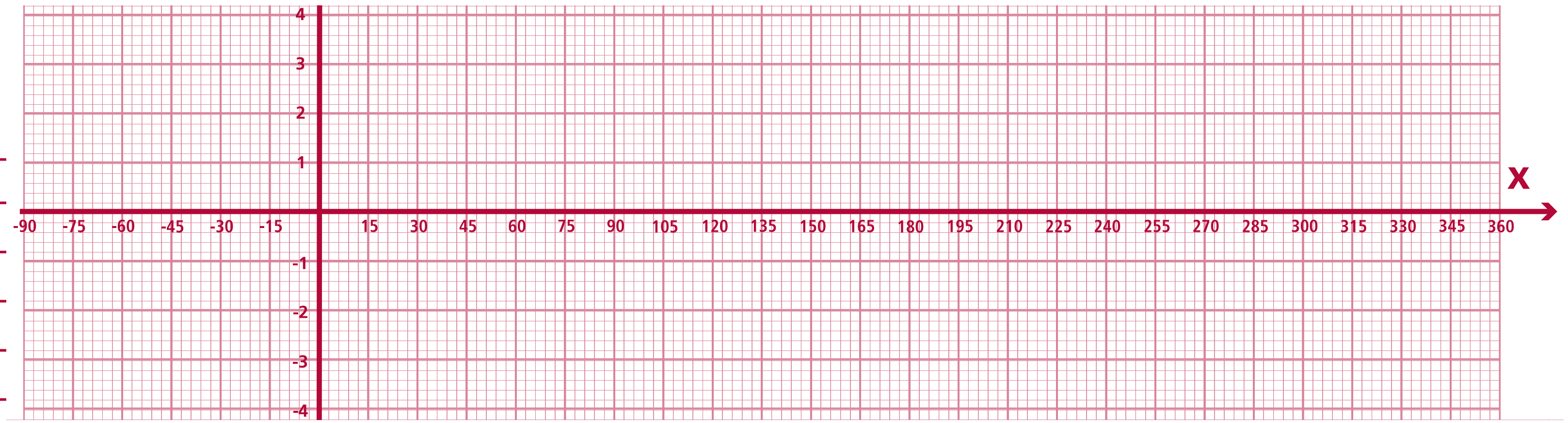
Student Activity 5

Project Maths Tionscadal Mata Development Team

By filling in a table first, and using the same axes but different colours for each graph, draw the graphs of

$x/^o$	0	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360
$\cos x$																									
$2x$																									
$\cos 2x$																									
$3x$																									
$\cos 3x$																									

	Period	Range
$y = \cos x$		
$y = \cos 2x$		
$y = \cos 3x$		
$y = \cos bx$		



In the graph of $y = a\cos bx$, what is the effect on the graph of varying b in $\cos bx$? ______________________

____

____

Student Activity 6

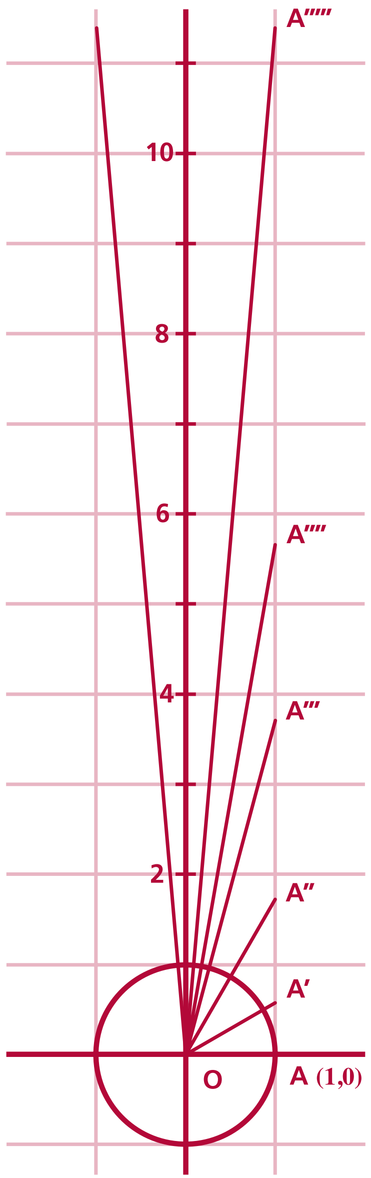
Project Maths Tionscadal Mata Development Team

Sketch each of the following graphs: $(0° \leq x \leq 360°)$ $y = 4\sin x$ Period =________________ Range =________________

Sketch each of the following graphs: $(0° \leq x \leq 360°)$ $y = \cos 4x$ Period =________________ Range =________________

Sketch each of the following graphs: $(0° \leq x \leq 360°)$ $y = 2\sin 3x$ Period =________________ Range =________________

Student Activity 7



Student Activity 7A

$\lvert\angle AOA'\rvert$	$\lvert\angle AOA''\rvert$	$\lvert\angle AOA'''\rvert$	$\lvert\angle AOA''''\rvert$	$\lvert\angle AOA'''''\rvert$
30.00°	60.00°	75.00°	80.00°	85.00°

$y = \tan x$

Using the diagram of the unit circle, read the approximate value of the tan of the angles in the table using the trigonometric ratios.

Angle θ/°	0	30	60	75	80	85
$\tan\theta$						

 www.projectmaths.ie

Student Activity 7

Student Activity 7B

By filling in a table first, and using the same axes but different colours for each graph, draw the graphs of

$x/^o$	-90	-75	-60	-45	-30	0	30	45	60	75	90	105	120	135	150	180	210	225	240	255	270	285	300	330	360
$\tan x$																									

Student Activity 7C

$y = \tan x$	Period	Range

