Linear Patterns

ARITHMETIC SEQUENCES AND SERIES

Project Maths
Tionscadal Mata
Development Team

Linear Patterns

Linear Patterns

Junior Certificate 4.1 to 4.4

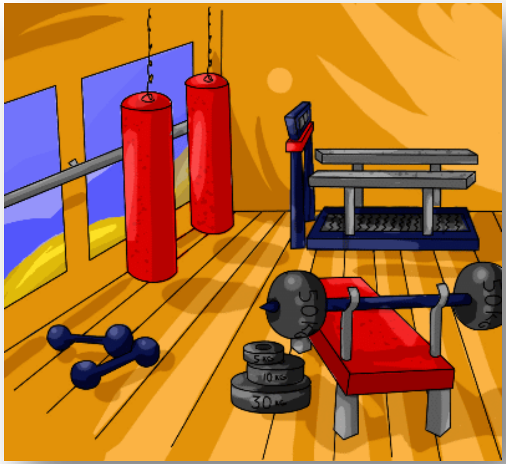
Arithmetic Sequences and Series

Leaving Certificate Section 3.1

Project Maths
Tionscadal Mata
Development Team

Gym Costs

Gym	Cost
Membership fee	€50
Cost per visit	€10



A. Is there a pattern to the cost of this gym? Why?

B. Can you predict how much the gym will cost for 9 visits?

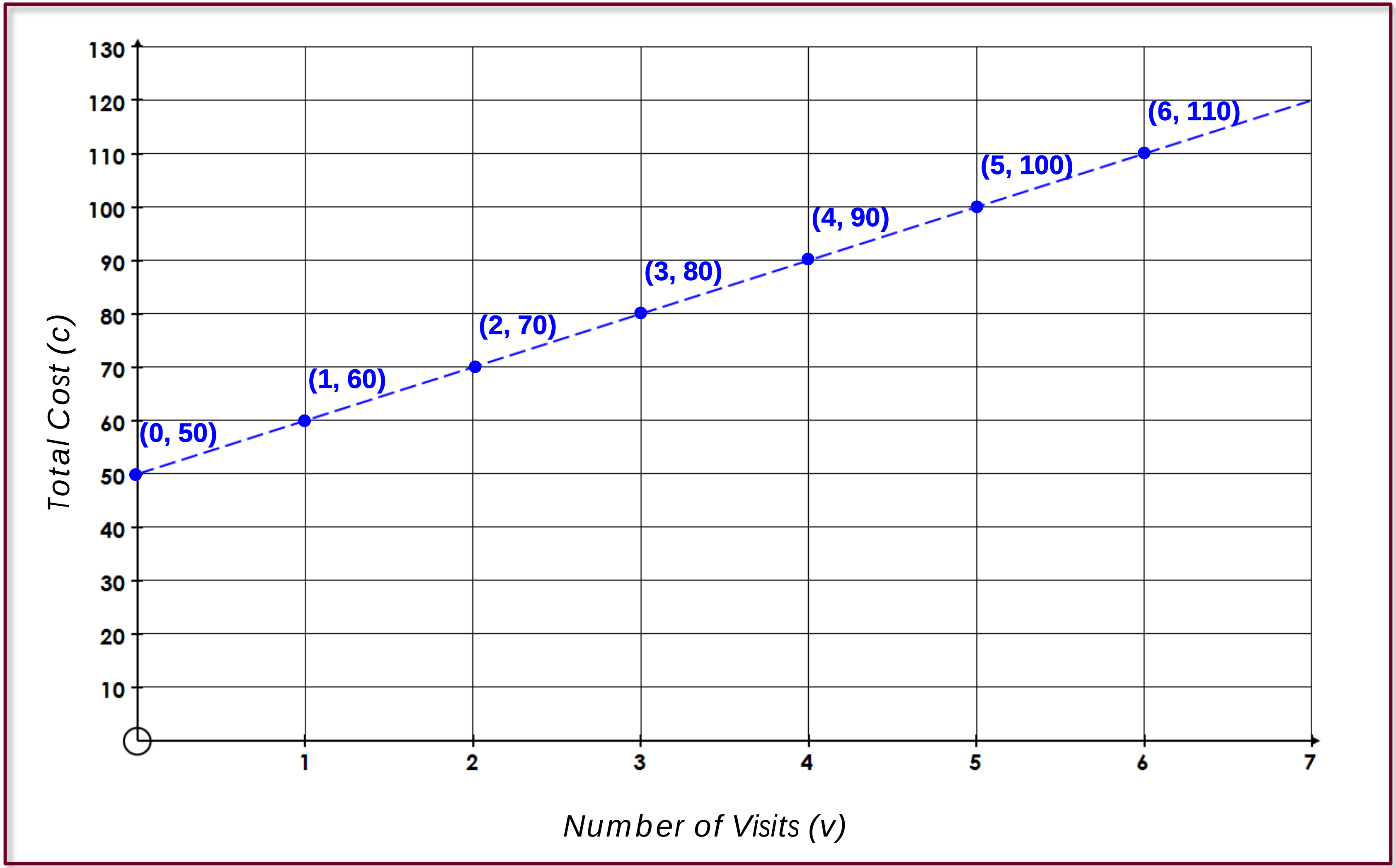
C. How can we investigate if a pattern exists?

Project Maths Tionscadal Mata Development Team ©

Gym Costs

Number of visits to Gym	*Total cost c in €*	*Total cost c in €*
0	50	50
1	50 + 10	60
2	50 + 10 + 10	70
3	50 + 10 + 10 + 10	80
4	50 + 10 + 10 + 10 + 10	90
5	50 + 10 + 10 + 10 + 10 + 10	100
6	50 + 10 + 10 + 10 + 10 + 10 + 10	110

Graph of Gym Costs



Which is the independent variable and which is the dependent variable?

What is the Pattern of the Gym costs?

Number of visits	***Total cost c***	***Pattern***
Total cost of 0 Visits	50	50 + 0(10)
Total cost of 1 Visit	50 + 10	50 + 1(10)
Total cost of 2 Visits	50 + 10 +10	50 +2(10)
Total cost of 3 Visits	50 + 10 +10 +10	50 + 3(10)
Total cost of 4 Visits	50 + 10 +10 +10 +10	50 + 4(10)
Total cost of 6 Visits	50 + 10 + 10 + 10 + 10 +10 + 10	50 + 6(10)

Constant

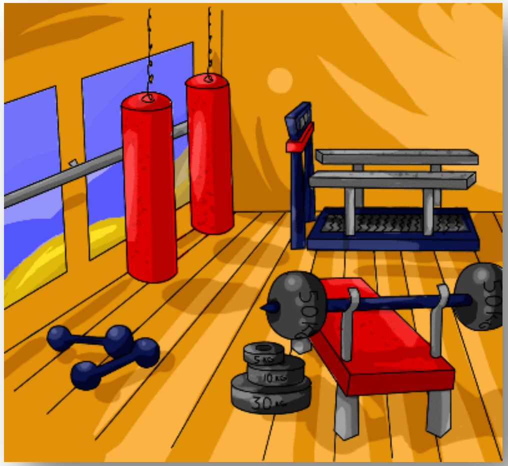
$$\text{Total Cost } c\text{, of } v \text{ visits} = 50 + 10v$$

Variable

The total cost *c*, of *v* visits is €50 plus €10 per visit

Cost Pattern for the Gym

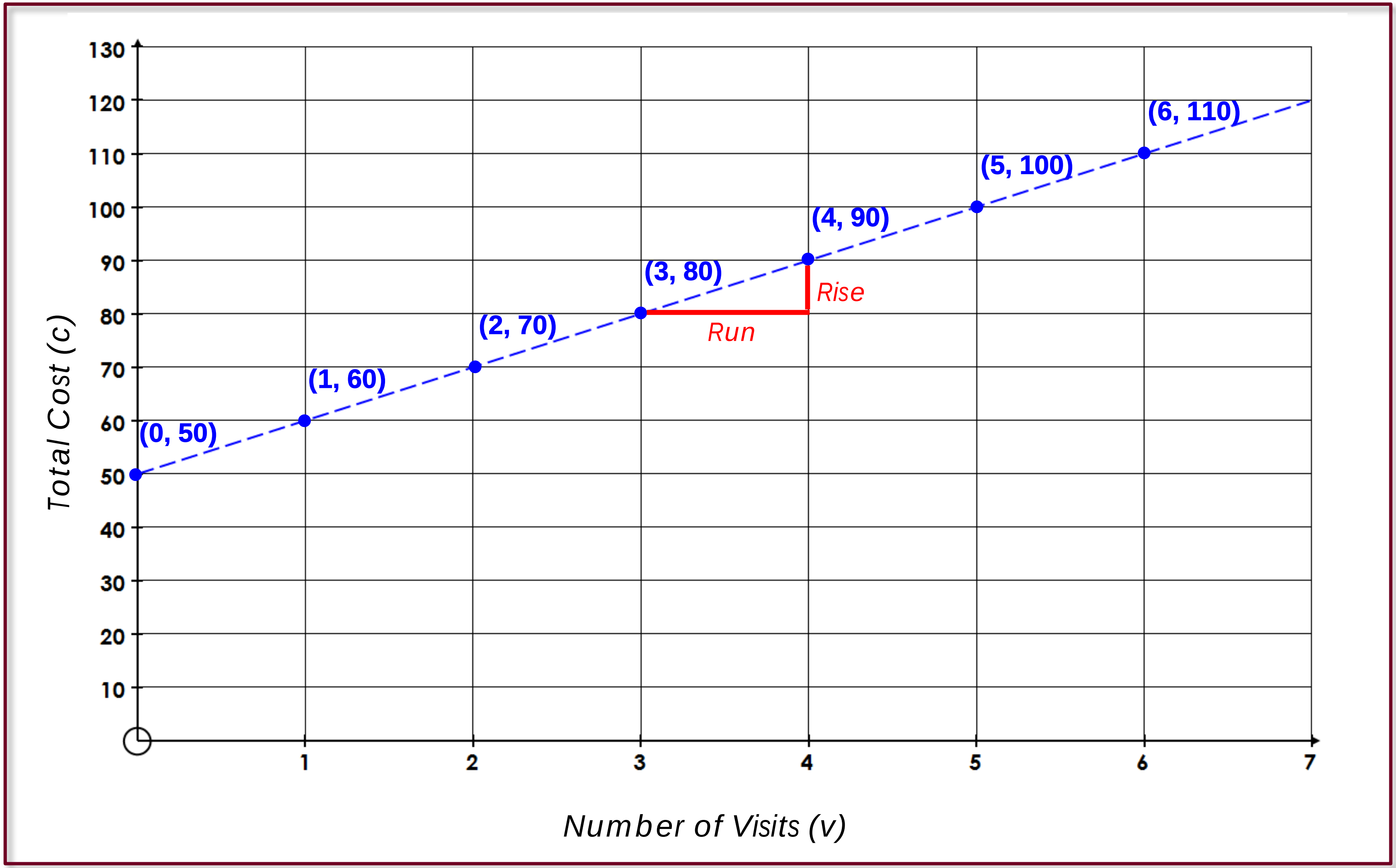
Visits	*Total cost c in €*	*Change*
0	50	
1	60	+10
2	70	+10
3	80	+10
4	90	+10
5	100	+10
6	110	+10



What type of pattern is modelled and why?

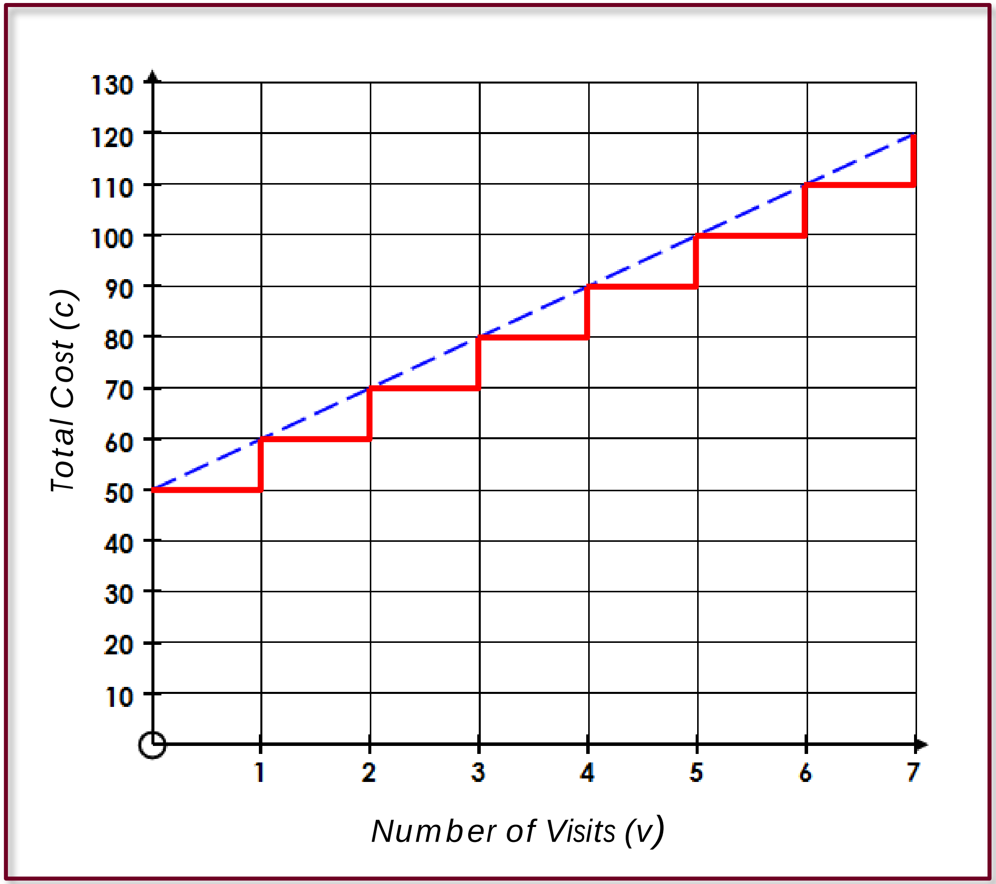
Linear Pattern

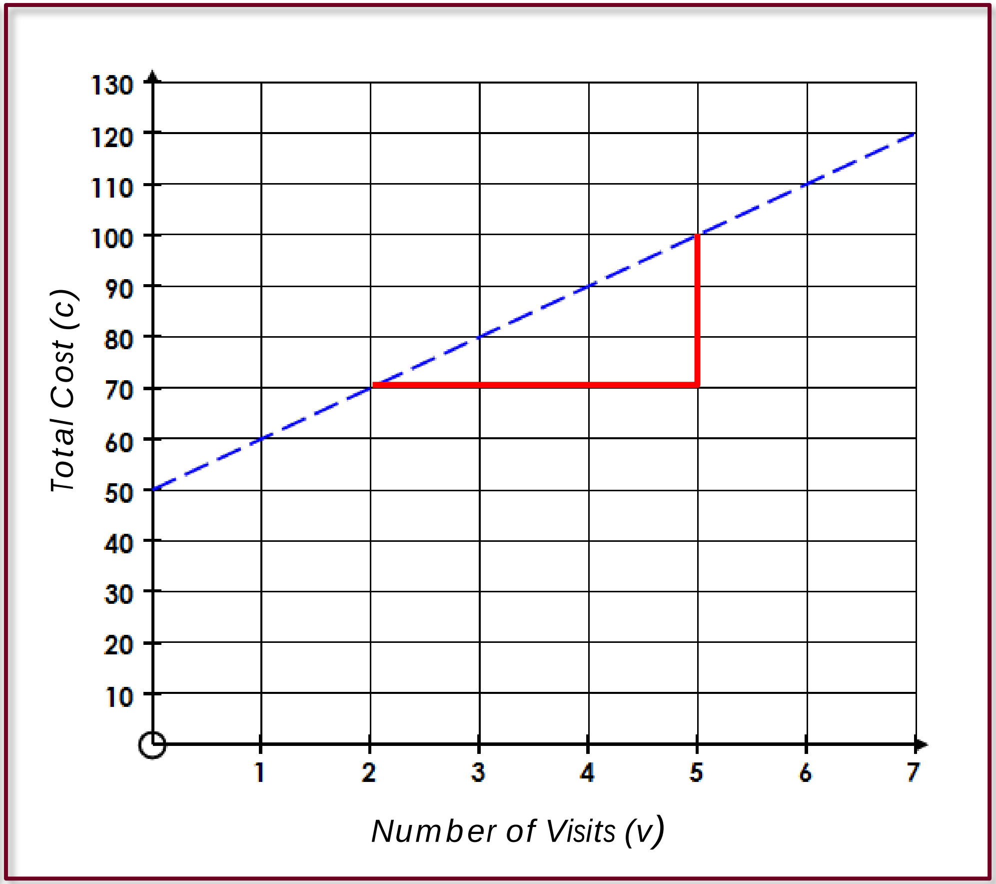
Rate of Change of the Gym costs



$$\text{Rate of change} = \text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{10}{1} = 10$$

For Linear Patterns Slope is the Same in each Interval

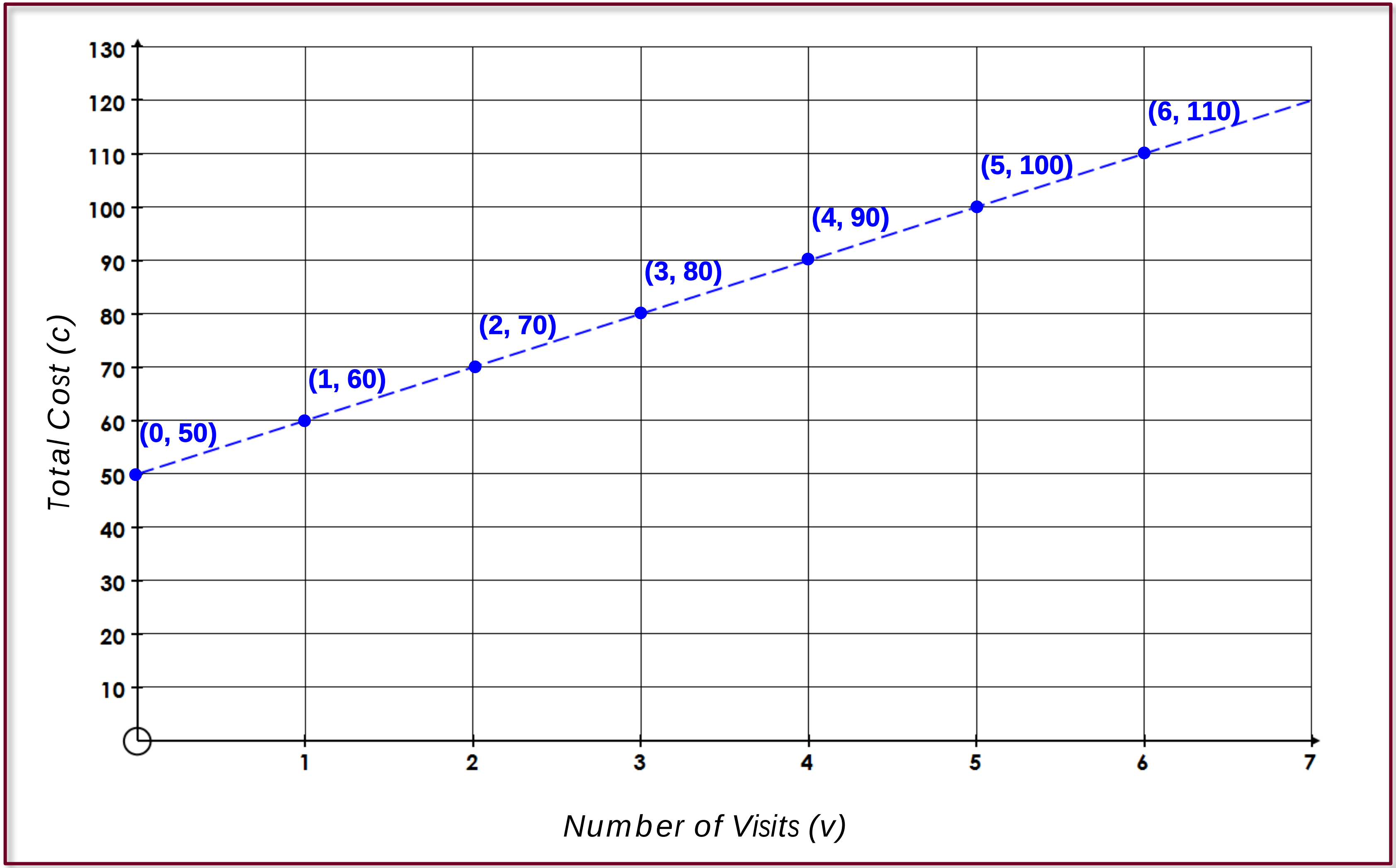




$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{10}{1} = 10$$

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{30}{3} = 10$$

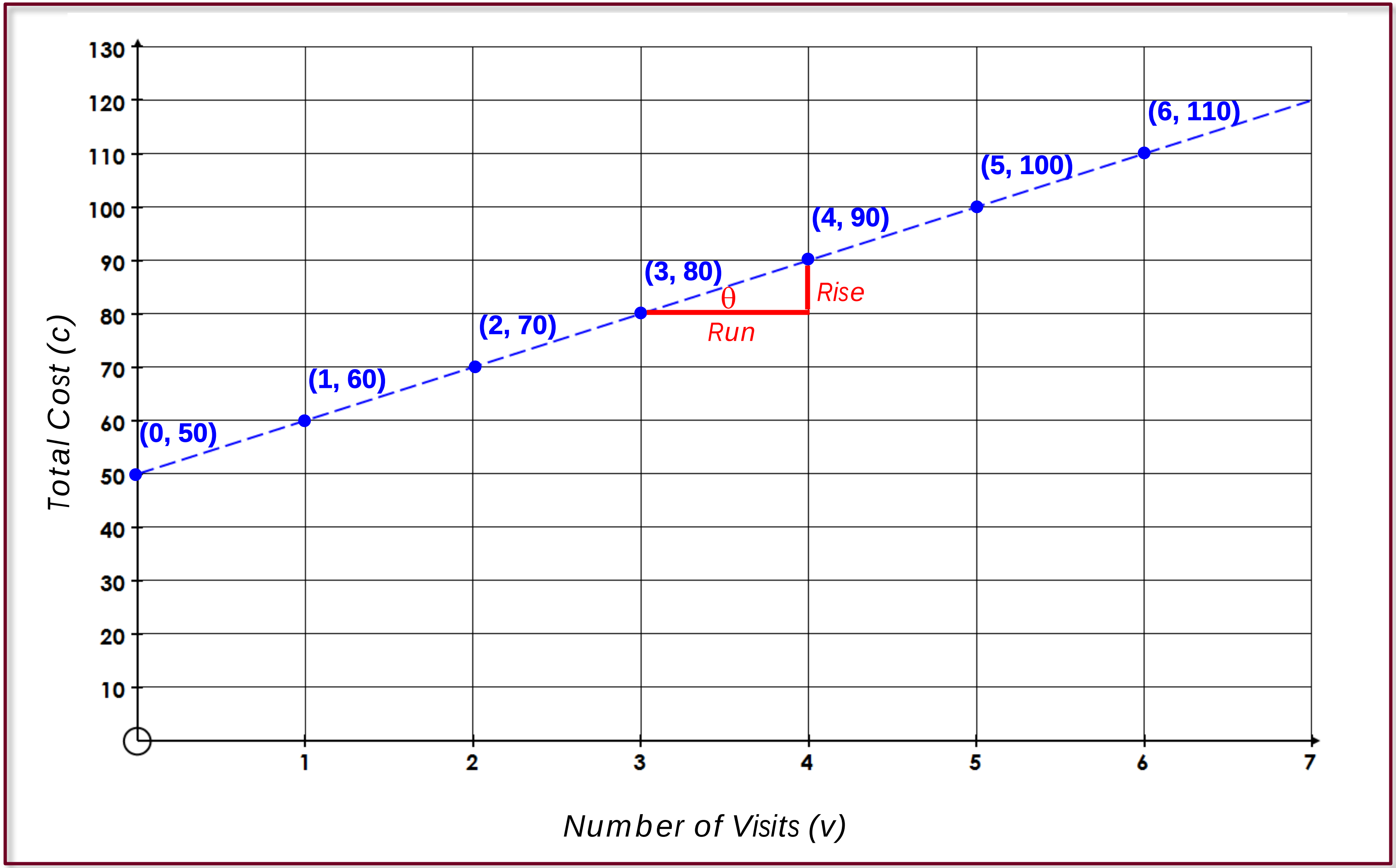
Pattern 50 + 10v



Total Cost $c = 50 + 10v$

Where is the 50 and the 10 represented on this graph?

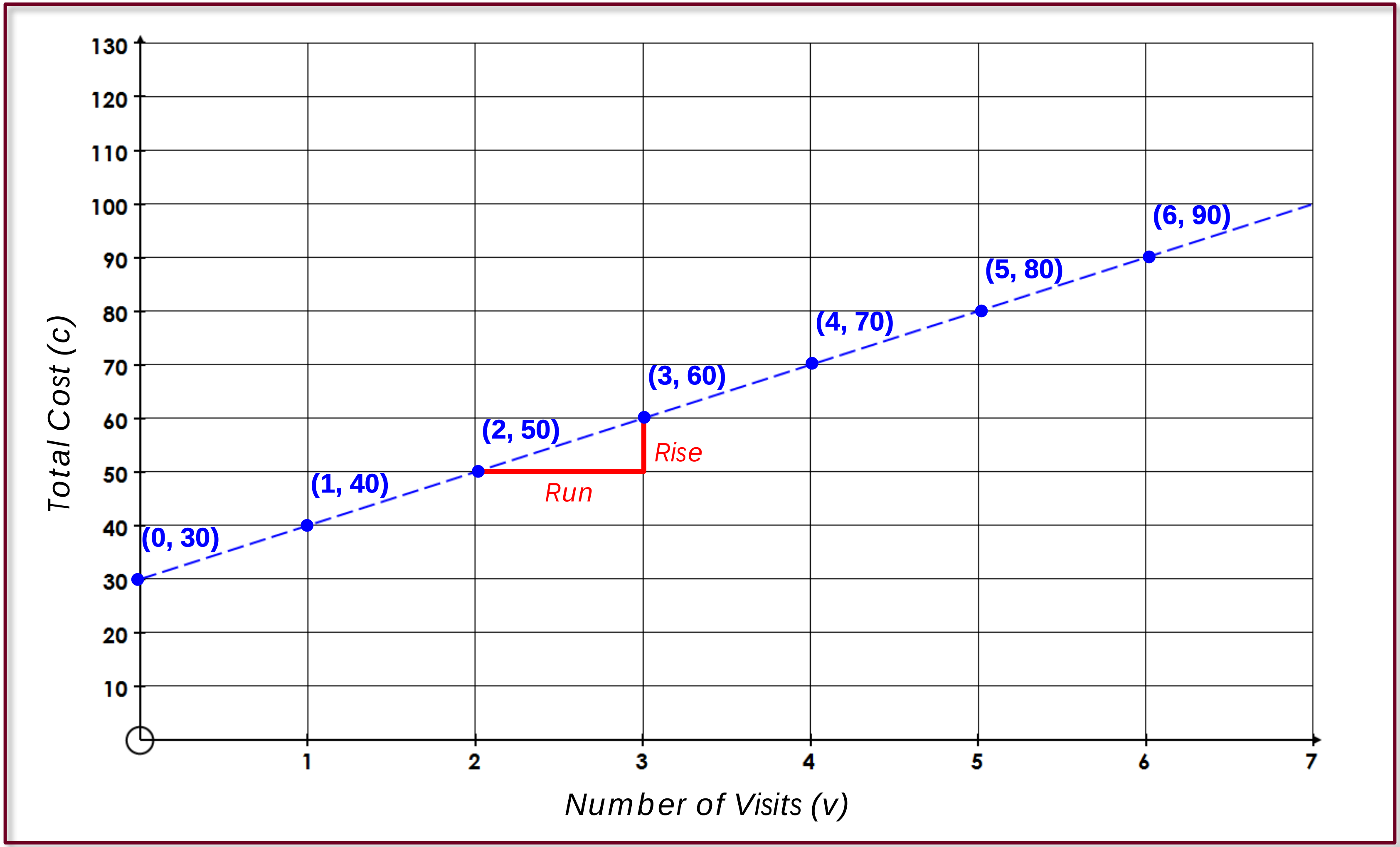
Alternative Ways of Finding the Slope or Rate of Change



$$\text{Slope} = \text{Rate of change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{90 - 80}{4 - 3} = 10$$

$$\text{Slope} = \tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\text{Rise}}{\text{Run}} = \frac{10}{1}$$

Determine the Costs of Gym B

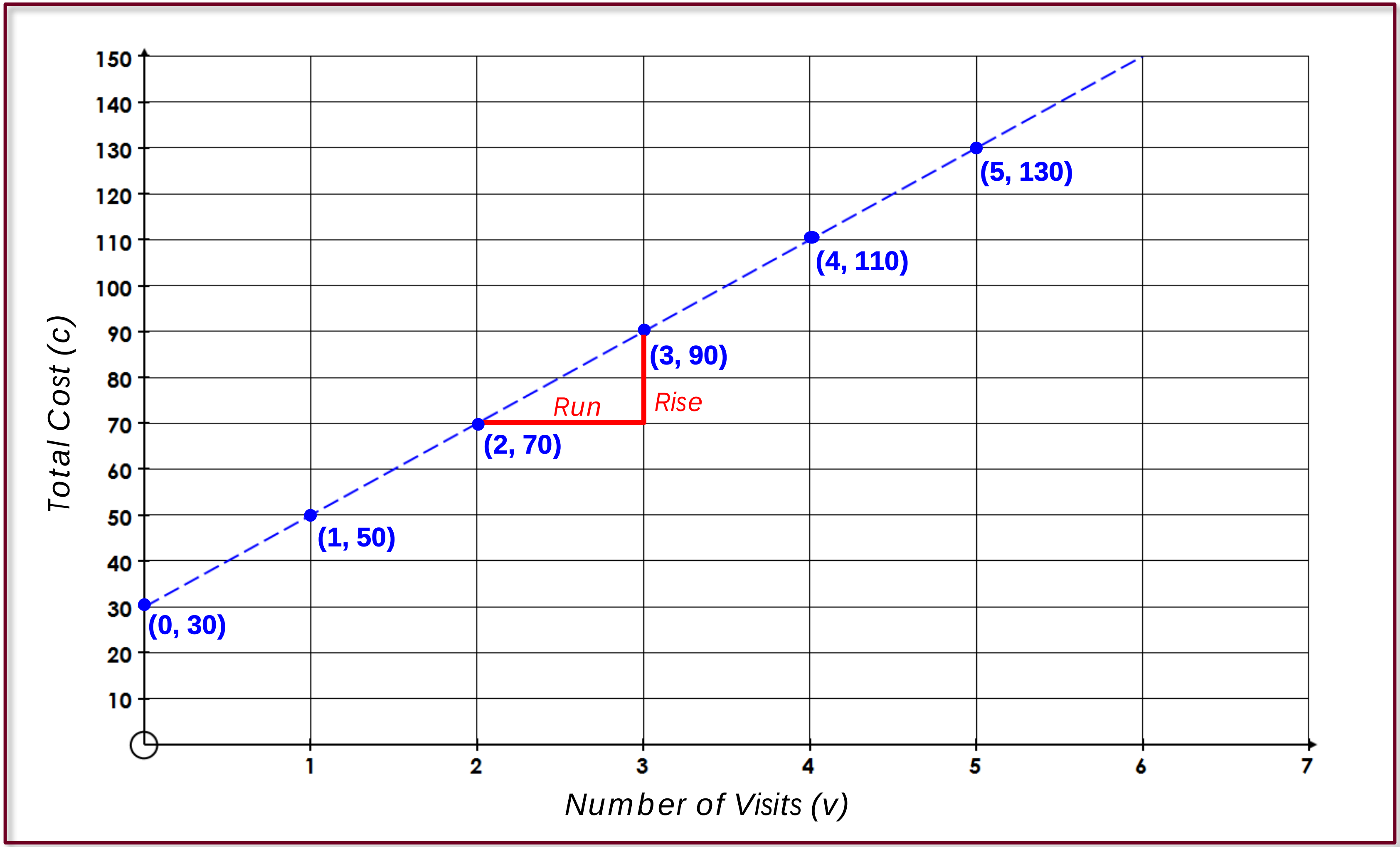


Membership fee of €30 and cost per visit of €10

Total cost in euro equals initial cost (30) + 10(number of visits)

$c = 30 + 10v$

Determine the Costs of Gym C



Membership fee of €30 and cost per visit of €20

$c = 30 + 20v$

Total cost in euro equals initial cost (30) + 20(number of visits)

Tree Ring Dating

Using tree ring dating does the growth rate of the circumference of this tree model a linear pattern? Explain your reasoning.

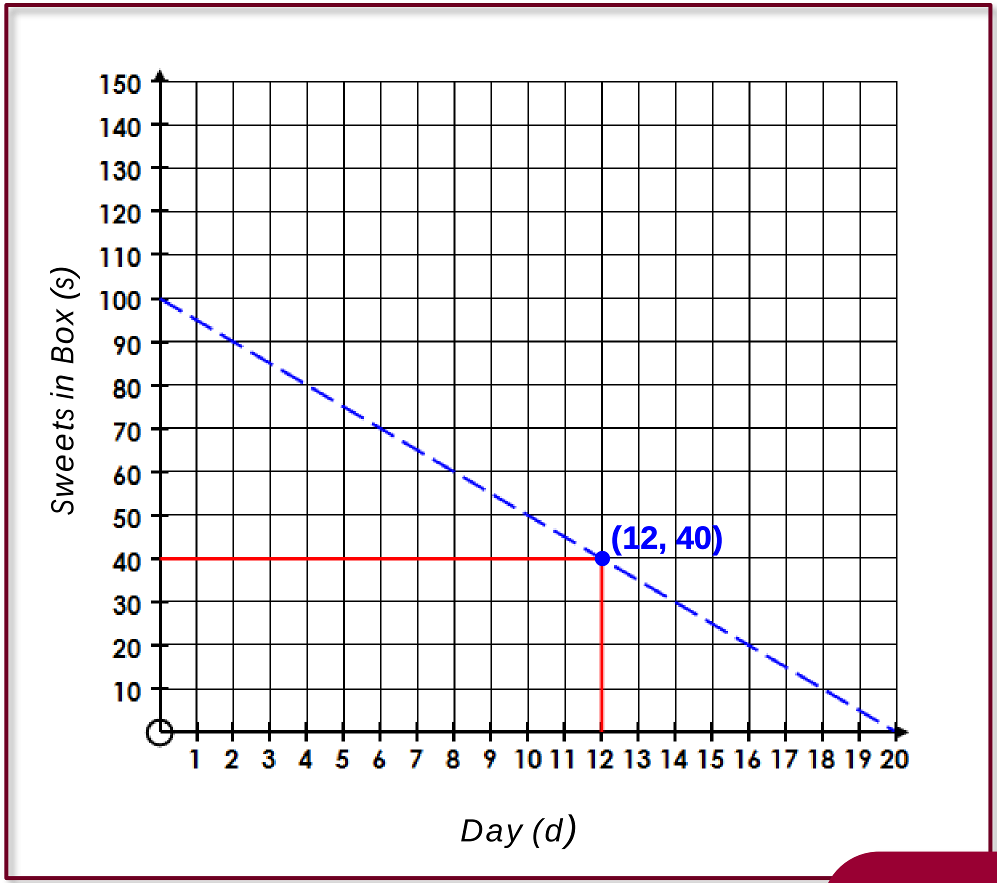
Joan's Sweet Box

Joan got a present of a box of 100 sweets.
She decides to eat 5 sweets per day from the box.
Represent this information in a table, a graph and a formula.
After how many days will she have exactly 40 sweets left?

Day Number	Sweets
0	100
1	95
2	90
3	85
4	80

Sweets *s*, in box on day, *d*.

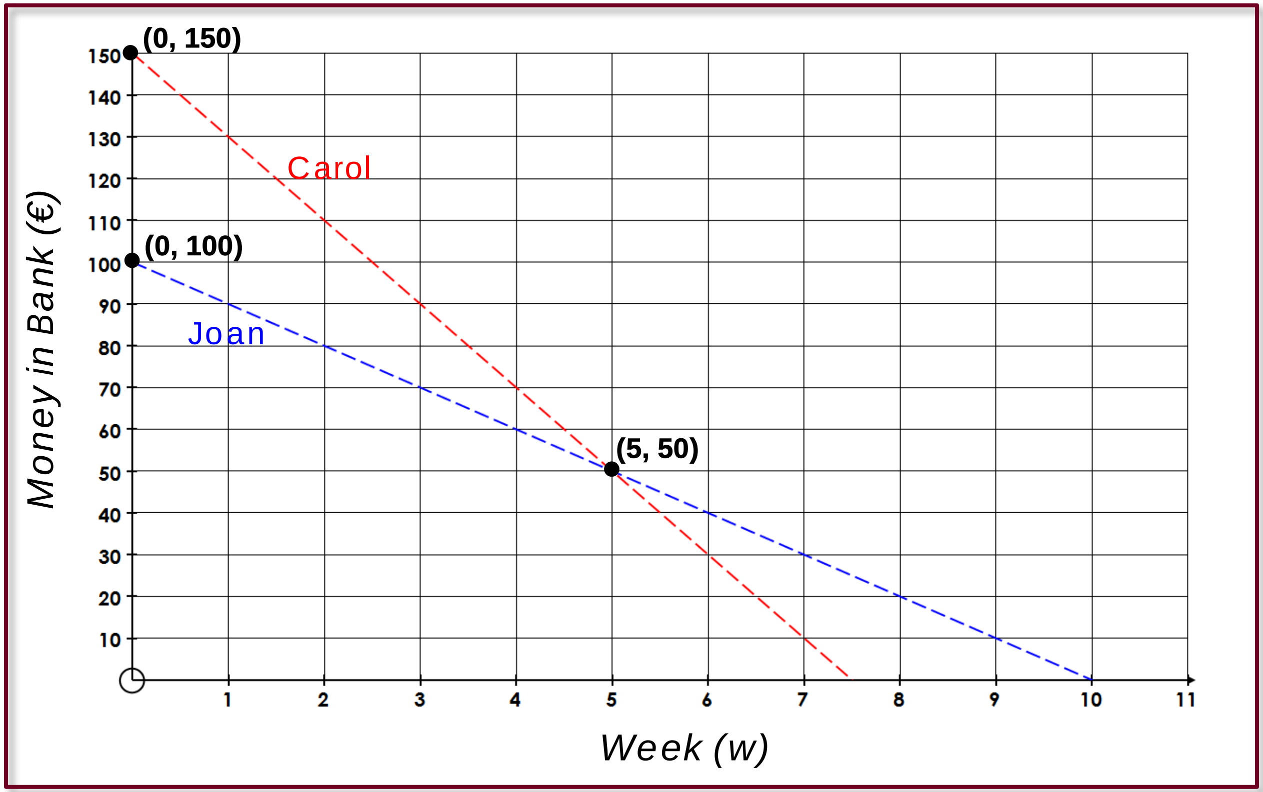
$s = 100 - 5d$



Project Maths Tionscadal Mata Development Team

Spending Habits

Given Carol's and Joan's spending habits are modelled by the pattern in the diagram below describe each of their spending habits and represent this with relevant formulas?



Carol has €150 to start with and she spends €20 per week.

$s(w) = 150 - 20w \quad f(x) = 150 - 20x$

Joan has €100 to start with and she spends €10 per week.

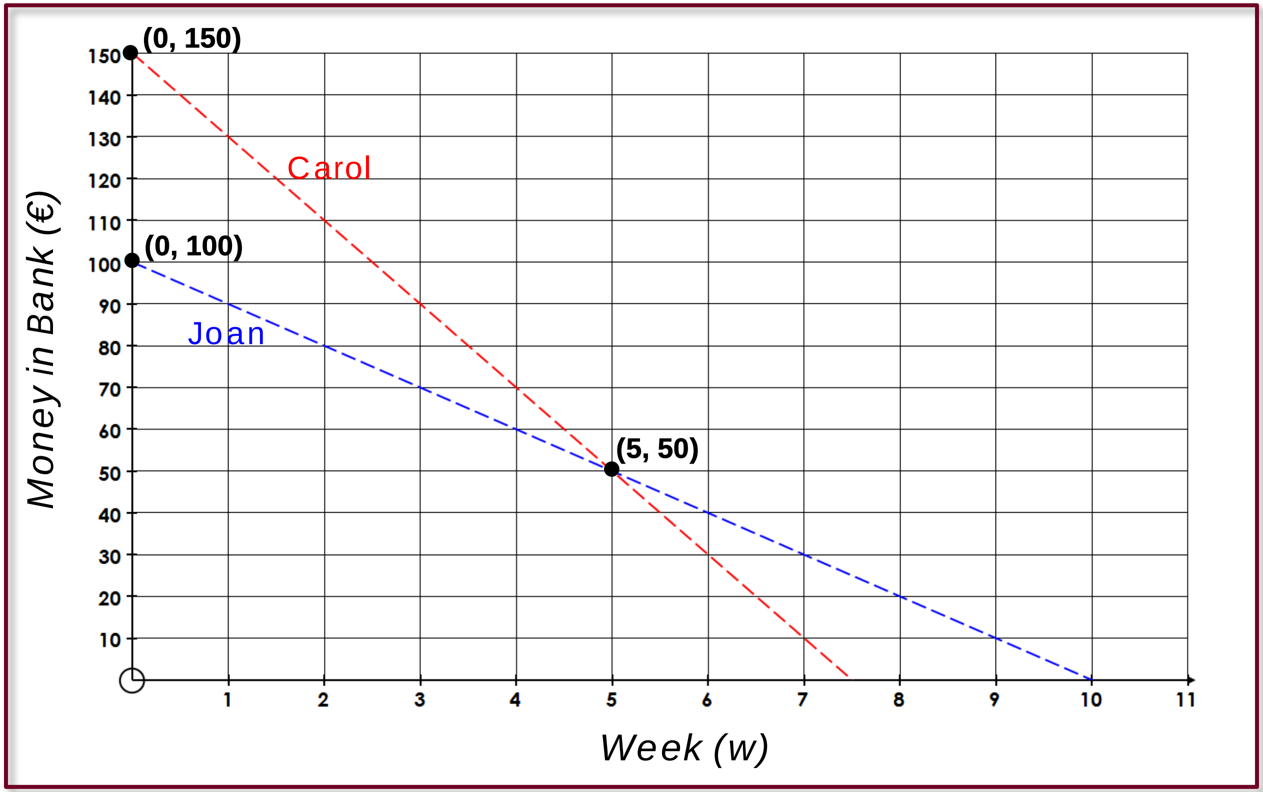
$s(w) = 100 - 10w \quad f(x) = 100 - 10x$

w represents weeks and $s(w)$ is the savings per week.

Project Maths Tionscadal Mata Development Team

Spending Habits

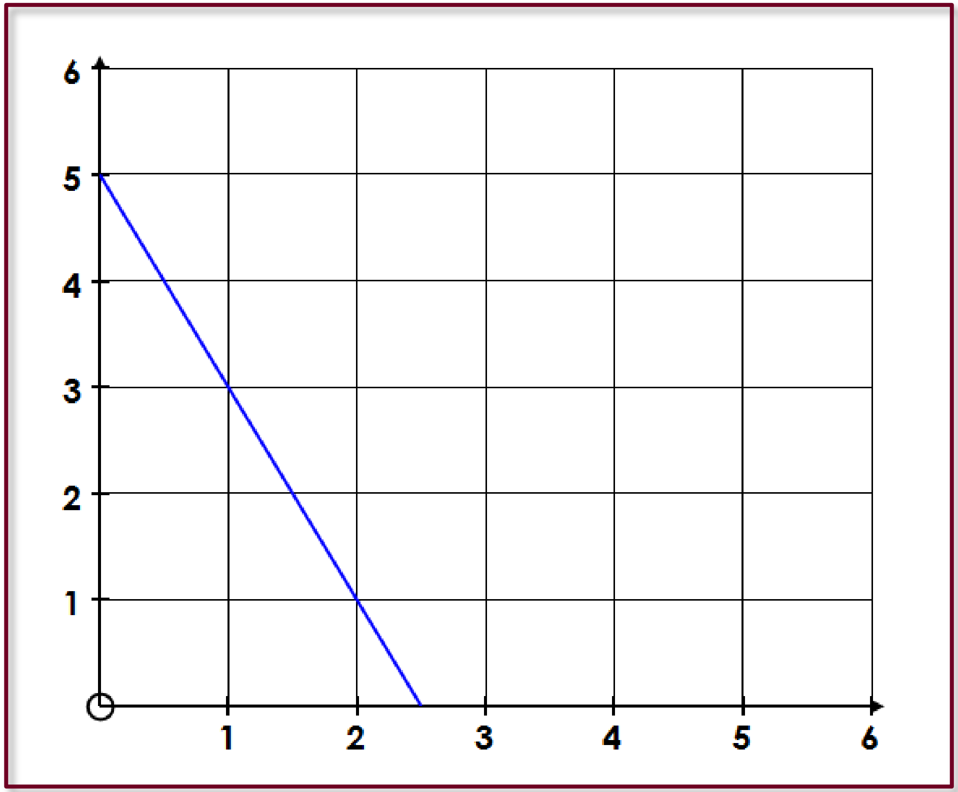
At the beginning of which week will Joan have no money and at the beginning of which week will Carol have no money?

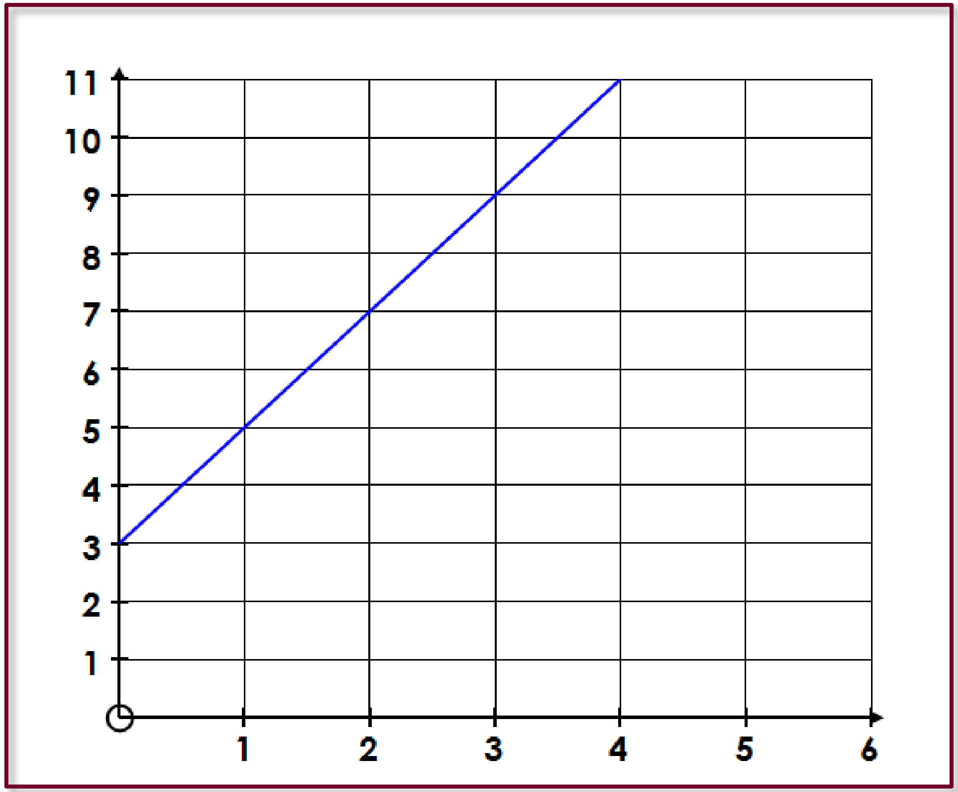


Joan has no money at the beginning of week 11

Carol has no money at the beginning of week 8, in fact she only has €10 during week 7

Give the Equation of the Following Lines





$y = 5 - 2x$

$y = -2x + 5$

$2x + y = 5$

$f(x) = 5 - 2x$

$y = 3 + 2x$

$y = 2x + 3$

$2x - y = -3$

$f(x) = 2x + 3$

$f(x) = ax + b$

x	$f(x) = ax + b$	Change
0	$f(0) = b$	
1	$f(1) = a + b$	a
2	$f(2) = 2a + b$	a
3	$f(3) = 3a + b$	a
4	$f(4) = 4a + b$	a
5	$f(5) = 5a + b$	a

If $f(x) = ax + b$, the rate of change(slope) is always equal to a.

Project Maths Tionscadal Mata Development Team

Examples of Linear Patterns

- Constant savings patterns with no interest
- Constant spending patterns
- ESB bills
- Mobile phone standing charge + charge per texts
- Call out charge for a workman + hourly rates
- Distance travelled by a car travelling at a constant speed
- Plant growing a constant amount per day
- Level of water in a tank as water is pumped from the tank at a constant pace
- Level of water in a tank when water enters the tank at a constant pace
- Vehicle depreciating in value by a constant amount each year (Straight Line depreciation) (This is not the reducing balance method.)

Project Maths
Tionscadal Mata
Development Team

What do you notice about these pictures?

Are these pictures in proportion?

How could you tell if they are in proportion?

If you double the length you must double the width

If you multiply the length by n you must multiply the width also by n

Enlargements are proportional relationships

Proportional and non Proportional Situations

$$f : x \to 2x$$

$$f : x \to 3x$$

$$f : x \to nx$$

Notice when $x = 0$, $y = 0$ so these functions always go through (0, 0)

Project Maths Tionscadal Mata Development Team ©

Proportional and non Proportional Situations

$$f(x) = x + 4$$

$$f(1) = 5$$

$$f(2) = 6$$

This function is non proportional

Proportional always linear and of the form y=mx

Proportional and non Proportional Situations

- It is linear of the form $y = mx$
- It passes through the origin, has no ‘start-up’ value
- If x is doubled (or increased by any multiple) then y is doubled (or increased by the same multiple)

Multi-Representational Approach

Story	Table	Graph	Formula	Which of the following apply? Linear, non linear, proportional or non proportional? Justify

Project Maths
Tionscadal Mata
Development Team

Learned so Far

- Tables, Graphs, Words and Formula (Multi Representational Approach)
- Linear Patterns
- Rate of change $= \dfrac{\text{Rise}}{\text{Run}} =$ Slope
- Equation of the line is $y = mx + c$, where m is the rate of change (slope) and c is where the line cuts the y–axis (y intercept)
- Variables and Constants
- Independent and dependent variables
- Proportional and non proportional situations

Project Maths
Tionscadal Mata
Development Team

Original Gym A Problem: Membership €50 and €10 per visit

	Total cost
0	50
1	60
2	70
3	80
4	90

Term	Pattern	Formula
1	50	a
2	50 +10	$a + d$
3	50 +10 + 10	$a + d + d$
4	50 +10 + 10+10	$a + d + d + d$
5	50 +10 + 10+10 +10	$a + d + d + d + d$

Sequence = {50, 60, 70, 80, 90, ….} (a = first term) (d = common difference)

Can you predict what the 10th term will be in terms a and d?

$a + 9d$

Can you predict what the nth term of the sequence will be in terms of a, d and n?

$T_n = a + (n - 1)d$

Formula and Tables booklet page 22

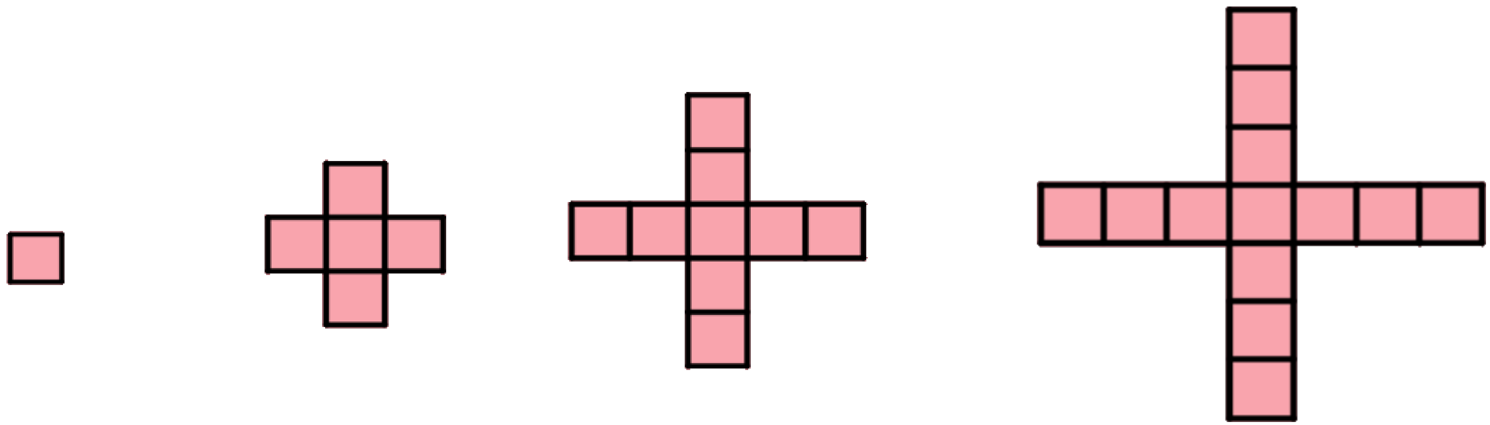
n is the position of the term in the sequence, not the number of visits to the gym

Arithmetic Sequences (Linear Patterns)

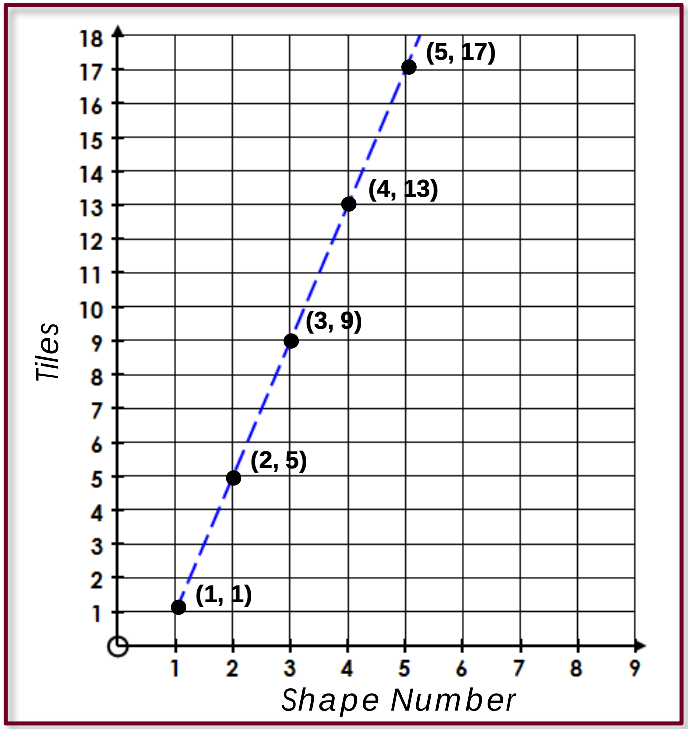
Sequences that have a first term, *a* and you add a common difference, *d* to the previous term to get the next term are known as arithmetic sequences.

1, 2, 3, 4, 5,…	Yes
1, 2, 4, 8, 16,…	No

Complete the next term of the following pattern and complete a table, graph and formula for the pattern.



Shape Number	Tiles
1	1
2	5
3	9
4	13
5	17



$T_n = 1 + (n-1)4$

$T_n = 4n - 3$

n is the shape number, $n \in N$

Joan joins a DVD club. It costs €12.00 to join the club and any DVD she rents will cost an extra €2. Jonathan joins a different DVD club where there is no initial charge, but it costs €4 to rent a DVD. Represent these two situations in a table. List the sequence that represents the cost of the DVDs to Joan and Jonathan. Are these sequences arithmetic sequences and why? In terms of *a* the first term, *d* the common difference and *n* the number of the terms, derive a formula for T_n for these sequence.

DVDs	**Cost to Joan**	**Cost to Jonathan**	**Term**
0	12	0	1
1	14	4	2
2	16	8	3
3	18	12	4
4	20	16	5
5	22	20	6

Joan	**Jonathan**
$T_n = a + (n-1)d$	$T_n = a + (n-1)d$
$T_n = 12 + (n-1)2$	$T_n = 0 + (n-1)4$
$T_n = 10 + 2n$	$T_n = 4n - 4$

n is the position of the term in the respective sequence

Joan {12, 14, 16, 18, 20, 22, 24, ...}

Jonathan {4, 8, 12,16, 20, 24, 28, ...}

Given the nth term of a arithmetic sequence is $T_n = 2n+3$.
What is the $(n + 1)^{th}$ term?

$$T_{n+1} = 2(n+1)+3$$
$$T_{n+1} = 2n+5$$

Given we are dealing with an arithmetic sequence in terms of a, d and n what should $T_{n+1} - T_n$ equal?
Answer: d

Was this true for this sequence?

$T_{n+1} - T_n$	$T_2 - T_1$
$= 2n+5-[2n+3]$	$T_1 = 5$ and $T_2 = 7$
$= 2$	$= 7-5 = 2$
	$= d$

To prove $T_{n+1} - T_n = d$ for all arithmetic sequences

$$T_{n+1} - T_n$$
$$= a+(n+1-1)d-[a+(n-1)d]$$
$$= a+nd-a-nd+d$$
$$= d$$

Project Maths
Tionscadal Mata
Development Team

Three Consecutive Terms

An arithmetic sequence is such that $T_n = 4n + 3$.
Is it possible to find three consecutive terms of this sequence such that their sum is equal to 117 and if so find these terms.

$$T_n = 4n + 3$$

$$T_{n+1} = 4(n+1) + 3$$

$$T_{n+2} = 4(n+2) + 3$$

$$T_n + T_{n+1} + T_{n+2} = 117$$

$$4n + 3 + 4(n+1) + 3 + 4(n+2) + 3 = 117$$

$$12n + 21 = 117$$

$$12n = 96$$

$$n = 8$$

$$T_8 = 4(8) + 3 = 35$$

$$T_9 = 4(9+1) + 3 = 39$$

$$T_{10} = 4(10+2) + 3 = 43$$

Project Maths Tionscadal Mata Development Team ©

Emma earns €300 during her first week in the job and each week after that she earns an extra €20 per week.

(a) How much does she earn during her tenth week in this job?

(b) How much does she earn in total during the first ten weeks in this job?

(a)

$$T_{10} = 300 + 9(20)$$

$$T_{10} = 300 + 180$$

$$T_{10} = 480$$

Hence on her tenth week she earns €480

(b) The sequence is as follows: 300, 320, 340, 360, 380, 400, 420, 440, 460, 480

Hence she earns €3900 in the first ten weeks in the job.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2(300) + (10-1)20\right]$$

$$S_{10} = 3900$$

Project Maths Tionscadal Mata Development Team

The first ten terms of the series is:

$$300 + 320 + 340 + 360 + 380 + 400 + 420 + 440 + 460 + 480 = 3900$$

Proof of Formula

The sum of the first *n* terms of an arithmetic series

(Students will not be required to prove this formula.)

$$S_n = T_1 + T_2 + T_3 + T_4 + \cdots + T_{n-1} + T_n$$

$$S_n = a + a + d + a + 2d + a + 3d + \cdots + a + (n-2)d + a + (n-1)d \qquad \textbf{[1]}$$

Note S_n can also be written as: $T_n + T_{n-1} + \cdots + T_4 + T_3 + T_2 + T_1$

Writing Sn in reverse:

$$S_n = a + (n-1)d + a + (n-2)d + \cdots + a + 3d + a + 2d + a + d + a \qquad \textbf{[2]}$$

Adding **[1]** and **[2]**

$$S_n = a + a + d + a + 2d + a + 3d + \cdots + a + (n-2)d + a + (n-1)d$$

$$S_n = a + (n-1)d + a + (n-2)d + \cdots + a + 3d + a + 2d + a + d + a$$

$$2S_n = \{2a + (n-1)d\} + \{2a + (n-1)d\} + \{2a + (n-1)d\} + \cdots + \{2a + (n-1)d\} + \{2a + (n-1)d\}$$

$$2S_n = n\{2a + (n-1)d\}$$

$S_n = \frac{n}{2}\{2a + (n-1)d\}$ Formula as per formula and tables booklet

Summing the Natural Numbers

Find the sum of the first 100 natural numbers.

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$a = 1 \quad d = 1 \quad n = 100$

$$S_{100} = \frac{100}{2}[2(1) + (100-1)1]$$

$$S_{100} = 50[101]$$

$$S_{100} = 5050$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_n = \frac{n}{2}[a + a + (n-1)d]$$

$$S_n = \frac{n}{2}[\text{first term} + \text{nth term}]$$

Gauss Method:

$\{1, 2, 3, 4, 5, 6, 7, \cdots, 99, 100\}$

$1 + 100 = 101$

$2 + 99 = 101$

$3 + 98 = 101$

$\vdots \quad \vdots \quad \vdots$

$50 + 51 = 101$

$S_n = 50(101) = 5050$

Project Maths
Tionscadal Mata
Development Team

Find the sum of all integers, from 5 to 1550 inclusive that are divisible by 5.

$a = 5 \quad d = 5 \quad T_n = 1550$

$T_n = a + (n-1)d$

$5 + 5(n-1) = 1550$

$5n = 1550$

$n = 310$

$$S_{310} = \frac{310}{2}\left[2(5) + (310-1)5\right]$$

$$S_{310} = 155\left[1555\right] = 241025$$

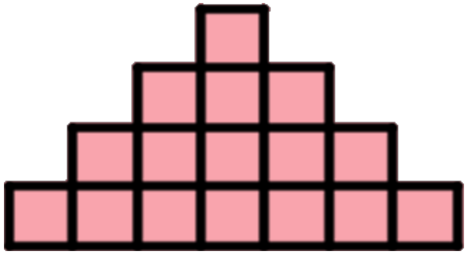
$$S_{310} = \frac{310}{2}\left[\text{First} + \text{last}\right]$$

$$S_{310} = 155\left[5 + 1550\right]$$

$$S_{310} = 241025$$

Project Maths
Tionscadal Mata
Development Team

Caroline bought 400 designer tiles at an end of line sale. She wants to use them as a feature in her new bathroom. If she decides on a pattern of the format shown, how many tiles will be on the bottom row of the design if she uses all the tiles.



$$S_n = \frac{n}{2}\left[2(1) + (n-1)2\right] = 400$$

$$n\left[2 + 2n - 2\right] = 800$$

$$2n^2 = 800$$

$$n^2 = 400$$

$$n = 20$$

$$T_{20} = 1 + (20-1)2 = 39$$

39 bricks needed for the bottom row of the design

Project Maths Tionscadal Mata Development Team ©

Denise has no savings and wants to purchase a car costing €6000. She starts saving in January 2012 and saves €200 that month. Every month after January 2012 she saves €5 more than she saved the previous month. When will Denise be able to purchase the car of her choice? Ignore any interest she may receive on her savings.

$a = 200 \qquad d = 5$

$$s_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_n = \frac{n}{2}\left[400 + (n-1)5\right]$$

$$S_n = 6000$$

$$n\left[400 + 5n - 5\right] = 12000$$

$$400n + 5n^2 - 5n = 12000$$

$$5n^2 + 395n - 12000 = 0$$

$$n = \frac{-395 \pm \sqrt{(395)^2 - 4(5)(-12000)}}{2(5)} = \frac{-395 \pm \sqrt{396025}}{10} = \frac{-395 \pm 629.31}{10} = 23.431$$

Hence after 23.43 months (December 2013) Denise will be able to purchase her car.

Project Maths Tionscadal Mata Development Team ©

Does the following graph model a sequence of numbers that form an arithmetic sequence?
Explain your reasoning.

It does ***not*** form a linear pattern.

A teacher is distributing sweets to 120 students. She gives each child a unique number starting at 1 and going to 120. She then distributed the sweets as follows, if the student has an odd on their ticket she gives them twice the number of sweets as their ticket number and if the number on their ticket is even she gives then three times the number of sweets as their ticket number.

How many sweets does she distribute?

Student	1	2	3	4	5	6	7	8
Sweets	2	6	6	12	10	18	14	24

Does the data in the table represent an arithmetic sequence?

No. The change not the same

Odd Numbered Students	**Even Numbered Students**
{2, 6, 10, 14,...}	{6,12,18, 24,...}
$S_{60} = \frac{60}{2}\left[2(2)+(60-1)4\right]$	$S_{60} = \frac{60}{2}\left[2(6)+(60-1)6\right]$
$S_{60} = 30\left[4+236\right] = 7200$	$S_{60} = 30\left[12+354\right] = 10980$

Total sweets distributed $7200+10980=18180$

Joe had been saving regularly for some months when he discovered he had lost his savings records. He found two records that showed he saved €260 in the fifth month and had a total of €3300 in the eleventh month. He knows he increased the amount he saved each month by a constant amount and had some savings before he commence this savings plan. How much did he increase his savings by each month and how much had he in his account when he started this savings plan?

$$T_5 = a + (5-1)d = a + 4d = 260$$

$$S_{11} = \frac{11}{2}\left[2a + (11-1)d\right] = 3300$$

$$11\left[2a + 10d\right] = 6600$$

$$2a + 10d = 600$$

$$a + 5d = 300$$

$$a + 5d = 300$$

$$a + 4d = 260$$

$$d = 40$$

$$a + 5(40) = 300$$

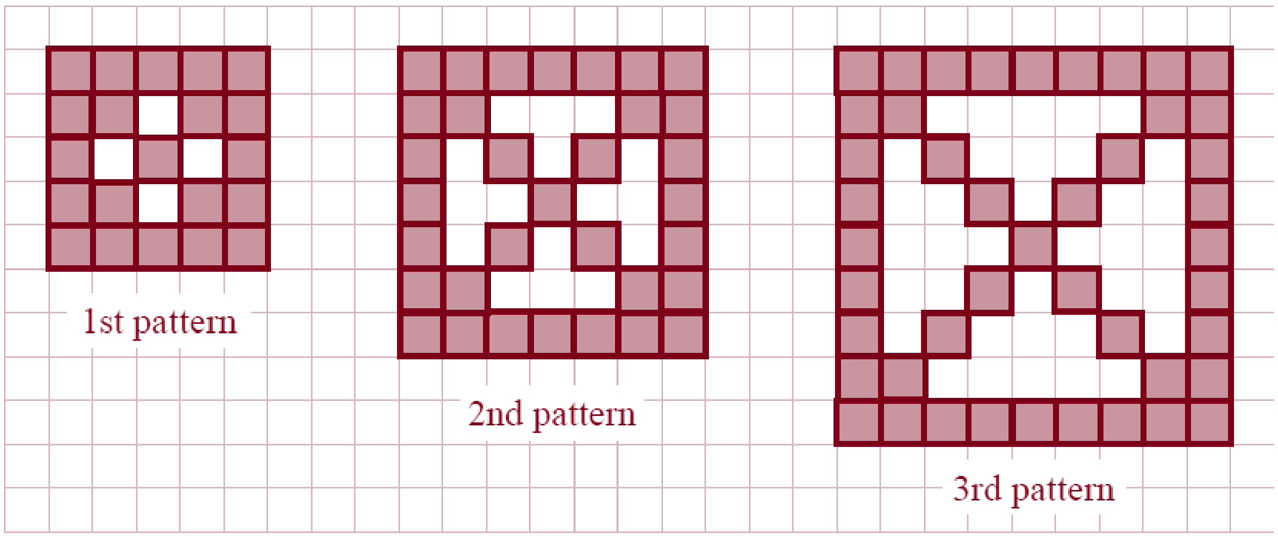
$$a = 100$$

He increased his savings by €40 per month and had €100 when he started this savings plan.

Project Maths Tionscadal Mata Development Team ©

Sample Paper Phase 2 P1 OL Q5

Síle is investigating the number of square grey tiles needed to make patterns in a sequence. The first three patterns are shown below, and the sequence continues in the same way. In each pattern, the tiles form a square and its two diagonals. There are no tiles in the white areas in the patterns – there are only the grey tiles.



(a) In the table below, write the number of tiles needed for each of the first five patterns:

Patterns	1	2	3	4	5
No. of tiles	21	33			

(a) In the table below, write the number of tiles needed for each of the first five patterns:

Patterns	1	2	3	4	5
No. of tiles	21	33	**45**	**57**	**69**

Change 12 12 12 12

Shape 1: $1+4+16=21$

Shape 2: $1+4+4+24$

$1+2(4)+3(8)=33$

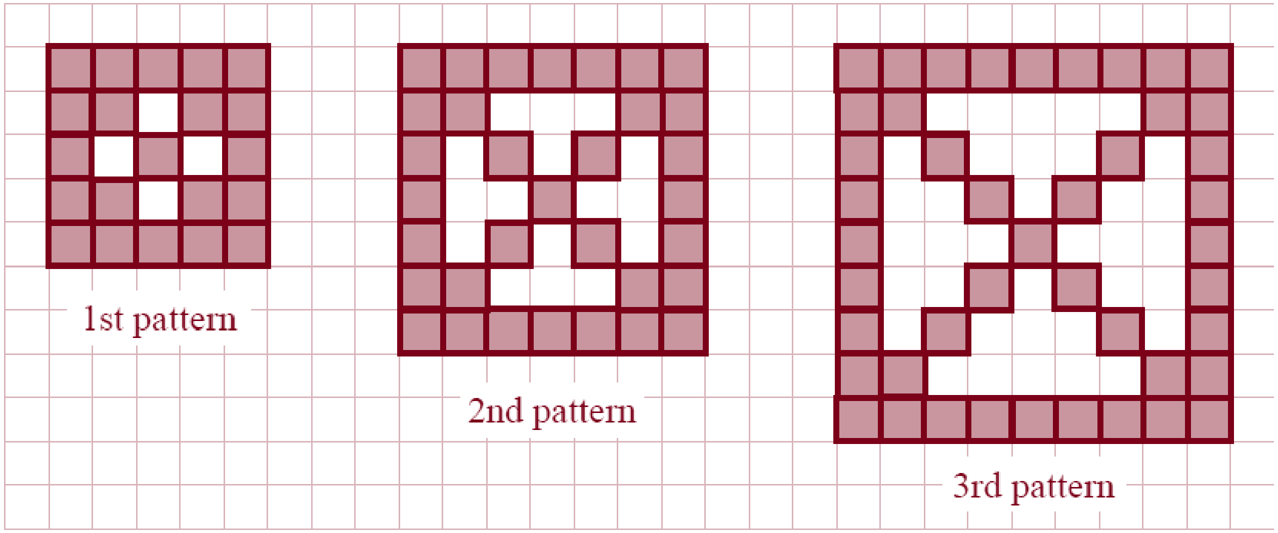
Shape 3: $1+4+4+4+32$

$1+3(4)+4(8)=45$

Shape 4: $1+4(4)+5(8)=57$

Shape 5: $1+5(4)+6(8)=69$

(b) Find, in terms of n, a formula that gives the number of tiles needed to make the nth pattern



Shape 1: $1+(1)4+2(8)$

Shape 2: $1+(2)4+3(8)$

Shape 3: $1+3(4)+4(8)$

Shape 4: $1+4(4)+5(8)$

Shape 5: $1+5(4)+6(8)$

Shape n: $1+n(4)+(n+1)(8)$

$9 + 12n$

or

Use T_n formula

$T_n = a+(n-1)d$

$T_n = 21+(n-1)12$

$T_n = 21+12n-12$

$T_n = 9+12n$

(c) Using your formula, or otherwise, find the number of tiles in the tenth pattern.

$$T_n = 9 + 12n$$

$$T_{10} = 9 + 12(10)$$

$$T_{10} = 129$$

(d) Síle has 399 tiles. What is the biggest pattern in the sequence that she can make?

$$9 + 12n = 399$$

$$12n = 390$$

$$n = 32.5$$

So 32nd pattern

(e) Find in terms of n, a general formula for the total number of tiles in the patterns.

Patterns	1	2	3	4	5
No. of tiles	21	33	45	57	69
Change		12	12	12	12

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_n = \frac{n}{2}\left[2(21) + (n-1)12\right]$$

$$S_n = n(15 + 6n)$$

(f) Síle starts at the beginning of the sequence and makes as many of the patterns as she can. She does not break up the earlier patterns to make the new ones. For example, after making the first two patterns, she has used up 54 tiles, (21 + 33). How many patterns can she make in total with her 399 tiles?

$$21 + 33 + 45 + 57 + 69 + 81 + 93 = 399$$

so exactly 7 patterns

or use S_n

$$S_n = n(15 + 6n) = 399$$

$$15n + 6n^2 = 399$$

$$2n^2 + 5n - 133 = 0$$

$$(2n + 19)(n - 7) = 0$$

Solution $n = 7$

Project Maths
Tionscadal Mata
Development Team

Sample Paper Phase 2 P1 OL Q6

John is given two sunflower plants. One plant is 16 cm high and the other is 24 cm high. John measures the height of each plant at the same time every day for a week. He notes that the 16 cm plant grows 4 cm each day, and the 24 cm plant grows 3·5 cm each day.

(a) Draw up a table showing the heights of the two plants each

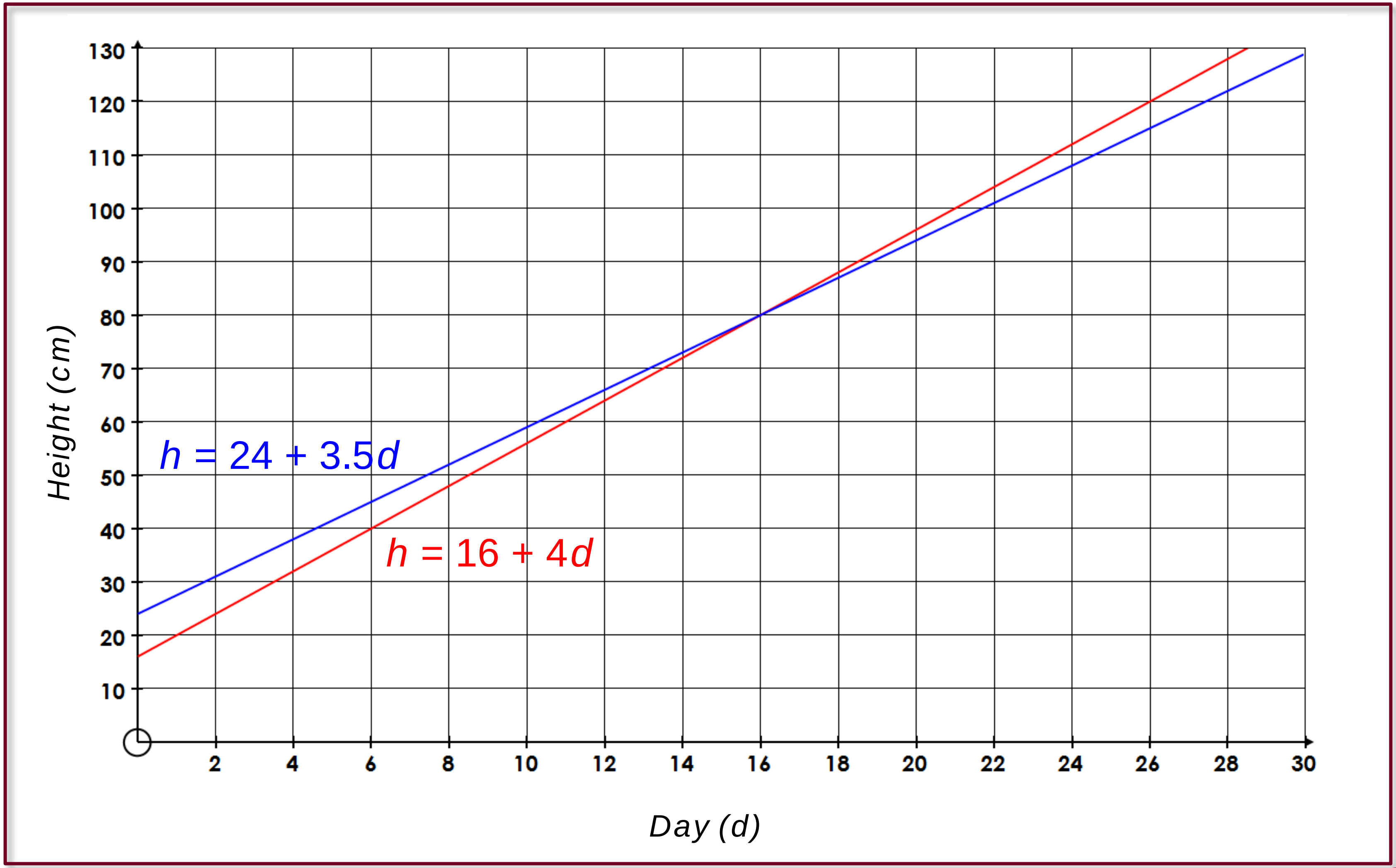
Day	Height of Plant A	Height of Plant B
0	16	24
1	20	27.5
2	24	31
3	28	34.5
4	32	38
5	36	41.5

(b) Write down two formulas – one for each plant – to represent the plant's height on any given day. State clearly the meaning of any letters used in your formulas.

Plant A : $h = 16 + 4d$, where d is the number of days John has had the plant and h is the height of the plant on day d.

Plant B: $h = 24 + 3.5d$, where d is the number of days John has had the plant and h is the height of the plant on day d.

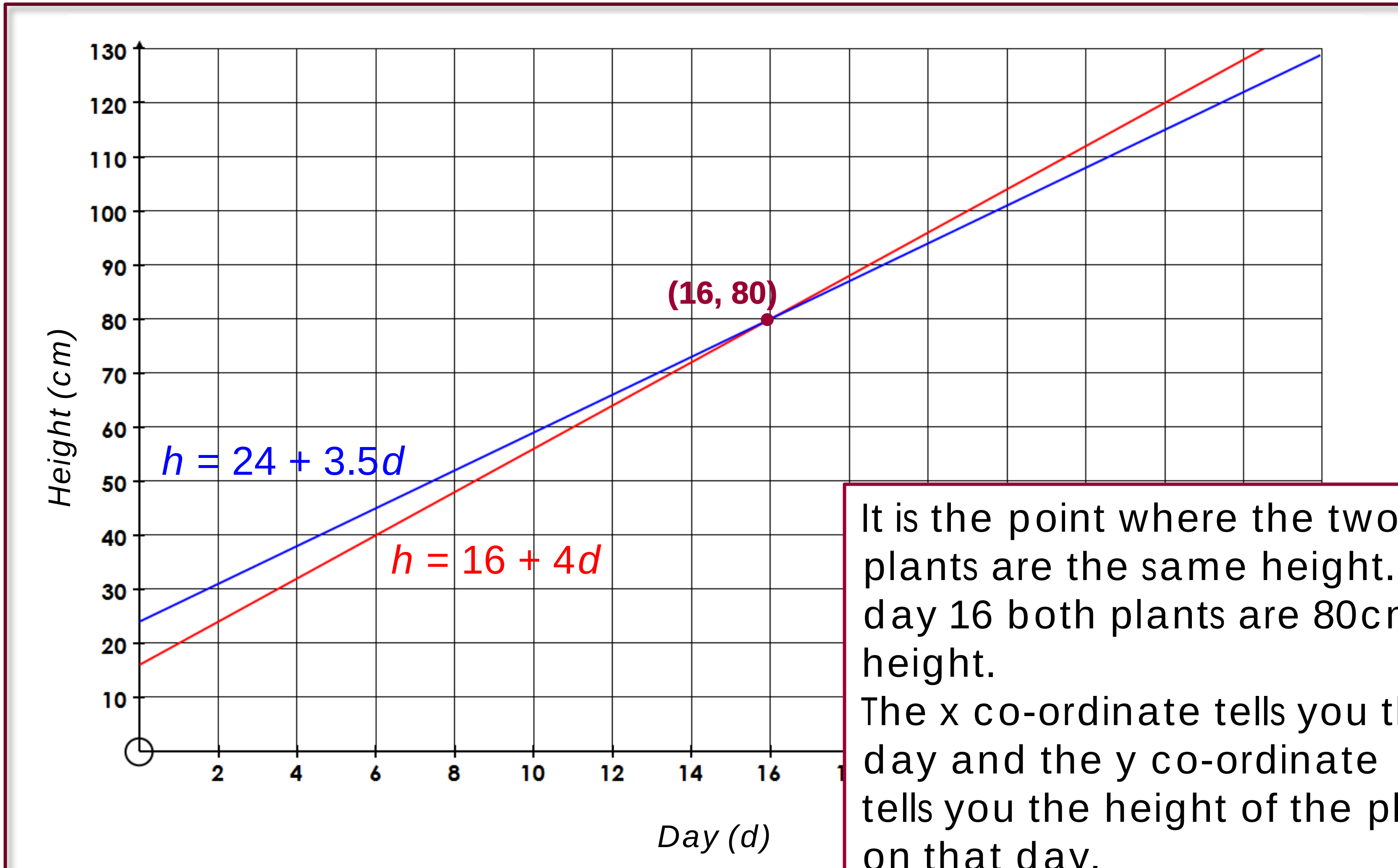
(c) John assumes that the plants will continue to grow at the same rates. Draw graphs to represent the heights of the two plants over the first *four weeks*.



Project Maths Tionscadal Mata Development Team

(d) (i) From your diagram, write down the point of intersection of the two graphs.

(ii) Explain what the point of intersection means, with respect to the two plants. Your answer should refer to the meaning of both co-ordinates.



It is the point where the two plants are the same height. On day 16 both plants are 80cm in height.
The x co-ordinate tells you the day and the y co-ordinate tells you the height of the plant on that day.

(e) Check your answer to part **(d)(i)** using your formulae from part **(b)**.

$16 + 4x = 24 + 3.5x$

$4x - 3.5x = 24 - 16$

$0.5x = 8$

$x = 16$

When $x = 16$

$f(x) = 16 + 4(16) = 80$

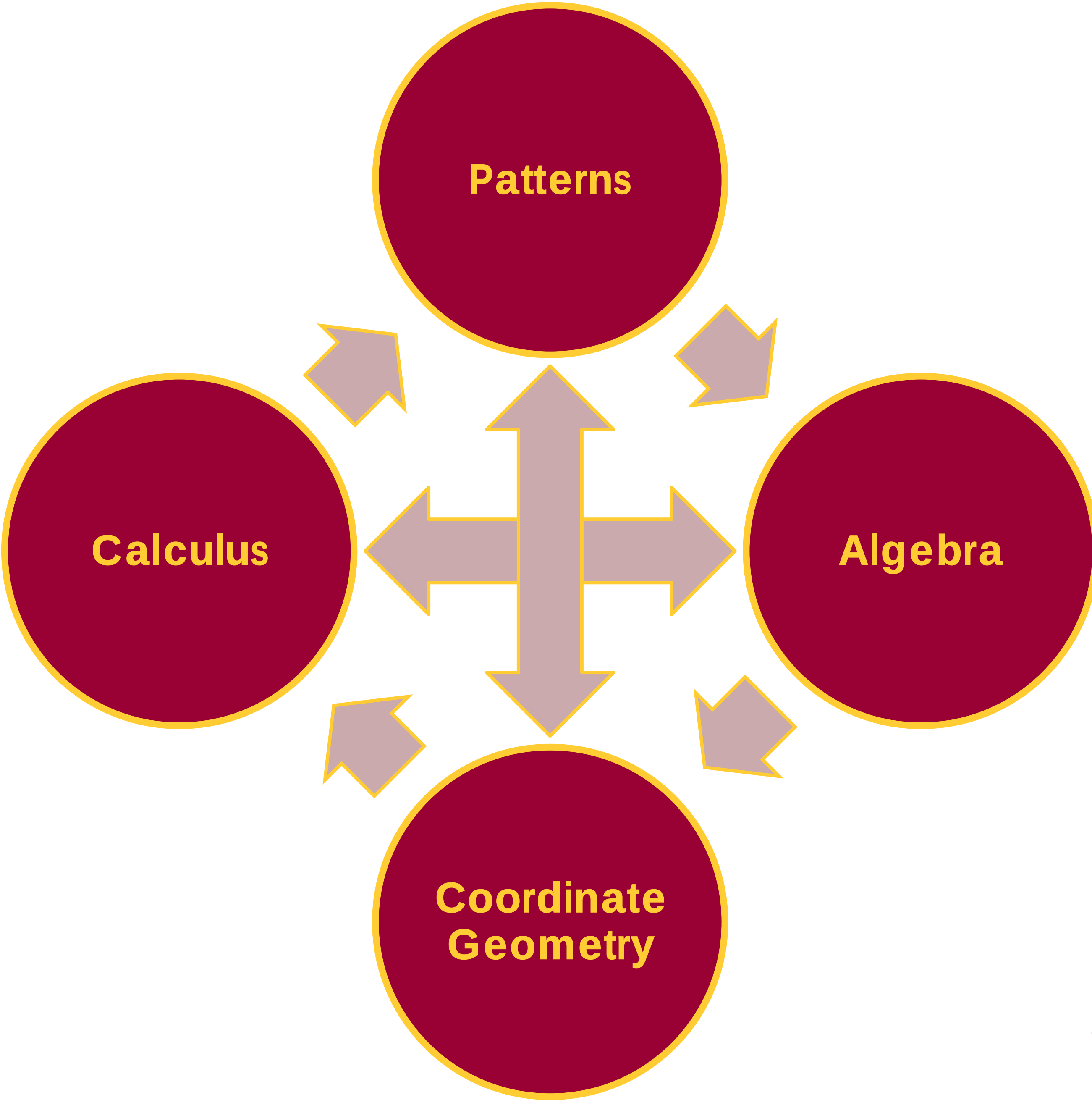
$g(x) = 24 + 3.5(16) = 80$

(f) The point of intersection can be found either by reading the graph or by using algebra. State one advantage of finding it using algebra.

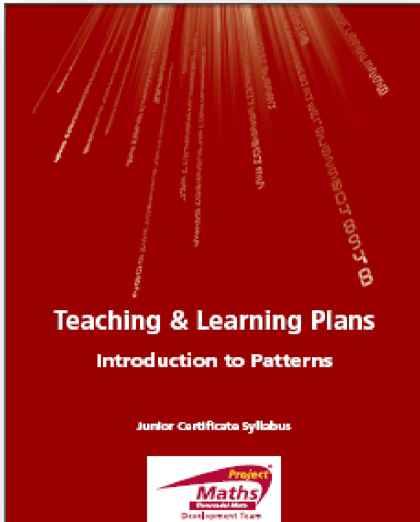
More accurate answer by Algebra.

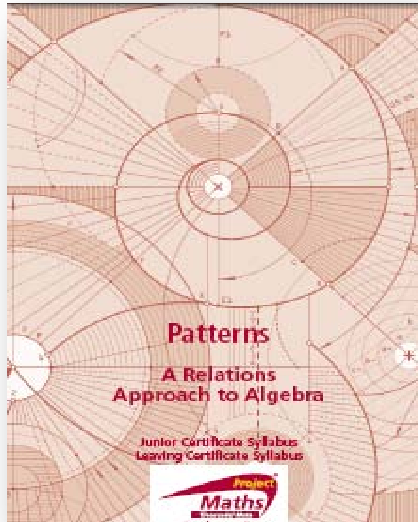
(g) John's model for the growth of the plants might not be correct. State one limitation of the model that might affect the point of intersection and its interpretation.

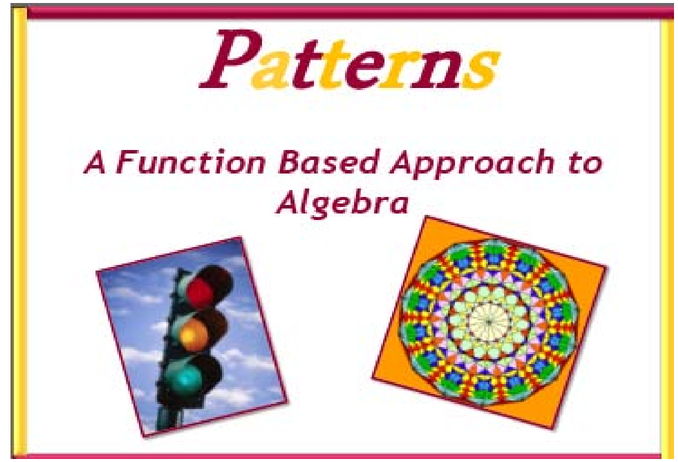
Plant's growth may not follow a regular pattern.


Patterns
Calculus
Algebra
Coordinate Geometry

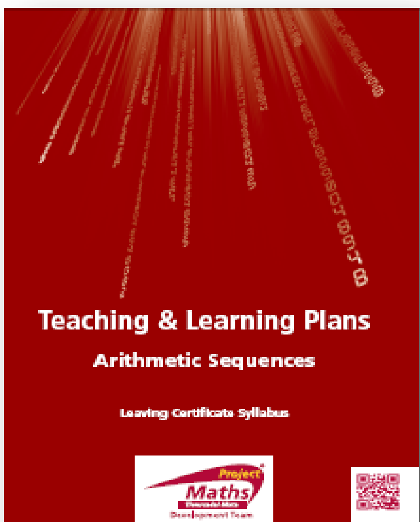
Resources

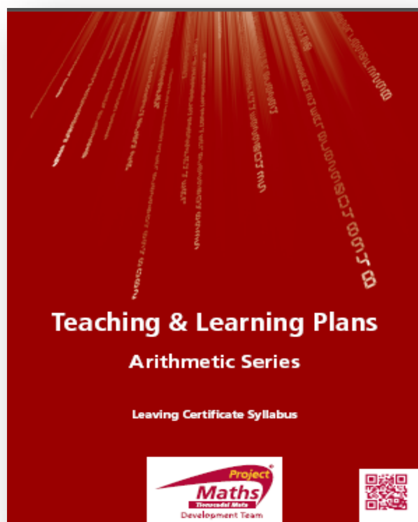


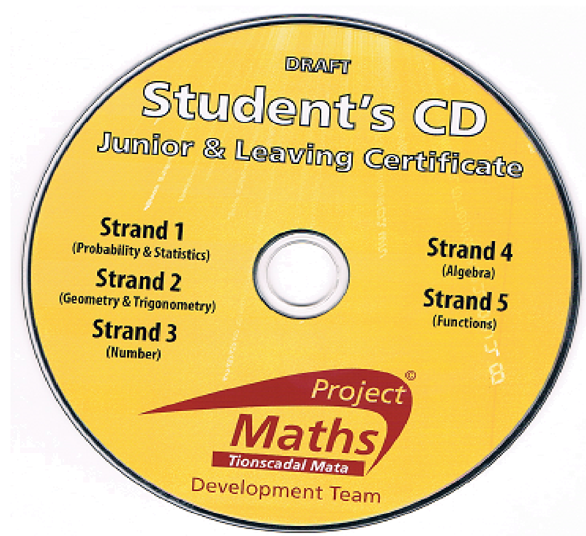




Show 2 Workshop 4







Each hour, a clock chimes the number of times that corresponds to the time of day. For example, at three o'clock, it will chime 3 times.
How many times does the clock chime in a day (24 hours)?

Day: $\{1, 2, 3, \ldots, 10, 11, 12\}$

Night: $\{1, 2, 3, \ldots, 10, 11, 12\}$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2(1) + (12-1)1\right]$$

$$S_{12} = 6\left[13\right]$$

$$S_{12} = 78$$

Total chimes in 24 hours is $2(78) = 156$

A theatre has 15 seats on the first row, 20 seats on the second row, 25 seats on the third row, and so on and has 24 rows of seats. How many seats are in the theatre?

{15, 20, 25,...}

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{24} = \frac{24}{2}\left[2(15) + (24-1)5\right]$$

$$S_{24} = 12\left[145\right]$$

$$S_{24} = 1740$$

Total number of seats in this theatre is 1740.

The green T-shaped figure in the table below is called T_{13} as this T-shaped figure has 13 as initial value.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	48	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Find the value of T_{26}

$26 + 27 + 28 + 37 + 47 = 165$

Find the sum of the numbers in the n^{th} T in terms of n.

$n + n + 1 + n + 2 + n + 11 + n + 21 = 5n + 35$

Find the set of numbers from which we can choose n.

Project Maths
Tionscadal Mata
Development Team