Junior Certificate Syllabus

3.3 Applied arithmetic	Solving problems involving, e.g., mobile phone tariffs, currency transactions, shopping, VAT and meter readings. Making value for money calculations and judgments. Using ratio and proportionality.	– solve problems that involve finding profit or loss, % profit or loss (on the cost price), discount, % discount, selling price, compound interest for not more than 3 years, income tax (standard rate only), net pay (including other deductions of specified amounts) – **solve problems that involve cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price) compound interest, income tax and net pay (including other deductions)**

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Leaving Certificate Syllabus

- $\mathbf{Z}$ of integers
- $\mathbf{Q}$ of rational numbers
- $\mathbf{R}$ of real numbers

and represent these numbers on a number line

– appreciate that processes can generate sequences of numbers or objects

– investigate patterns among these sequences

– use patterns to continue the sequence

– generate rules/formulae from those patterns

– develop decimals as special equivalent fractions strengthening the connection between these numbers and fraction and place value understanding

– consolidate their understanding of factors, multiples, prime numbers in $\mathbf{N}$

– express numbers in terms of their prime factors

– appreciate the order of operations, including brackets

– express non-zero positive rational numbers in the form $a \times 10^n$, where $n \in \mathbf{N}$ and $1 \leq a < 10$ and perform arithmetic operations on numbers in this form

– interpret the modulus as distance from the origin on an Argand diagram and calculate the complex conjugate

– generalise and explain patterns and relationships in algebraic form

– recognise whether a sequence is arithmetic, geometric or neither

– find the sum to n terms of an arithmetic series

– express non-zero positive rational numbers in the form $a \times 10^n$, where $n \in \mathbf{Z}$ and $1 \leq a < 10$ and perform arithmetic operations on numbers in this form

– investigate geometric sequences and series

– prove by induction

- simple identities such as the sum of the first n natural numbers and the sum of a finite geometric series
- simple inequalities such as

  $n! > 2^n$

  $2^n > n^2 \; (n \geq 4)$

  $(1+x)^n \geq 1+nx \; (x > -1)$
- factorisation results such as 3 is a factor of $4^n - 1$

– apply the rules for sums, products, quotients of limits

– find by inspection the limits of sequences such as

$$\lim_{n \to \infty} \frac{n}{n+1}; \quad \lim_{n \to \infty} r^n \; |r| < 1$$

– solve problems involving finite and infinite geometric series including applications such as recurring decimals and financial applications, e.g. deriving the formula for a mortgage repayment

– derive the formula for the sum to infinity of geometric series by considering the limit of a sequence of partial sums

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Leaving Certificate Arithmetic

3.3 Arithmetic	– check a result by considering whether it is of the right order of magnitude and by working the problem backwards; round off a result – make and justify estimates and approximations of calculations; calculate percentage error and tolerance – calculate average rates of change (with respect to time) – solve problems involving • finding depreciation (reducing balance method) • costing: materials, labour and wastage • metric system; change of units; everyday imperial units (conversion factors provided for imperial units) – estimate of the world around them, e.g. how many books in a library	– accumulate error (by addition or subtraction only) – solve problems that involve calculating cost price, selling price, loss, discount, mark up (profit as a % of cost price), margin (profit as a % of selling price), compound interest, depreciation (reducing balance method), income tax and net pay (including other deductions)	– use *present value* when solving problems involving loan repayments and investments

LC Syllabus page 26

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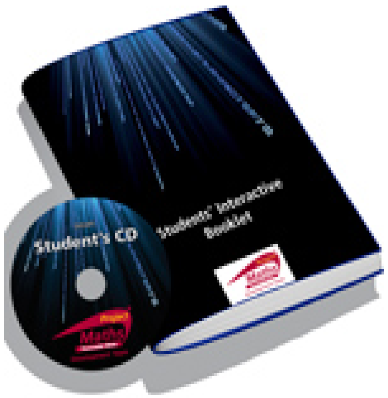
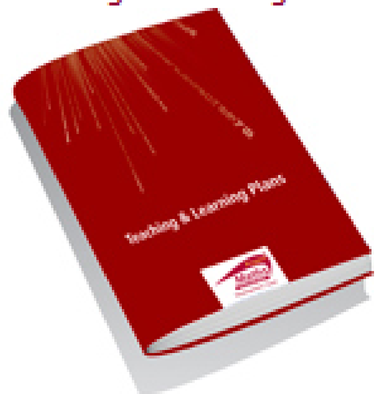
Resources

Students' CD



Teaching & Learning Plans



Teacher Handbooks

Algebra

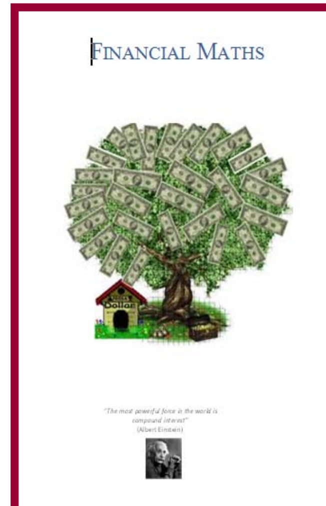
Leaving Certificate Strand 3
Number

Series and Sequences

- Arithmetic Sequence and Series
- Student Activity Arithmetic Sequence and Series
- Arithmetic Sequence Quiz
- Arithmetic Series Quiz
- Geometric Sequence and Series
- Student Activity Sequence and Series
- Geometric Sequence Quiz
- Geometric Series Quiz
- Sum to infinity of a GP

Arithmetic

- ESB Bill
- Student Activity ESB Bill
- VAT
- Student Activity on VAT
- Exchange Rate
- Student Activity on Exchange Rate
- Amortisation of a loan



http://www.projectmaths.ie

Linking Depreciation and Compounding to Prior Knowledge on Exponential Functions using Tables, Graphs and Formulae

2007 JC HL Q1 (b)

✍ A snowman has a mass of 12 kg.

It melts at a rate of 0·2% of its mass per minute.

What will be the mass of the snowman after 3 minutes?

Give your answer correct to 2 decimal places.

Poorly answered.

Common errors: ***1.*** *Ignoring cumulative loss of mass.*

2. *Mistake in % or decimal.*

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Solution

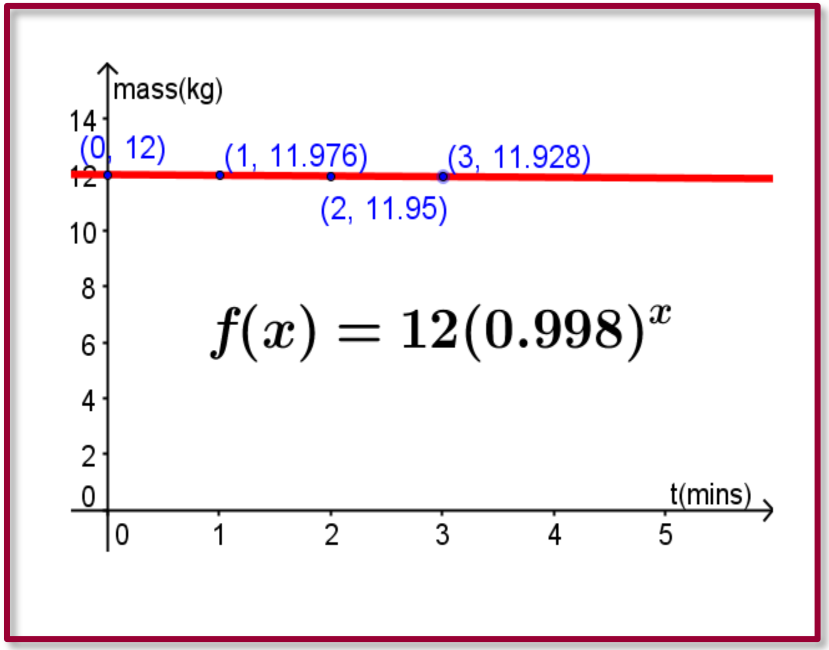
Method 1

Starting mass	12 kg
Mass lost min 1	12(0.002) **= 0.024 kg**
Mass after 1 min	12 – 0.024 **= 11.976 kg**
Mass lost min 2	11.976(0.002) **= 0.023952 kg**
Mass after 2 mins	11.976 – 0.023953 **= 11.952048**
Mass lost min 3	11.952048(0.002) **= 0.023904**
Mass after 3 mins	11.952048 – 0.023904 **= 11.928144**

Method 2

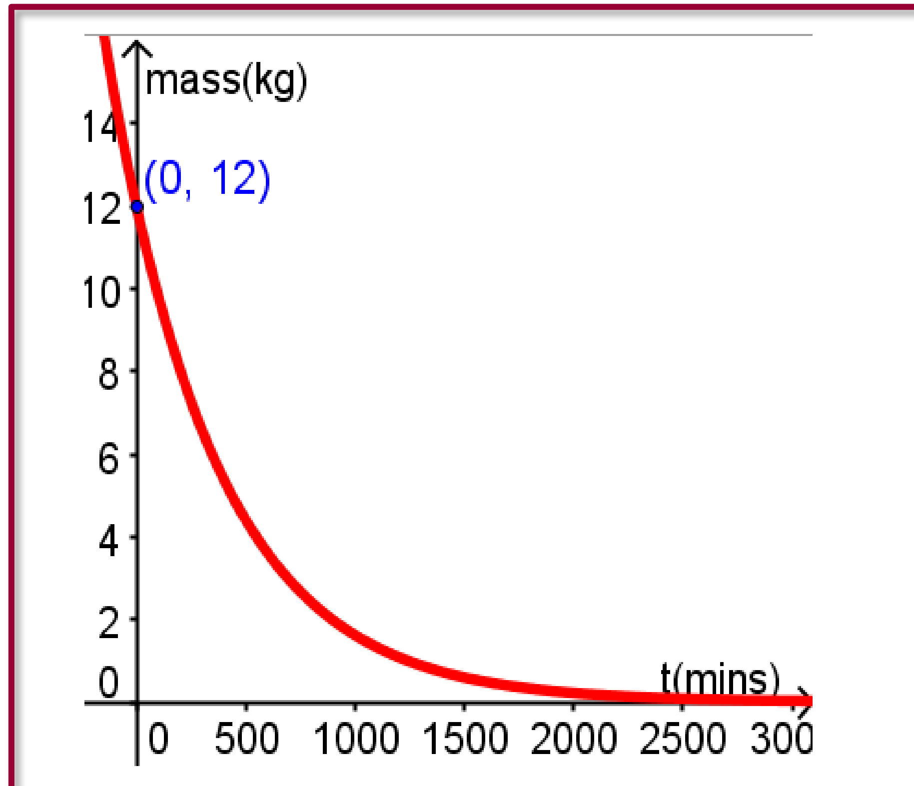
time / minutes	mass remaining (after time *t*) / kg
0	12
1	$12\left(\frac{99.8}{100}\right)=11.976$
2	$11.976\left(\frac{99.8}{100}\right)=12\left(\frac{99.8}{100}\right)^2=11.952$
3	$11.952\left(\frac{99.8}{100}\right)=12\left(\frac{99.8}{100}\right)^3=11.928$

Answer: 11.93 kg



Looks linear........???

The bigger picture.........



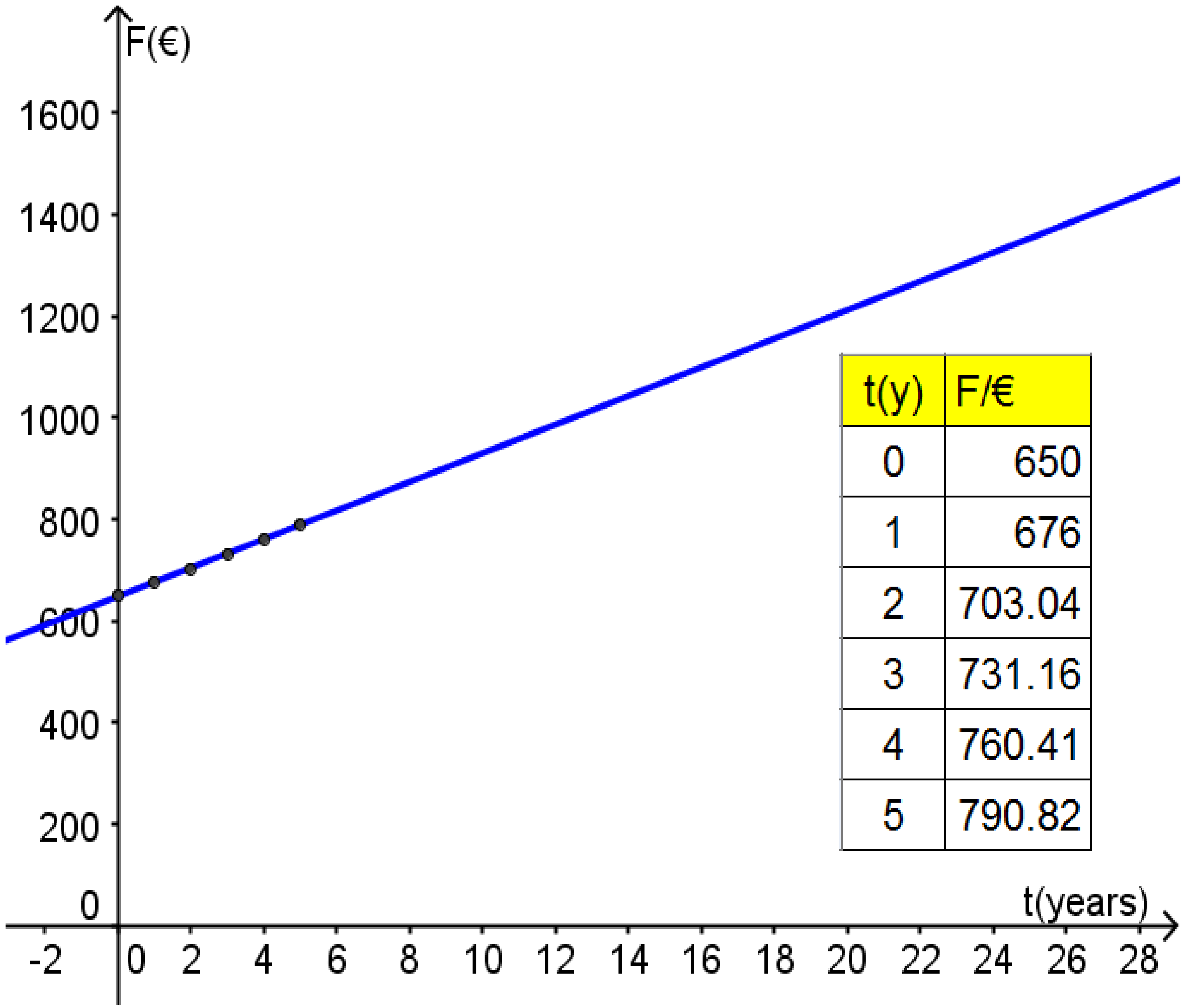
€650 is deposited in a fixed interest rate bank account. The amount in the account at the end of each year is shown in the following table.

End of year	1	2	3	4	5
Final value/€	676	703.04	€731.16	760.41	790.82

(a) Explain whether or not the relationship between final value and time can be modelled by a linear, quadratic or exponential function or by none of these?

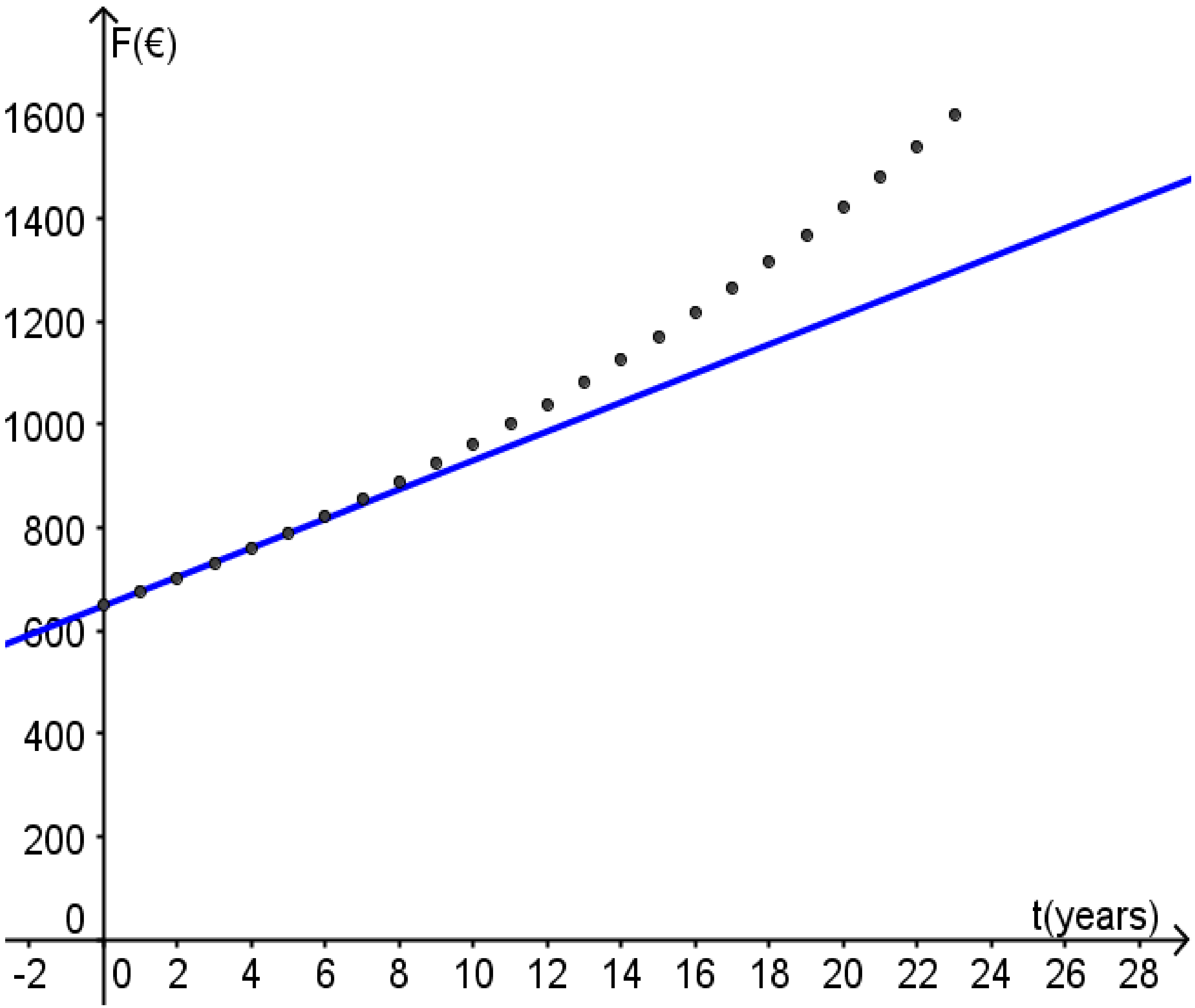
(b) If you plot a graph of final value against time what does the graph look like for this limited range of times?

Looks linear......................



t(y)	F/€
0	650
1	676
2	703.04
3	731.16
4	760.41
5	790.82

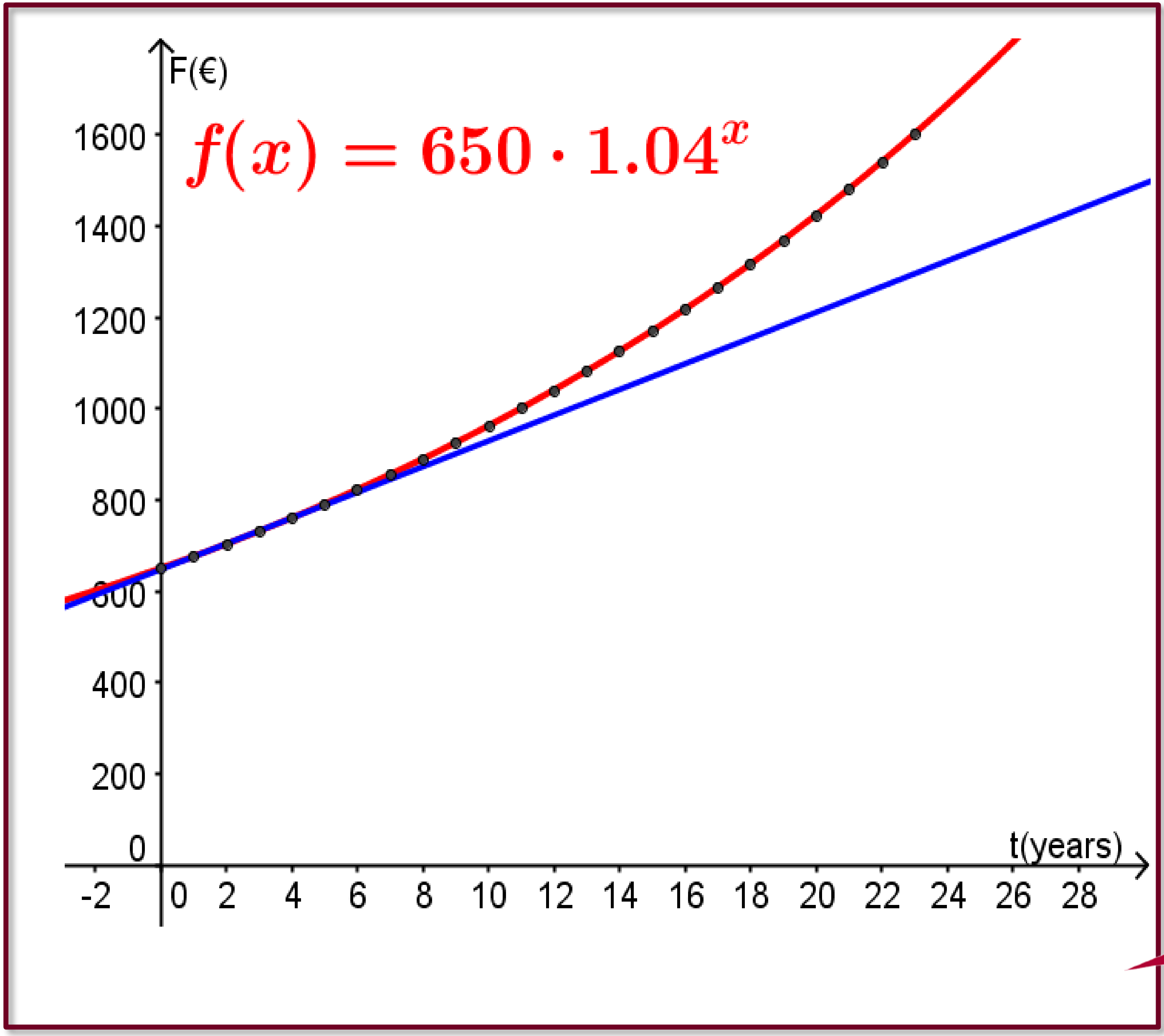
On further investigation......................



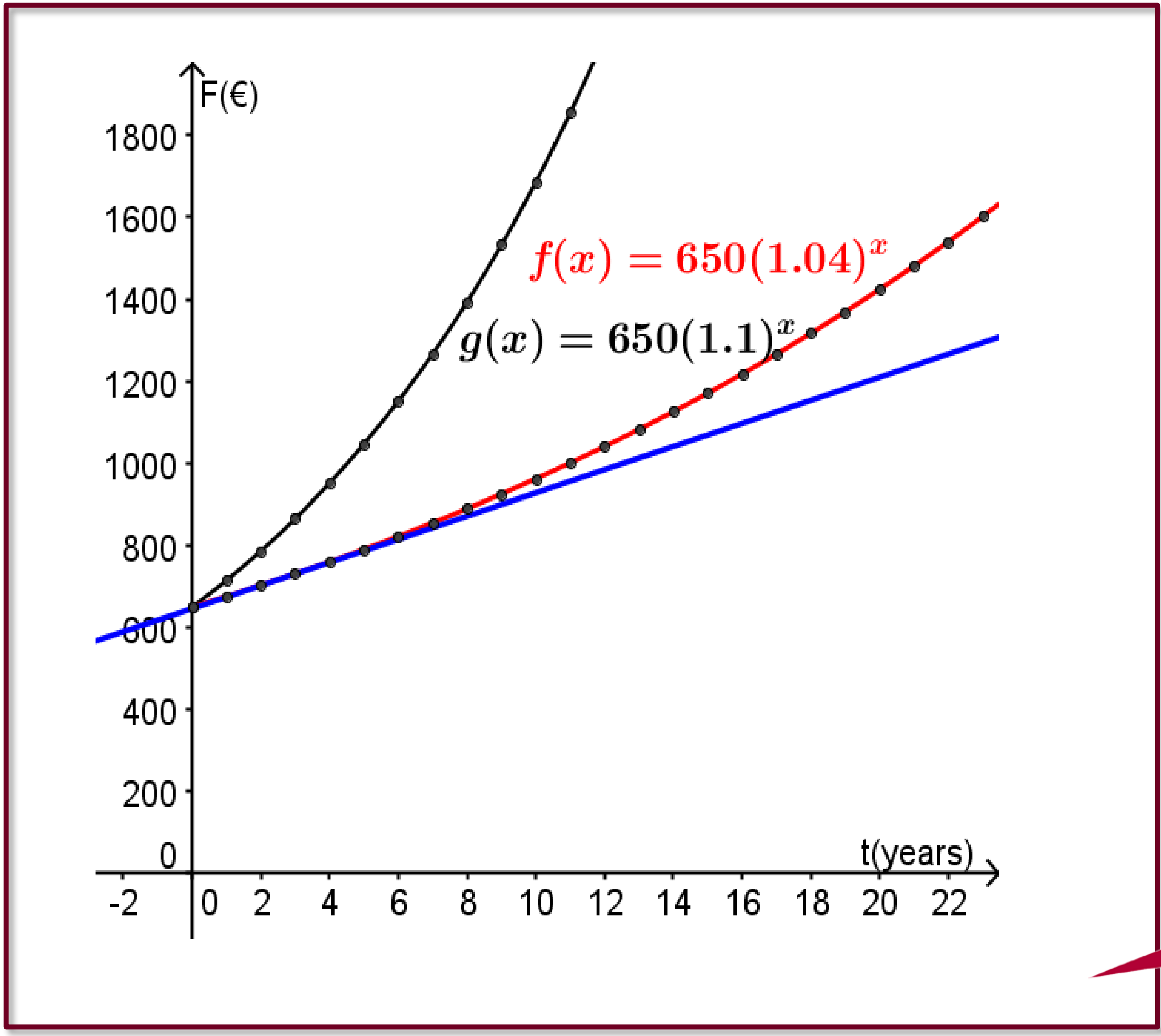
t(y)	F/€
0	650
1	676
2	703.04
3	731.16
4	760.41
5	790.82
6	822.46
7	855.36
8	889.57
9	925.15
10	962.16
11	1000.65
12	1040.67
13	1082.3
14	1125.59
15	1170.61
16	1217.44
17	1266.14
18	1316.78
19	1369.45
20	1424.23
21	1481.2
22	1540.45
23	1602.07

What formula expresses the final value in *t* years given an initial value of €650?

Final value is growing exponentially



Comparing a 10% interest rate to a 4% interest rate



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What is the prior knowledge for compound interest and depreciation?

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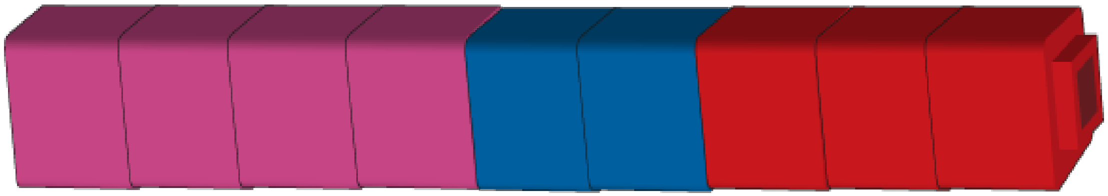
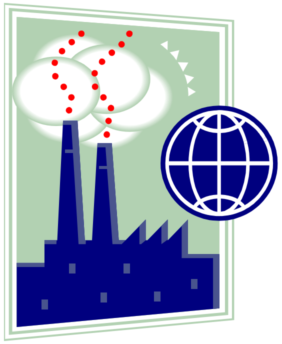
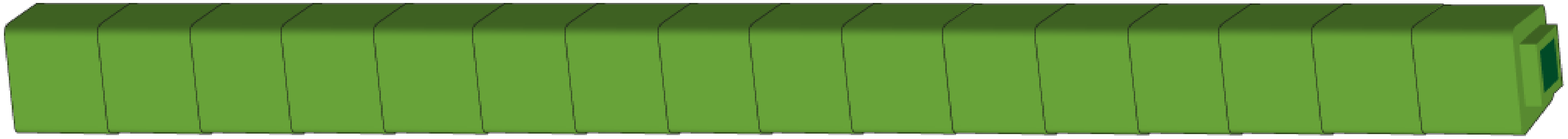
Joan gets a new job as a trainee. She starts on €40 per day. She is told that in 6 months she will get a 50% rise and in another 6 months she will get another 50% rise. She says “Great, in one year’s time I will have doubled my money.” Discuss.



Agus sin é compounding!



An item was being produced for €16 twenty years ago. Due to technology innovations it was reduced by 50% ten years ago and reduced again by 50% recently. Is the item now free?



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Understanding Terms and Notation

John put €200 into the bank for 1 year and got 10% interest during that year. At the end of the year he had €220. This means that he had gained €20 on his original money.
Match John's figures to each of the words in the table below.

Principal	***Interest Rate as a %***	***Interest Rate as a decimal***	***Final value***	***Number of years***	**Interest**
P	r	i	F	T	
200	**10**	**0.1**	**220**	**1**	**20**

Express interest in terms of final value and principal.

$$\text{Interest} = F - P$$

Variables denoting money are in capital letters!

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Apply Terms Used for Investments

The following figures represent a certain amount of money invested for a certain number of years at a certain interest rate.

€472.05, 4 yrs, €300, 12%, 0.12, €172.05

Using all of the terms:

Principal, Final value, interest rate as a %, interest rate as a decimal, number of years, interest,

write out a brief "story" to included all of the above figures.

Evaluate the Missing Figures

Name	Principal	Interest rate % (p.a.)	Final Value	Number of years	Interest
Anne	€1 000	6%	€1338.23	5	**€338.23**
Michael	€1 000	7%	**€1838.46**	9	€838.46
Dominic	**€4000**	8%	€5 038.85	3	€1 038.85

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Interest Rates as Percentages and Decimals

Below are some annual interest rates expressed as percentages. Convert these to annual rates expressed as decimals.

r	*i*
6%	**0.06**
2.3%	**0.023**
103.2%	**1.032**
15%	**0.15**
0.35%	**0.0035**
0.0246%	**0.000246**
0.00035%	**0.0000035**

Below are some annual interest rates expressed as decimals. Convert these to annual rates expressed as percentage rates.

i	*r*
0.07	**7%**
0.045	**4.5%**
0.12	**12%**
0.18	**18%**
0.0375	**3.75%**
0.00278	**0.287%**
0.00035	**0.035%**

Applying Rules for Indices

Complete the following table, without the use of a calculator.

Leave your answers in index form:

Before multiplying	After multiplying	Before multiplying	After multiplying
1. $a^7 \times a^3$		6. $1.02^3 \times 1.02^3$	
2. $8^3 \times 8^2$		7. $1.14^5 \times 1.14^2$	
3. $8.2^5 \times 8.2^2$		8. $1.06^4 \times 1.06$	
4. $(6.4)^5\ (6.4)^2$		9. $(1.08)^5\ (1.08)$	
5. $1.3^2 \times 1.3^5$		10. 1.07×1.07^5	

Principal to Final Value After 1 Year in one Step

Patrick invests €100 in a savings account for 1 year at a fixed annual equivalent rate (AER) of 4.3%. At the end of the year he asks the bank how much money he has in total and how much interest he was paid. Fill out the table below for the figures which the bank will give him.

Method 1	
Principal (*P*)	**€100**
Interest for the year	**€4.3**
Final value (*F*)	**€104.3**

What percentage of the principal is the final value ? **104.3%**

Express as a ratio: $F:P = a:1,\ a \in R$ **$F:P$ = 1.043:1**

Find the final value from the principal in one step.

Method 2

$F = P(1.043) = 100(1.043) =$ €104.30

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Evaluate Using the Calculator (prior knowledge for method 2)

1. 7^4

2. 4.5^4

3. 1.8^5

4. 1.06^6

5. 1.325^5

6. $\left(\frac{3}{2}\right)^7$

7. $\left(\frac{1}{2}\right)^4$

8. $10(3)^4$

9. $100(6)^3$

10. $1000(2.5)^3$

11. $300(1.03)^6$

12. $2000(1.025)^5$

13. $250(1.16)^4$

14. $400(1.08)^4$

Compound Interest Over a Number of Periods of Time

Mary has received a gift of €5000.
She is hoping to buy a car for €6000 with her savings in three years' time.

She intends to save the gift money until then.
The bank is offering her 4% AER if she invests the money for the three years. Will she be able to afford to buy the car from her savings at the end of the three years ?

Final Value Using Compounding (two methods)

Method 1	
Value of the gift (*P*)	**€5000**
Interest for the 1st year (I_1) (4% of €5 000)	**€200**
F_1 =Final value (end of year 1)	**€5200**
Interest for the 2nd year (I_2)	**€208**
F_2 = Final Value (end of year 2)	**€5408**
Interest for the 3rd year (I_3)	**€216.32**
F_3 =Final Value (end of year 3)	**€5624.32**

$0.04 = i$

$1.04 = 1 + i$

Write an expression for the final value *F* in terms of *P* , *i* and *t*.

Method 2

Value at end of year 1 €5000 × **1.04** = €5200	=	Value at end of year 1 €5000 × **1.04**
Value at end of year 2 €5200 × **1.04** = €5408	=	Value at end of year 2 €5000 × $\mathbf{1.04^2}$ **Check with a calculator**
Value at end of year 3 €5408 × **1.04** = €5624.32	=	Value at end of year 2 €5000 × $\mathbf{1.04^3}$ **Check with a calculator**

What % of *P* is F_1? 104% Express this as a number and use it to calculate F_1 1.04

What % of F_1 is F_2? 104% Express this as a number and use it to calculate F_2 1.04

What % of F_2 is F_3 ? 104% Express this as a number and use it to calculate F_3 1.04

Compound Interest Formula

$$F = P(1+i)^t$$

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Iontu seo a leanas, is é t an fad ama ina bhlianta agus is é i an ráta bliantúil úis, dímheasa nó fáis, agus é sloinnte mar dheachúil nó mar chodán (ionas go seasann $i = 0.08$ do ráta 8%, mar shampla)*.

In all of the following, t is the time in years and i is annual rate of interest, depreciation or growth, expressed as a decimal or fraction (so that, for example, $i = 0.08$ represents a rate of 8%)*.

Ús iolraithe F = luach deiridh, P = príomhshuim	$F = P(1+i)^t$	**Compound interest** F = final value, P = principal
Luach láithreach P = luach láithreach, F = luach deiridh	$P = \dfrac{F}{(1+i)^t}$	**Present value** P = present value, F = final value
Dímheas **– modh an chomhardaithe laghdaithigh** F = luach déanach, P = luach tosaigh	$F = P(1-i)^t$	**Depreciation** **– reducing balance method** F = later value, P = initial value
Dímheas **– an modh dronlíneach** A = méid an dímheasa bhliantúil P = luach tosaigh, S = dramhluach t = saolré eacnamaíoch fhónta	$A = \dfrac{P-S}{t}$	**Depreciation** **– straight line method** A = annual depreciation amount P = initial value, S = scrap value t = useful economic life

*Bionn feidhm ag na foirmlí sin freisin nuair a bhítear ag athiolrú i gceann eatraimh chothroma seachas blianta. Sa chás sin, déantar t a thomhas sa tréimhse chuí ama, agus is é i an ráta don tréimhse.

*The formulae also apply when compounding at equal intervals other than years. In such cases, t is measured in the relevant periods of time, and i is the period rate.

Calculating the Principal

John wants to have €10 000 saved in 10 years time to pay for his child's education. The bank is offering him an AER (annual equivalent rate) of 7%. How much money would he need to invest now in order to have €10000 in 10 years time?

P	*i*	(1+*i*)	*t*	*F*
€5083.49	0.07	1.07	10	€10000

Finding Roots

Revision using the Calculator

$2 \times 2 \times 2 \times 2 \times 2 = 32$, therefore the 5th root of 32 is 2.

Verify this using the root key $\left[\sqrt[b]{a}\right]$

Evalaute the following:

(a) $625^{\frac{1}{4}}$

(b) $\sqrt[12]{1.043}$

(c) $1.043^{\frac{1}{365}}$

(d) $\sqrt[10]{\frac{5000}{3750}}$

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Finding Periodic Interest Rate

Fiona has put €7 000 into a savings account.
She would like €10 000 in 4 years time in order to build an extension to her house. She decides to ask a few banks, building societies etc. what rate of interest they are willing to offer. What annual rate of interest does Fiona need in order to have enough saved to build the extension?

P	***i***	**(1+*i*)**	***t***	***F***
€7000			4	€10,000

$$10000 = 7000(1+i)^4$$

$$\frac{10}{7} = (1+i)^4$$

$$\sqrt[4]{\frac{10}{7}} = (1+i) = 1.0933 \Rightarrow i = 0.0933$$

$$r = 9.3\%$$

or

$$10000 = 7000(1+i)^4$$

$$\log_{10} 10000 = \log_{10} 7000 + 4\log(1+i)$$

$$\frac{4 - \log_{10} 7000}{4} = \log(1+i)$$

$$1.0932 = (1+i) \Rightarrow i = 0.093$$

$$r = 9.3\%$$

Finding Number of Years (or other time periods) LCHL

Fiona has put €5000 into a savings account at 7% AER.
She needs €10 000 in order to build an extension to her house.
How many years will it take for Fiona to reach her target of €10 000 ?
Give your answer correct to one decimal place.

P	*i*	(1+*i*)	*t*	*F*
€5000	0.07	1.07		€10000

N.B. Prior knowledge of logarithms required

$10000 = 5000(1.07)^t$

$2 = (1.07)^t$

$\log_{1.07} 2 = t$

$t = 10.24$ years

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Reducing Balance

Jillian and Noel are each going to buy a games console. It costs €500 and they are getting a loan from the credit union . Jillian says "I am making a lot of money at the moment so I can afford to pay €100 per month." Noel says that he can only afford €80 per month. The credit union is charging them a monthly interest rate of 1% to be paid at the end of each month.
Find their outstanding balances at the end of each month.

(a) A loan is taken out

(b) After 1 month interest is added on

(c) The person then makes his/her monthly repayment.
This process is then repeated until the loan is fully paid off.

Reducing Balance

Jillian's First 3 Months

Initial Loan	
Interest 1	
Total (500 x 0.01)	
Payment	
Balance	

Interest 2	
Total	
Payment 2	
Balance 2	

Interest 3	
Total	
Payment 3	
Balance 3	

Noel's first 3 months

Initial Loan	
Interest 1	
Total (500 x 0.01)	
Payment	
Balance	

Interest 2	
Total	
Payment 2	
Balance 2	

Interest 3	
Total	
Payment 3	
Balance 3	

Reducing Balance

Compare the total interest paid by Jillian and Noel.

Total interest J = €15.55
Total interest N = €18.98

Compare the time taken by Jillian and Noel to pay off the loan.

t_J = 6 months
t_N = 7 months

Jillian

Initial loan	€500.00
Interest 1	€5.00
Total	€505.00
Payment 1	€100.00
Balance 1	€405.00

Interest 2	4.05
Total	€409.05
Payment 2	€100.00
Balance 2	€309.05

Interest 3	3.0905
Total	€312.14
Payment 3	€100.00
Balance 3	€212.14

Interest 4	2.121405
Total	€214.26
Payment 4	€100.00
Balance 4	€114.26

Interest 5	1.14261905
Total	€115.40
Payment 5	€100.00
Balance 5	€15.40

Interest 6	0.154045241
Total	€15.56
Payment 6	€15.56
Balance 6	€0.00

Noel

Initial loan	€500.00
Interest 1	€5.00
Total	€505.00
Payment 1	€80.00
Balance 1	€425.00

Interest 2	4.25
Total	€429.25
Payment 2	€80.00
Balance 2	€349.25

Interest 3	3.4925
Total	€352.74
Payment 3	€80.00
Balance 3	€272.74

Interest 4	2.727425
Total	€275.47
Payment 4	€80.00
Balance 4	€195.47

Interest 5	1.95469925
Total	€197.42
Payment 5	€80.00
Balance 5	€117.42

Interest 6	1.174246243
Total	€118.60
Payment 6	€80.00
Balance 6	€38.60

Interest 7	0.385988705
Total	€38.98
Payment 7	€38.98
Balance 7	€0.00

Depreciation

A company buys a new lorry for €50 000. After 4 years it needs to sell the lorry. The value of the lorry reduces by 15% each year.
What is the value of the lorry after 4 years?

Depreciation

Method 1	
Original Value of Lorry *P*	**€50000**
Depreciation in year 1 =€50000(0.15)	**€7500**
F_1 =Final value (end of year 1)	**€42500**
Depreciation in year 2	**€6375**
F_2 = Final Value (end of year 2)	**€36125**
Depreciation in year 3	**€5418.75**
F_3 =Final Value (end of year 3)	**€30706.25**

Method 2

Value at end of year 1 €50000 × **0.85** = €42500	=	Value at end of year 1 €50000 × **0.85**
Value at end of year 2 €42500 × **0.85** = €36125	=	Value at end of year 2 €50000 × **0.85^2** **Check with a calculator**
Value at end of year 3 €536125 × **0.85** = €30706.25	=	Value at end of year 3 €50000 × **0.85^3** **Check with a calculator**

$0.04 = i$

$1.04 = 1 + i$

Write an expression for the final value *F* in terms of *P* , *i* and *t*.

What % of *P* is F_1?	85%	Express this as a number and use it to calculate F_1	0.85
What % of F_1 is F_2?	85%	Express this as a number and use it to calculate F_2	0.85
What % of F_2 is F_3 ?	85%	Express this as a number and use it to calculate F_3	0.85

Depreciation – Reducing Balance Method

$$F = P(1 - i)^t$$

Matamaitic an airgeadais		Financial mathematics
Iontu seo a leanas, is é t an fad ama ina bhlianta agus is é i an ráta bliantúil úis, dímheasa nó fáis, agus é sloinnte mar dheachúil nó mar chodán (ionas go seasann $i = 0.08$ do ráta 8%, mar shampla)*.		In all of the following, t is the time in years and i is annual rate of interest, depreciation or growth, expressed as a decimal or fraction (so that, for example, $i = 0.08$ represents a rate of 8%)*.
Ús iolraithe F = luach deiridh, P = príomhshuim	$F = P(1+i)^t$	**Compound interest** F = final value, P = principal
Luach láithreach P = luach láithreach, F = luach deiridh	$P = \dfrac{F}{(1+i)^t}$	**Present value** P = present value, F = final value
Dímheas **– modh an chomhardaithe laghdaithigh** F = luach déanach, P = luach tosaigh	$F = P(1-i)^t$	**Depreciation** **– reducing balance method** F = later value, P = initial value
Dímheas **– an modh dronlíneach** A = méid an dímheasa bhliantúil P = luach tosaigh, S = dramhluach t = saolré eacnamaíoch fhónta	$A = \dfrac{P-S}{t}$	**Depreciation** **– straight line method** A = annual depreciation amount P = initial value, S = scrap value t = useful economic life

*Bíonn feidhm ag na foirmlí sin freisin nuair a bhítear ag athiolrú i gceann eatraimh chothroma seachas blianta. Sa chás sin, déantar t a thomhas sa tréimhse chuí ama, agus is é i an ráta don tréimhse.

*The formulae also apply when compounding at equal intervals other than years. In such cases, t is measured in the relevant periods of time, and i is the period rate.

Depreciation – Reducing Balance Method

LCOL NCCA 2011

A machine depreciates in value by 40% in its first year of use. During its second year it depreciates by 25% of its value at the beginning of that year. Thereafter, for each year, it depreciates by 10% of its value at the beginning of the year.

Calculate

(i) the value after eight years of equipment costing €500 new

(ii) The value when new, of equipment, valued at €100 after five years of use.

Solution

(i) $F = P(1-i)^t$

$F = 500(0.6)(0.75)(0.9)^6 = €119.57$

(ii) $F = P(1-i)^t$

$100 = P(0.6)(0.75)(0.9)^3 \Rightarrow P = €304.83$

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AER, EAR, CAR
&
Interest Rates other than Annual


Deposit Account is Back
Earn up to 4% EAR Variable
Instant access to your funds at standard variable rate of 3%
EBS
Home
Mortgages
Fixed Term Accounts
You are here > Home / Saving and
EBS 15 Month Fixed Term Savings Account
5.25% FIXED OVER 15 MONTHS
4.18% AER* FIXED
A savings rate you'll really warm to!
MARKET LEADING RATE
Click here for more
6.85% (AER 4.52%)
for 18 months
Ireland State Savings
NATIONAL TREASURY MANAGEMENT AGENCY
The main NTMA State Savings™ products are:
Actual BEFORE TAX
Total Return
Gross AER¹
NTMA State Savings™ Products
Actual AFTER Tax
Net AER¹
Total Return
10% 3.23% 3 year Savings Bonds² 3.23% 10.00%
15% 3.56% 4 year National Solidarity Bonds³ 3.31% 13.92%
21% 3.53% 5½ year Savings Certificates²
20% 3.37% 6 year Instalment Savings
50% 4.14%
Interest First Deposit Account
Why wait for your interest? Get it all in the first month!
with the Interest First Deposit Account
10% GROSS FIXED
4.7% AER
26 Month Term
› More Info

Savings and Investments

AER (annual equivalent/effective rate) tells you what interest you will earn annually, which depends on how often interest is added.

- ✓ Used for savings and investments
- ✓ It may or may not include charges;
- ✓ Allows investors to make comparisons between savings accounts which pay interest at different intervals
- ✓ Takes into consideration the effect of compounding interest

The financial regulator's office considers the terms AER/EAR and CAR all to be equivalent. The term CAR is approved for use in relation to tracker bonds – for other investment products the regulator considers the acronym AER or EAR should be used.

AER

Leaving Certificate 2010 Sample Paper 1 Foundation Level Q2

A sum of €5000 is invested in an eight-year government bond with an annual equivalent rate **(AER)** of 6%.
Find the value of the investment when it matures in eight years' time.

$$F = P(1+0.06)^8 = €7969.24$$

Leaving Certificate 2010 Sample Paper 1 Ordinary Level Q2

(a) A sum of €5000 is invested in an eight-year government bond with an annual equivalent rate **(AER)** of 6%. Find the value of the investment when it matures in eight years' time.

(b) A different investment bond gives 20% interest after 8 years. Calculate the AER for this bond.

$$6000 = 5000(1+i)^8 \Rightarrow (1+i) = 1.2^{\frac{1}{8}} = 1.02305 \Rightarrow i = 0.02305$$

$$\text{AER} = 2.305\%$$

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Converting Monthly Rate to AER

€100 earns 0.287% per month compound interest.

i. What is its final value after 1 year?

ii. If interest was paid and compounded annually what is the annual equivalent rate?

Solution:

After 12 months, $i =$ monthly interest rate

(i) $F = P(1+i)^{12} = 100(1.00287)^{12} = 100(1.034988) \approx 100(1.035) = €103.5$

(ii) AER = 3.5%

Interest Rates other than AER

The €100 is left on deposit for 15 months at 0.287% per month compound interest.

i. Calculate the final value. Give the answer to the nearest 10 c.
ii. What is the rate of interest for the 15 months?
iii. What is this interest rate called?

Solution:

After 15 months, $i =$ monthly interest rate

(i) $F = P(1+i)^{15} = 100(1.00287)^{15} = 100(1.043925) = €104.39 \approx €104.4$

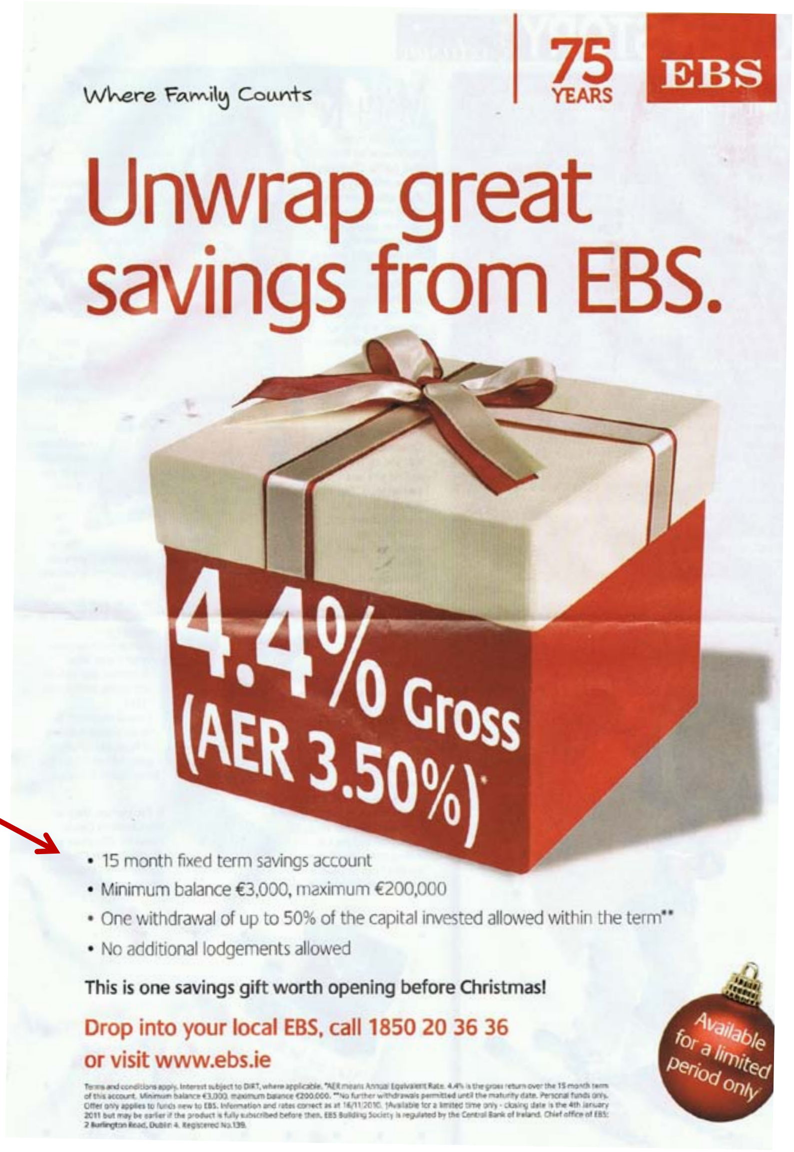
(ii) The interest rate is 4.4%

(iii) 4.4% is the gross interest rate

Unwrap the Terminology

Advertisement Christmas 2010 with AER

15 month fixed term savings account



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We Really do Want your Money

2011

15 month fixed term savings account

Justify that that 5.25% fixed for 15 months is the same as 4.18% AER

Show that 5.25% fixed over 15 months is equal to 4.18% AER

2011

$$105.25 = 100(1+i)^{1.25}$$

$$105.25 = 100(1+i)^{\frac{15}{12}}$$

$$1.0525 = (1+i)^{\frac{15}{12}}$$

$i =$ AER (annual equivalent rate) which is the interest rate for 12 months

$$(1+i) = (1.0525)^{\frac{12}{15}} = 1.0418$$

$$\Rightarrow i = 0.0418$$

Hence AER= 4.18%

AER to Monthly to Daily Rate

Find, correct to three significant figures, the rate of interest per month that would, if paid and compounded monthly, be equivalent to an effective annual rate of 3.5%?

$F = P(1+i)^{12}$ (i is the monthly interest rate)

$\Rightarrow 103.5 = 100(1+i)^{12}$

$\Rightarrow 1.035 = (1+i)^{12}$

$\Rightarrow \sqrt[12]{1.035} = 1+i$

$\Rightarrow 1.00287 = 1+i \Rightarrow i = 0.00287$

This is a monthly rate of 0.287%

Find, correct to three significant figures, the rate of interest per day, that would, if paid and compounded **daily,** be equivalent to an effective annual rate of 3.5%?

Daily interest rate $= ((1.035)^{1/365} - 1) = 0.00009425$

This is a percentage rate of 0.009425%

9 Month Fixed Term Converted to AER

A bank has offered a 9 month fixed term reward account paying 2.55% on maturity , for new funds from €10,000 to €500,000.
(You get your money back in 9 months time, along with 2.55% interest.)
Confirm that this is , as advertised, an EAR of 3.41%.

Solution 1 (Assuming 3.41% AER)

$F = P(1+i)^t$

Assuming €100 is invested, what will it amount to in $\frac{3}{4}$ year at 3.41% AER?

$F = 100(1.0341)^{\frac{3}{4}}$ (matching periodic rate and period)

$F = 102.55$

If €100 becomes €102.55 in 9 months, this represents an interest rate for the 9 months of 2.55%.

Solution 2

Alternatively, calculate the AER given that the interest rate for 9 months is 2.55%

$(1+i)^{\frac{1}{9}} = (1.0255)^{\frac{1}{9}} = 1.002801728 \Rightarrow$ monthly interest rate $i = 0.002801728$

$(1.002801728)^{12} = 1.0341 = 1 + i \Rightarrow$ AER = annual equivalent rate = 3.41%

or calculate $(1.0255)^{\frac{4}{3}}$