Purpose

- Using Problem Solving as a methodology to explore geometry construction in context
- Geometry construction: connections within Strand 2

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Section A: Geometry 2012/2013/2014

Higher

- Constructions 1 – 15 and 22
- Terms: is equivalent to, if and only if, proof by contradiction
- Prove theorems 11,12 and 13

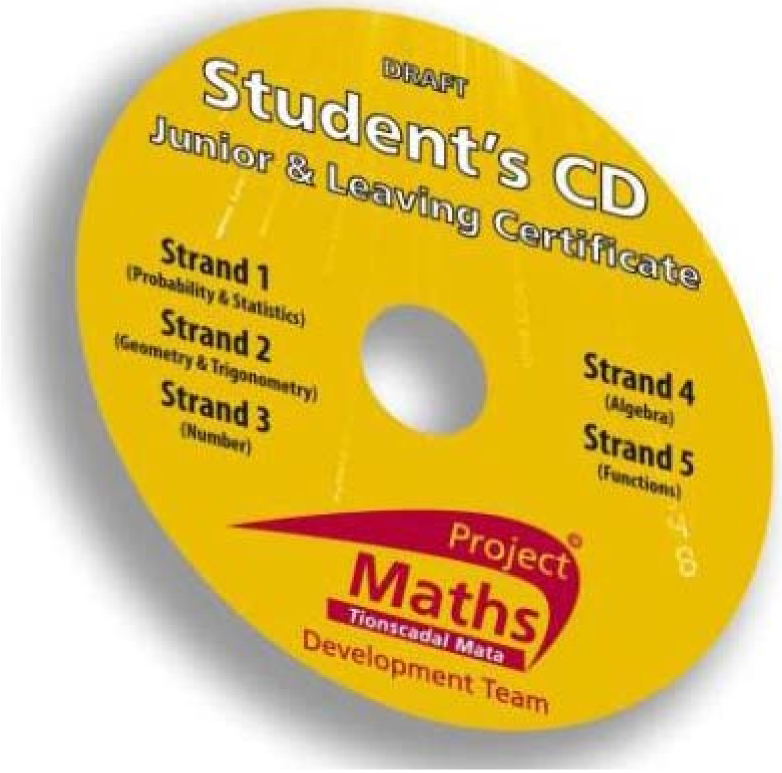
Ordinary

- Constructions 16, 17, 21
- Terms: theorem, proof, axiom, corollary, converse, implies
- Investigate and use theorems 7,8,11,12,13,16, 17,18,20,21 and corollary 6

Foundation

- Constructions 18, 19,20

	Constructions (Supported by 46 definitions, 20 propositions, 5 axioms and 21 theorems)	CMN Introd. Course	JC ORD	JC HR	LC FN	LC ORD	LC HR
1	Bisector of an angle, using only compass and straight edge.	√	√	√	√	√	√
2	Perpendicular bisector of a segment, using only compass and straight edge.	√	√	√	√	√	√
3	Line perpendicular to a given line l, passing through a given point not on l.			√			√
4	Line perpendicular to a given line l, passing through a given point on l.	√	√	√	√	√	√
5	Line parallel to given line, through a given point.	√	√	√	√	√	√
6	Division of a line segment into 2 or 3 equal segments without measuring it.	√	√	√	√	√	√
7	Division of a line segment into any number of equal segments, without measuring it.			√			√
8	Line segment of a given length on a given ray.	√	√	√	√	√	√
9	Angle of a given number of degrees with a given ray as one arm.		√	√	√	√	√
10	Triangle, given lengths of 3 sides.		√	√	√	√	√
11	Triangle, given SAS data.		√	√	√	√	√
12	Triangle, given ASA data		√	√	√	√	√
13	Right-angled triangle, given length of hypotenuse and one other side		√	√	√	√	√
14	Right-angled triangle, given one side and one of the acute angles.		√	√	√	√	√
15	Rectangle given side lengths.		√	√	√	√	√
16	Circumcentre and circumcircle of a given triangle, using only straight edge and compass.					√	√
17	Incentre and incircle of a triangle of a given triangle, using only straight edge and compass.					√	√
18	Angle of 60° without using a protractor or set square.				√	√	√
19	Tangent to a given circle at a given point on it.				√	√	√
20	Parallelogram, given the length of the sides and the measure of the angles.				√	√	√
21	Centroid of a triangle.					√	√
22	Orthocentre of a triangle.						√



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7 Constructions to Study

The instruments that may be used are:

straight-edge: This may be used (together with a pencil) to draw a straight line passing through two marked points.

compass: This instrument allows you to draw a circle with a given centre, passing through a given point. It also allows you to take a given segment $[AB]$, and draw a circle centred at a given point C having radius $|AB|$.

ruler: This is a straight-edge marked with numbers. It allows you measure the length of segments, and to mark a point B on a given ray with vertex A, such that the length $|AB|$ is a given positive number. It can also be employed by sliding it along a set square, or by other methods of sliding, while keeping one or two points on one or two curves.

protractor: This allows you to measure angles, and mark points C such that the angle $\angle BAC$ made with a given ray $[AB$ has a given number of degrees. It can also be employed by sliding it along a line until some line on the protractor lies over a given point.

set-squares: You may use these to draw right angles, and angles of $30°$, $60°$, and $45°$. It can also be used by sliding it along a ruler until some coincidence occurs.

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Students learn about	Students should be able to
2.5 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

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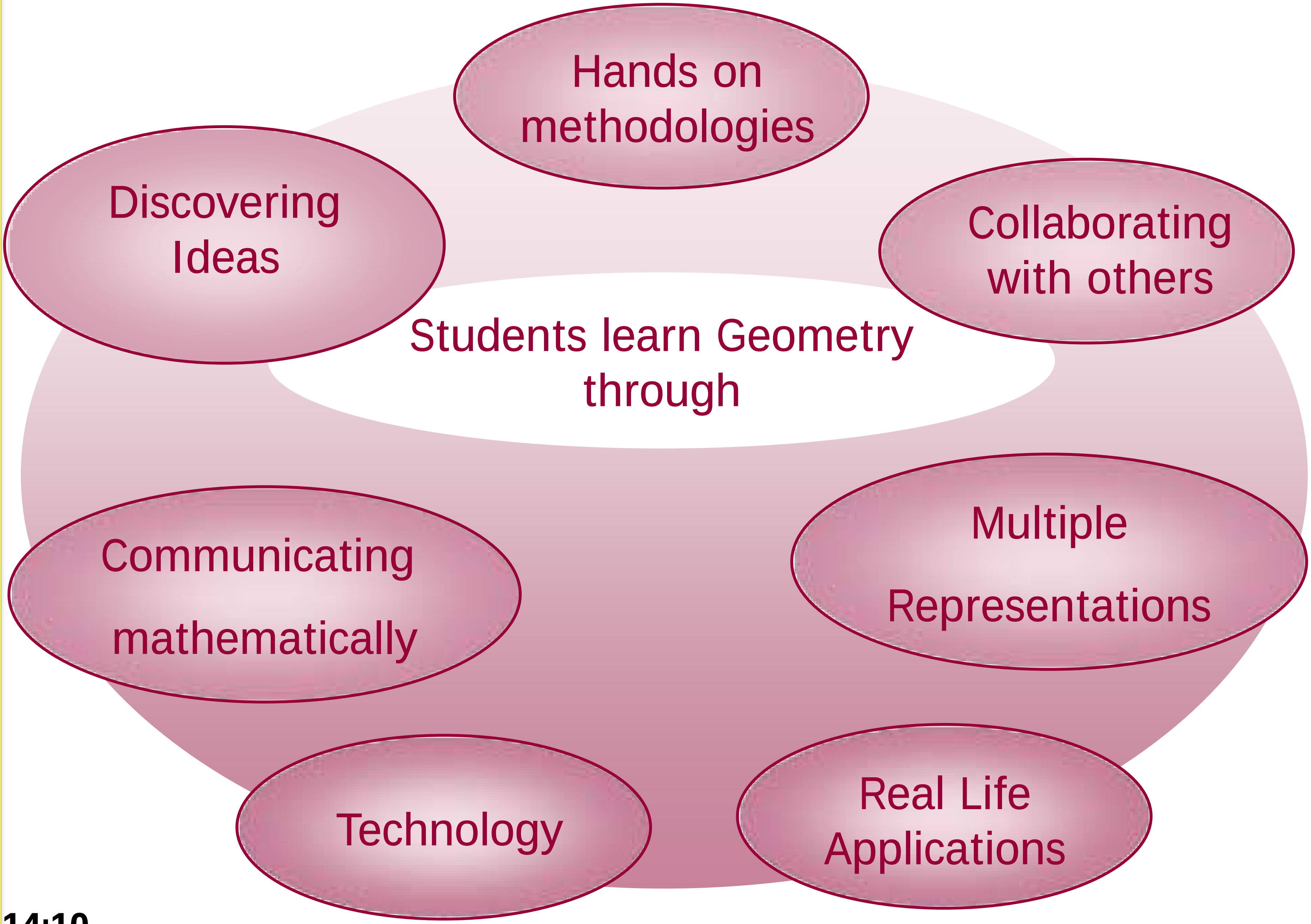
Hands on methodologies
Discovering Ideas
Collaborating with others
Students learn Geometry through
Communicating mathematically
Multiple Representations
Technology
Real Life Applications

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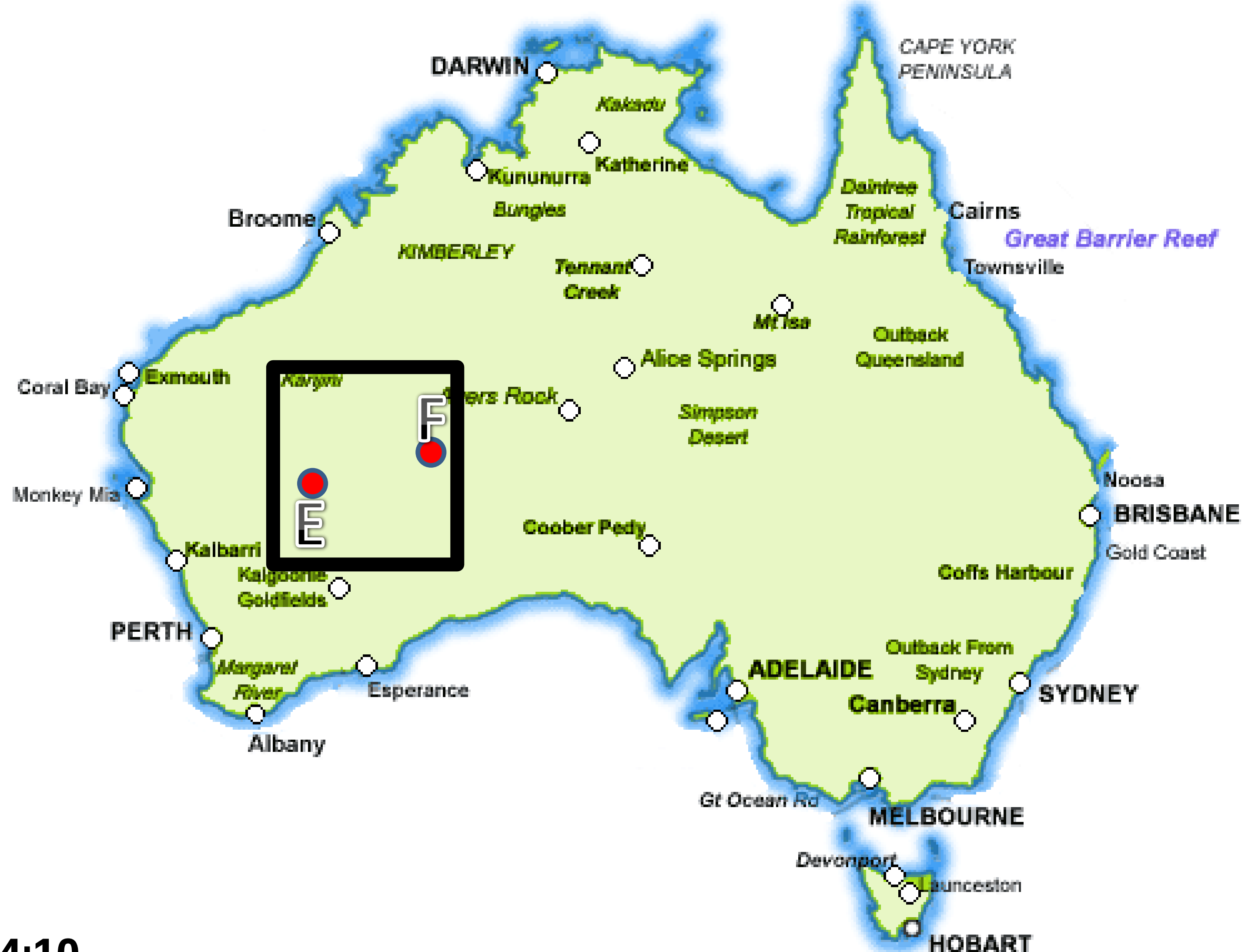
In a remote area of Australia the Royal Flying Doctor Service has a base located at E and another base located at F.



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In a remote area of Australia the Royal Flying Doctor Service has a base located at E and another base located at F.

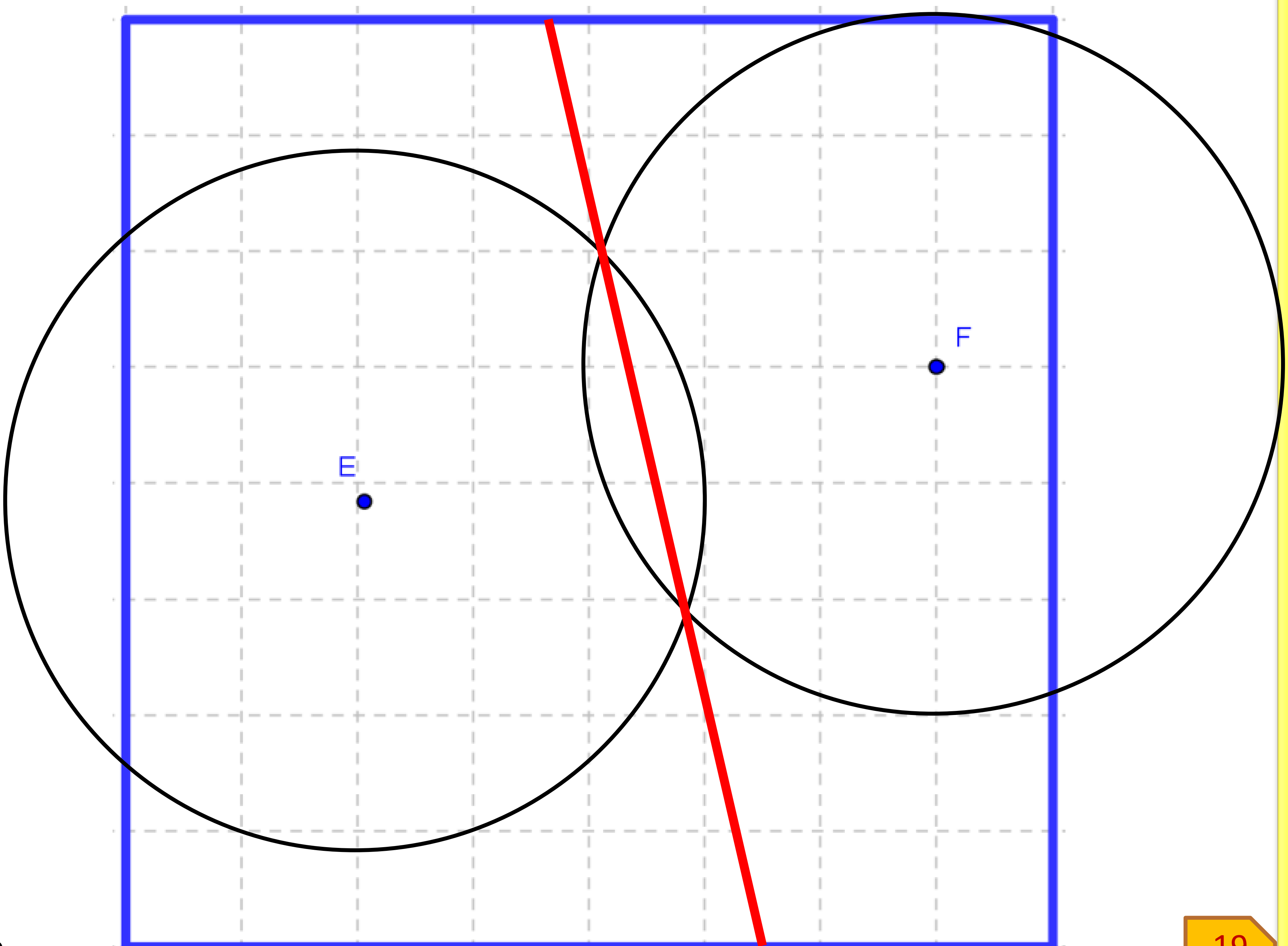
All emergency calls are received at a central call centre and are then transferred to the closest base.

The map of the area shows the position of the two bases.

You need to divide the area into two regions so that any emergency is responded to from the nearer base.

The scale on the map is “grid square side = 20km”.

See Workbook: Page 19



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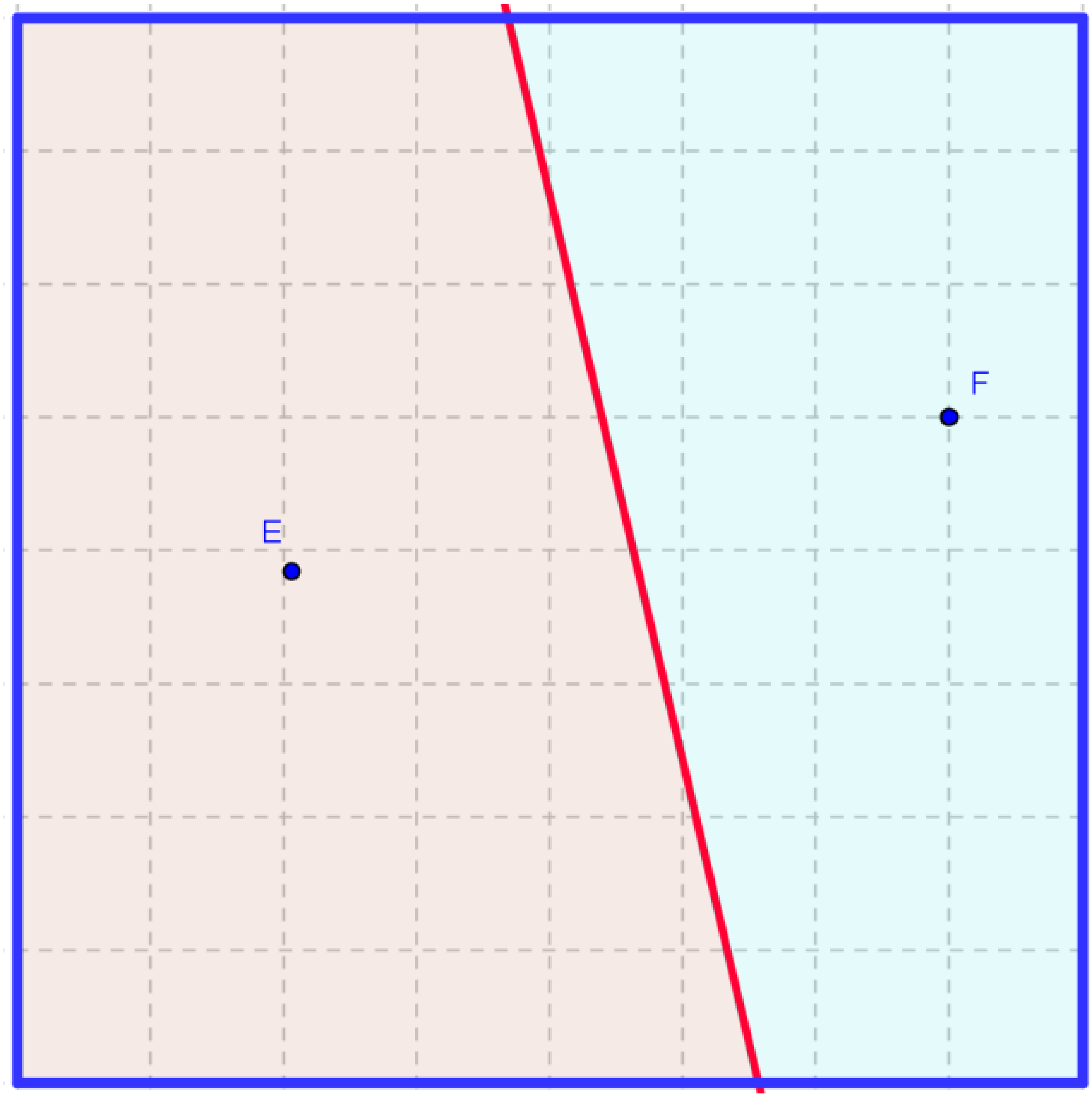

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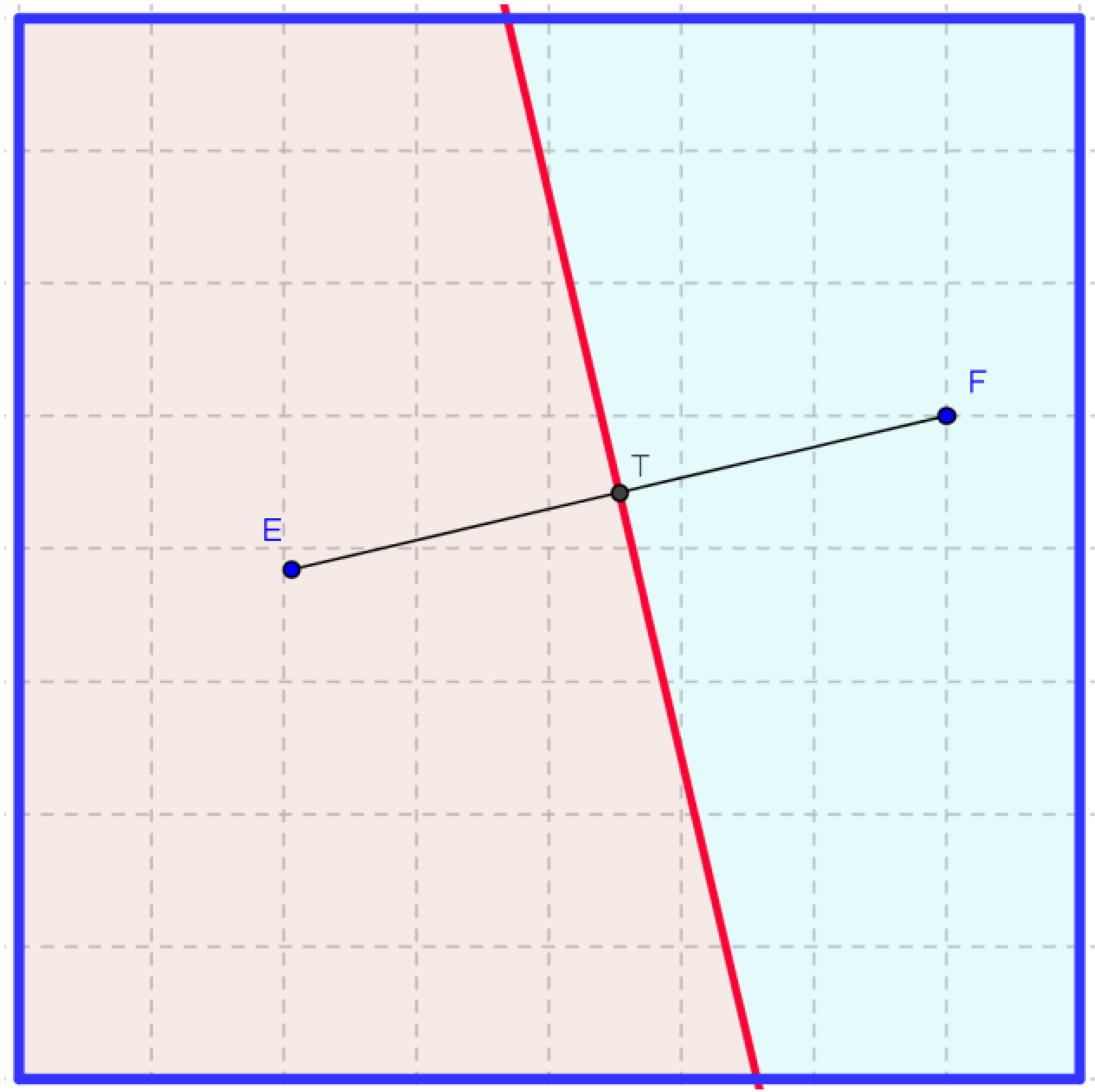

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Hint 1: Mark in and name a point which is the same distance from both aircraft bases.

T = Mid Point [EF]

Hint 2: Is/are there other point(s) which is/are the same distance from both aircraft bases and if there is/are where would it/they be?

Infinite number of points through T perpendicular to [EF]

(i) Is there a special name for the line which you have drawn?

Perpendicular bisector (Mediator) [EF]

(ii) What is the approximate area of each region?

E: *14,800 km²*   F: *10,800 km²*

(iii) The number of emergency calls is consistent for all of the area. You have 56 people overall to staff the two aircraft bases. How many staff do you need at each base?

E: *32 people*   F: *24 people*

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(ii) What is the approximate area of each region?

E: 14,800 km^2 F: 10,800 km^2

(iii) The number of emergency calls is consistent for all of the area. You have 56 people overall to staff the two aircraft bases. How many staff do you need at each base?

E: 32 people F: 24 people

(iv) The area has recently received government funding to set up one specialist emergency neurological team which must service both bases. Assuming the number of emergency calls is consistent for all of the area, where is the best location to place this team? Give a reason for your choice.

At T the mid point of [EF]

as this is equidistant from both bases and is also the shortest distance of all the equidistant points.

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Reset

Construction 2. Perpendicular bisector of a line segment.

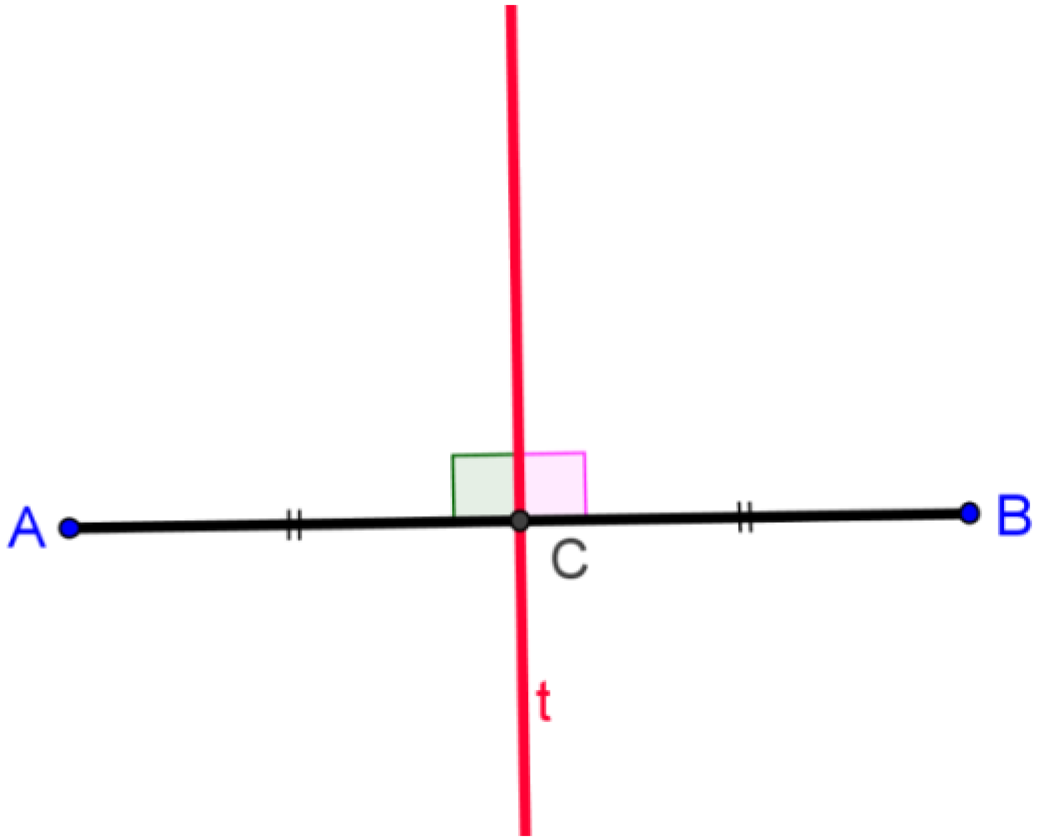
Instructions: Click on the Tick Boxes in sequence to show construction steps.

Tick Boxes

- [ ] A
- [ ] B
- [ ] C
- [ ] D
- [ ] E
- [ ] F
- [ ] G

See Student CD

In the diagram below t is the perpendicular bisector [AB] and the point of intersection is C



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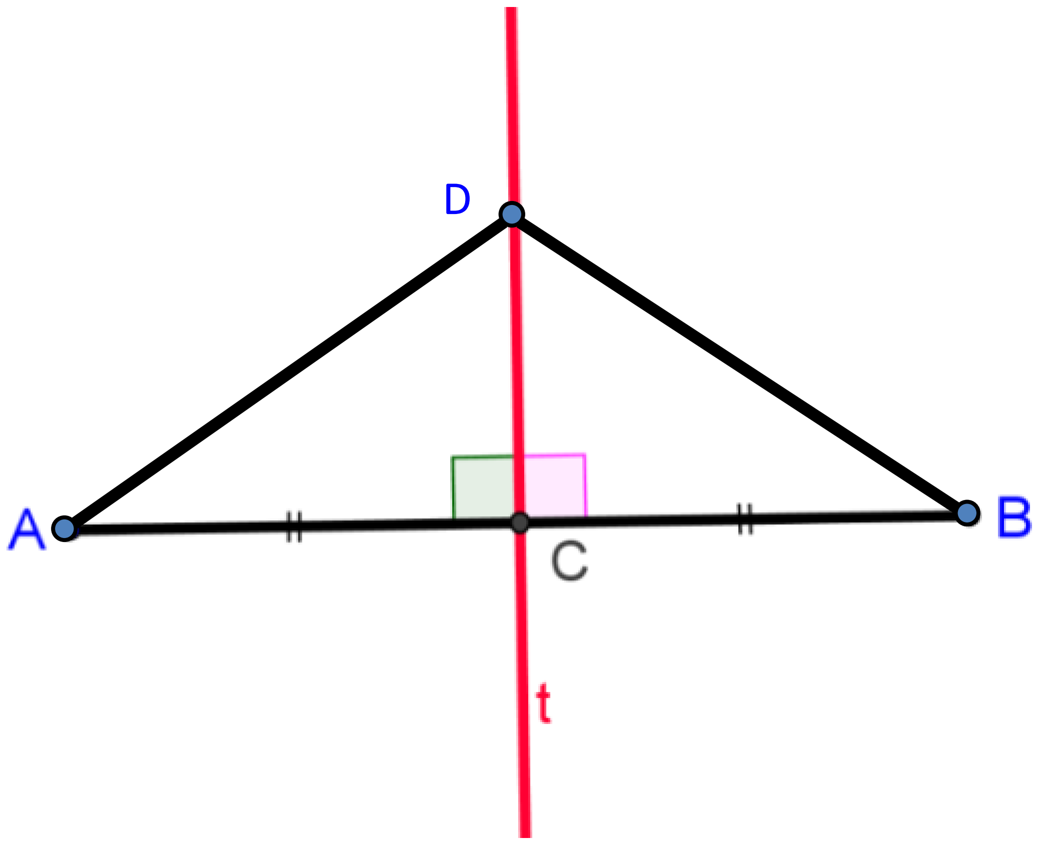
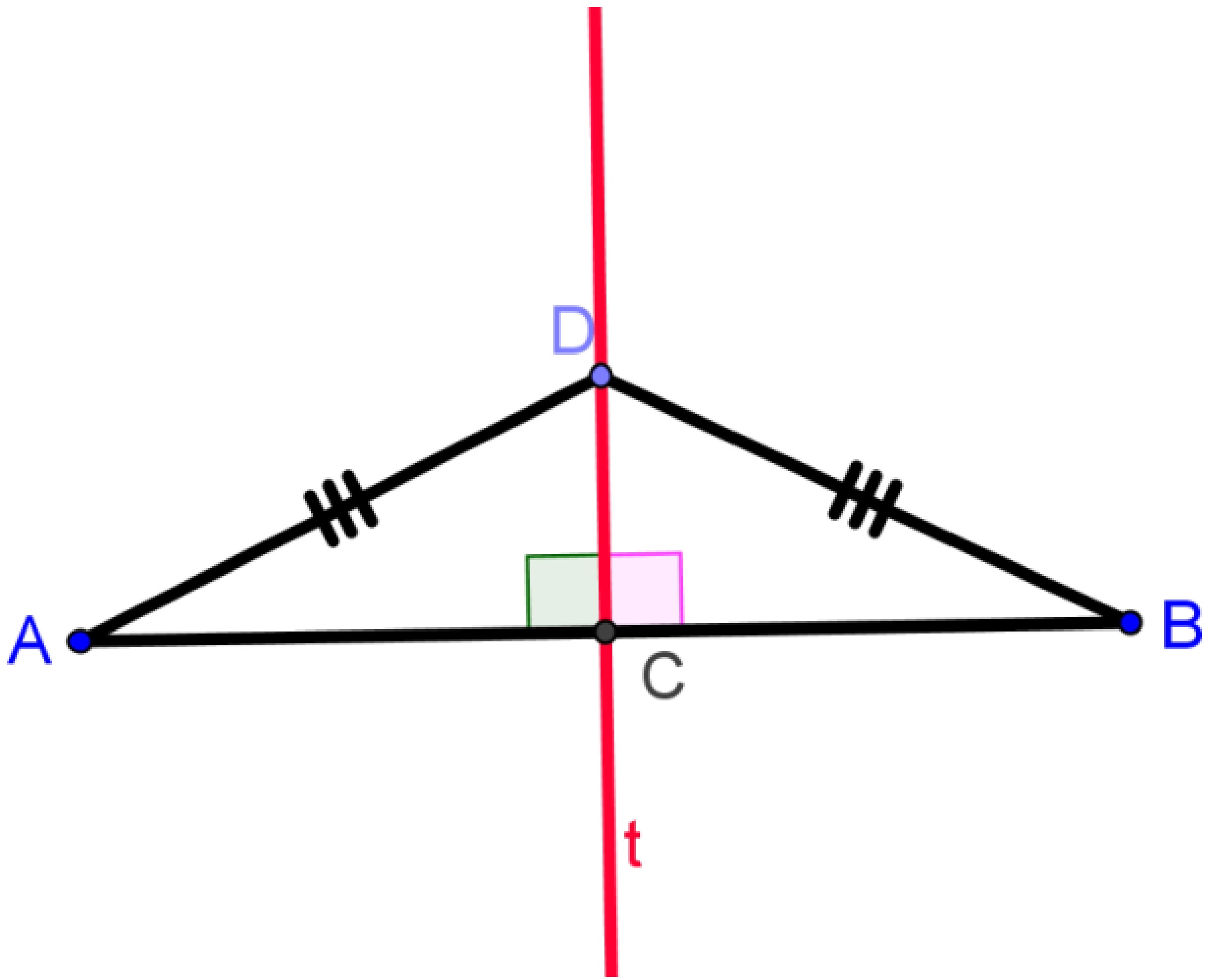
By letting D be a point on t what can you say about triangles ACD and BCD, giving reasons for your answer?



Now look at triangles ACD and BCD. What can you say about these triangles, giving reasons for your answer?



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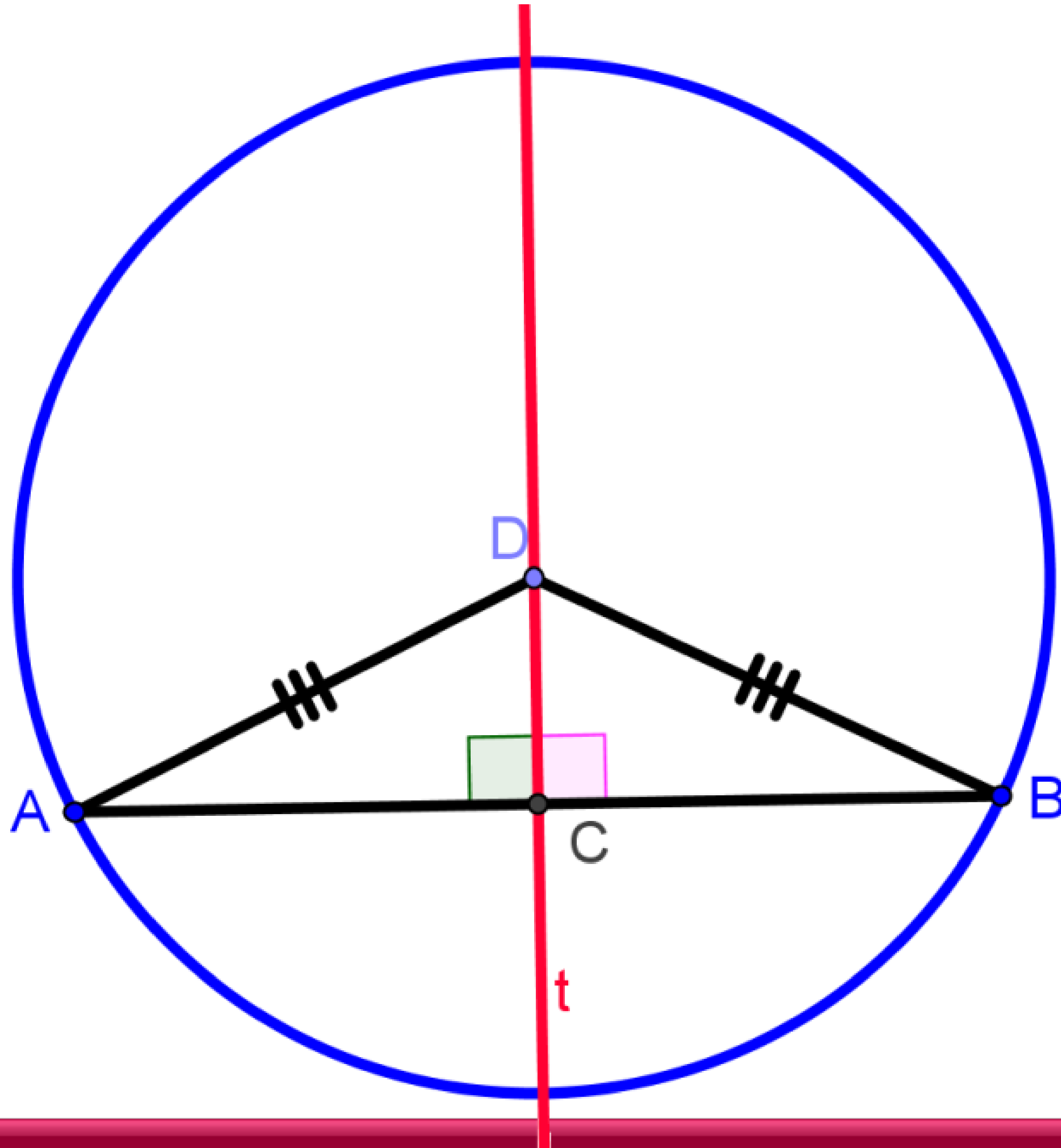

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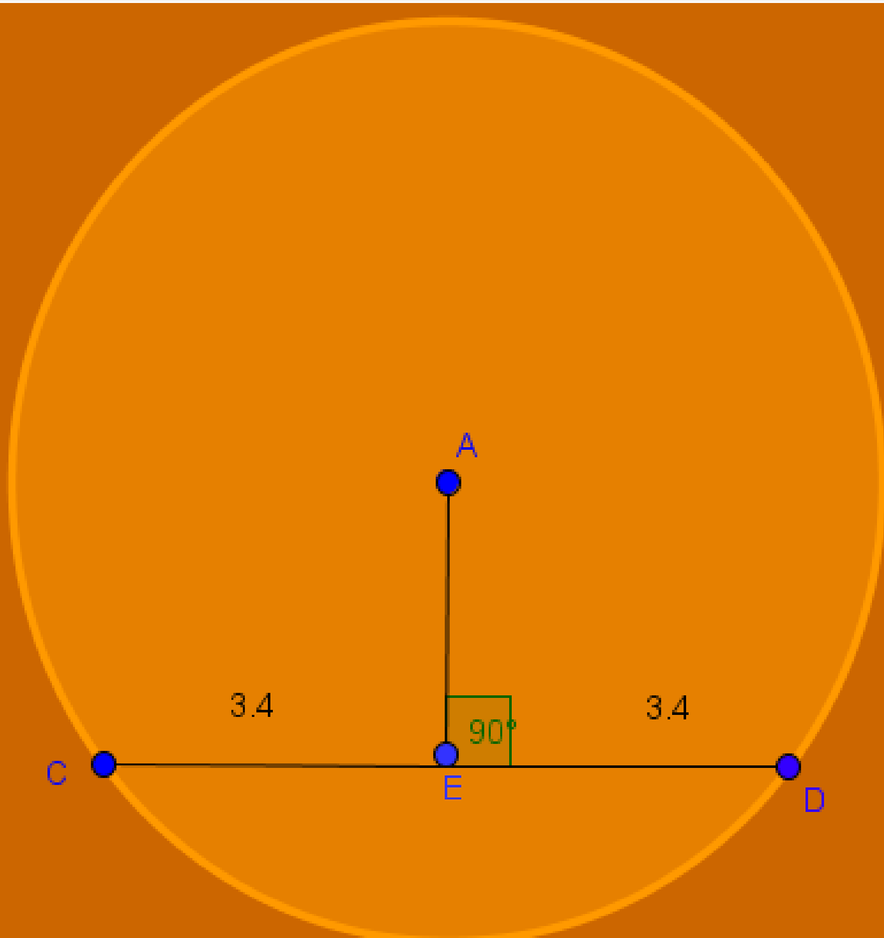

Theorem 21 (i) The perpendicular from the centre to a chord bisects the chord.

(ii) The perpendicular bisector of a chord passes through the centre.

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In a different remote area of Australia the Royal Flying Doctor Service has a base located at P another base located at Q and another base located at S.

All emergency calls are received at a central call centre and are then transferred to the closest base.

This map of the area shows the position of the three aircraft bases.

You need to divide the area into three regions so that any emergency is responded to from the nearest base.

The scale on the map is

"grid square side = 20km".

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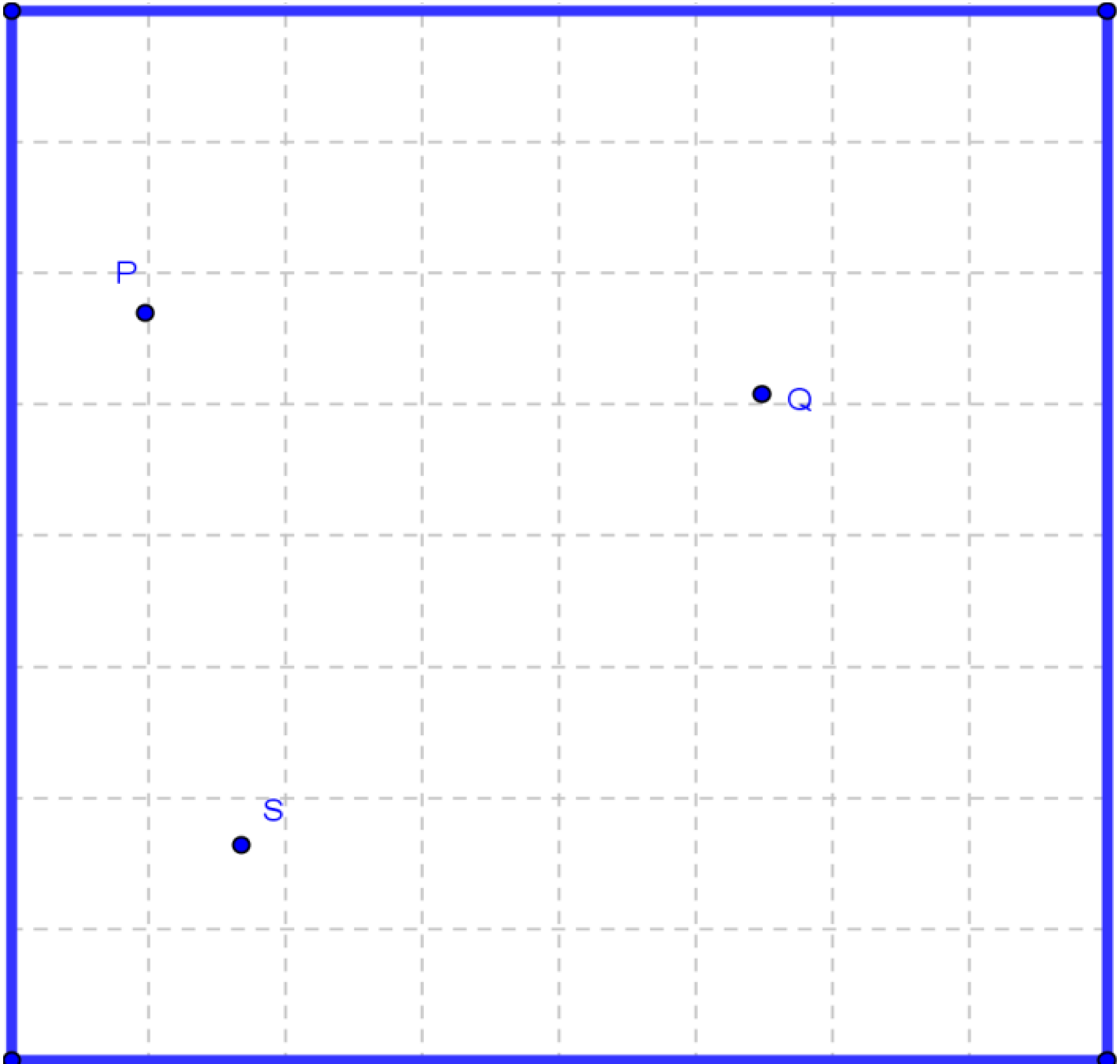

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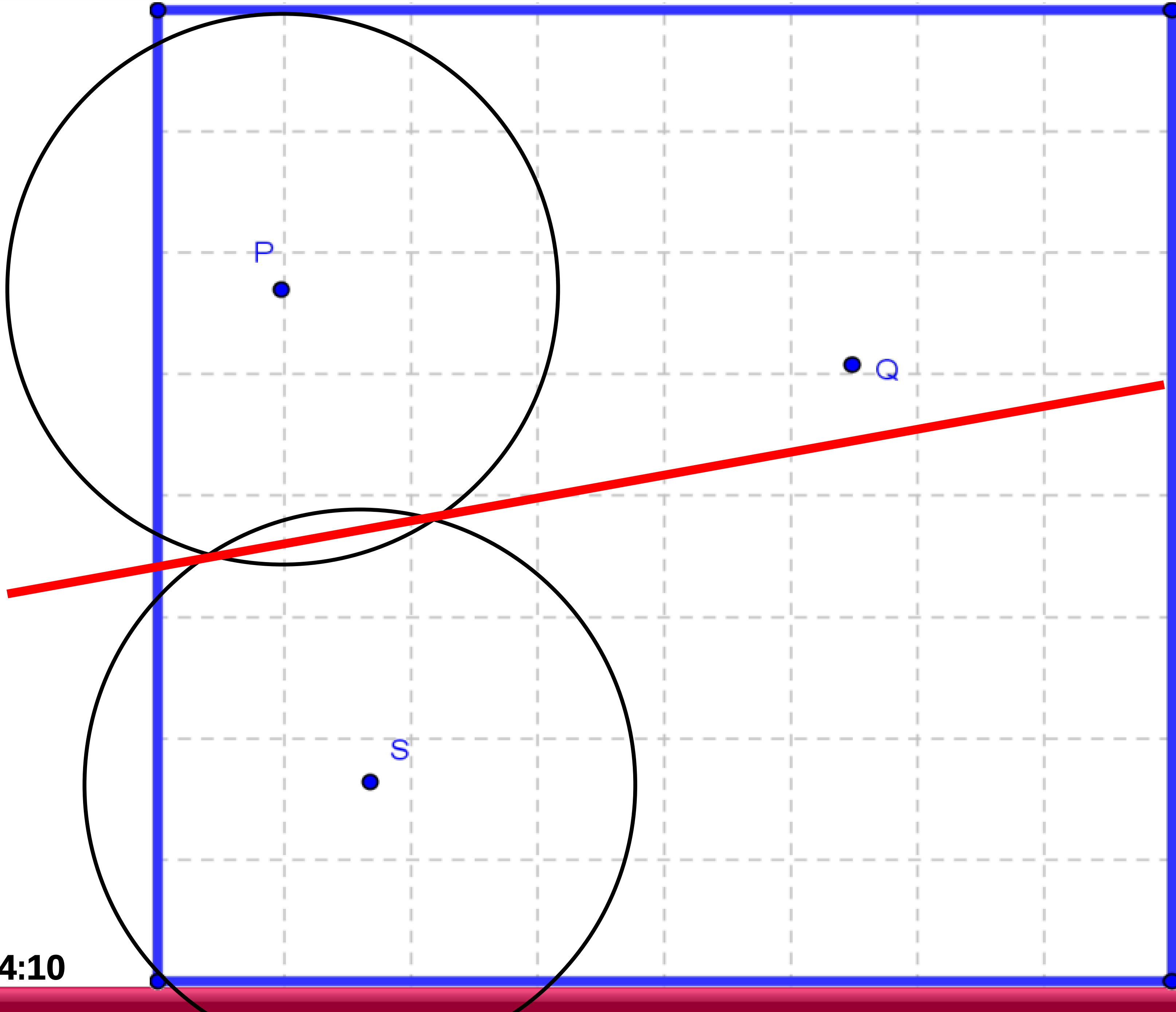

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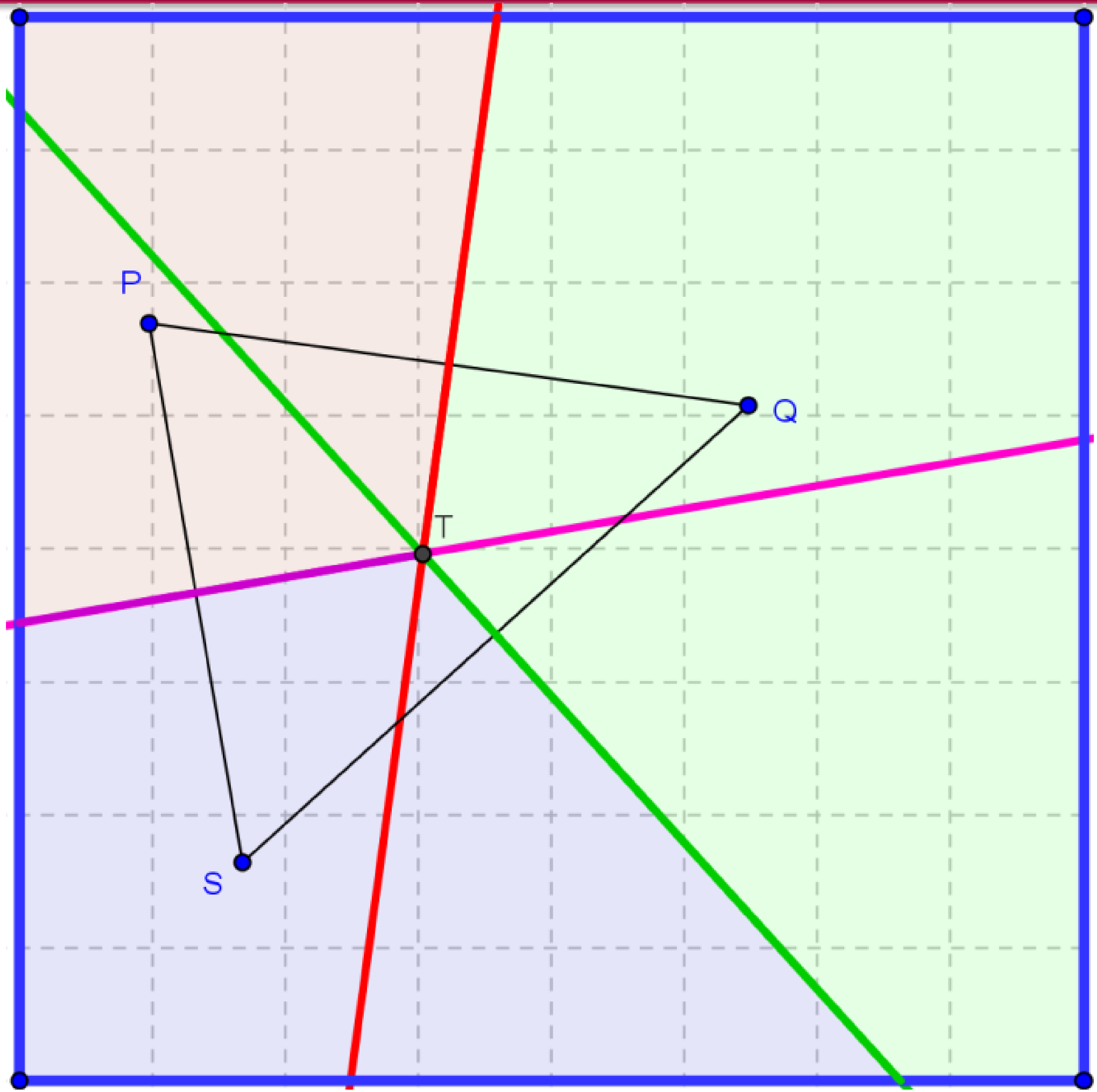

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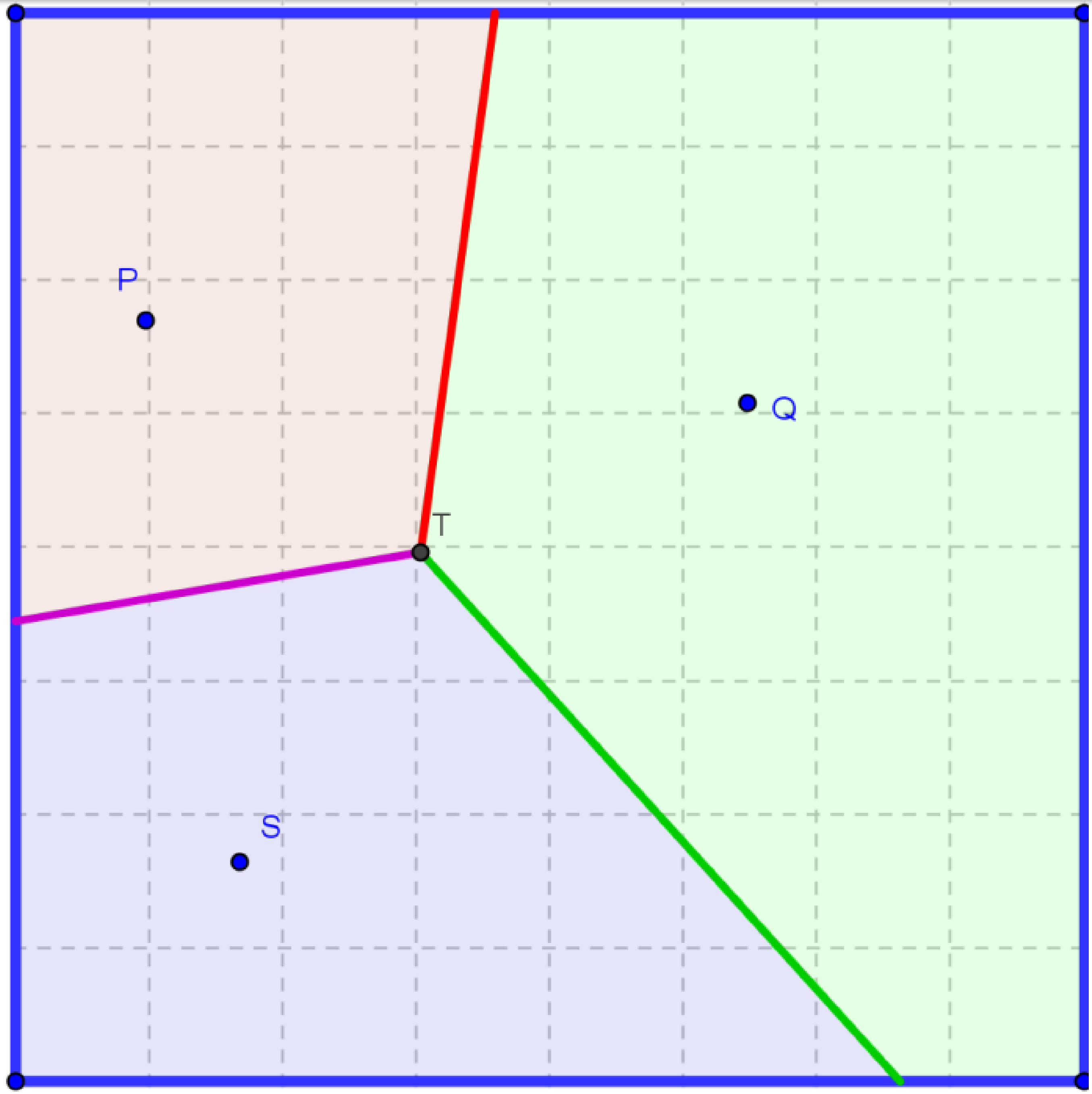

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(i) What three lines have you drawn?

Perpendicular Bisectors of [PQ] , [QS] and [SP]

(ii) Have these three lines intersected at a common point?

Yes at T

(iii) What do you notice about the distances from the point of intersection to each aircraft base.

They are all equal

(iv) What does the above result indicate to you?

A circle can be drawn through all of the points P , Q and S
with centre at T
T is the circumcentre
and
the circle is the circumcircle of the triangle PQS

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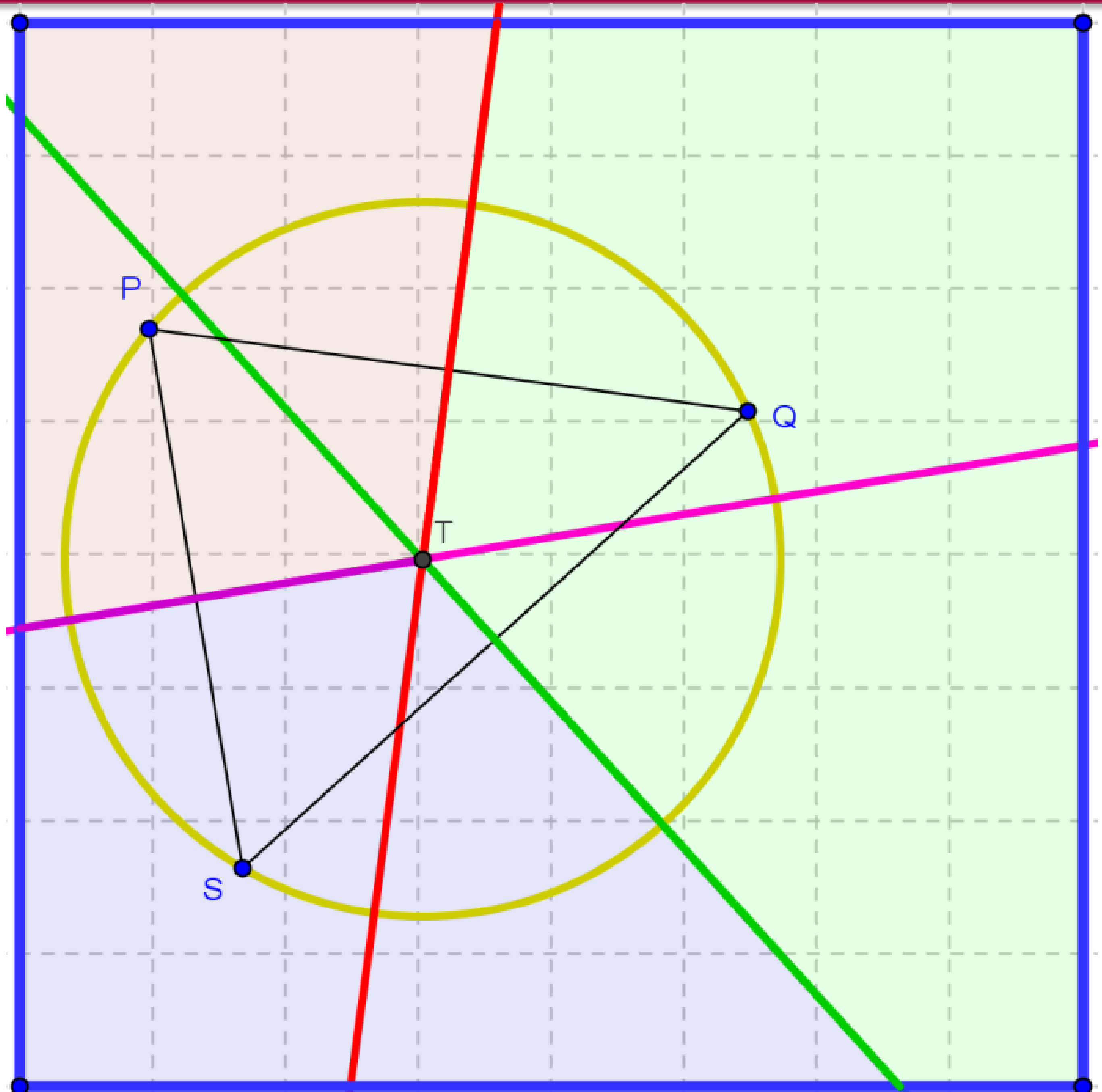

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(v) What is the approximate area of each region?

P: 5,600 km^2 Q: 12,800 km^2 S: 7,200 km^2

(vi) The number of emergency calls is consistent for all of the area and you have 168 people overall to staff the three bases. How many staff members do you need at each aircraft base.

P: 37 people Q: 84 people S: 47 people

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(vii) The area has recently received government funding to set up one specialist emergency neurological team which must service all three aircraft bases. Assuming the number of emergency calls is consistent for all of the area, where is the best location to place this team. Give a reason for your choice?

At T

as it is equidistant from all three hospitals so response time to each hospital is equal.

Therefore no hospital is disadvantaged

(viii) Would the location you selected above always be the best location regardless of the relative positions of the three aircraft bases? Give a reason for your choice.

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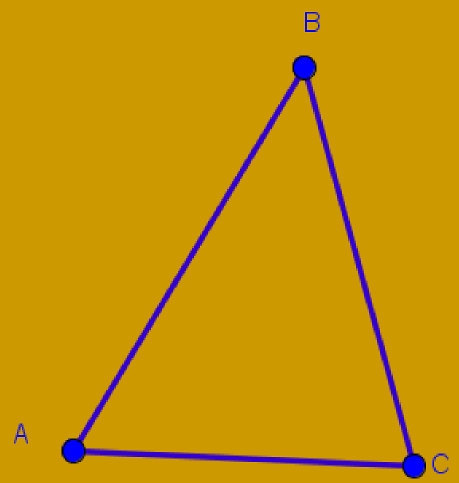
Construction 16. Circumcentre and circumcircle of a given triangle, using only straight edge and compass. Use in connection with Student Activity construction16. Click the boxes in alphabetical order.

Reset



- A. Click to show mid points of the sides
- B. Click to show perpendicular bisectors (2 is sufficient)
- C. Click to show the circumcentre
- D. Click to show circumcircle and a circumradius

See student CD

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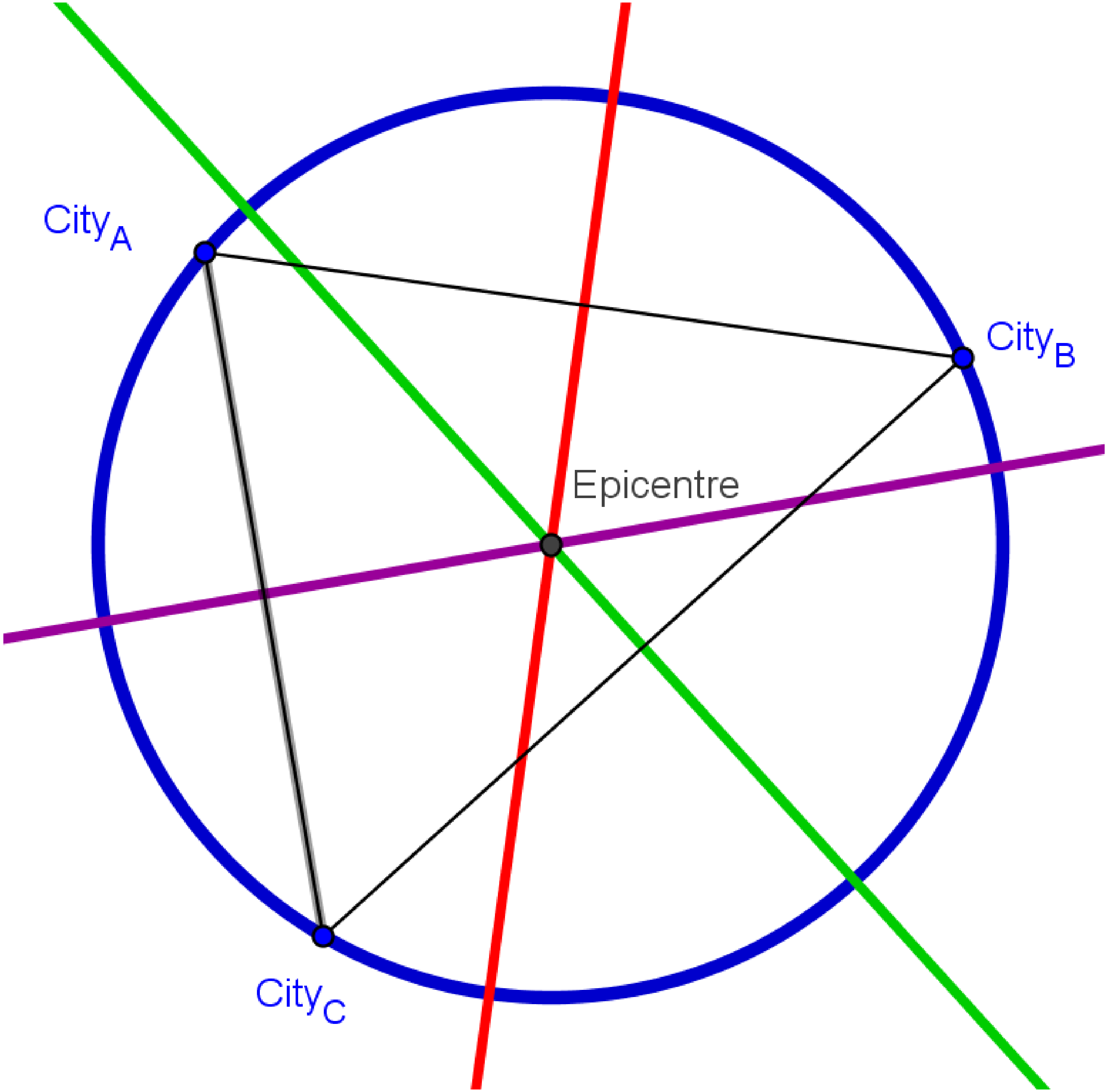
During a recent earthquake in the western United States, three cities recorded identical measures of intensity from seismographic readings taken at the same time.

Since the seismographic readings were the same, each city was the same distance from the epicentre of the earthquake.

In the space below, using only a straight edge and compass show how the location of the epicentre of the earthquake can be determined.



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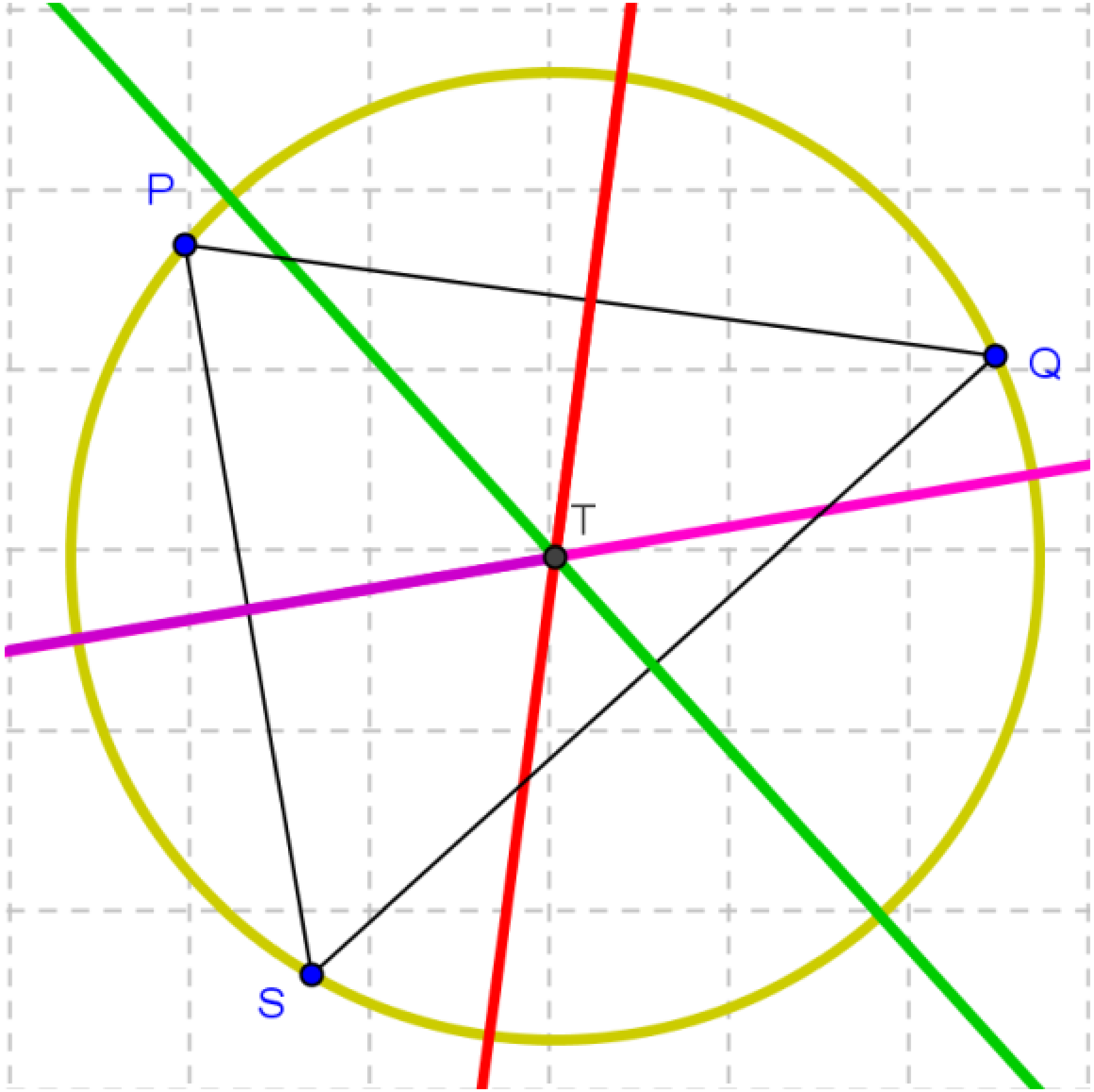
Location of the circumcentre of a triangle



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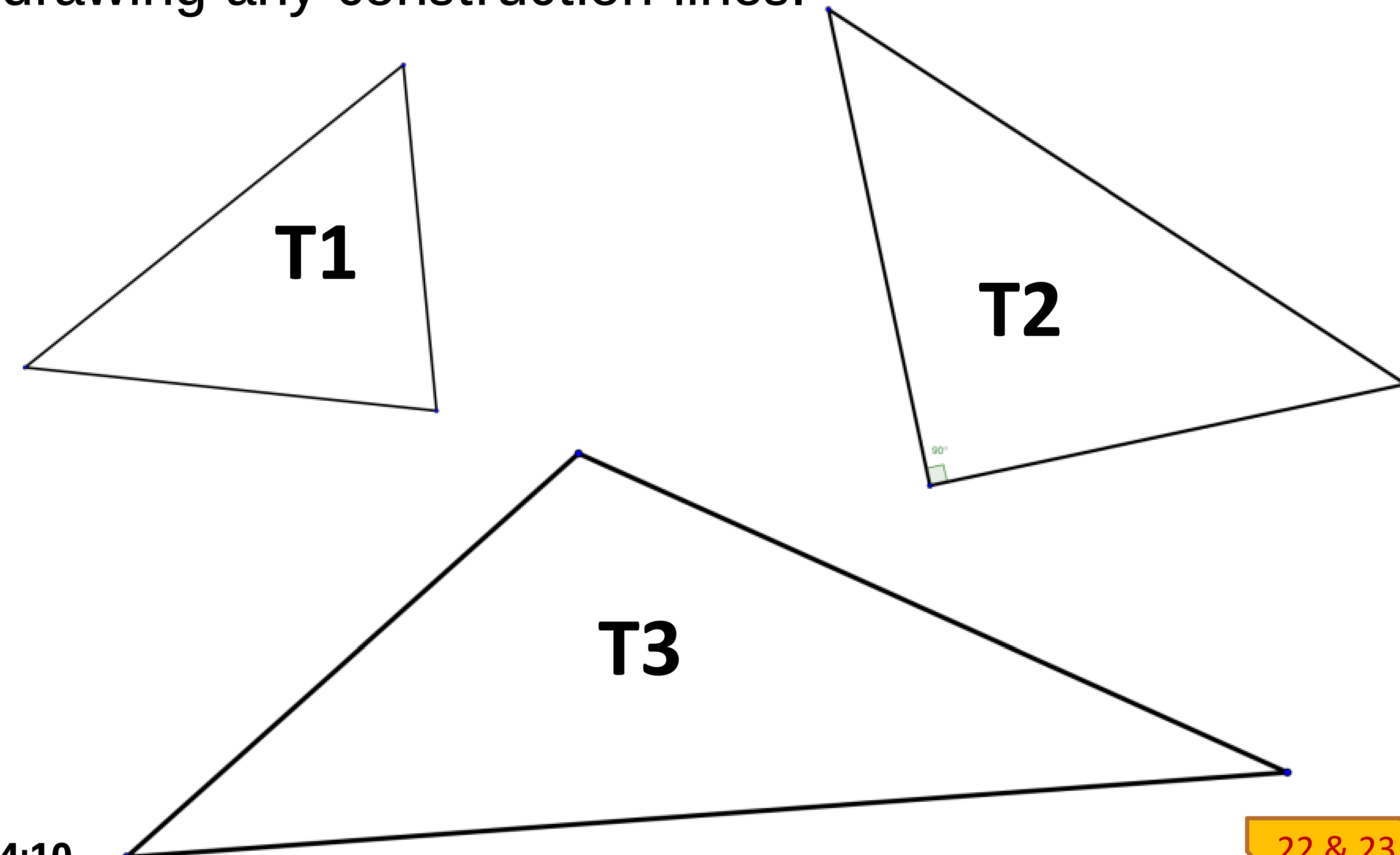
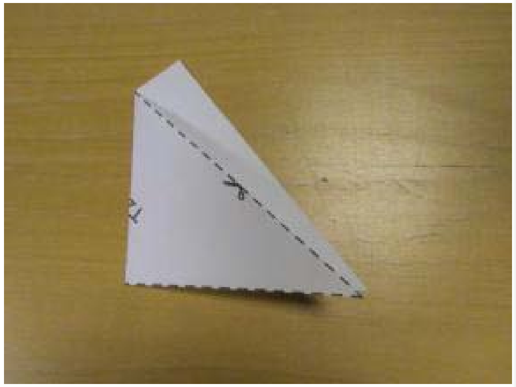
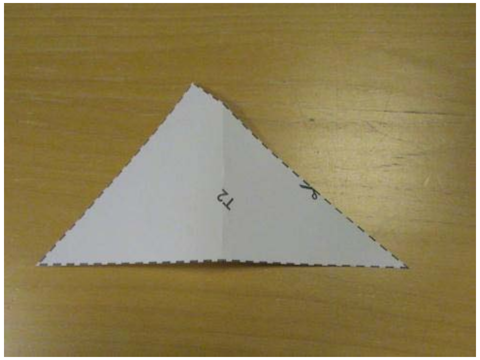
Take each of the three triangles, cut them out and locate the circumcentre of each triangle without drawing any construction lines.



Fold from corner to corner resulting in the fold line being the perpendicular bisector of the side between the two corners.
Do twice, then the circumcentre is the intersection of the two folds.





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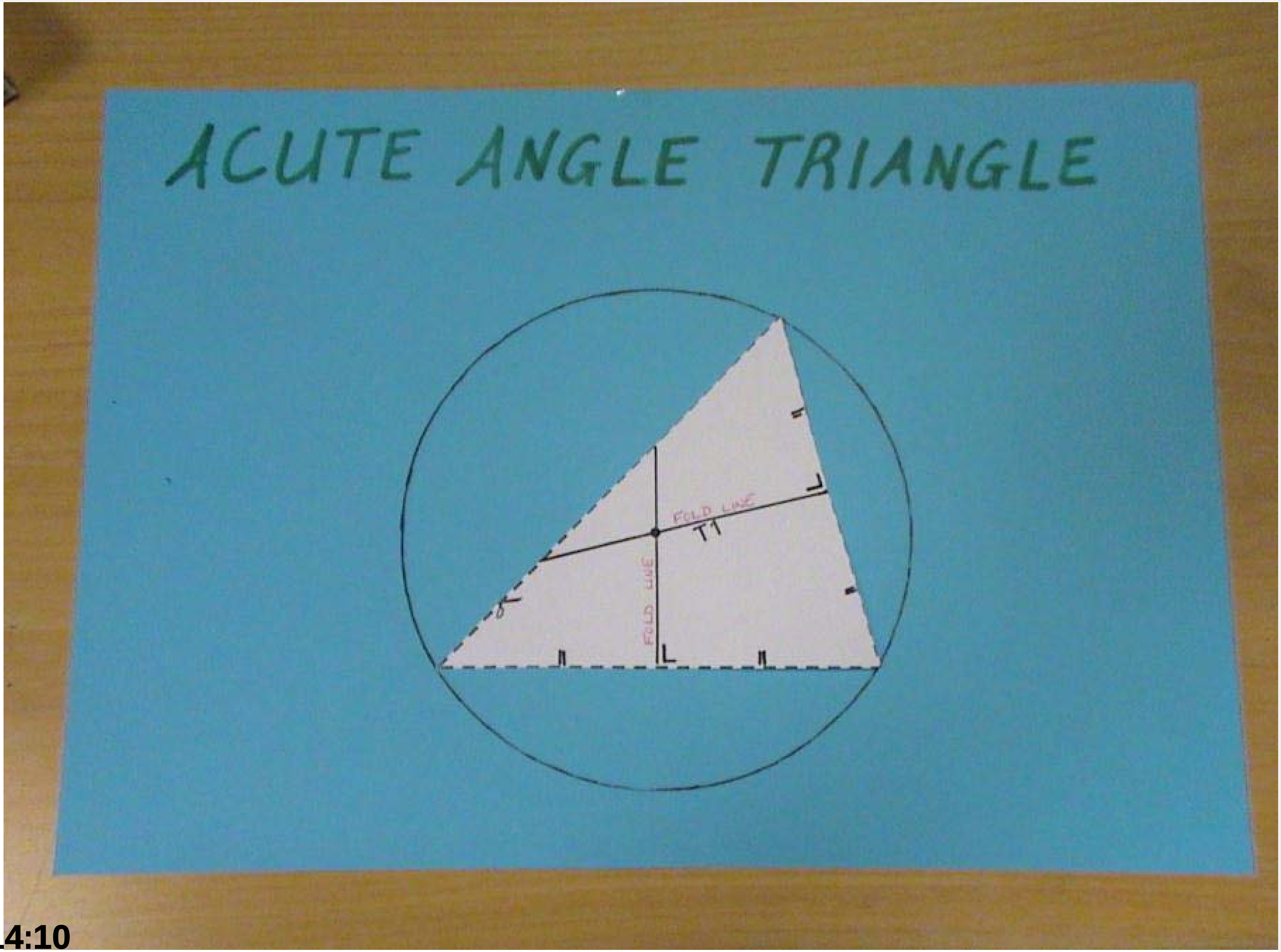

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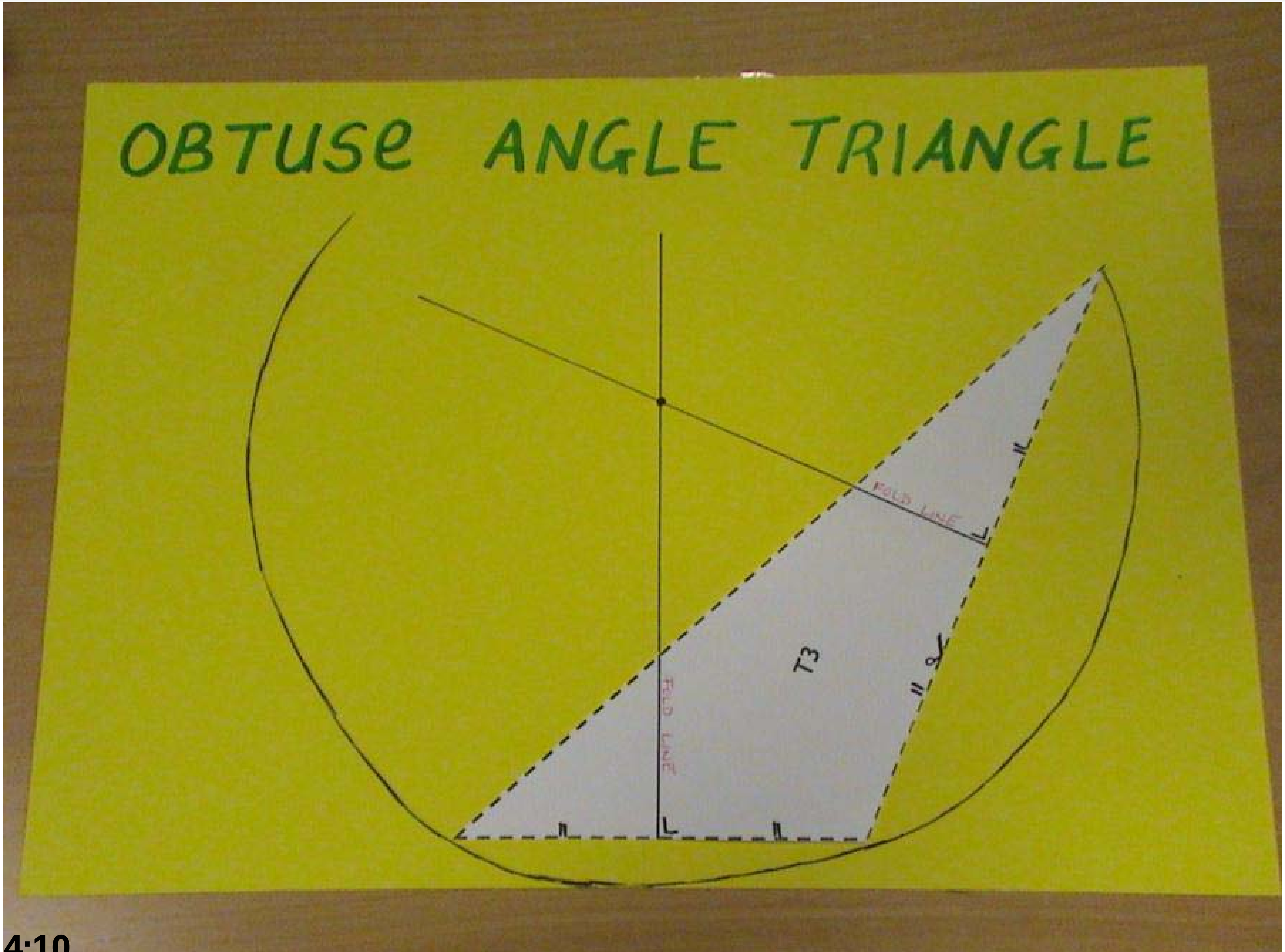

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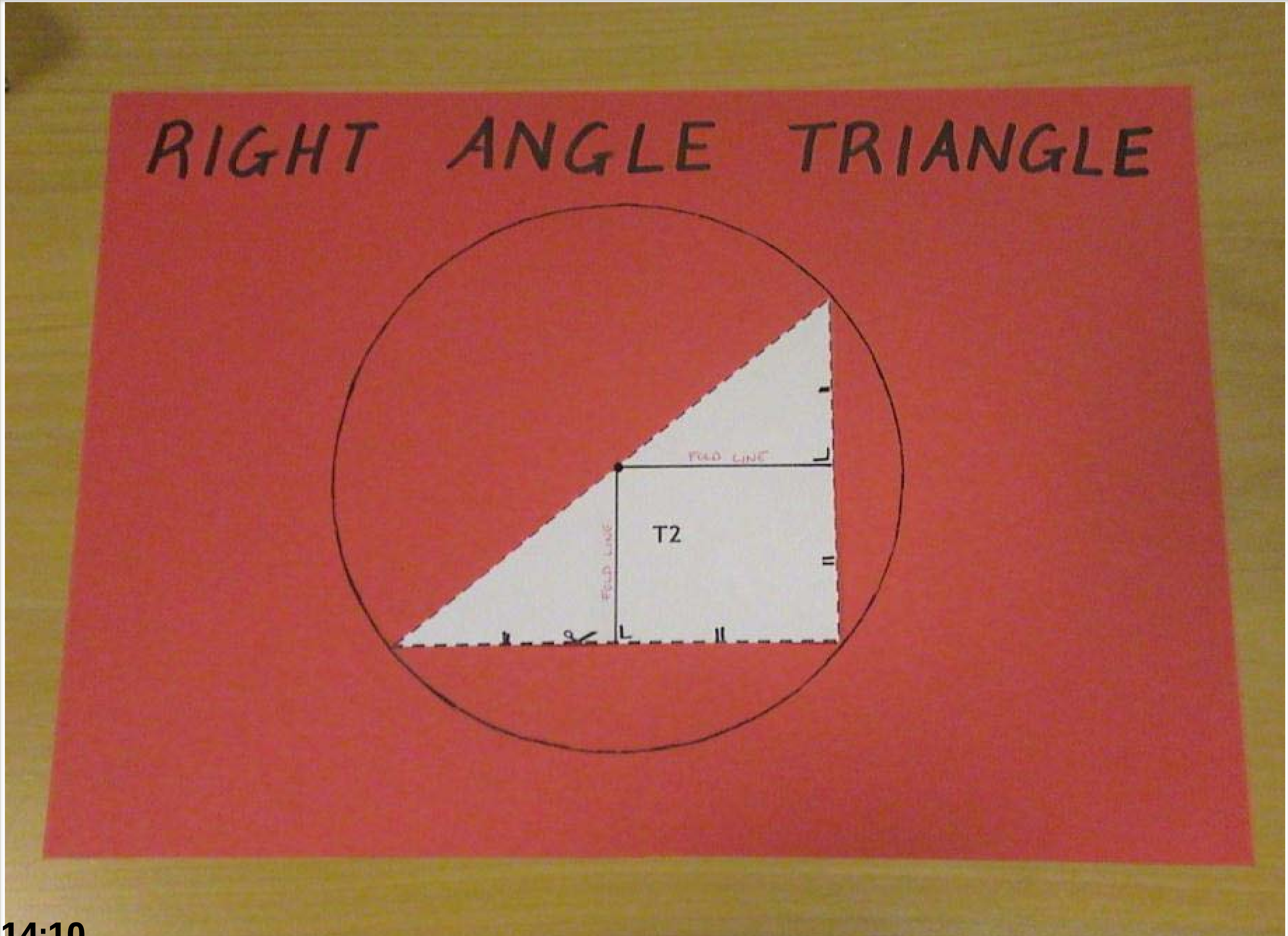

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Task 3

(i) Take each of the three triangles on the handout, cut them out and locate the circumcentre of each without drawing any construction lines.

(ii) In triangle *T1*, what can you say about the size of all of the angles?

All acute angles

(iii) In relation to triangle *T1*, where is the circumcentre located?

Inside the triangle

(iv) Do you think this will be the case for all triangles of this type?

?

(v) What type of triangle is *T2*?

Right angled triangle

(vi) What name is given to the side opposite the 90° angle?

Hypotenuse

(vii) In relation to triangle *T2*, where is the circumcentre located?

On the hypotenuse (mid point)

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(vii) In relation to triangle *T2*, where is the circumcentre located?

On the hypotenuse (mid point)

(viii) Do you think this will be the case for all triangles of this type?

?

(ix) What name is given to the largest angle in *T3*?

Obtuse angle

(x) Could you locate the circumcentre of *T3*?

No

(xi) Give a reason for your response to the previous question?

The bisectors will intersect outside the triangle

(xii) Do you think this will be the case for all triangles of this type?

?

(xiii) Formulate a conjecture to indicate where the circumcentre of any triangle is located?

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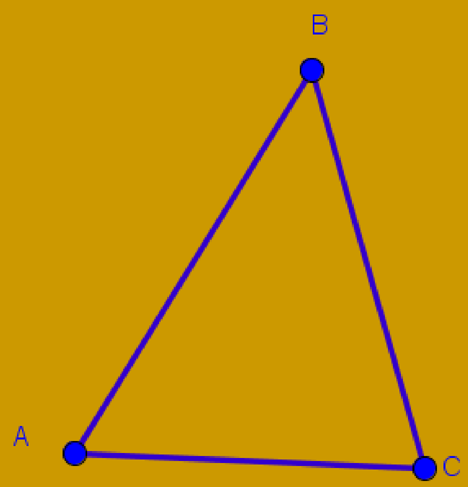
Construction 16. Circumcentre and circumcircle of a given triangle, using only straight edge and compass. Use in connection with Student Activity construction16. Click the boxes in alphabetical order.

Reset



- ☐ A. Click to show mid points of the sides
- ☐ B. Click to show perpendicular bisectors (2 is sufficient)
- ☐ C. Click to show the circumcentre
- ☐ D. Click to show circumcircle and a circumradius

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Geometry construction: connections within Strand 2

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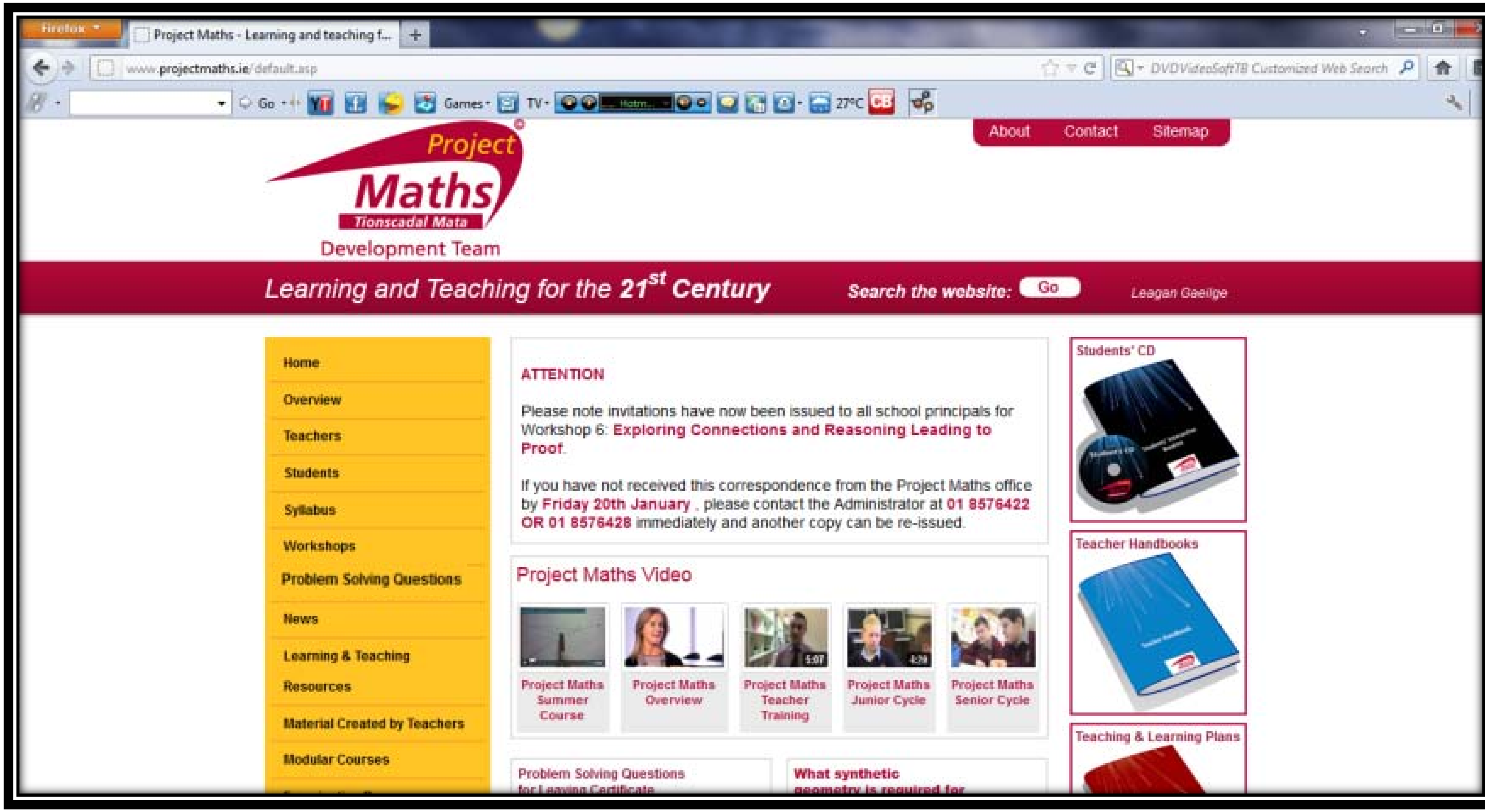
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Please note invitations have now been issued to all school principals for Workshop 6: Exploring Connections and Reasoning Leading to Proof.
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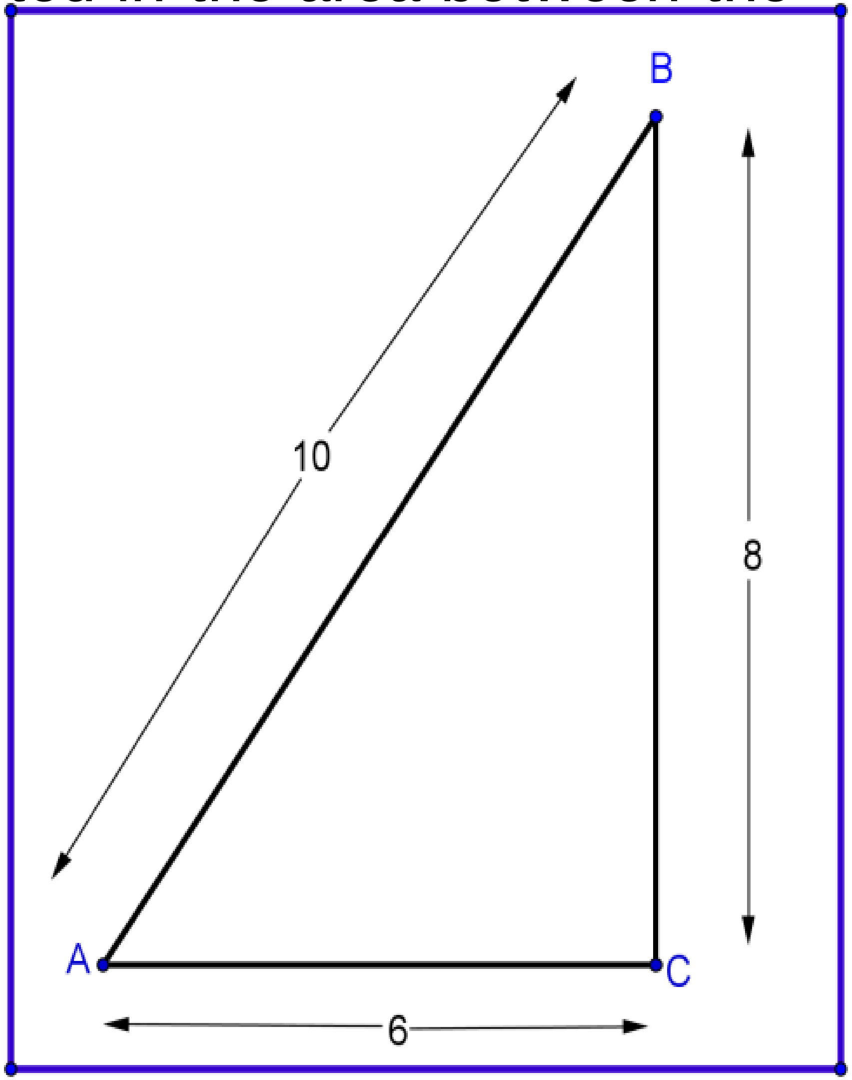
Three roads, as shown, join three villages A, B and C

A mobile phone mast is to be erected in the area between the villages.

It was suggested that it would be fair to erect it at a point equidistant from the three villages.

Why was it not possible to do so?



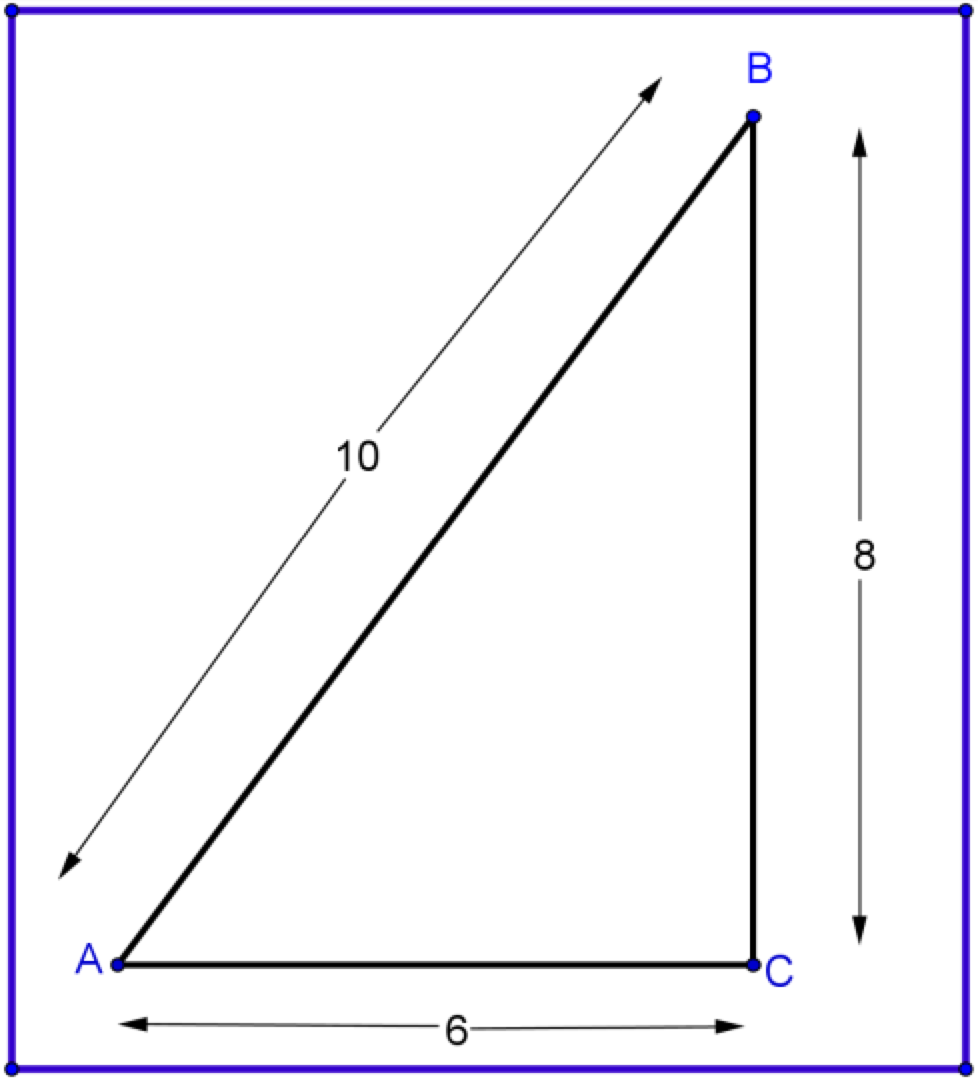
See workbook page 24

Not possible

That point is the circumcentre.

In a right angled triangle the circumcentre is the mid point of the hypotenuse.

In this case that would be on the road from A to C exactly mid way between those two towns



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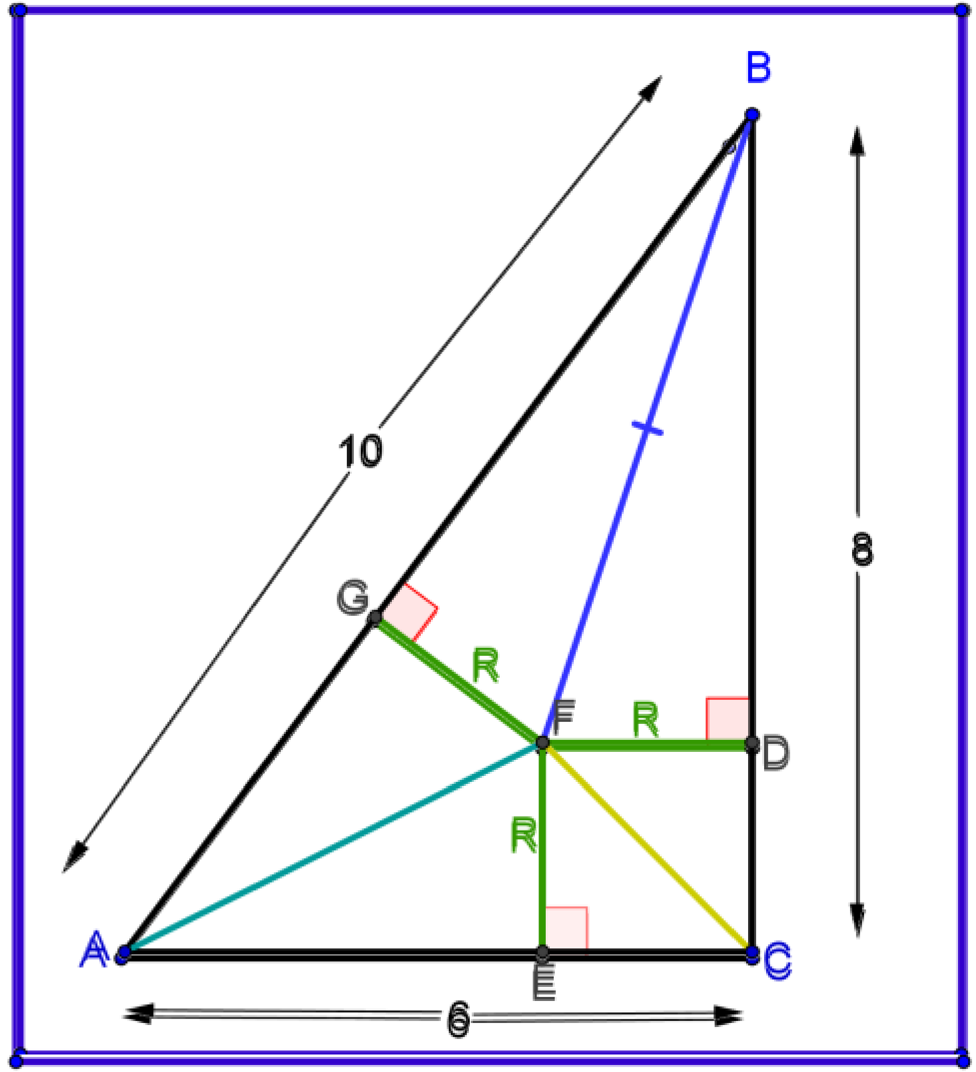
It was then decided to erect the mast at F, which is equidistant from the three roads.

How far is the mast from each road?



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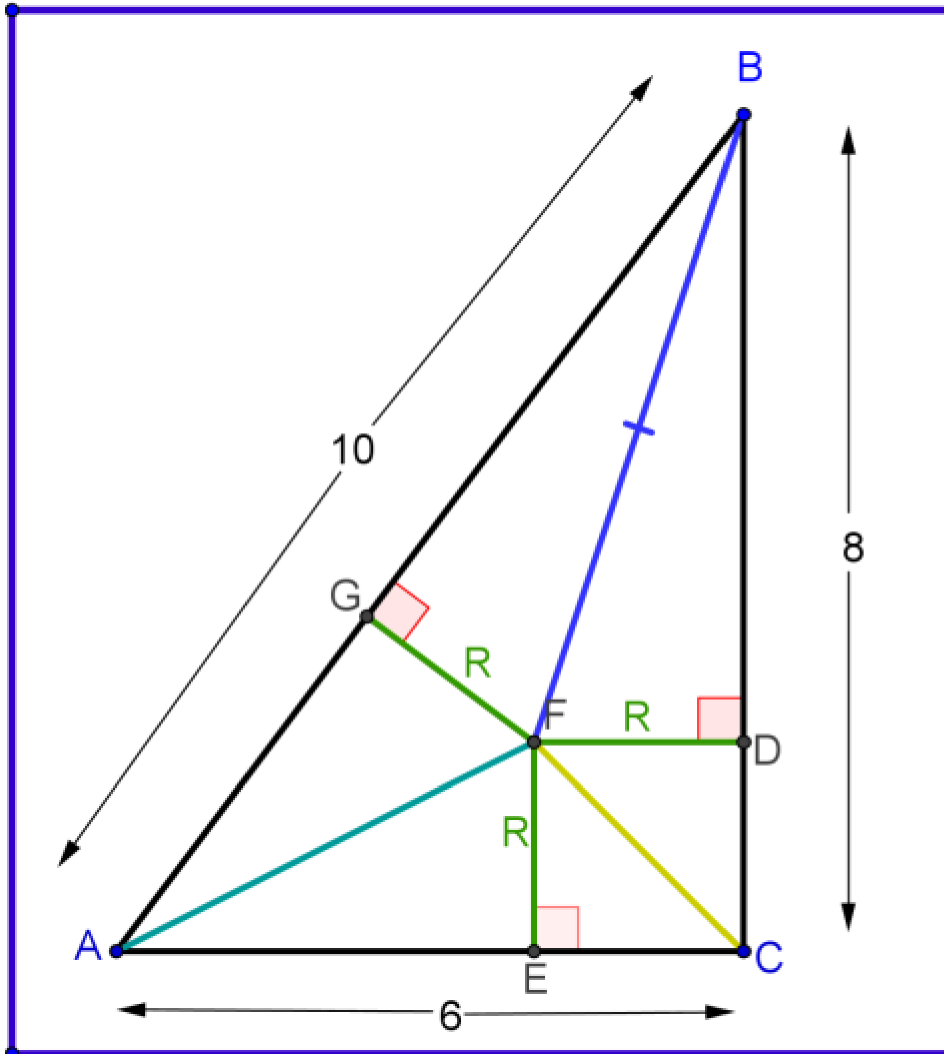
How far is the mast from each village (as the crow flies)?



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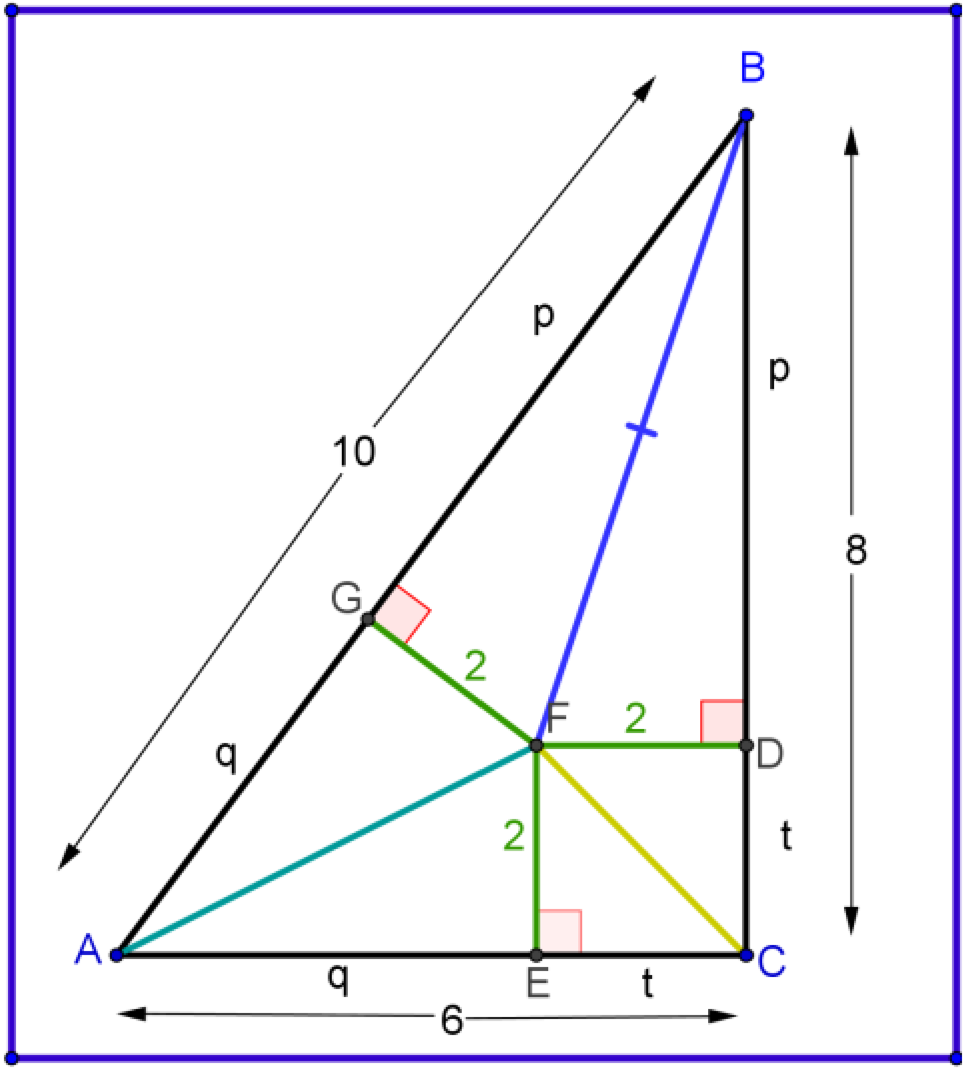
How far is the mast from each village (as the crow flies)?

$$p + q = 10 \quad (i)$$

$$p + t = \ 8 \quad (ii)$$

$$q + t = \ 6 \quad (iii)$$



$sub\ (iv)$ $\qquad$ $sub\ in\ (iii)$

$q = 4$ $\qquad$ $t = 2$

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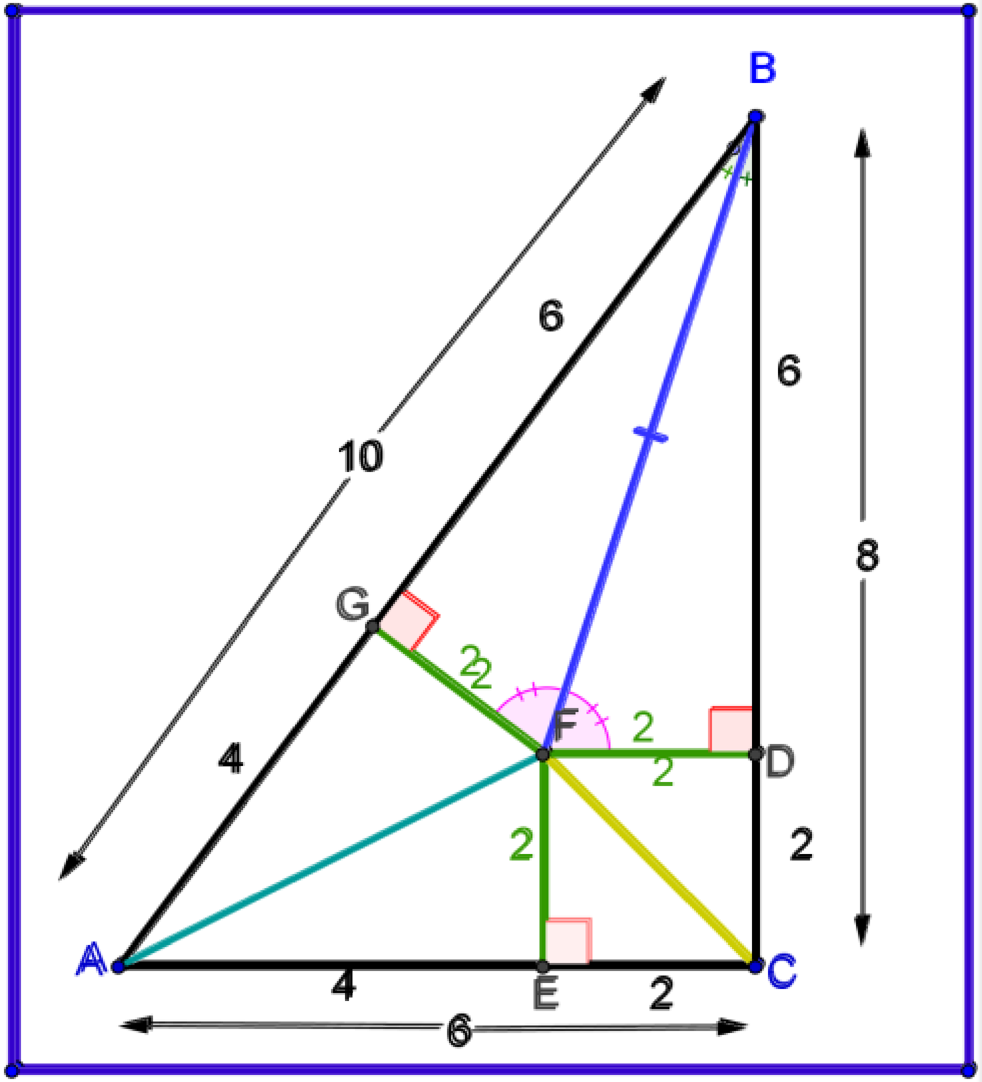
How far is the mast from each village (as the crow flies)?



$In\ \triangle BFD$

$$|BF|^2 = 2^2 + 6^2$$

$$|BF| = \sqrt{40}$$

$$|BF| = 6.325$$

$In\ \triangle CFD$

$$|CF|^2 = 2^2 + 2^2$$

$$|CF| = \sqrt{8}$$

$$|CF| = 2.828$$

$In\ \triangle AFE$

$$|AF|^2 = 2^2 + 4^2$$

$$|AF| = \sqrt{20}$$

$$|AF| = 4.472$$

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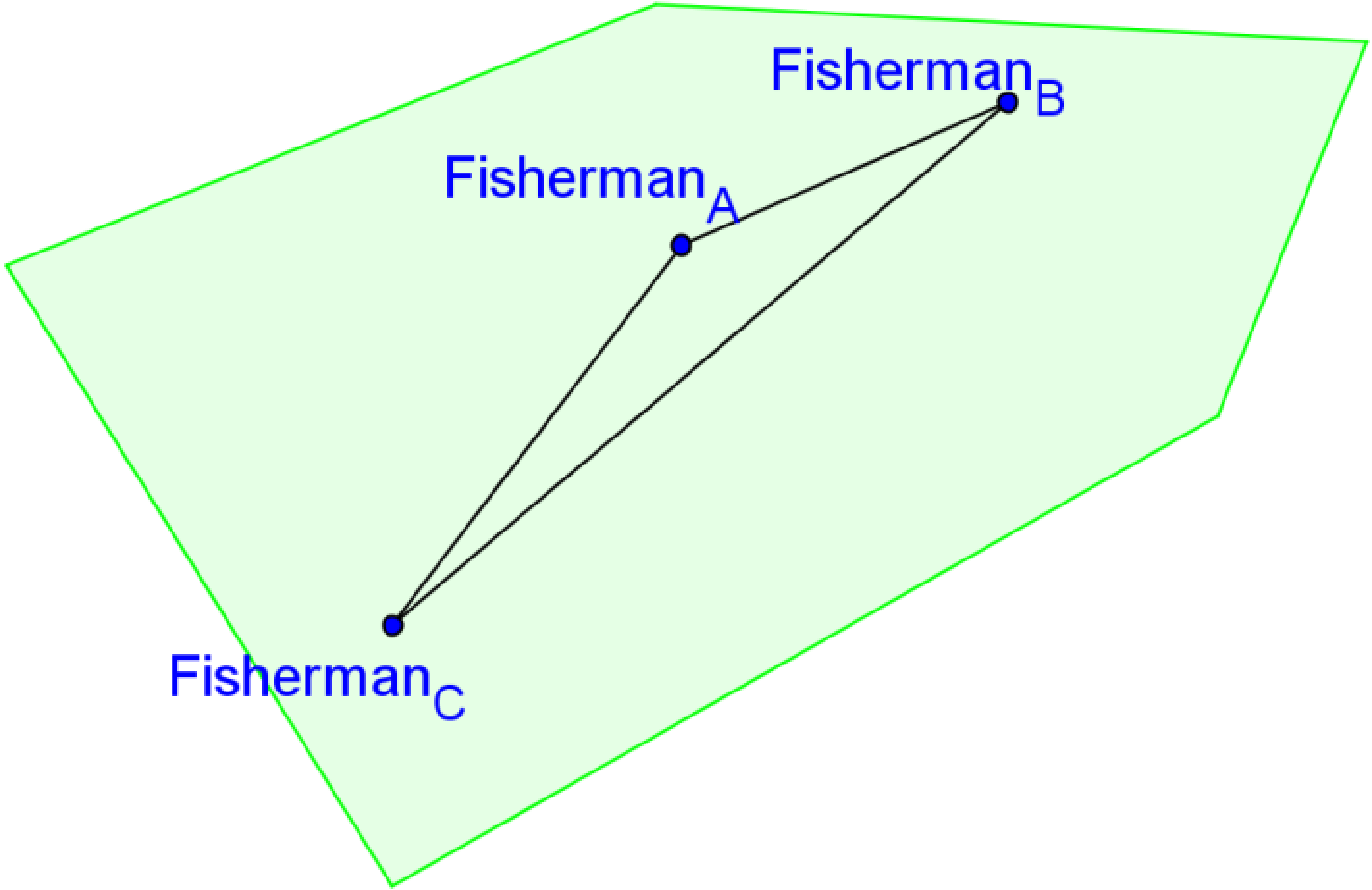
A fishing community has applied to the County Council for planning permission to extend their memorial to those lost at sea.

The memorial is located in the park close to the pier (see diagram), and consists of bronze statues of three fishermen.

To complete the memorial the community wish to locate a bronze sculpture of a damaged fishing boat at a point which is the same distance away from the statues of each fisherman.

Suggest one reason why the County Council might refuse planning permission.



See workbook: Page 25

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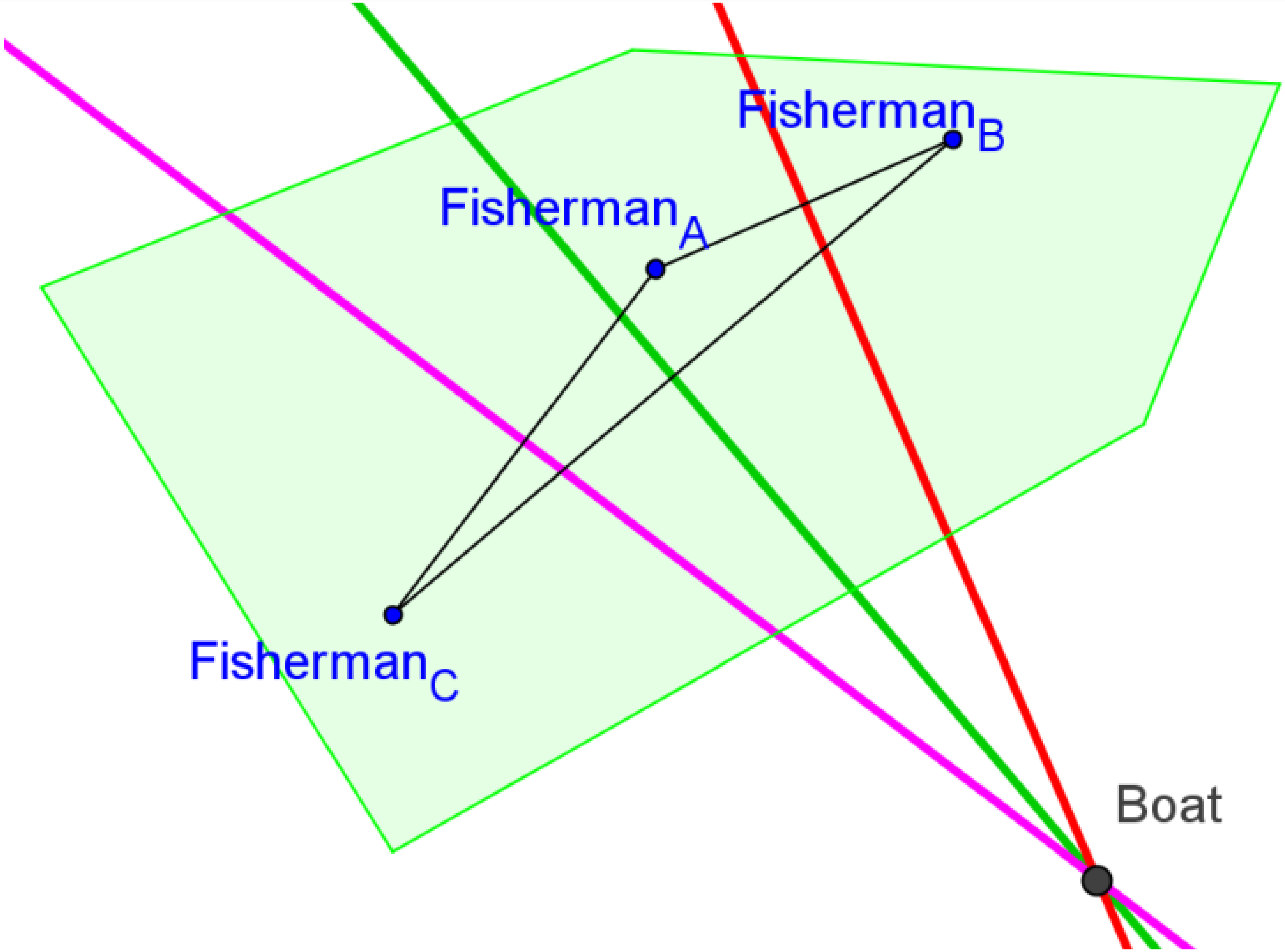

The extended memorial would not fit in the park

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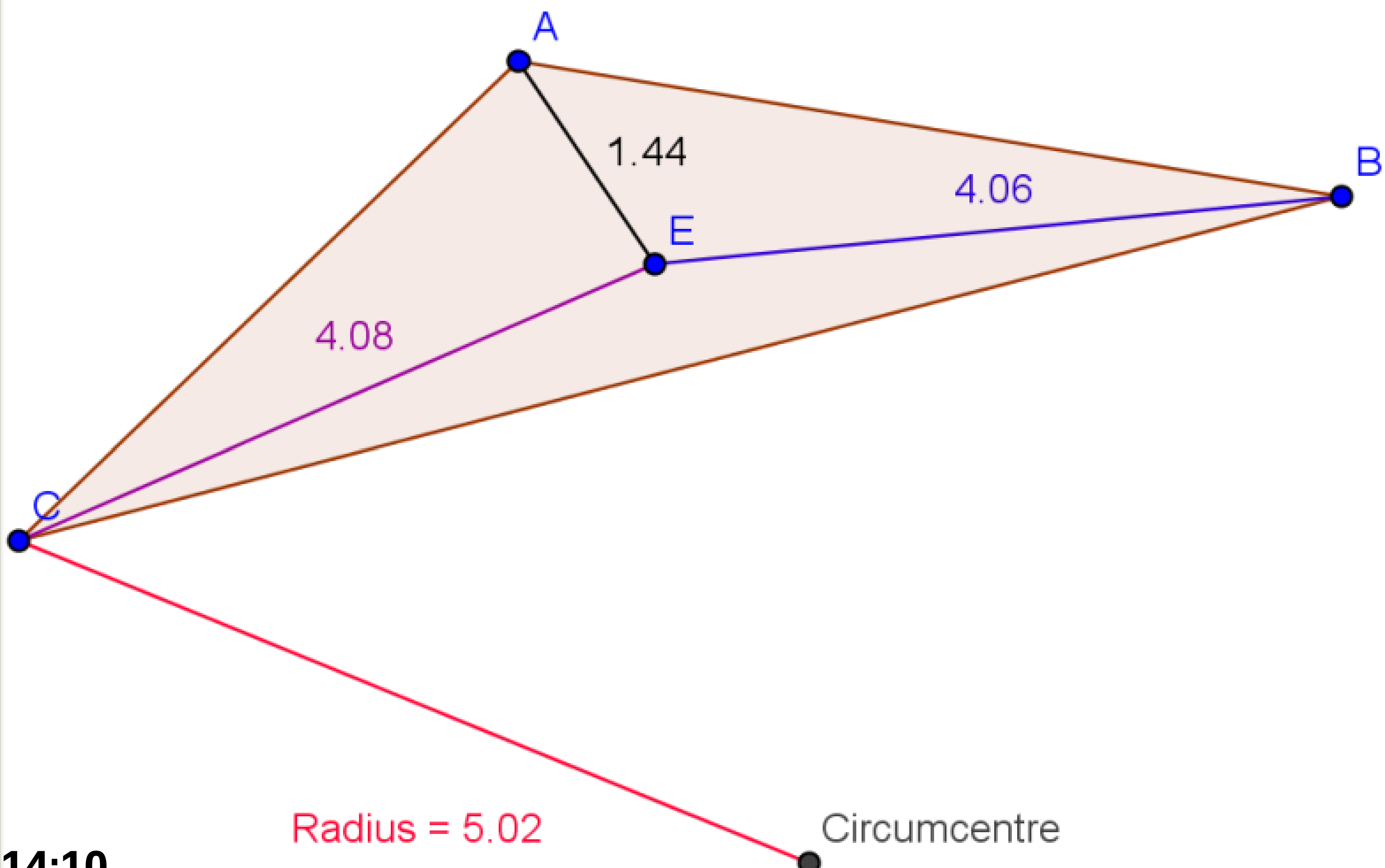
Note : - while the circumcentre is equidistant from all three vertices, there are points which are closer to all three vertices



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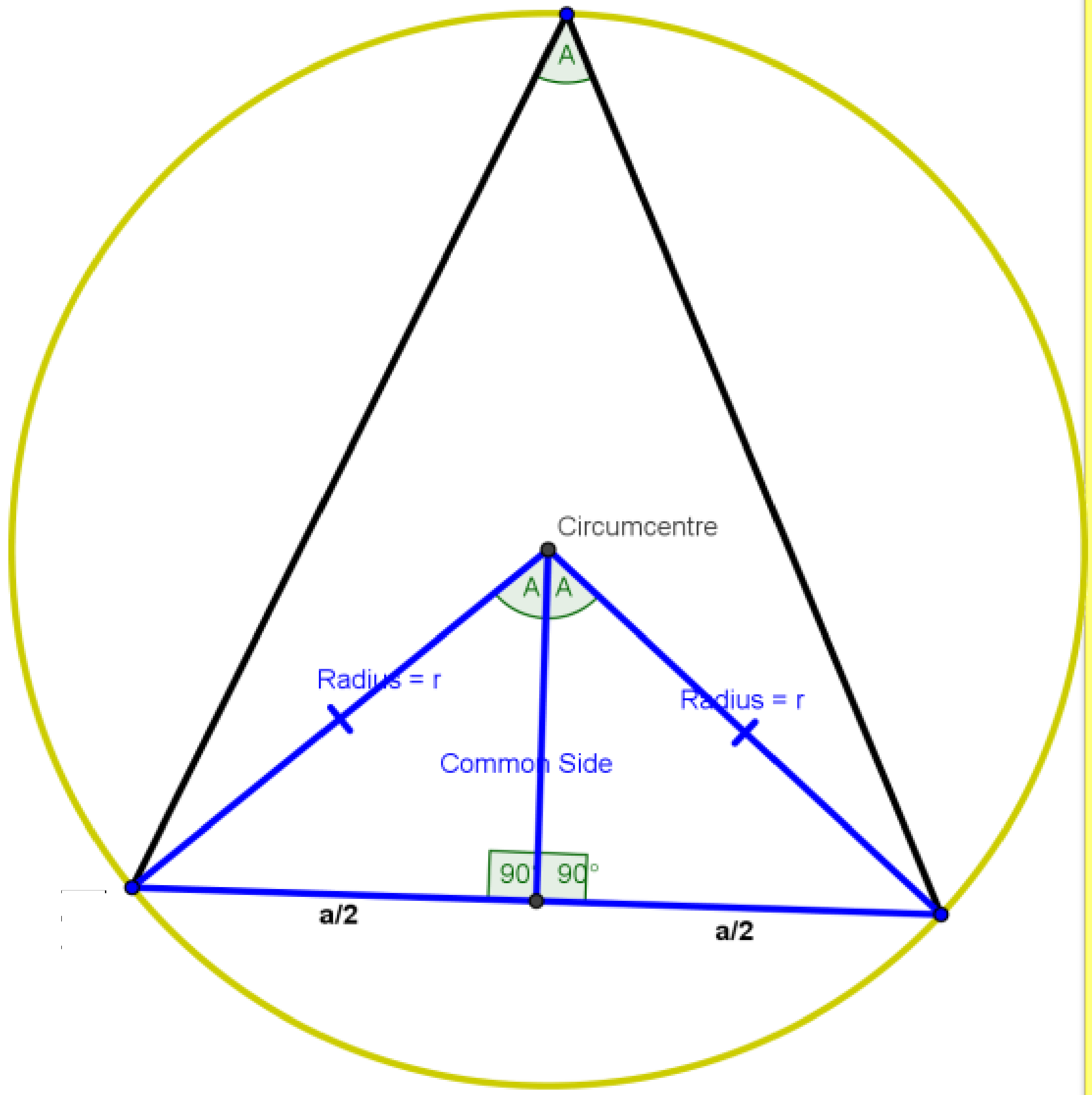
Radius and Area of Circumcircle



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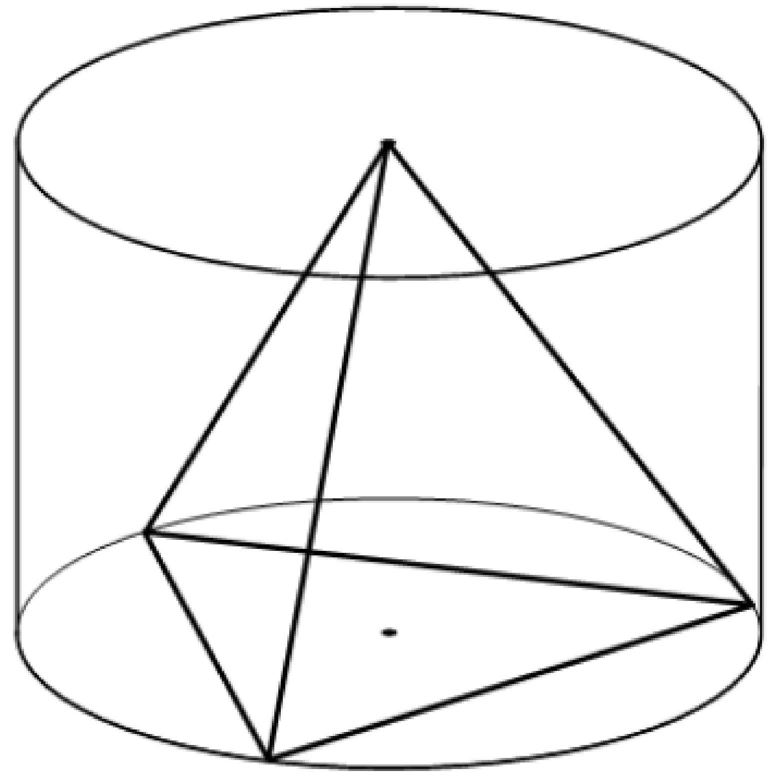
A regular tetrahedron has four faces, each of which is an equilateral triangle.

A wooden puzzle consists of several pieces that can be assembled to make a regular tetrahedron.

The length of one edge of the tetrahedron is 2a.

The manufacturer wants to package the assembled tetrahedron in a clear cylindrical container, with one face flat against the bottom and the apex just touching the top.

If the volume of the cylindrical container is $\frac{8\sqrt{6}}{9}\pi a^3$



See workbook: Page 27

Find the height of the puzzle

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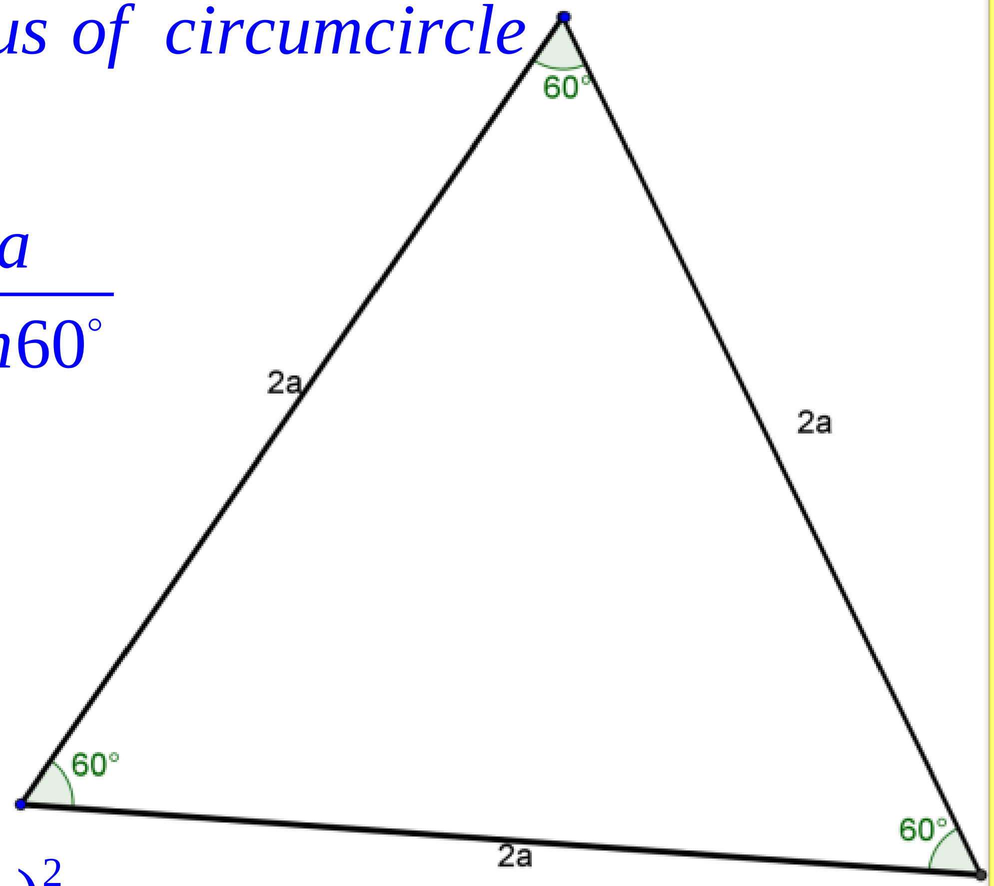
$$\text{Radius of cylinder} = \text{radius of circumcircle}$$

$$= \frac{2a}{2Sin60^\circ}$$

$$= \frac{2a}{\sqrt{3}}$$



$$\text{Area of circular base} = \pi\left(\frac{2a}{\sqrt{3}}\right)^2$$

$$= \frac{4\pi a^2}{3}$$

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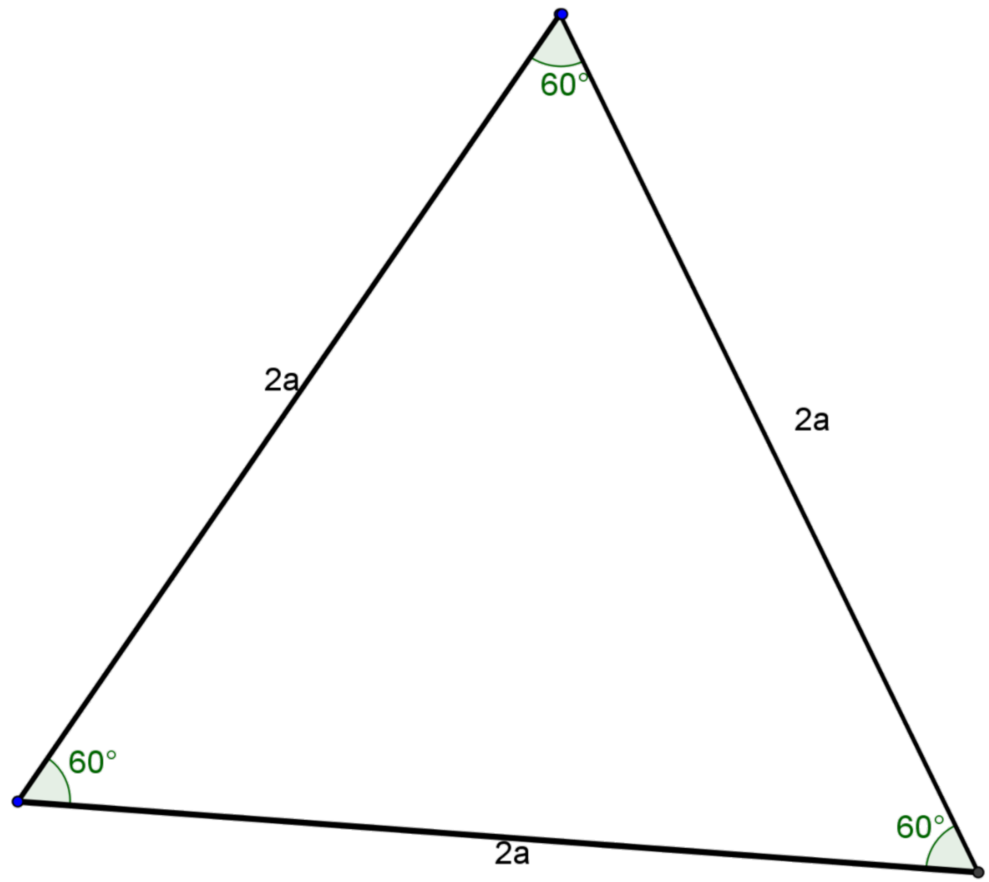
$$height = \frac{Volume}{Area}$$

$$= \frac{\dfrac{8\sqrt{6}\pi a^3}{9}}{\dfrac{4\pi a^2}{3}}$$

$$= \frac{8\sqrt{6}\pi a^3}{9} \cdot \frac{3}{4\pi a^2}$$

$$= \frac{2\sqrt{6}a}{3} = \frac{2\sqrt{2}a}{\sqrt{3}} = 2\sqrt{\frac{2}{3}}a$$



In a public park the gardener measures the distances between the only three trees using a measuring tape.

The distances are 31.83m, 25.75m, and 29.27m.

He wishes to fence off a circular play area in which to place swings and have the three trees as part of the perimeter of that play area.

What area of the park will he be fencing off?

$in\ \triangle ABC$

$$\beta = Cos^{-1}\frac{31.83^2+25.75^2-29.27^2}{2(31.83)(25.75)}$$

$$\beta = 60^o$$

$in\ \triangle BDE$

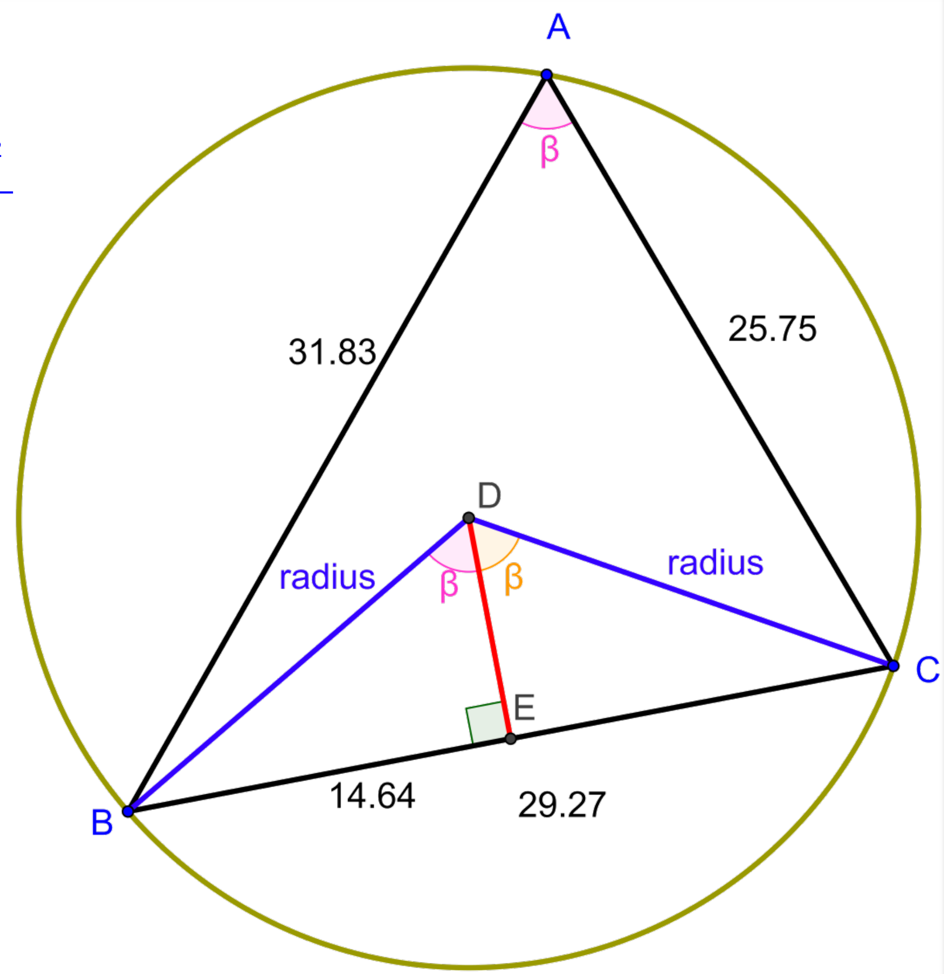
$$Sin60^o = \frac{14.64}{radius}$$

$$radius = \frac{14.64}{Sin60^o}$$

$$radius = 16.9m$$

$$Area\ \ play\ area = \pi(16.9^2)$$

$$Area\ \ play\ area = 897m^2$$



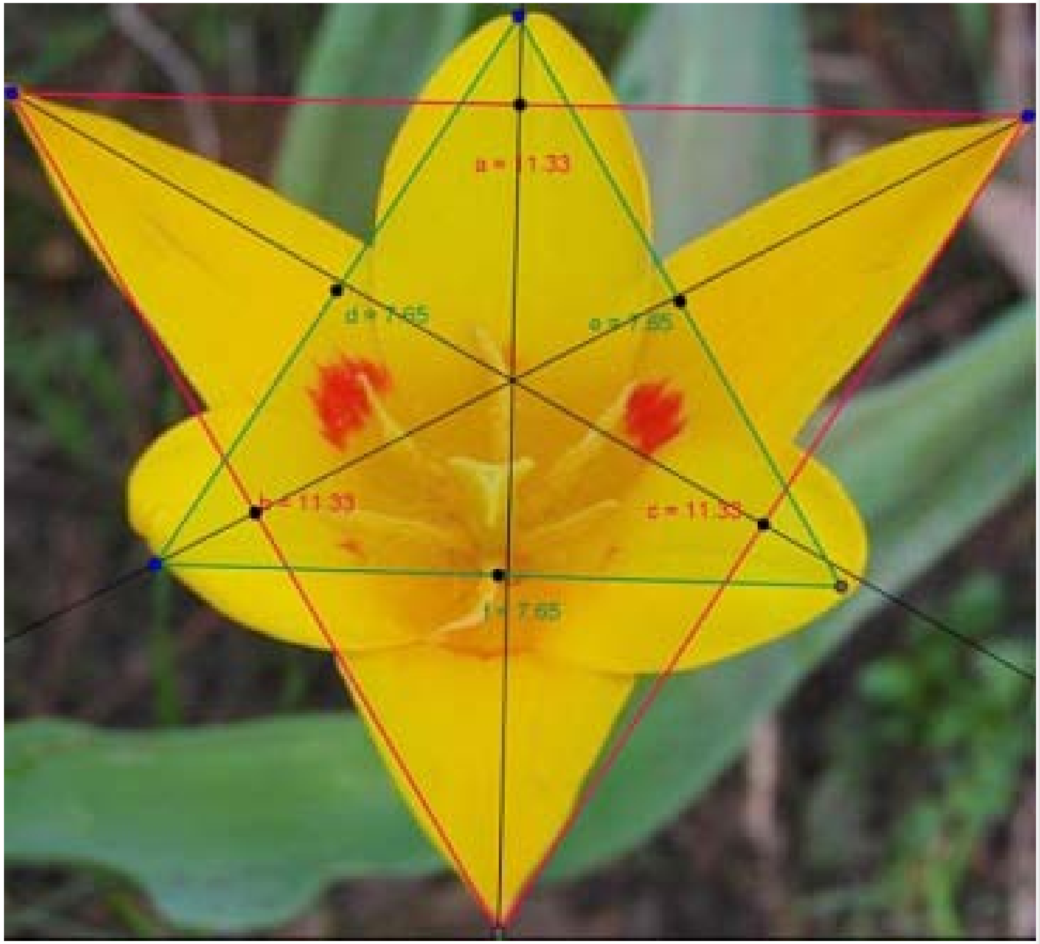
Circumcircle in Nature

Look at the tulip:

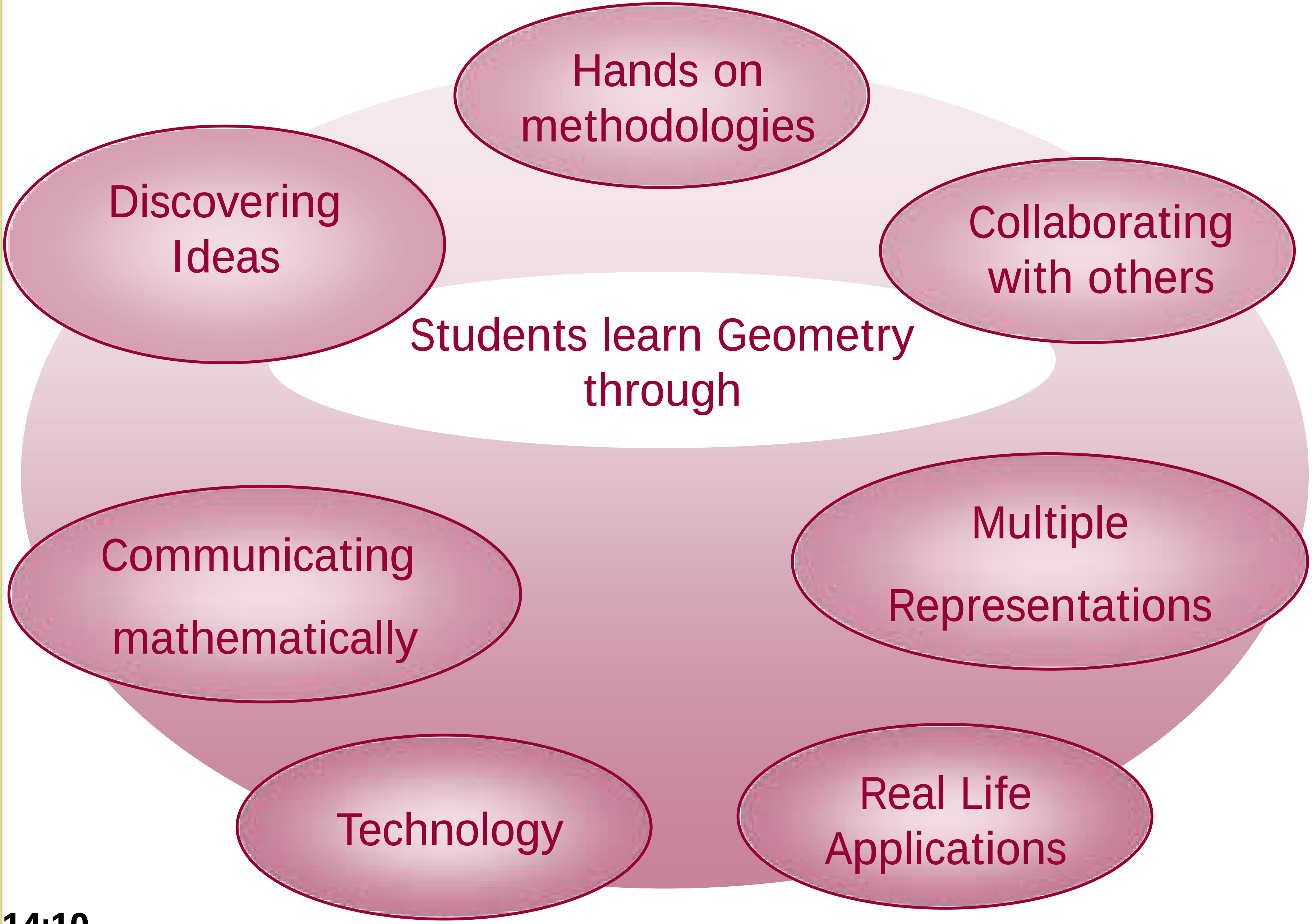
the longer petals make an **equilateral triangle**,

the short petals make another **equilateral triangle,** so that triangles have **the same perpendicular bisectors.**

Thus the c**entre of the circumcircles** is the same for the two triangles.



As the triangles are equilateral the common circumcentre is also the common incentre, centroid and orthocentre


Hands on methodologies
Discovering Ideas
Collaborating with others
Students learn Geometry through
Communicating mathematically
Multiple Representations
Technology
Real Life Applications

14:10

Summary

- Using Problem Solving as a methodology to explore geometry construction in context

- Geometry construction: connections within Strand 2

14:10

Have you accessed and/or used resources in class from the following websites.....

Have you accessed and/or used resources in class from the following websites.....

1. http://ww.projectmaths.ie:
- "*Patterns: A Relations Approach to Algebra*" document featured on the homepage which explores linear, quadratic, cubic, inverse and exponential relationships
- Project Maths News updates
- Teaching and Learning plans
- Teacher Handbooks which include a suggested sequence and a suggested number of classes for various lesson ideas
- Student's CD (which now includes all 5 strands) with the Student Activities in class
- Syllabus and Resources Documents which match resources to the syllabus
- Booklets, powerpoints, posters, video links from Workshops 1-5
- Booklets of Sample Papers relevant to LC 2012
- Problem Solving tab?

2. http://www.censusatschool.ie
- Census at School to generate a class set of data, source data from students in other countries and access lots of statistical analysis tools and teaching resources

3. http://www.ncca.ie/projectmaths
- NCCA Student Resource Materials which are books of questions for JC and LC Strand 1 and Strand 2 and include examples for all levels
- Pre-LC 2010 papers
- Copies of the LC 2012, LC 2013, JC 2013 and JC 2014 syllabuses

4. http://www.examinations.ie
- circular S77/2011 from the SEC which came with the 2012 sample papers
- LC 2010 and 2011 and 2012 Sample Papers
- LC 2010 and 2011 Project Maths Papers
- JC 2011 and 2012 Sample Papers
- "*Report on the Trial Final*", a report that includes model solutions, marking schemes, comments on the answering, exemplars of student work for the 2010 LC Sample Paper 2 with additional notes related to the assessment of candidate work

Have you also........
- encouraged your students to access the booklets of exam-style questions for Strands 1-4 in the student zone of the NCCA website [http://www.ncca.ie/projectmaths]
- encouraged your students to access the Students' tab [http://www.projectmaths.ie]
- attended the modular courses for ICT for (i) Strands 1 and 2 or (ii) Strands 3, 4 and 5 or accessed the resources for same on the website or attended the modular courses for content for (i) Strand 1 or (ii) Strand 2 or accessed the resources for same on the website [http://www.projectmaths.ie]
- shared more resources with the rest of the teachers in your maths team in recent months
- facilitated the use of concrete resources (e.g. unifix cubes, dice, geometry set, geostrips,clinometers etc.) more often in recent months
- considered the implementation of a common approach in your mathematics department for the teaching of certain topics e.g. algebraic skills, fractions

14:10