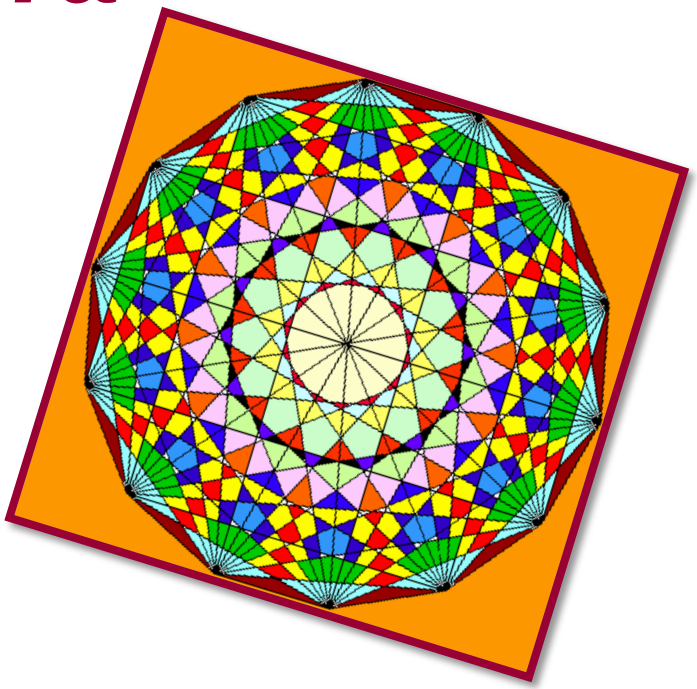
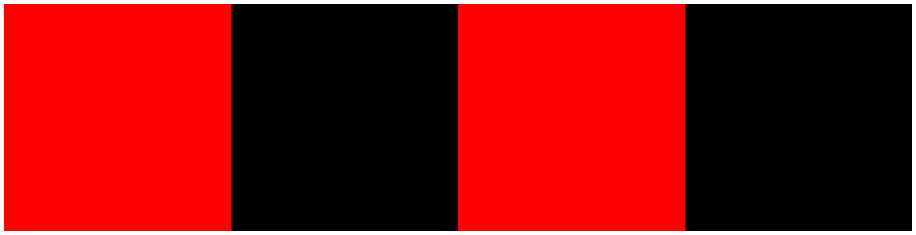
Patterns

A Function Based Approach to Algebra



Coloured Blocks Diagram



Is there a pattern to these colours?
Can you use the pattern to predict what colour will be at a particular point?
How can we investigate if there is a pattern?

Table

Block Position	Colour
1	
2	
3	
4	
5	
6	

Black	
Block	Position
1st	2
2nd	4
3rd	6
4th	8
5th	10

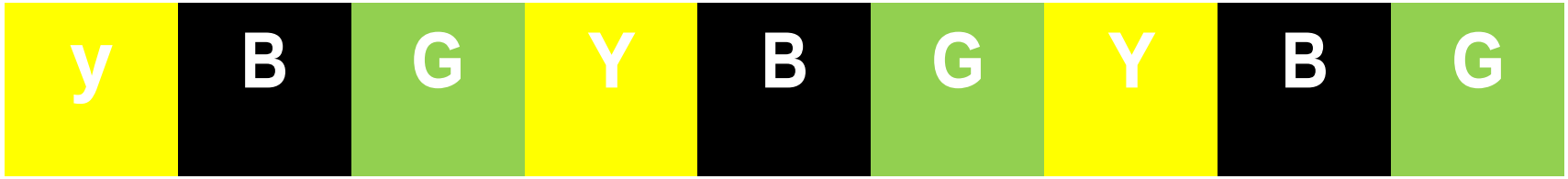
True: Black block is always even.

Words: You double the number of the black block

Black	
Block	***Position***
1	2
2	4
3	6
4	8
5	10

Red	
Block	***Position***
1	1
2	3
3	5
4	7
5	9

Coloured Blocks Diagram



Is there a pattern to these colours?
Can you use the pattern to predict the next colour?
Ways to investigate the pattern:

- table
- story
- general expression
- justify

Tables

Green	
Block	Position
1	3
2	6
3	9
4	12
5	15
n	3n

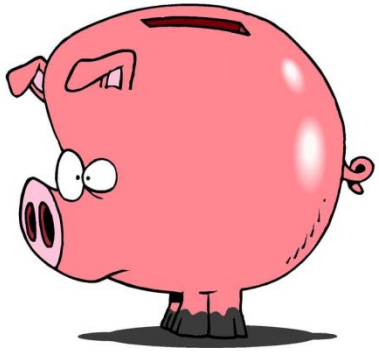
Black	
Block	Position
1	2
2	5
3	8
4	11
5	14
n	3n - 1

Yellow	
Block	Position
1	1
2	4
3	7
4	10
5	13
n	3n - 2

Money Box Problem

Mary	*Money Box*
Start	€0
Growth per day	€2

John	*Money Box*
Start	€3
Growth per day	€2



Mary

Time/days	Money in Box €
0	0
1	2
2	4
3	6
4	8
5	10

John

Time/days	Money in Box €
0	3
1	5
2	7
3	9
4	11
5	13

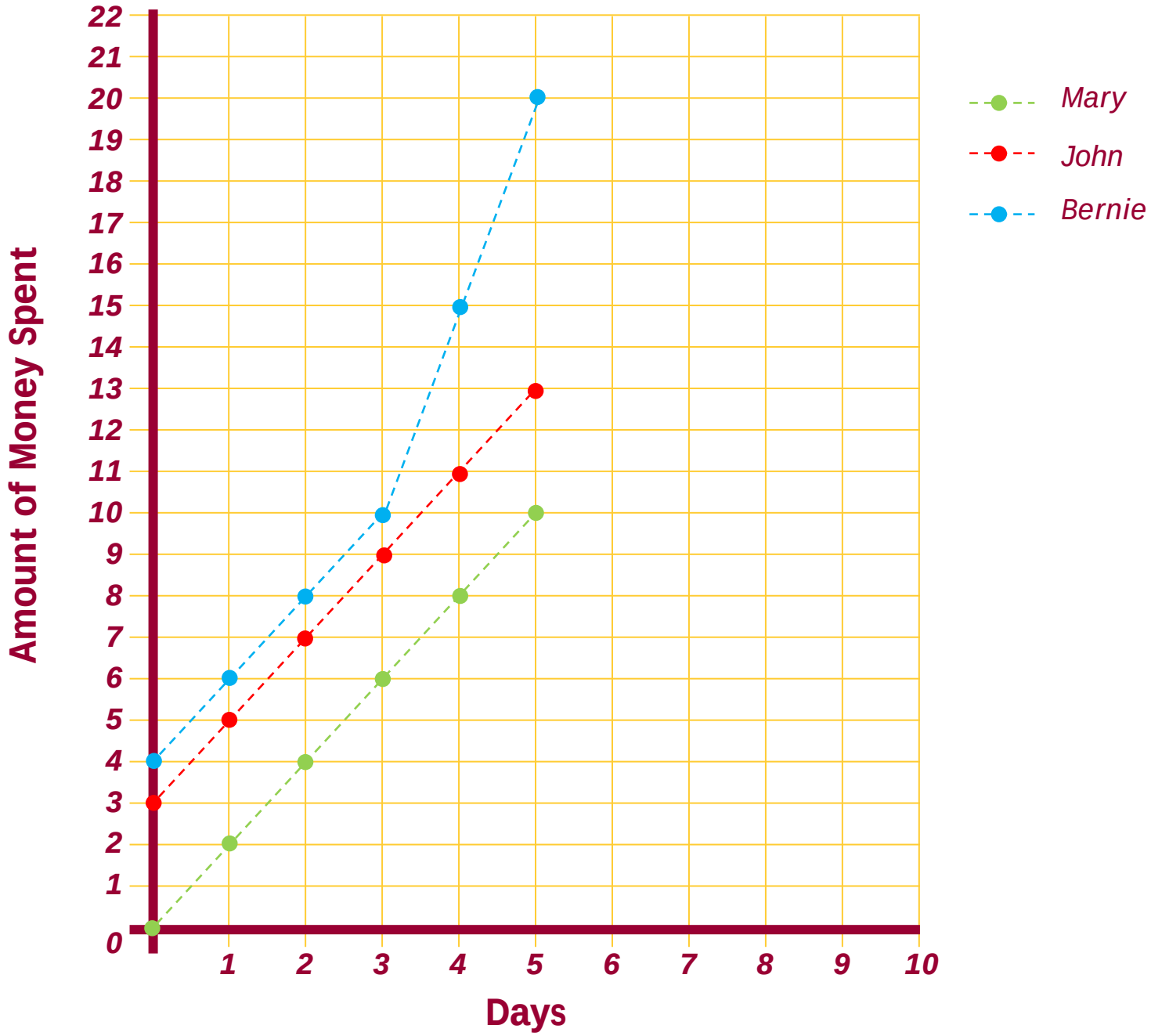
Time on Horizontal Axis
Total Money on Vertical Axis

Identifying variables and constants

Money Box	Varying	Constant
John		
Mary		

What is varying each Day?
What is constant?
Can you put this into words?

Draw a Graph


Amount of Money Spent
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
Days
Mary
John
Bernie

Pattern of Growth for John's Money

Time/days	Money/€	Change
0	3	
1	5	+2
2	7	+2
3	9	+2
4	11	+2
5	13	+2
6	15	+2

Sunflower growth

Sunflower	a	b	c	d
Start height/cm	3	6	6	8
Growth per day/cm	2	2	3	2

Is there a pattern to the growth of these sunflowers?

Can we use this pattern to predict height at some future time?

How can we investigate if a pattern exists?

Table for Each Sunflower

Time/days	Height/cm	Change
0		

Pattern of Growth for 4 Different Sunflowers

A

t/d	h/cm	
0	3	
1	5	
2	7	
3	9	
4	11	
5	13	
6	15	

B

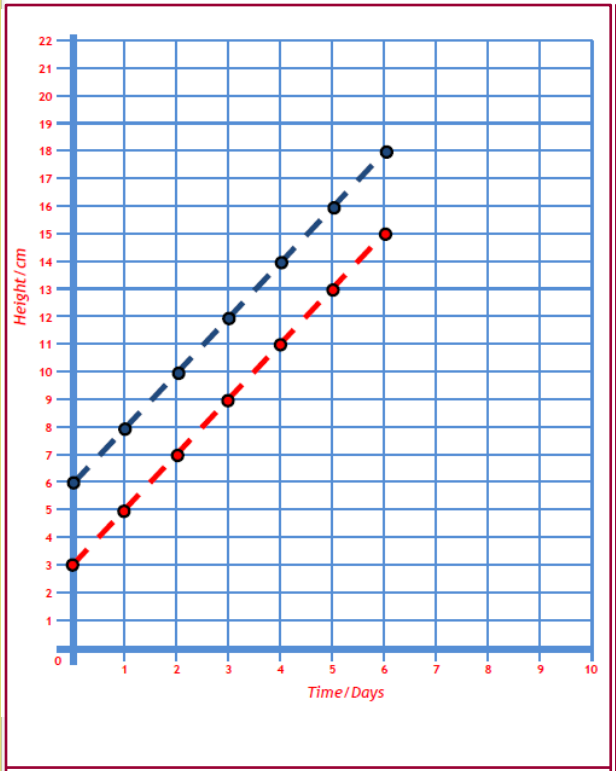
t/d	h/cm	
0	6	
1	8	
2	10	
3	12	
4	14	
5	16	
6	18	

C

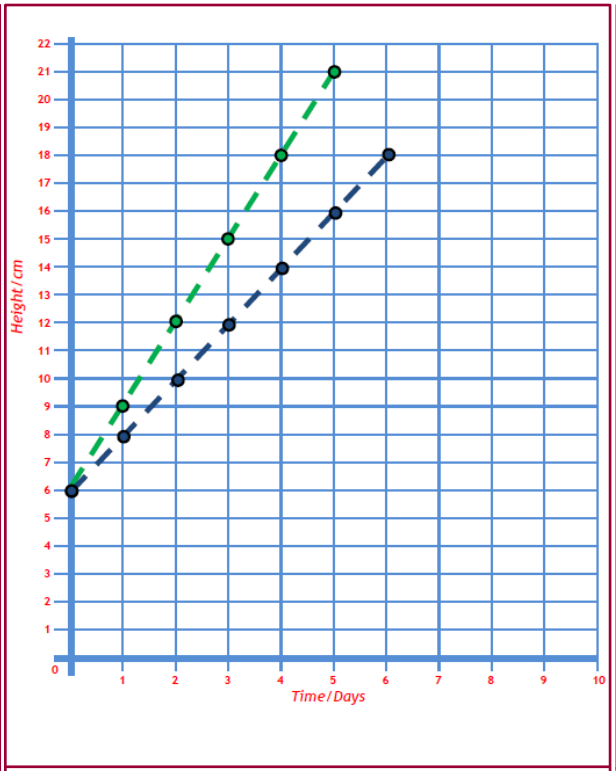
t/d	h/cm	
0	6	
1	9	
2	12	
3	15	
4	18	
5	21	
6	24	

D

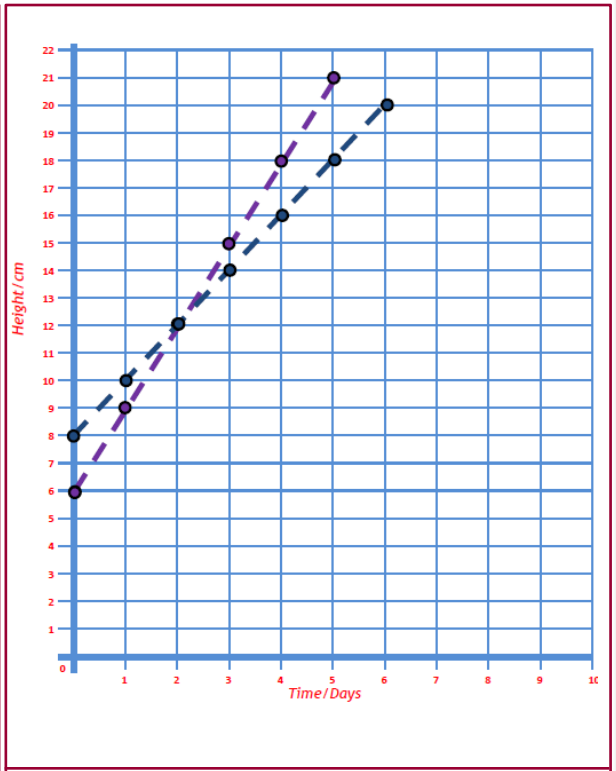
t/d	h/cm	
0	8	
1	10	
2	12	
3	4	
4	16	
5	18	
6	20	



Sunflower A and B



Sunflower B and C



Sunflower C and D

Formula Representation

- **Describe in words** the height of the sunflower a, on any day.
- **Describe in symbols** the height of the sunflower a, on any day.
- **Identify the variables and constants** in this formula.
- Where do the **y – intercept and the slope of the graph appear in the formula**?

Pattern of Growth for 4 Different Sunflowers

A

T days	H cm	
0	3	
1	5	
2	7	
3	9	
4	11	
5	13	
6	15	

B

T days	H cm	
0	6	
1	8	
2	10	
3	12	
4	14	
5	16	
6	18	

C

T days	H cm	
0	6	
1	9	
2	12	
3	15	
4	18	
5	21	
6	24	

D

T days	H cm	
0	8	
1	10	
2	12	
3	14	
4	16	
5	18	
6	20	

Describe in words the height of the sunflower a, on any day.

Describe in symbols the height of the sunflower a, on any day.

Identify the variables and constants in this formula.

Where do the y – intercept and the slope of the graph appear in the formula?

Pattern of Growth for 4 Different Sunflowers

A

T days	H cm	Pattern
0	3	3
1	5	3+2
2	7	3+2+2
3	9	3+2+2+2
4	11	3+2+2+2+2
5	13	3+2+2+2+2+...
6	15	3+2+2+2+2+...

B

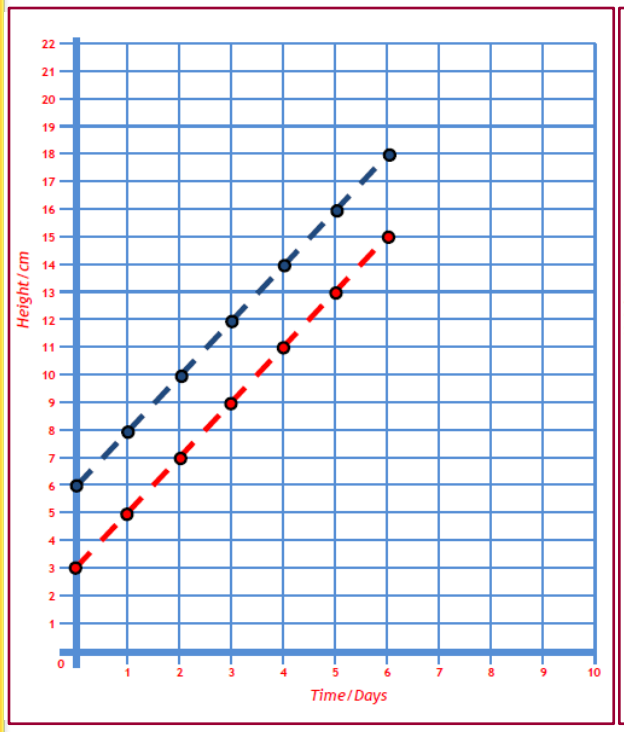
T days	H cm	Pattern
0	6	6
1	8	6+2
2	10	6+2+2
3	12	6+2+2+2
4	14	6+2+2+2+2
5	16	6+2+2+2+2+...
6	18	6+2+2+2+2+...

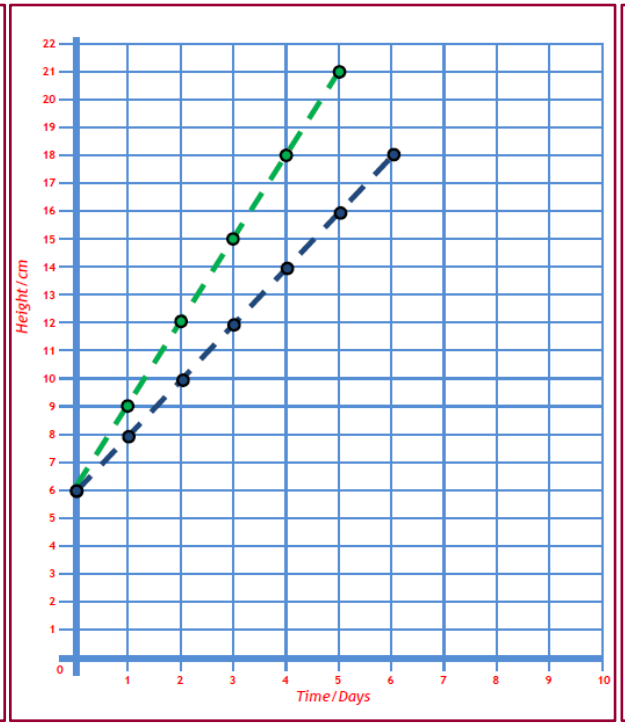
C

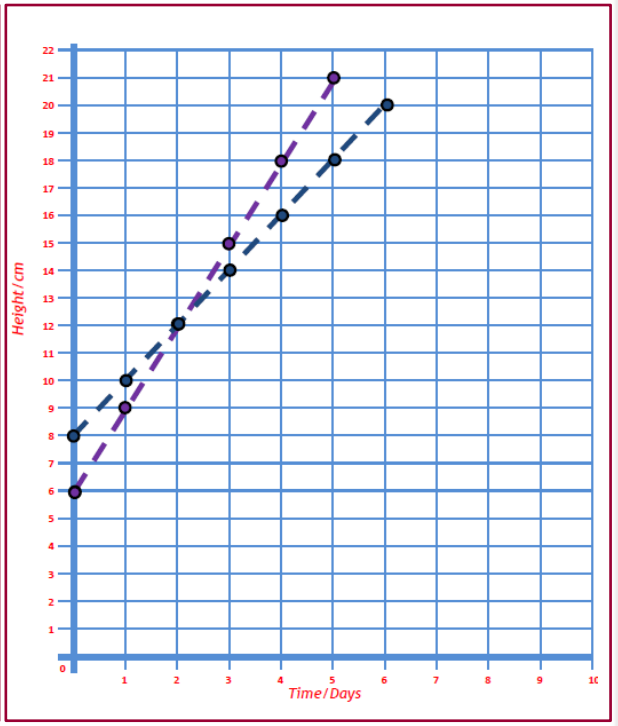
T days	H cm	Pattern
0	6	6
1	9	6+3
2	12	6+3+3
3	15	6+3+3+3
4	18	6+3+3+3+3
5	21	6+3+3+3+3+...
6	24	6+3+3+3+3+...

D

T days	H cm	Pattern
0	8	8
1	10	8+2
2	12	8+2+2
3	14	8+2+2+2
4	16	8+2+2+2+2
5	18	8+2+2+2+2+...
6	20	8+2+2+2+2+...







t/d	h/cm	
0	3	
1	5	
2	7	
3	9	
4	11	
5	13	
6	15	

$$h = 3 + 2d$$

Over to you on your White Boards

1. Draw a rough sketch of the graph: $y = 2x + 1$

2. I start off with 6 euro in my money box and put in 3 euro each day.
Draw a rough sketch of the graph.

3. The initial speed of a car is 10 m/s and the rate at which it increases its speed every second is 2m/s^2.
Write down a linear law for the speed of the car after t seconds.
Draw a graph of the law.

Solution :

Speed $= 10 + 2$(number of seconds travelling)

$v = 10 + 2t$

or $v = 2t + 10$

All the graphs you a have drawn on your white boards have been increasing functions.

Assess the learning:

Do this on your white board

Story :

Isabelle has a money box with 20 euro in it. She takes 2 euro out each day to buy sweets in the shop.

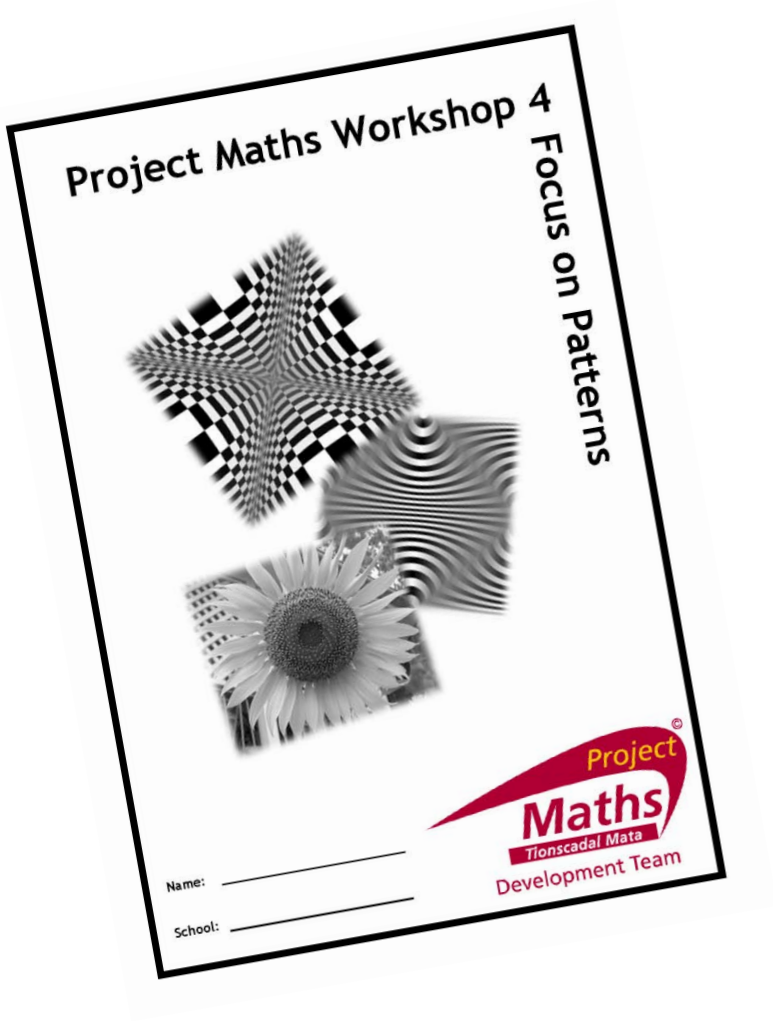
Draw a rough graph of how this might look.

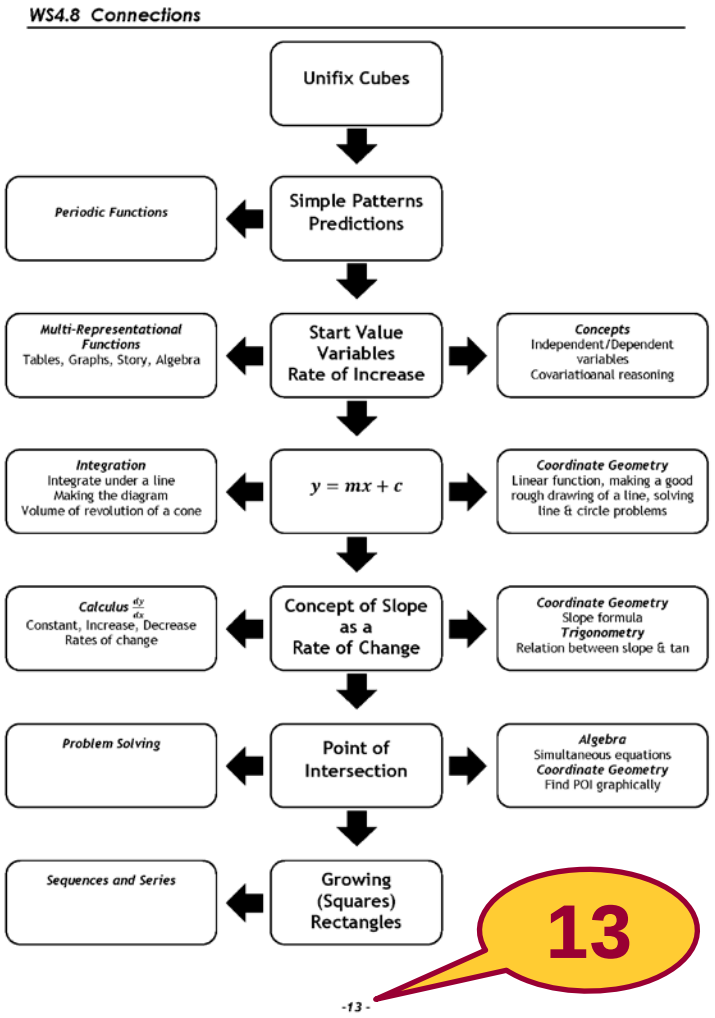
Investigate if your graph is close by doing a table.

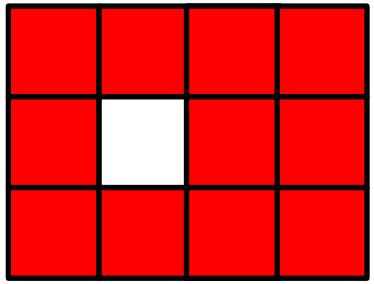
From your table, what is your slope/rate of change?

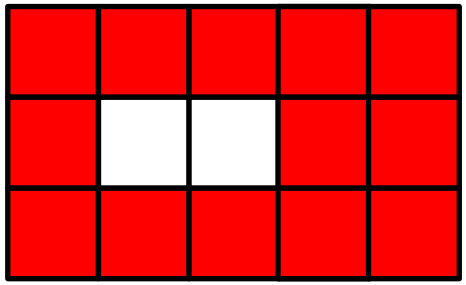
Conclusion: Decreasing graph has a negative slope

Connections



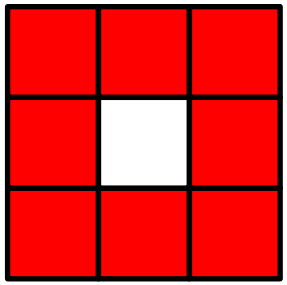


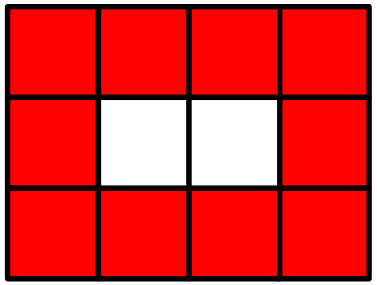


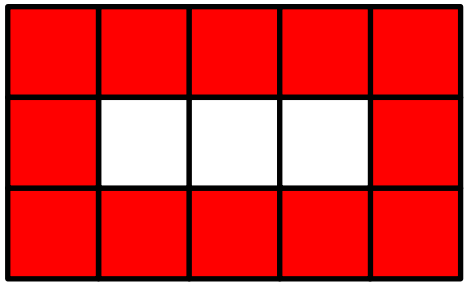


We have 6 red blocks always on the outside. 6 is a constant.

To make the next pattern we are adding two more red (+ 1 white)







1 White, 6 + 2 Red

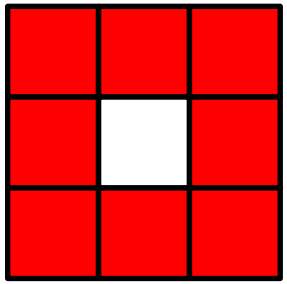
1 White, 8 Red

2 White, (6 + 2 + 2) Red

2 White, 10 Red

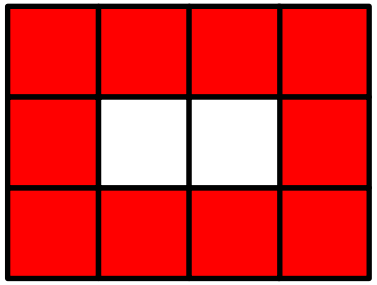
3 White, (6 + 2 + 2 + 2) Red

3 White, 12 Red



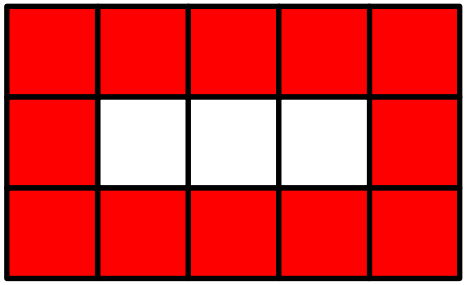
1 White, 6 + 2 Red

1 White, 8 Red



2 White, (6 + 2 + 2) Red

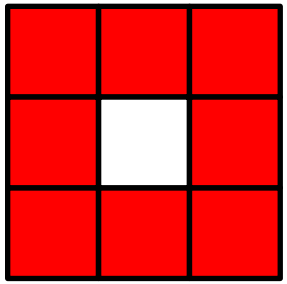
2 White, 10 Red



3 White, (6 + 2 + 2 + 2) Red

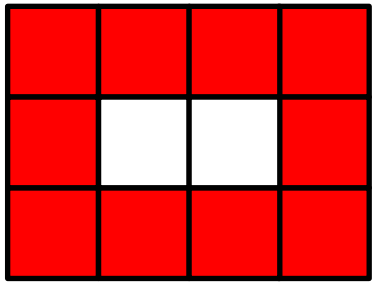
3 White, 12 Red

White	Red	Red
1	6 + 2	6 + 1(2)
2	6 + 2 + 2	6 + 2(2)
3	6 + 2 + 2 + 2	6 + 3(2)
4	6 + 2 + 2 + 2 + 2	6 + 4(2)
⋮	⋮	⋮



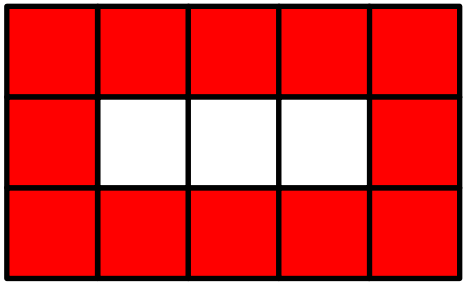
1 White, 6 + 2 Red

1 White, 8 Red



2 White, (6 + 2 + 2) Red

2 White, 10 Red

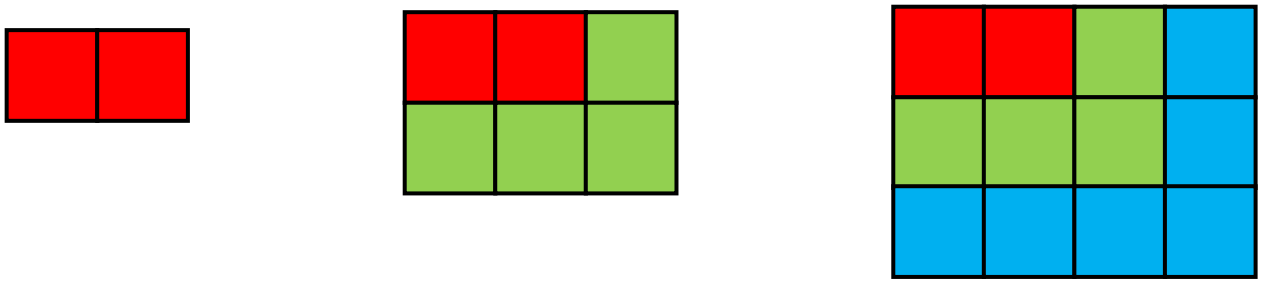


3 White, (6 + 2 + 2 + 2) Red

3 White, 12 Red

White	Red	Red
1	8	8 + 0(2)
2	8 + 2	8 + 1(2)
3	8 + 2 + 2	8 + 2(2)
4	8 + 2 + 2 + 2	8 + 3(2)
⋮	⋮	⋮

Growing Rectangles



Complete the next two rectangles in the above pattern.
There is squared paper in your handbooks.
Is there a relationship between the:

- draw a **table** ht, length,area, change, change
- draw the **graph** of height(x-axis) v area y-axis.
- find the **formula**
- **draw conclusions from all of the above**

Table

Height, *h*	Length, *l*	Area = *h* x *l*	Number of tiles n in each rectangle	Change	Change of change
1	2	2	2		
2	3	6	6	+4	
3	4	12	12	+6	+2
4	5	20	20	+8	+2
5	6	30	30	+10	+2
6	7	42	42	+12	+2
7	8	56	56	+14	+2

Investigate the first change and the second change.
What do we notice?

If we let n be the height, write a formula for the area in terms of n, on your white board.

Formula

$$Area = n(n+1)$$

At this stage they will have done the procedures for Algebra…

$$Area = n^2 + n$$

Observed so far :

- first change varies
- second change constant
- n^2 term

Characteristics :

- first change varies
- second change constant
- n^2 term
- curved graph

Are the characteristics of…..

A Quadratic Relationship

Story: How to ask for Pocket Money

"I only want you to give me pocket money for the month of July.

All I want is for you to give me 2 c on the first day of the month, double that for the second day, and double that again for the 3rd day… and so on.

On the first day I will get 2 c, on the 2nd day 4 c, on the 3rd day 8c and so on until the end of the month. That is all I want."

Is this a good deal for my parents or is it a good deal for me?

Investigate using a Table

Day	Money in cent
1	2
2	2 x 2
3	2 x 2 x 2
4	
5	
6	
7	
8	
9	
10	

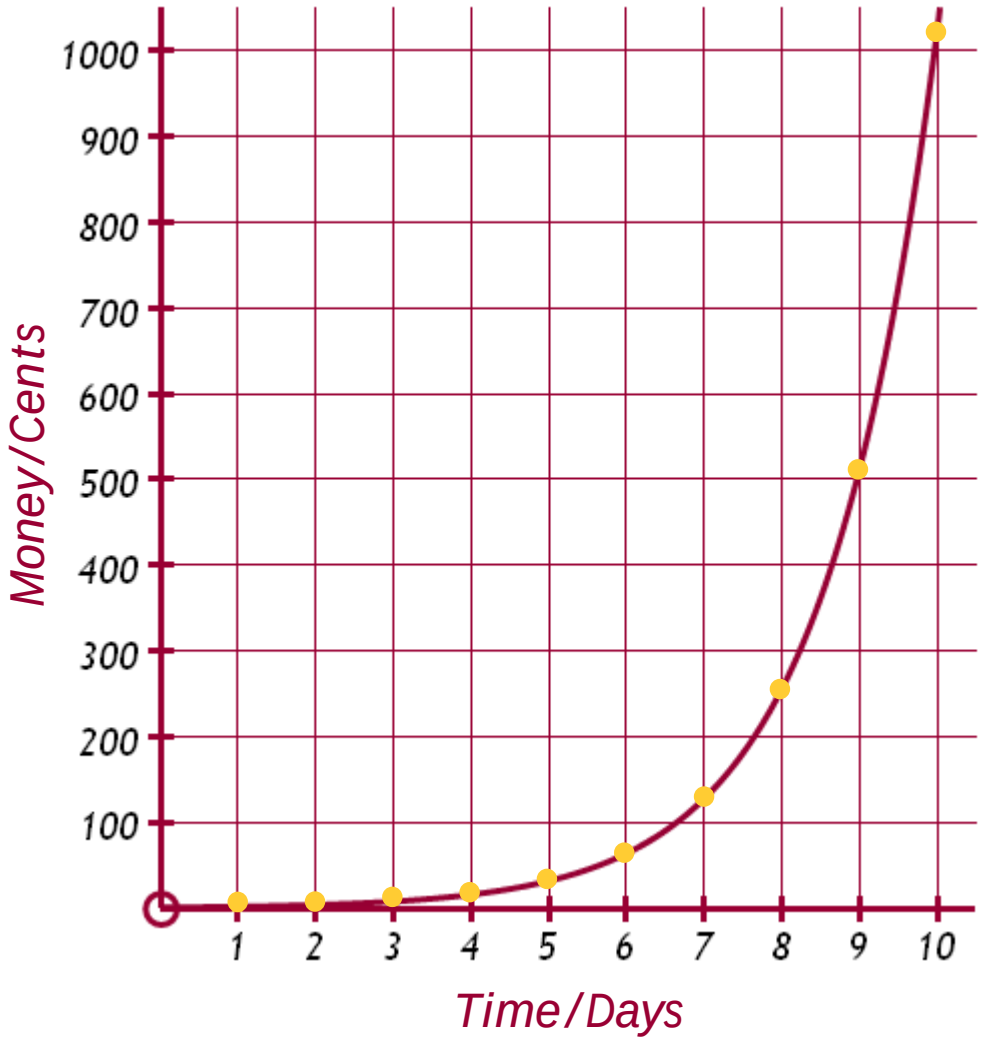
Do this on your white board

If we let n be the number of days, can we write a formula for the Amount of Pocket Money? Draw the graph.

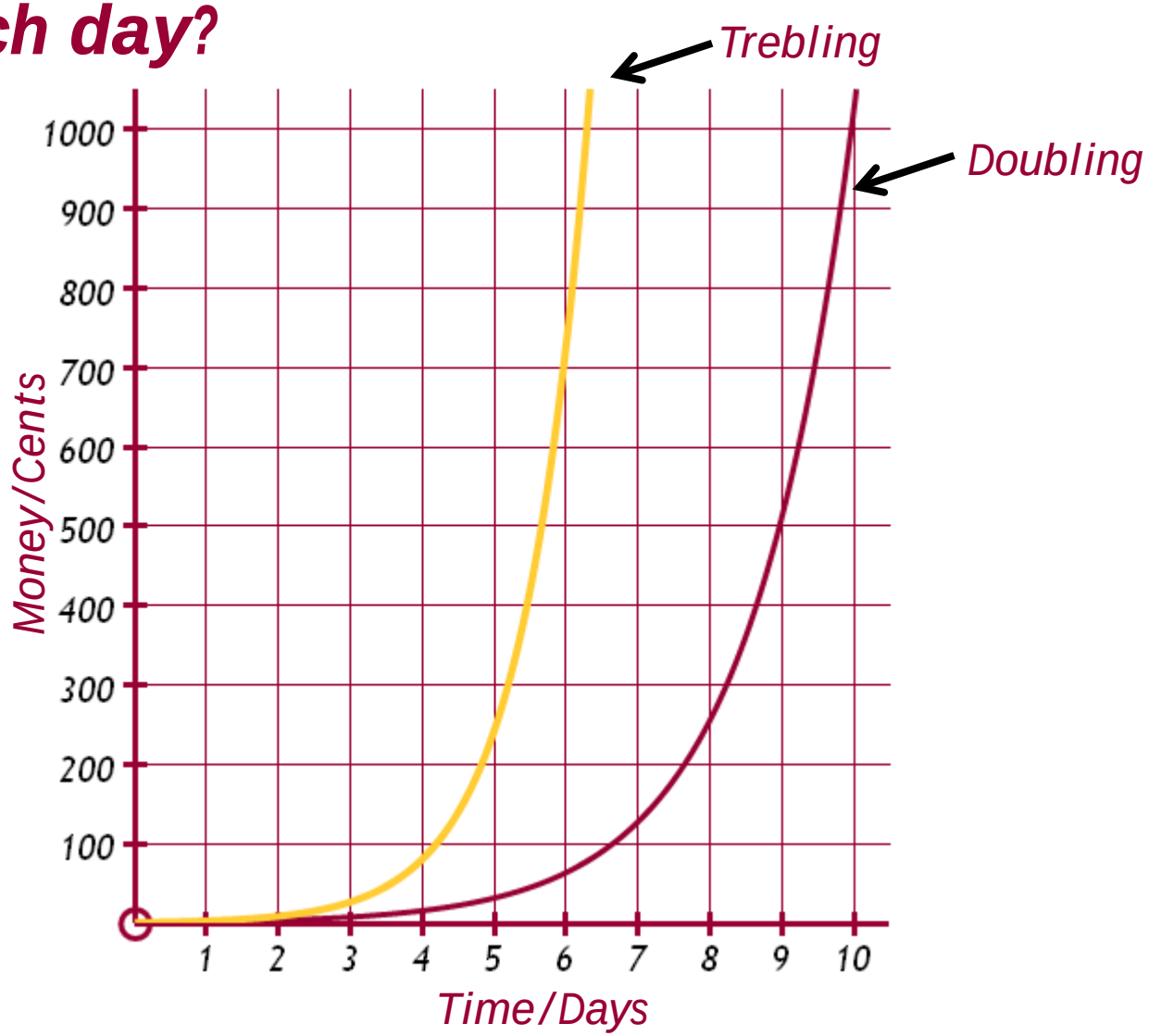
Formula : $Amount = 2^n$

Words : Doubling

Graph


1000
900
800
700
600
500
400
300
200
100
Money/Cents
1
2
3
4
5
6
7
8
9
10
Time/Days

What if your Dad trebled the amount of money each day?



The money would grow even quicker.

Lets look at the Changes in a Table

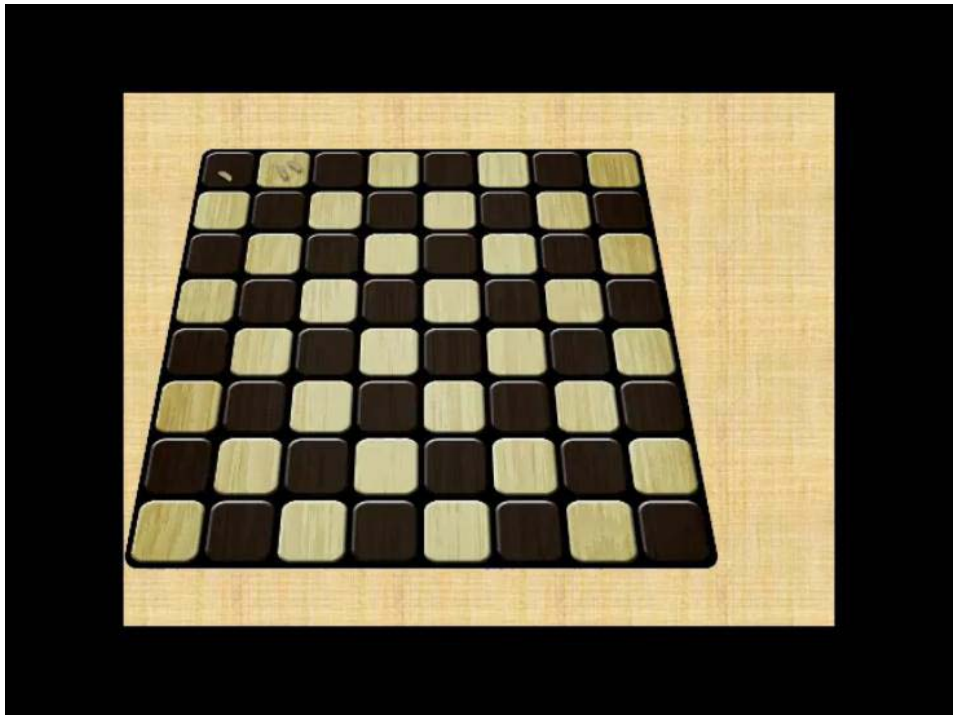
Days	Money in cent	Change	Change of change
1	2		
2	4	+2	
3	8	+4	+2
4	16	+8	+4
5	32	+16	+8
6	64	+32	+16
7	128	+64	+32
8	256	+128	+64
9	512	+256	+128
10	1024	+512	+256

What do you notice about the Change columns.....
They develop in a ratio.

Characteristics :

- change develops in a ratio
- Formula: 2^n or 3^n
- Words: Doubling or Trebling
- curved graph that grows very quickly

Are the characteristics of…..

An Exponential Relationship (video/chess ppt)

Grains of Rice Story