

# Exploration of Exponential Functions



Monday 11 June 2018

Exam reference number

Centre name

Exam type

Exam reference number

300 marks

Timing 9:30 – 12:00

Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	
21	
22	
23	
24	
25	
26	
27	
28	
29	
30	
<b>Total</b>	

Exam type

Mark

Check

# Prior Knowledge

If  $a$  is any positive number and  $x$  is any integer, then  $a^x > 0$

$$\text{e.g. } 3^4 = 81 > 0$$

$$4^{-2} = \frac{1}{4^2} = \frac{1}{16} > 0$$

- Understand and apply the laws of indices
- Functions

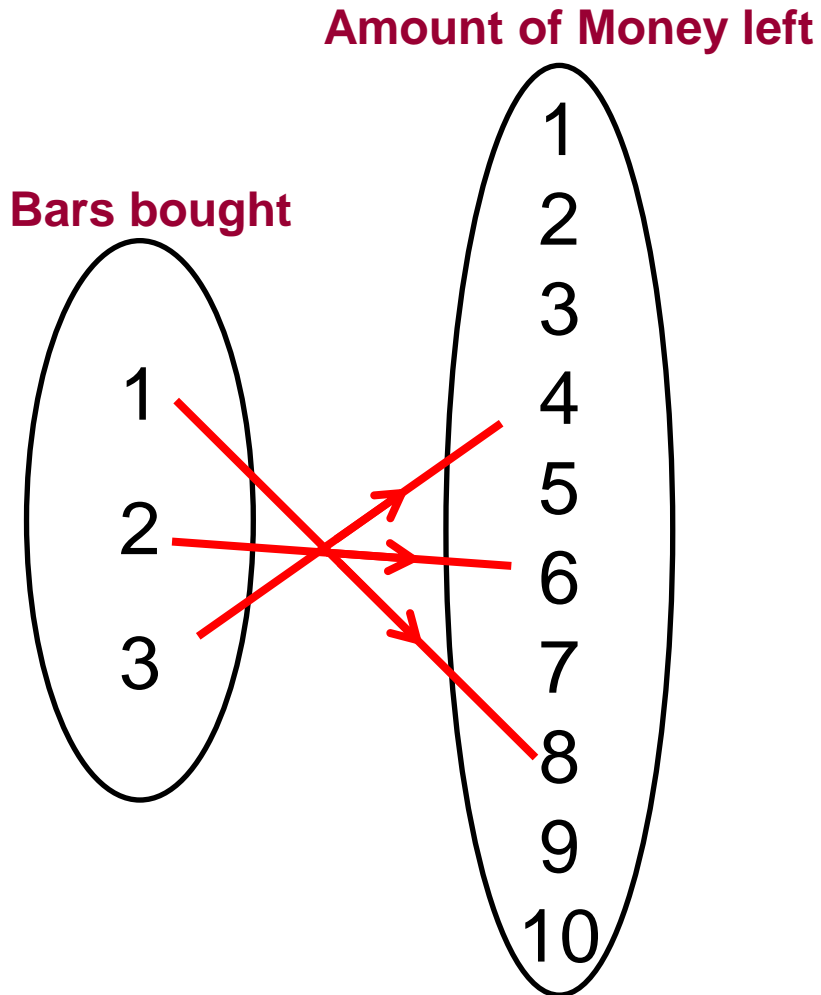
# Exponents/Indices/Powers

(base)<sup>exponent</sup>

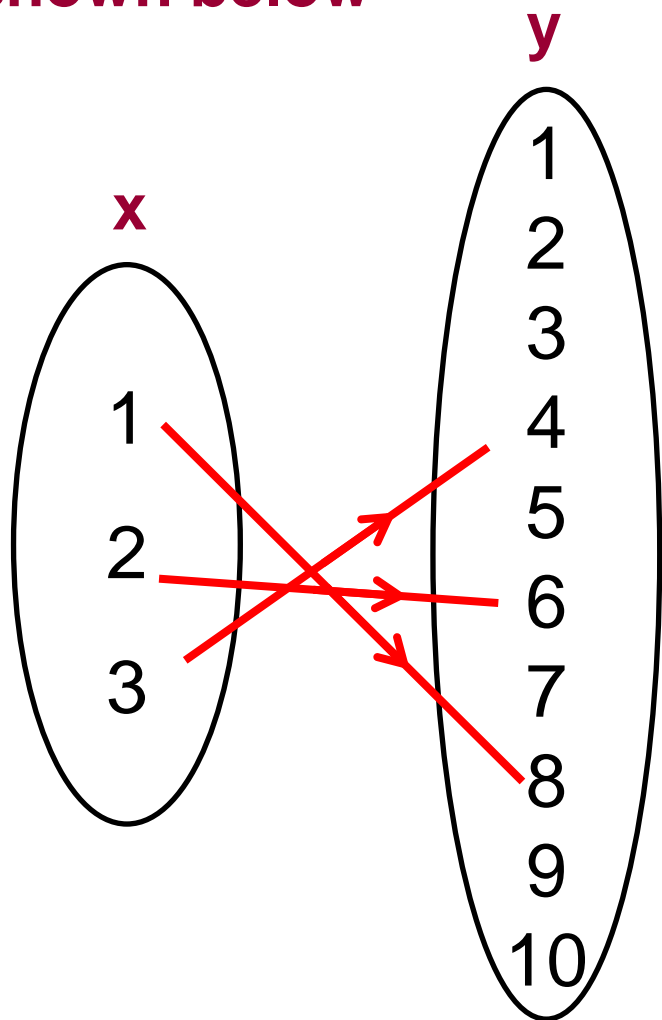


The exponent says how many of the base are being multiplied together.

John has €10 to buy bars of chocolate which cost €2 each.



John has €10 to buy bars of chocolate which cost €2 each. Let  $x$  be the number of chocolate bars we buy and  $y$  be the amount of money left. The relationship between  $x$  and  $y$  is shown below



1. Each value of  $x$  corresponds to values of  $y$

&

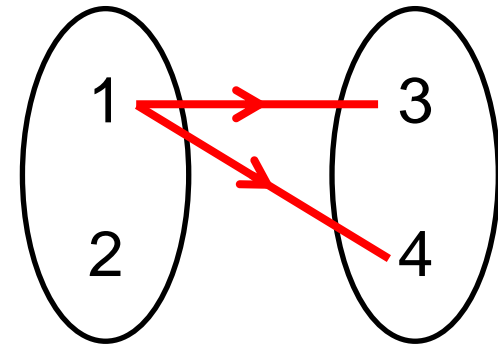
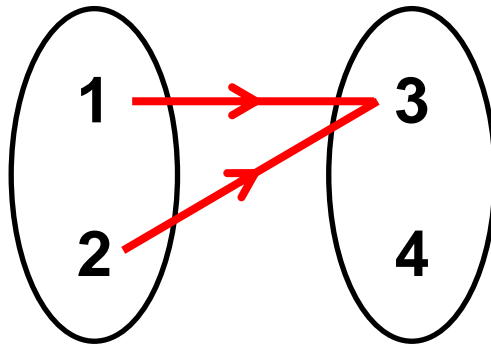
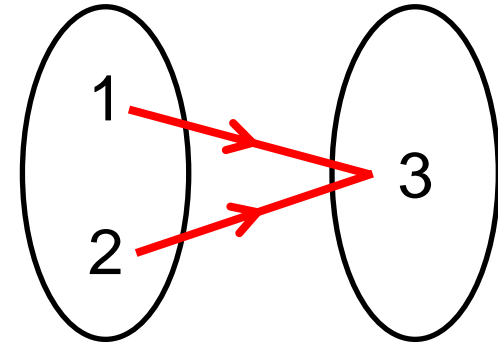
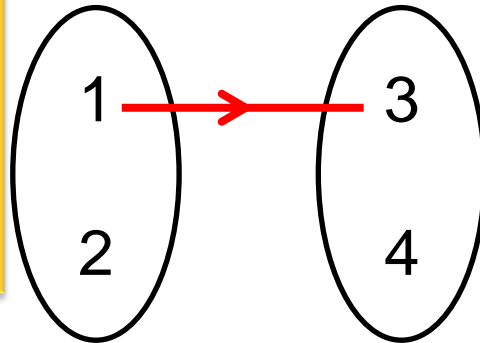
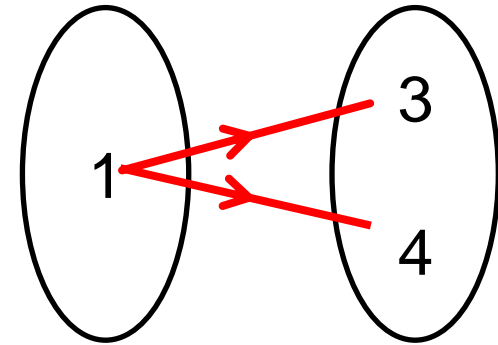
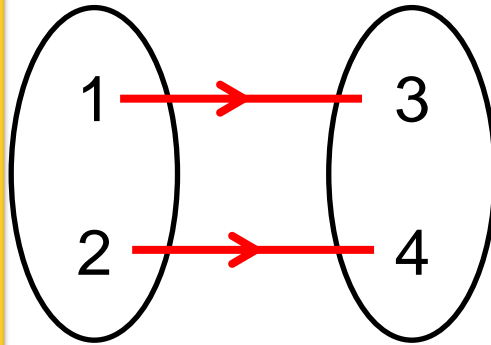
2. Each value of  $x$  corresponds to only one value of  $y$

$y$  is a function of  $x$  if both of these 2 conditions are true

1. Each value of  $x$  corresponds to values of  $y$

&

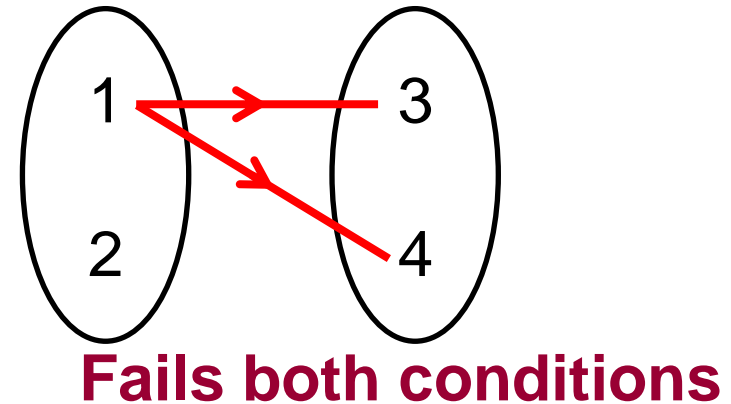
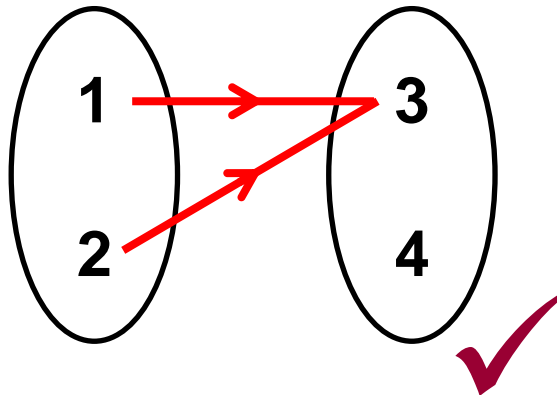
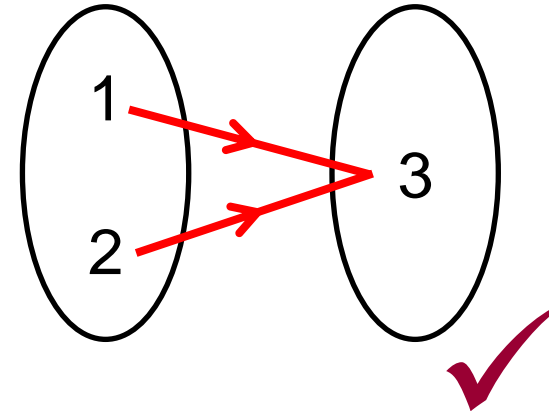
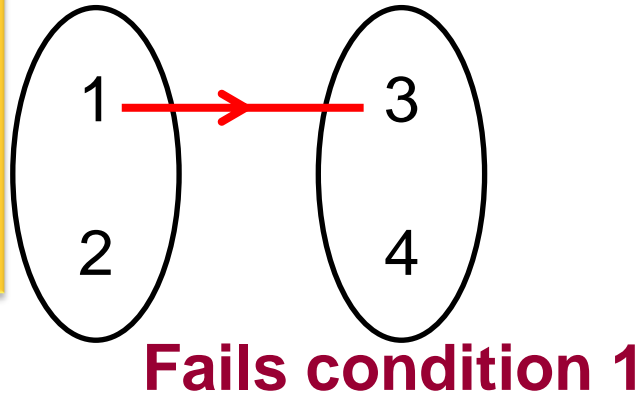
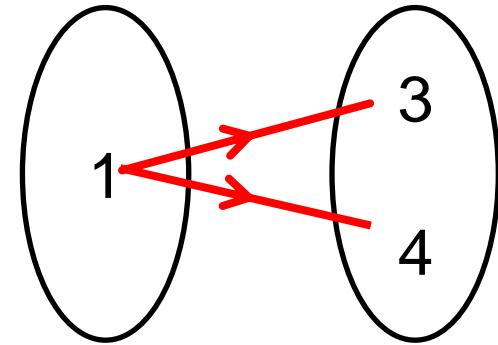
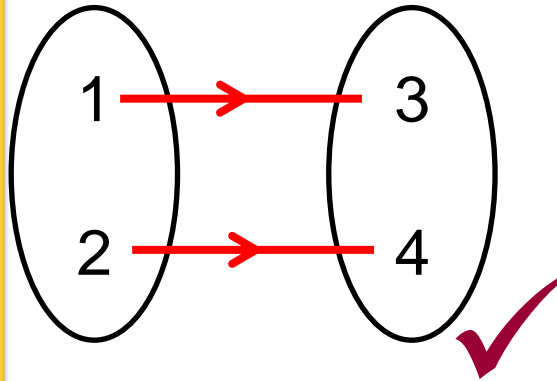
2. Each value of  $x$  corresponds to only one value of  $y$



1. Each value of  $x$  corresponds to values of  $y$

&

2. Each value of  $x$  corresponds to only one value of  $y$

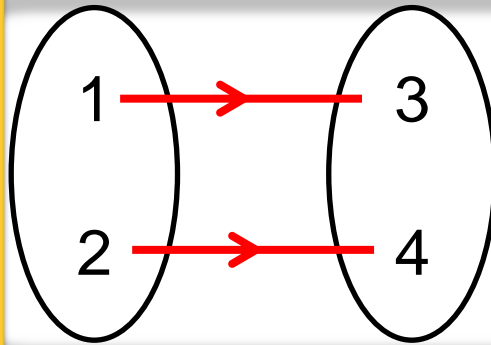




1. Each value of  $x$  corresponds to values of  $y$

&

2. Each value of  $x$  corresponds to only one value of  $y$



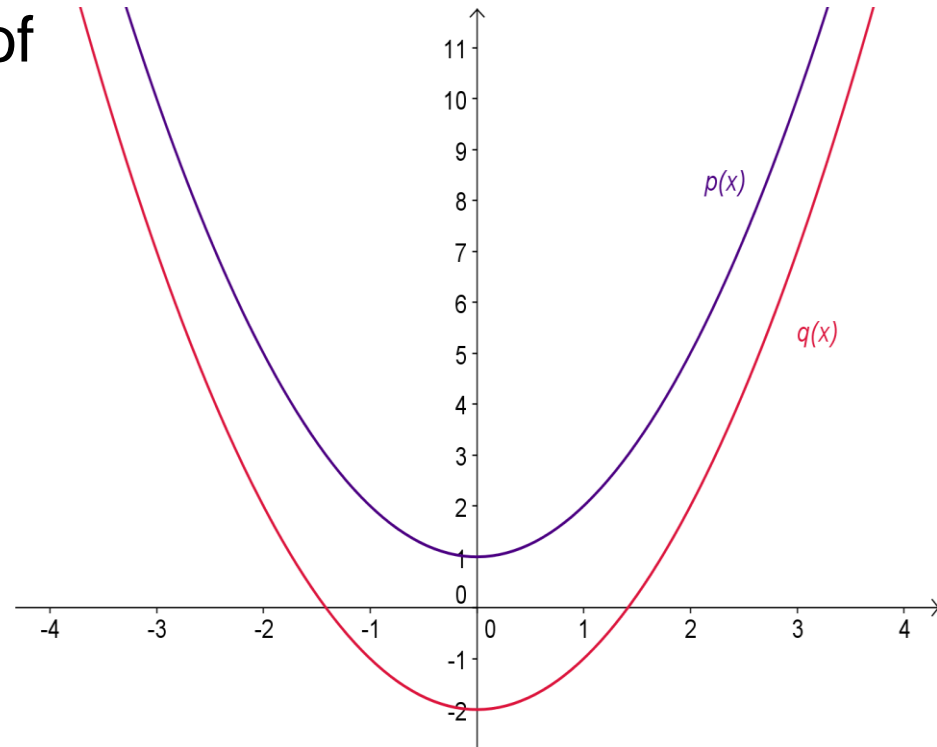
**Bijjective**

# Basic Technique

Read information from a graph

e.g. the figure shows the graph of  
 $p(x) = x^2 + 1$  and  $q(x) = x^2 - 2$   
in the domain  $-4 \leq x \leq 4$ ,  $x \in \mathbb{R}$

Discuss what are the similarities  
and the differences



# Basic Technique

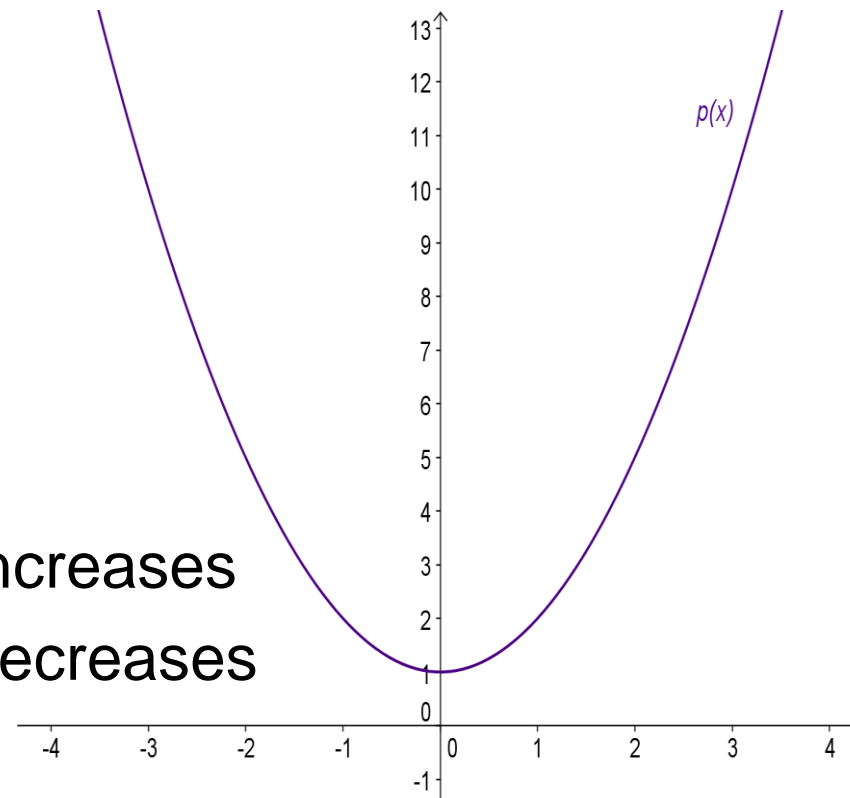
Read information from a graph

e.g. the figure shows the graph of

$$P(x) = x^2 + 1$$

in the domain  $-4 \leq x \leq 4$ ,  $x \in \mathbb{R}$

- The graph has no x- intercepts
- Its y intercept is 1
- For  $x \geq 0$ ,  $x^2 + 1$  increases as x increases
- For  $x \leq 0$ ,  $x^2 + 1$  increases as x decreases



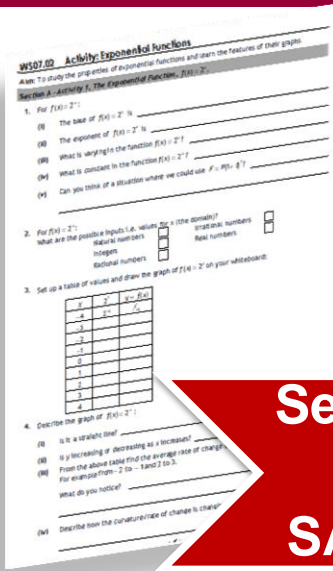
# Exponential Functions

## Learning Outcomes

After completing this session you will be able to:

1. Understand the properties of exponential functions
2. Learn the features of their graphs

# Exponential Functions Activity Sheets



## Section A SA:1-4

Page  
4 - 8

- $y = 2^x$  &  $y = 3^x$
- Compare
- Characteristics

## Section B SA:1-4

Page  
9 - 13

- $y = (1/2)^x$  &  $y = (1/3)^x$
- Compare
- Characteristics

## Section C SA: 1-4

Page  
14 - 16

- Compare  $y = (1/2)^x$  and  $y = 2^x$
- Compare  $y = (1/3)^x$  and  $y = 3^x$
- Now I see....

## Section 4

Page  
17-18

Problem Solving Questions

# Organisation

- Groups 1 & 5  
Complete Section A  
Student Activity 1  
page 4 - 5

- Groups 3, 7 & 9  
Complete Section B  
Student Activity 1  
page 9 - 10

- Groups 2 & 6  
Complete Section A  
Student Activity 2  
page 6 - 7

- Groups 4, 8 & 10  
Complete Section B  
Student Activity 2  
page 11 - 12

# Feedback

- Groups 1 & 5  
Section A  
Student Activity 1  
page 4 - 5

- Groups 2 & 6  
Section A  
Student Activity 2  
page 6 - 7

# Feedback

Q1

$$f(x) = 2^x$$

(i) Base

(ii) Exponent

(iii) Varying

(iv) Constant

Q2 Domain

$$g(x) = 3^x$$

Base

Exponent

Varying

Constant

Domain



# Feedback

Q1

$$f(x) = 2^x$$

(i) Base

2

(ii) Exponent

x

(iii) Varying

x & f(x)

(iv) Constant

2

Q2 Domain

$x \in R$

$$g(x) = 3^x$$

Base

3

Exponent

x

Varying

x & g(x)

Constant

3

Domain

$x \in R$

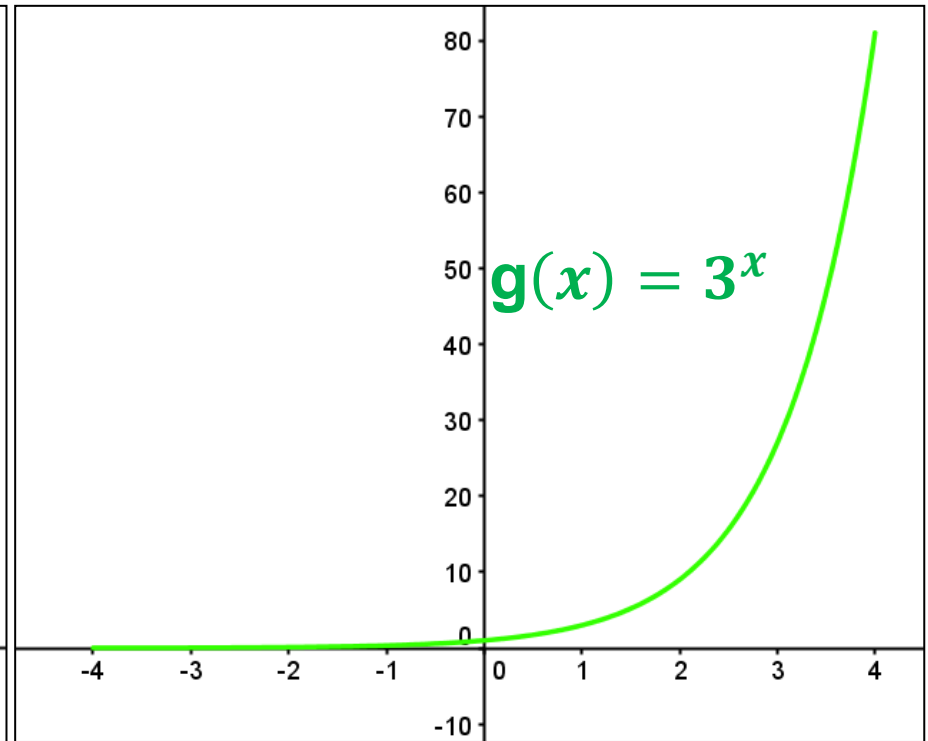
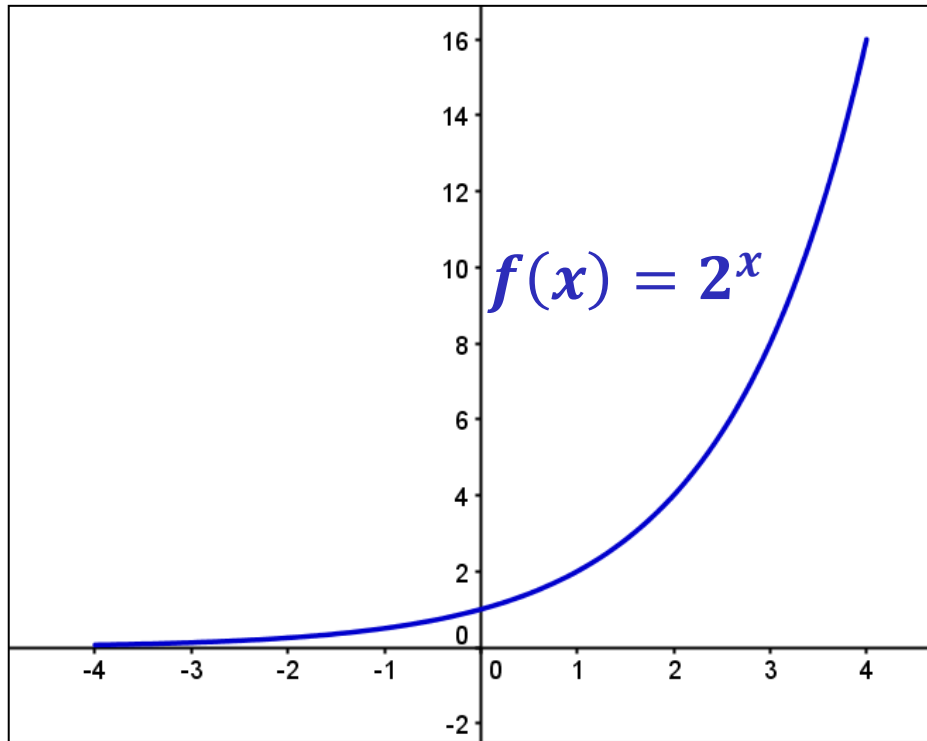
$x$	$(2)^x$	$f(x)$
-4	$(2)^{-4}$	1/16
-3	$(2)^{-3}$	1/8
-2	$(2)^{-2}$	1/4
-1	$(2)^{-1}$	1/2
0	$(2)^0$	1
1	$(2)^1$	2
2	$(2)^2$	3
3	$(2)^3$	8
4	$(2)^4$	16

$x$	$(3)^x$	$g(x)$
-4	$(3)^{-4}$	1/81
-3	$(3)^{-3}$	1/27
-2	$(3)^{-2}$	1/9
-1	$(3)^{-1}$	1/3
0	$(3)^0$	1
1	$(3)^1$	3
2	$(3)^2$	9
3	$(3)^3$	27
4	$(3)^4$	81

# Groups

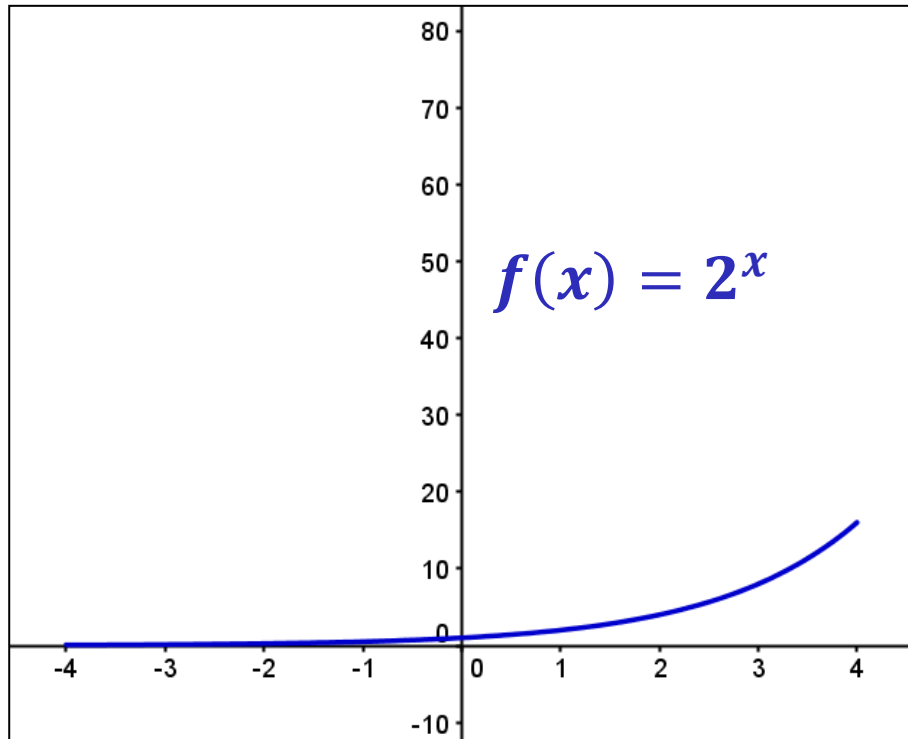
1 & 5

2 & 6

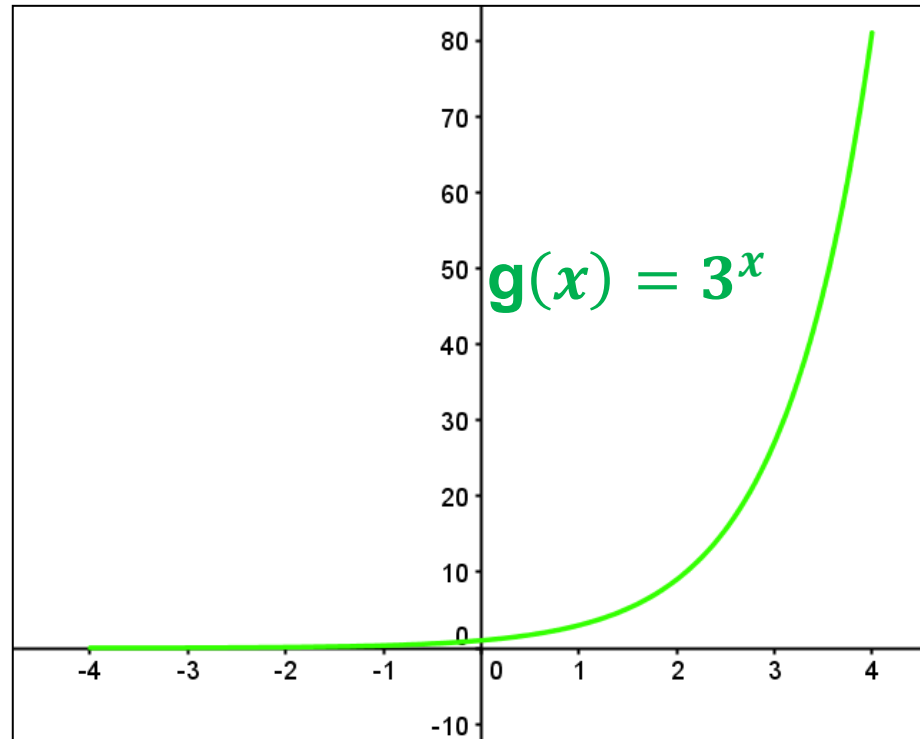


# Groups

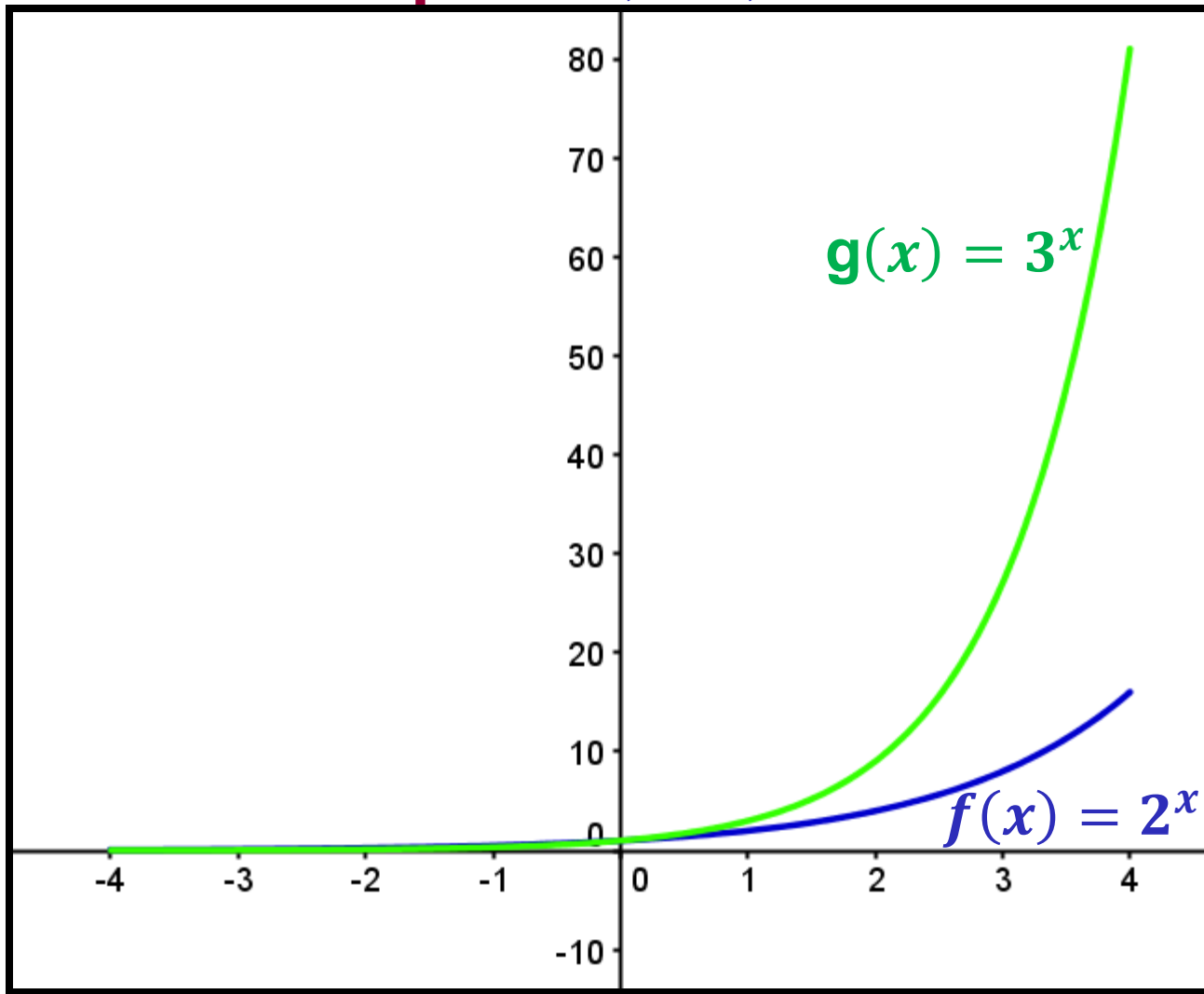
1 & 5



2 & 6



# Groups 1, 5, 2 & 6



# Questions 4 & 5

Q4

(i) Straight Line

**No**

(ii) Y increasing or decreasing as x increases

**Increasing**

(iii) Rate of change

**Not constant**

(iv) Describe how its curvature/rate of change is changing

**Increasing**

Q5

(i) Outputs (Range)

**$R^+$**

(ii) Negative Outputs, Why?

**No**

(iii) Outputs as x decreases

**Decreases**

(iv) An output of 0? Why?

**No**

(v) X-intercept

**None**

(vi) Y-intercept

**1**

# Questions 4 & 5

Q4

(i) Straight Line

(ii) Y increasing or decreasing as x increases

(iii) Rate of change

(iv) Describe how its curvature/rate of change is changing

Q5

(i) Outputs (Range)

(ii) Negative Outputs, Why?

(iii) Outputs as x decreases

(iv) An output of 0? Why?

(v) X-intercept

(vi) Y-intercept

# Feedback

- Groups 3, 7 & 9  
Section B  
Student Activity 1  
page 9 - 10

- Groups 4,8 & 10  
Section B  
Student Activity 2  
page 11 - 12



# Feedback

$$f(x) = \left(\frac{1}{2}\right)^x$$

$$g(x) = \left(\frac{1}{3}\right)^x$$

Q1

(i) Base

Base

(ii) Exponent

Exponent

(iii) Varying

Varying

(iv) Constant

Constant

Q2 Domain

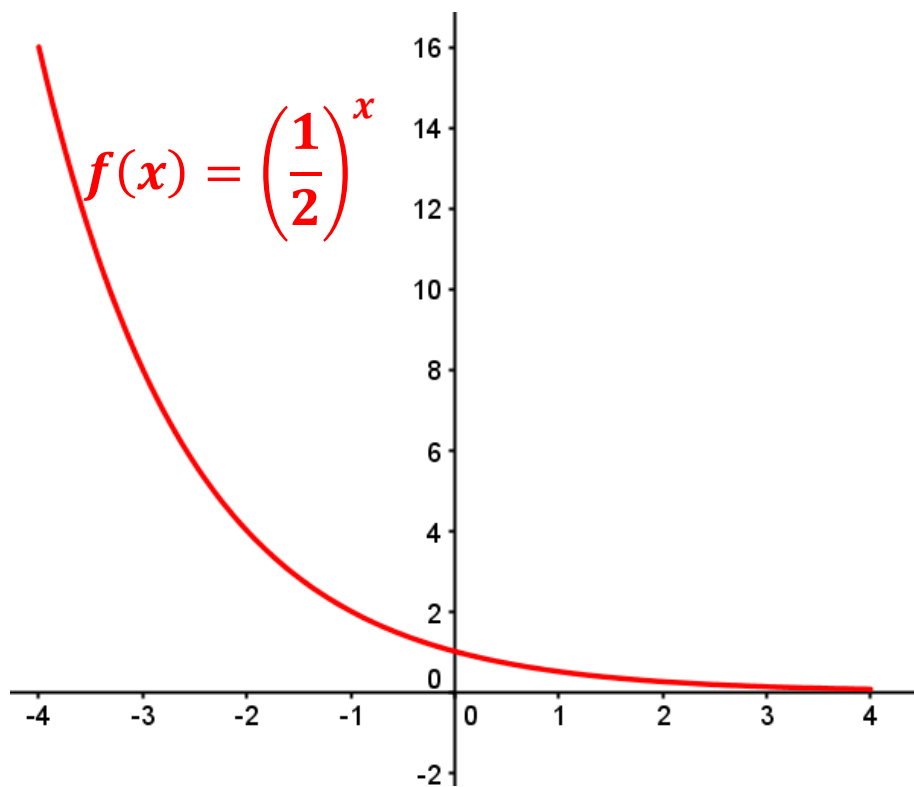
Domain

<b>x</b>	<b><math>\left(\frac{1}{2}\right)^x</math></b>	<b>f(x)</b>
-4	$\left(\frac{1}{2}\right)^{-4}$	16
-3	$\left(\frac{1}{2}\right)^{-3}$	8
-2	$\left(\frac{1}{2}\right)^{-2}$	4
-1	$\left(\frac{1}{2}\right)^{-1}$	2
0	$\left(\frac{1}{2}\right)^0$	1
1	$\left(\frac{1}{2}\right)^1$	$\frac{1}{2}$
2	$\left(\frac{1}{2}\right)^2$	$\frac{1}{4}$
3	$\left(\frac{1}{2}\right)^3$	$\frac{1}{8}$
4	$\left(\frac{1}{2}\right)^4$	$\frac{1}{16}$

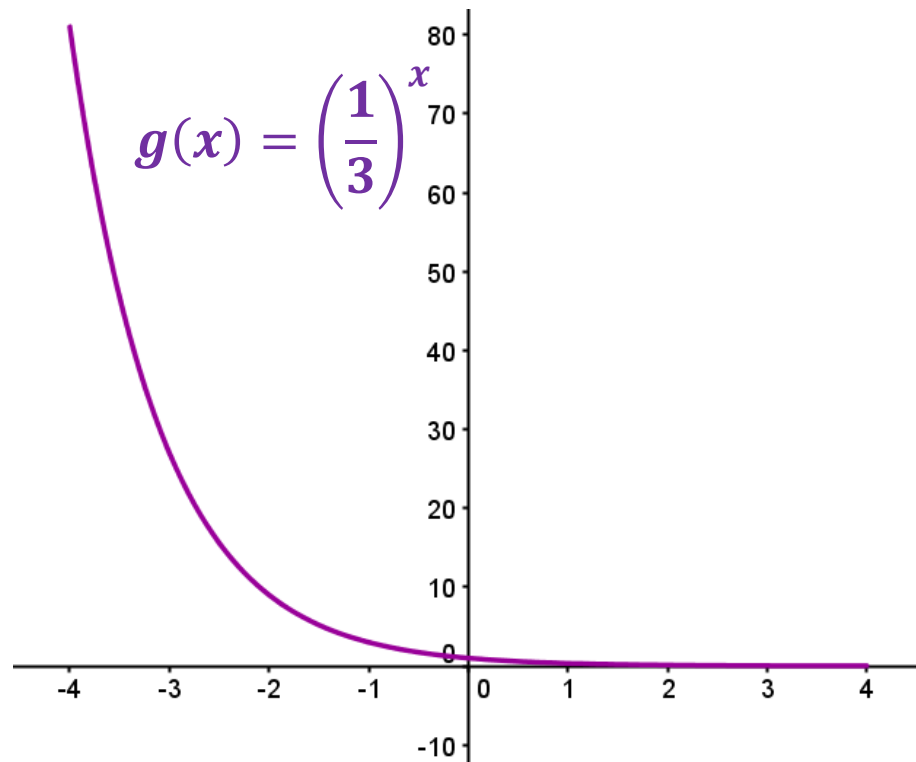
<b>x</b>	<b><math>\left(\frac{1}{3}\right)^x</math></b>	<b>g(x)</b>
-4	$\left(\frac{1}{3}\right)^{-4}$	81
-3	$\left(\frac{1}{3}\right)^{-3}$	27
-2	$\left(\frac{1}{3}\right)^{-2}$	9
-1	$\left(\frac{1}{3}\right)^{-1}$	3
0	$\left(\frac{1}{3}\right)^0$	1
1	$\left(\frac{1}{3}\right)^1$	$\frac{1}{3}$
2	$\left(\frac{1}{3}\right)^2$	$\frac{1}{9}$
3	$\left(\frac{1}{3}\right)^3$	$\frac{1}{27}$
4	$\left(\frac{1}{3}\right)^4$	$\frac{1}{81}$

# Groups

3,7 & 9



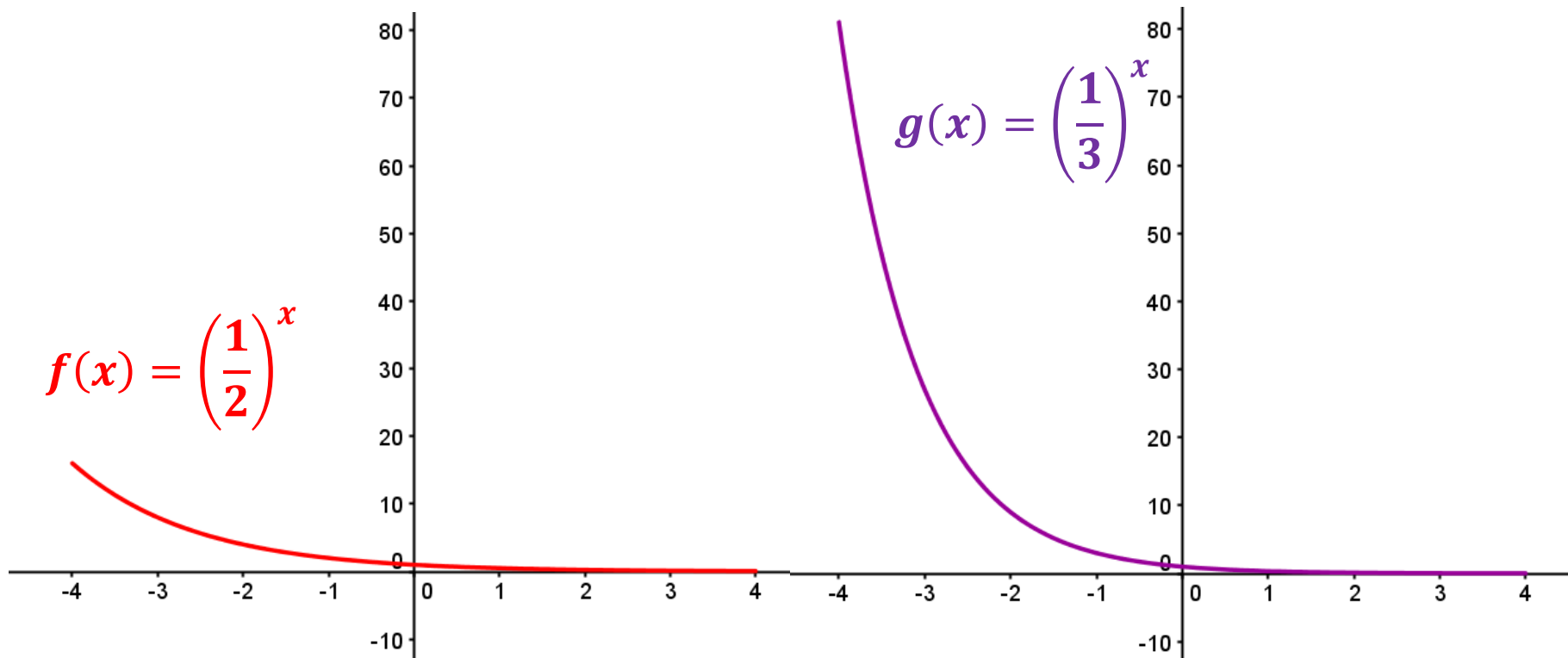
4,8 & 10



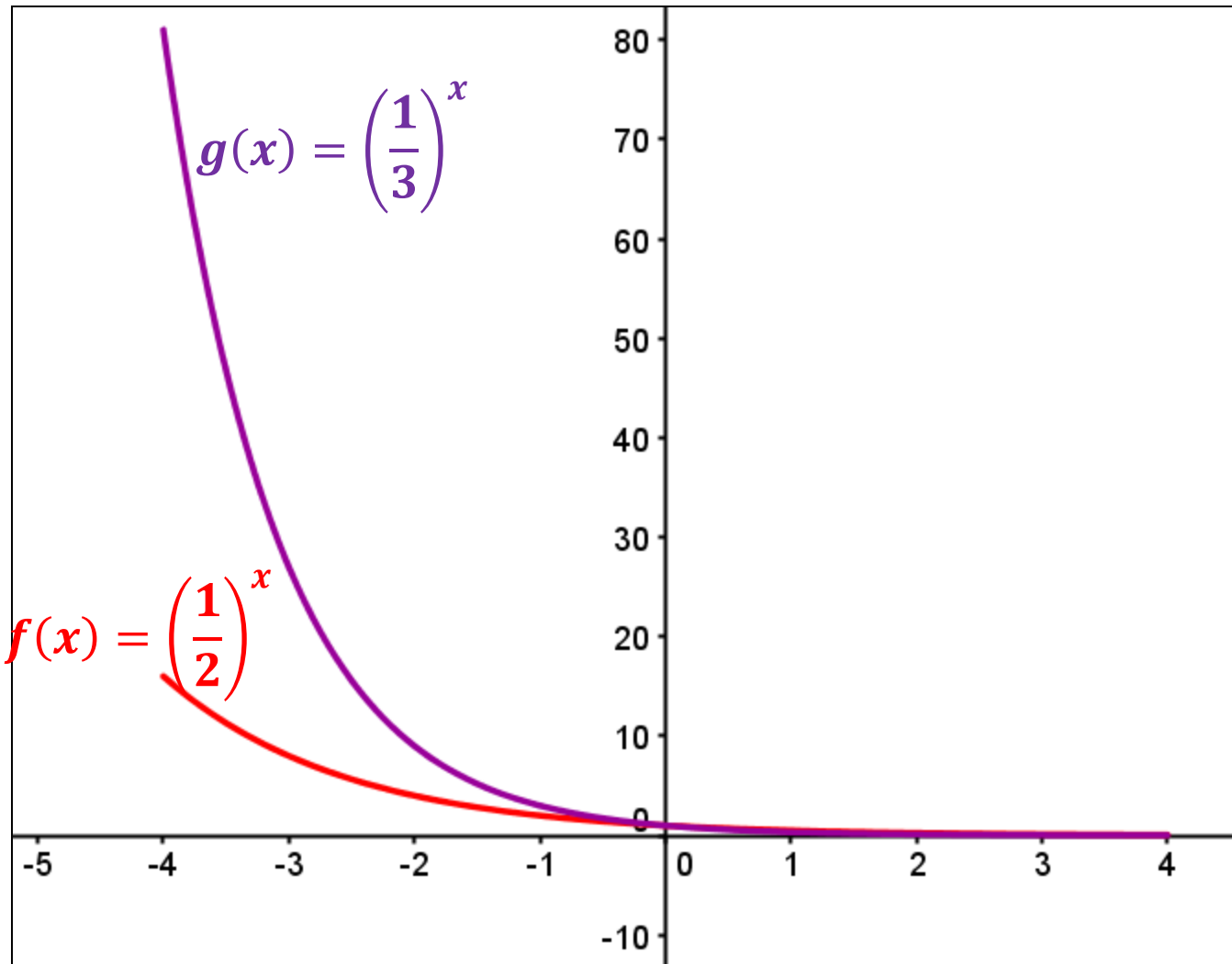
# Groups

3,7 & 9

4,8 & 10



# Groups 3,7,9,4,8 & 10



# Questions 4 & 5

Q4

(i) Straight Line

(ii) Y increasing or decreasing as x increases

(iii) Rate of change

(iv) Describe how its curvature/rate of change is changing

Q5

(i) Outputs (Range)

(ii) Negative Outputs, Why?

(iii) Outputs as x decreases

(iv) An output of 0? Why?

(v) X-intercept

(vi) Y-intercept

# Questions 4 & 5

Q4

(i) Straight Line

**No**

(ii) Y increasing or decreasing as x increases

**Decreasing**

(iii) Rate of change

**Not constant**

(iv) Describe how its curvature/rate of change is changing

**Decreasing**

Q5

(i) Outputs (Range)

**$R^+$**

(ii) Negative Outputs, Why?

**No**

(iii) Outputs as x decreases

**Increases**

(iv) An output of 0? Why?

**No**

(v) X-intercept

**None**

(vi) Y-intercept

**1**

# Organisation

- Groups 1, 5, 2 & 6
- Draw either  $2^x$  or  $3^x$   
Complete Section A  
Activities 3 & 4  
Page 8

Groups 3,7,9,4,8,& 10

- Draw either  $\left(\frac{1}{2}\right)^x$  or  $\left(\frac{1}{3}\right)^x$   
Complete Section B  
Activities 3 & 4  
Pages 13 & 14



# Compare $2^x$ and $3^x$

Section A Activity 3 page 8 (Groups 1,2,5,& 6)

1. How are they the same and how are they different?
2. Are they functions?
3. Name this type of function and why?

# Understand the characteristics of

$$f(x) = a^x, a > 1$$

## Section A

## Activity 4

## page 8

- Domain
- Straight Line
- Is y increasing or decreasing as x increases
- Maximum/  
Minimum value

- Curvature

- Range

- X-intercept

- Y-intercept

# Understand the characteristics of

$$f(x) = a^x, a > 1$$

## Section A

## Activity 4 page 8

- |                                                |            |               |            |
|------------------------------------------------|------------|---------------|------------|
| • Domain                                       | $x \in R$  | • Curvature   | Increasing |
| • Straight Line                                | No         | • Range       | $R^+$      |
| • Is y increasing or decreasing as x increases | Increasing | • X-intercept | None       |
| • Maximum/<br>Minimum value                    | None       | • Y-intercept | 1          |

# Compare $\left(\frac{1}{2}\right)^x$ and $\left(\frac{1}{3}\right)^x$

Section B Activities 3 & 4 page 13

1. How are they the same and how are they different?
2. Are they functions?
3. Name this type of function and why?

# Understand the characteristics of

$$f(x) = a^x, 0 < a < 1$$

Section B

Activity 4 page 13

• Domain

• Straight Line

• Is y increasing or decreasing as x increases

• Maximum/  
Minimum value

• Curvature

• Range

• X-intercept

• Y-intercept

# Understand the characteristics of

$$f(x) = a^x, 0 < a < 1$$

## Section B Activity 4 page 13

- |                                                |            |               |            |
|------------------------------------------------|------------|---------------|------------|
| • Domain                                       | $x \in R$  | • Curvature   | Decreasing |
| • Straight Line                                | No         | • Range       | $R^+$      |
| • Is y increasing or decreasing as x increases | Decreasing | • X-intercept | None       |
| • Maximum/Minimum value                        | None       | • Y-intercept | 1          |

# Organisation

- Groups 1,3,5,7,& 9
- Section C Activity 1
- Page 14

- Groups 2,4,6,8 & 10
- Section C Activity 2
- Page 14

# Compare $2^x$ and $\left(\frac{1}{2}\right)^x$

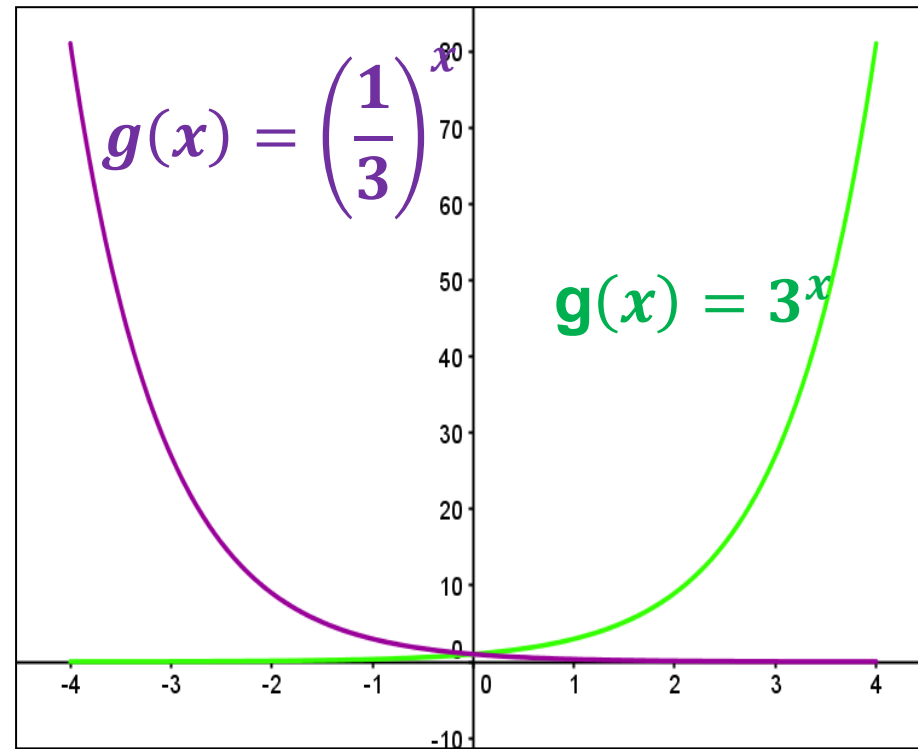
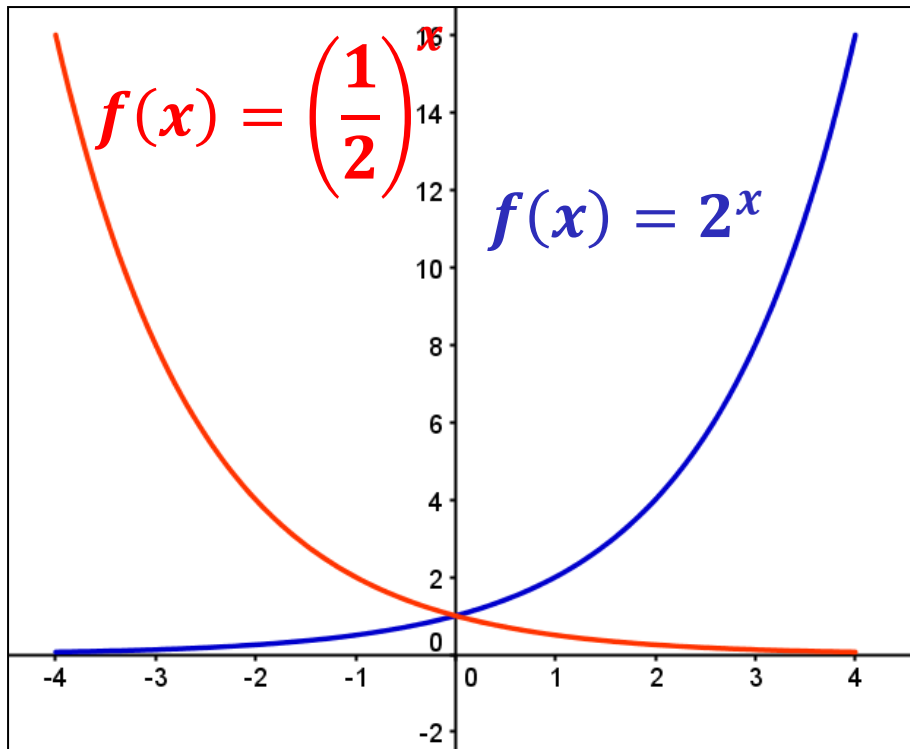
1. Same
2. Different
3. Write  $f(x) = \left(\frac{1}{2}\right)^x$  using a base of 2
4. What transformation maps the graph of  $f(x) = 2^x$  onto the graph of  $f(x) = \left(\frac{1}{2}\right)^x$  ?



# Compare $3^x$ and $\left(\frac{1}{3}\right)^x$

1. Same
2. Different
3. Write  $g(x) = \left(\frac{1}{3}\right)^x$  using a base of 3
4. What transformation maps the graph of  $g(x) = 3^x$  onto the graph of  $g(x) = \left(\frac{1}{3}\right)^x$  ?

# Compare



# All Groups

Complete Section C

Activity 3

page 15

# Section C

1. If  $f(x) = a^x, a \in \mathbb{R}, a > 1$ , then the properties of the exponential function are:

---

---

2. If  $f(x) = a^x, a \in \mathbb{R}, a > 1$ , then the features of the exponential graph are:

---

---

3. If  $f(x) = a^x, a \in \mathbb{R}, 0 < a < 1$ , then the properties of the exponential function are:

---

---

4. If  $f(x) = a^x, a \in \mathbb{R}, 0 < a < 1$ , then the features of the exponential graph are:

---

---

# All Groups

Section C - Activity 4: Which of the following equations represent exponential functions?

Equation	Is it an exponential Function? Yes/No	Explain why
$f(x) = \left(\frac{1}{2}\right)^x$		
$f(x) = x^2$		
$f(x) = (-2)^x$		
$f(x) = 2(3)^x$		
$f(x) = -2^x$		
$f(x) = 3(x)^{\frac{1}{2}}$		
$f(x) = (0.9)^x$		

## Problem Solving Questions on Exponential Functions

**Note:** Extension Activities are required to strengthen students' abilities in the following areas from the syllabus:

<i>Level</i>	<i>Syllabus</i>	<i>Page</i>
JCHL	$f(x) = a2^x$ and $f(x) = a3^x$ , where $a \in \mathbb{N}, x \in \mathbb{R}$ .	Page 31
LCFL	$f(x) = a2^x$ and $f(x) = a3^x$ , where $a \in \mathbb{N}, x \in \mathbb{R}$ .	Page 32
LCOL	$f(x) = ab^x$ , where $a \in \mathbb{N}, b, x \in \mathbb{R}$ .	Page 32
LCHL	$f(x) = ab^x$ , where $a, b, x \in \mathbb{R}$ .	Page 32

## Exponential Functions (properties)

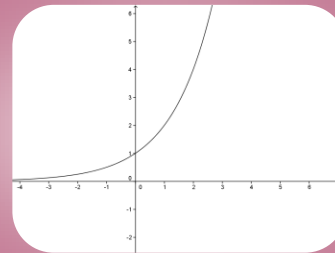
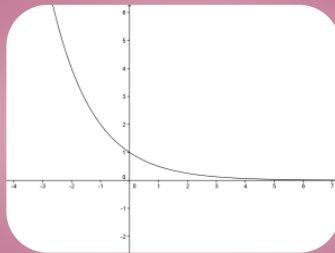
$$f(x) = a^x, 0 < a < 1$$

$$f(x) = a^x, a > 1$$

## Exponential Graphs (features)

$$\left(\frac{1}{2}\right)^x \text{ and } \left(\frac{1}{3}\right)^x$$

$$2^x \text{ and } 3^x$$



**Underlying Principles**

Prior Knowledge

Connections

Effective  
questioning

What if  
questions

Group work

Misconceptions

Discussion &  
Communication

Methods rather  
than answers

Rich tasks