Patterns
&
A Relations/Functions based approach to Algebra

Does this look as familiar to you……. as it does to me?

Primary School	First Year	Third Year
$3 \times ? = 18$ *Ans: 6*	$3x = 18$ $\frac{3x}{3} = \frac{18}{3}$ $x = 6$ *Ans: x = 6*	$3x = 18$ $x = 18 - 3$ $x = 15$ *Ans: x = 15*
$? + 5 = 12$ *Ans: 7*	$x + 5 = 12$ *Ans: x = 7*	$x + 5 = 12$ $x = 12 + 5$ $x = 17$ *Ans: x = 17*
$2 \times ? = 6$ *Ans: 3*	$2x = 6$ $x = \frac{6}{2}$ $x = 3$ *Ans: x = 3*	$2x = 6$ $x = \frac{6}{-2}$ $x = -3$ *Ans : x = -3*

Does this look as familiar to you…….
as it does to me?

Primary School

$3 \times ? = 18$

Ans: 6

$? + 5 = 12$

Ans: 7

$2 \times ? = 6$

Ans: 3

First Year

$3x = 18$

$\frac{3x}{} = \frac{18}{}$

2

$x = 3$

Ans: x = 3

Third Year

$3x = 18$

$x = 18 - 3$

$x = 15$

: x = 15

$x + 5 = 12$

$x = 12 + 5$

$x = 17$

: x = 17

$2x = 6$

$x = \frac{6}{-2}$

$x = -3$

Ans : x = -3

Procedure overtaking Understanding

*"to cover their lack of understanding. It appears that students resort to memorising **rules and procedures** and they eventually come to believe that this activity represents the essence of algebra."*

Carolyn Kieran

Chief Examiners' Reports

JC OL - 2006

"Common mistakes were : combining terms leading to oversimplification and **transposition** errors."

JC HL - 2006

"Areas of weaknesses in candidates answers: Handling **transpositions** in algebraic type equations."

LC FL - 2005

"There were.......**transposition** errors".
"**Transposing** accurately and with understanding is certainly a difficulty. "

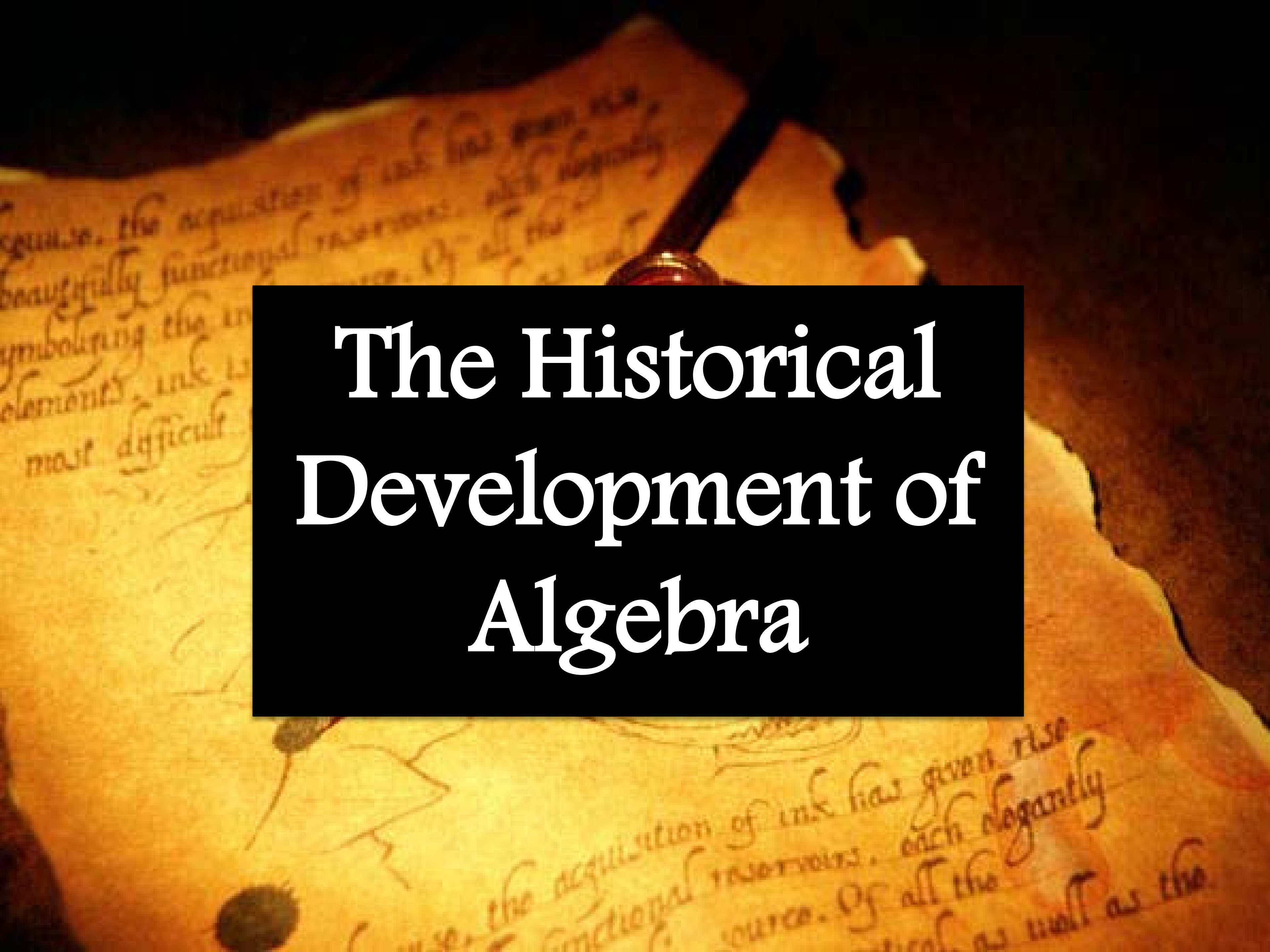
LC OL - 2005

"Average candidates experience difficulty with all but the most basic of algebraic manipulations and can cope only with basic routines in solving equations ."

How can we develop in our students,

a deeper understanding

of what Algebra is?



The Historical Development of Algebra

Before 250 A.D.

- Ordinary Language Descriptions used for Solving Problems
- No symbols or special signs to represent unknowns

c. 250 A.D.

- Diophantus introduced the use of letters for unknown quantities.
- There were no procedures on how to use algebra.
- Diophantus’ maths was still presented as a story of how to do things.

Also in late 16th Century

- It was now possible to use algebra as a tool for proving rules governing numerical relations.

3rd to 16th Century

- Algebra was used to get a result for the unknown: For example x = 9
- They were not using algebra to express the general case.

Late 16th Century

- Vieta read the work of Diophantus and began to use a letter for a given, as well as an unknown
- Now it became possible to express general solutions.

Latest Development

- Procedures for governing algebra.

Question

If the procedures were the latest development…

Are we doing the right thing, introducing students so early to the procedures?

Should we introduce Algebra in the same natural way it was discovered?

Question

`Moreover what psychological demands are made on a First Year Student when we introduce them to abstract algebra so quickly?

Concept of Equality

In Primary School:

The equal sign is seen as an instruction.

3 plus 5 equals what?

$3 + 5 =$

$23 \times 37 =$

In Post-Primary School:

Students need to see the equal sign as now meaning equality

$a + 3a = 4a$

$3(a+b) = 3a + 3b$

Project Maths Syllabus

Old Syllabus	New Syllabus
	?
Expressions	?
Procedures	?
Equations	?
Word Problems	?
	?
	Expressions
	Procedures
	Equations

Patterns

"Mathematics……the study of patterns"

Pattern → Conjecture → Proof → Formula Procedure

Pattern → Observe → Generalise

"algebra as a systematic way of expressing generality"

Syllabus

Concrete → Abstract

Words	→	Abbreviations
Numbers	→	Variables
Arithmetic properties of Numbers	→	Arithmetic Properties of Algebra

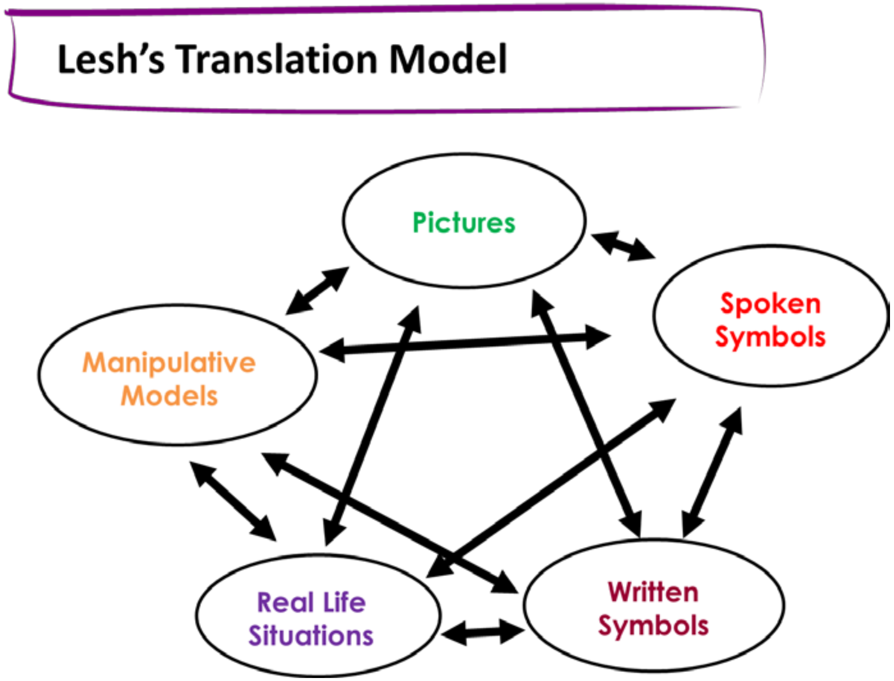
“algebra as generalised arithmetic”

Syllabus

Multi-Representations

We will discover algebra through the **Multi-Representational** approach:

- Story
- Tables
- Graphs
- Manipulate Models



"representational activities"

Syllabus

Key outcomes and words:

- Identify patterns and describe different situations using tables and graphs, words and formulae
- Generalise in words and symbols
- A variable
- Independent and dependent variables
- **Constant rate of change** as that which characterises a linear function
- Inputs and Outputs
- 'Start amount'
  - Table
  - Formula
  - y-intercept of a graph
- Rate of change of the dependent variable
  - Table
  - Graph as a 'slope'
  - Formula $y = mx + c$
- Constant rate of change
- Identify linear relationships as having constant first differences between successive *y* values (outputs)
- Know that parallel lines have the same slope (same rate of change of *y* with respect to *x*)
- Connect increasing functions with positive slope, decreasing functions with negative slope and constant functions with zero slope

Project Maths publication "A Functions based approach to Algebra"