Purpose

(1)To recap on rate of change and distinguish between average and instantaneous rates of change.

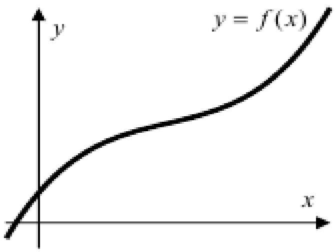
(2)To introduce the idea of the derivative.

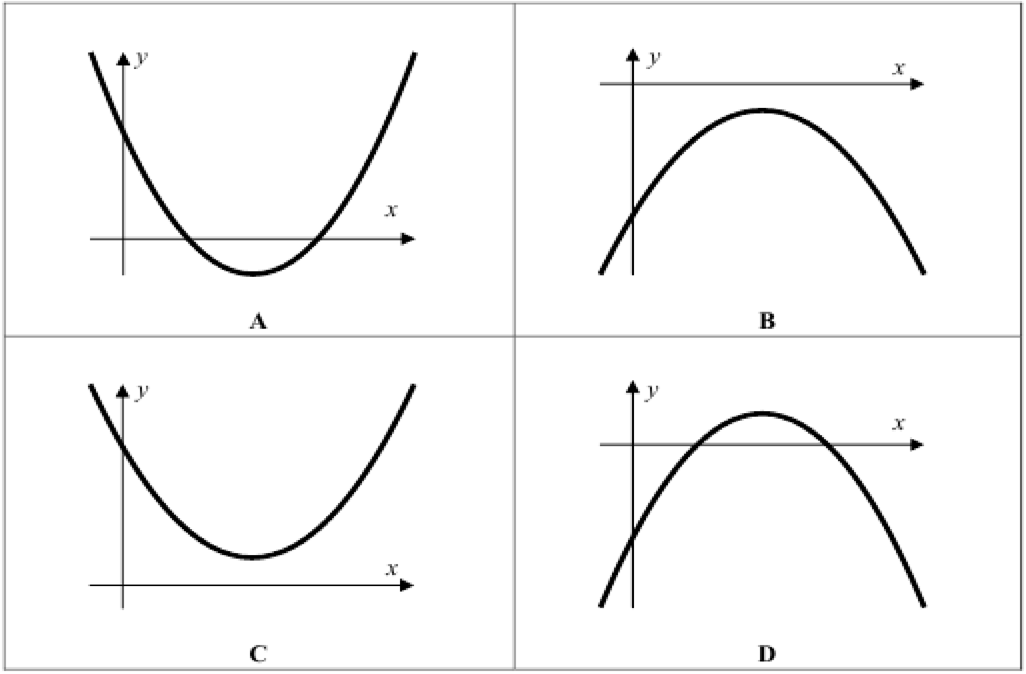
Things have changed.........

The graph of a cubic function f is shown on the right.

One of the four diagrams **A**, **B**, **C**, **D** below shows the graph of the derivative of f.

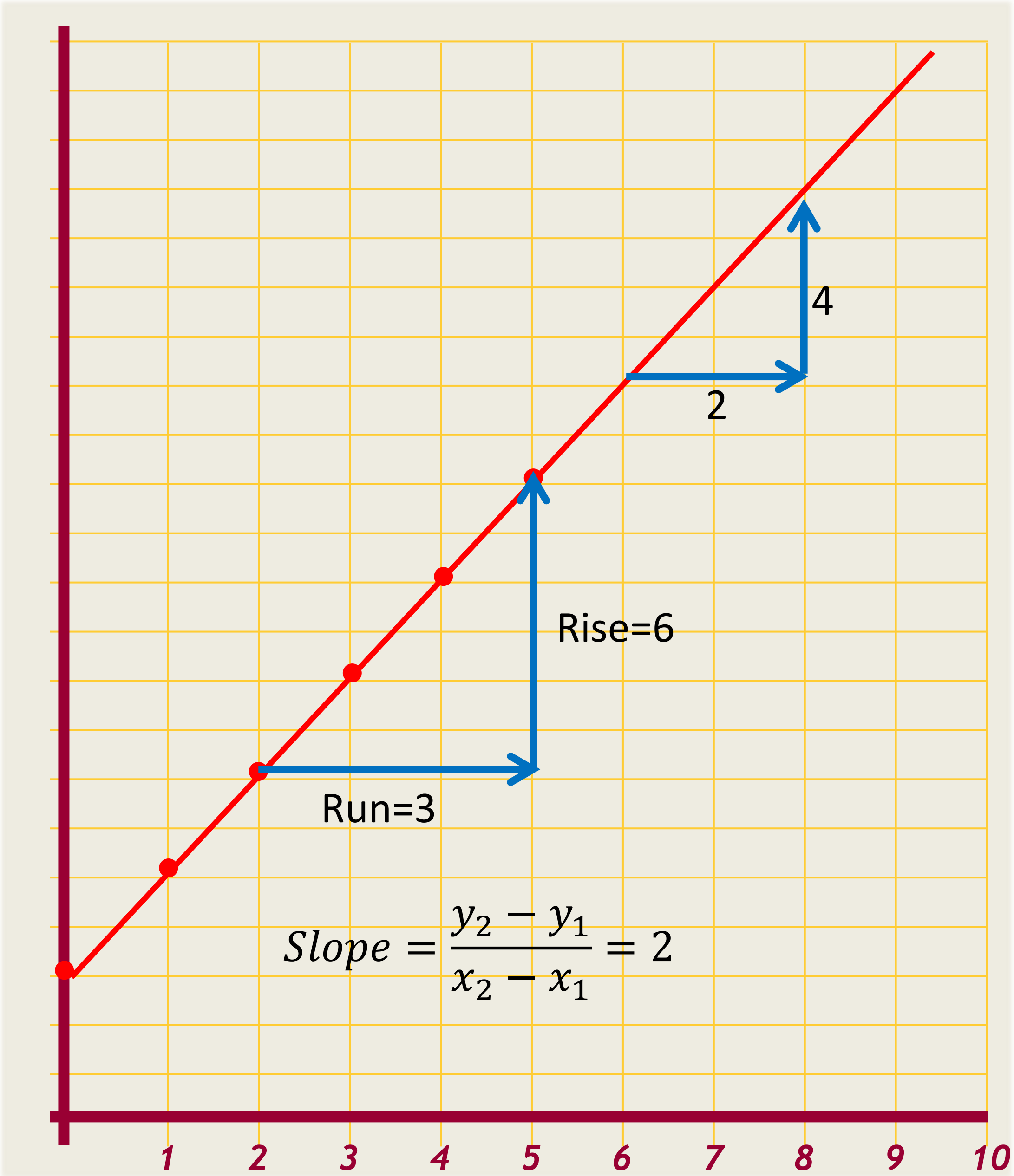
State which one it is, and justify your answer.



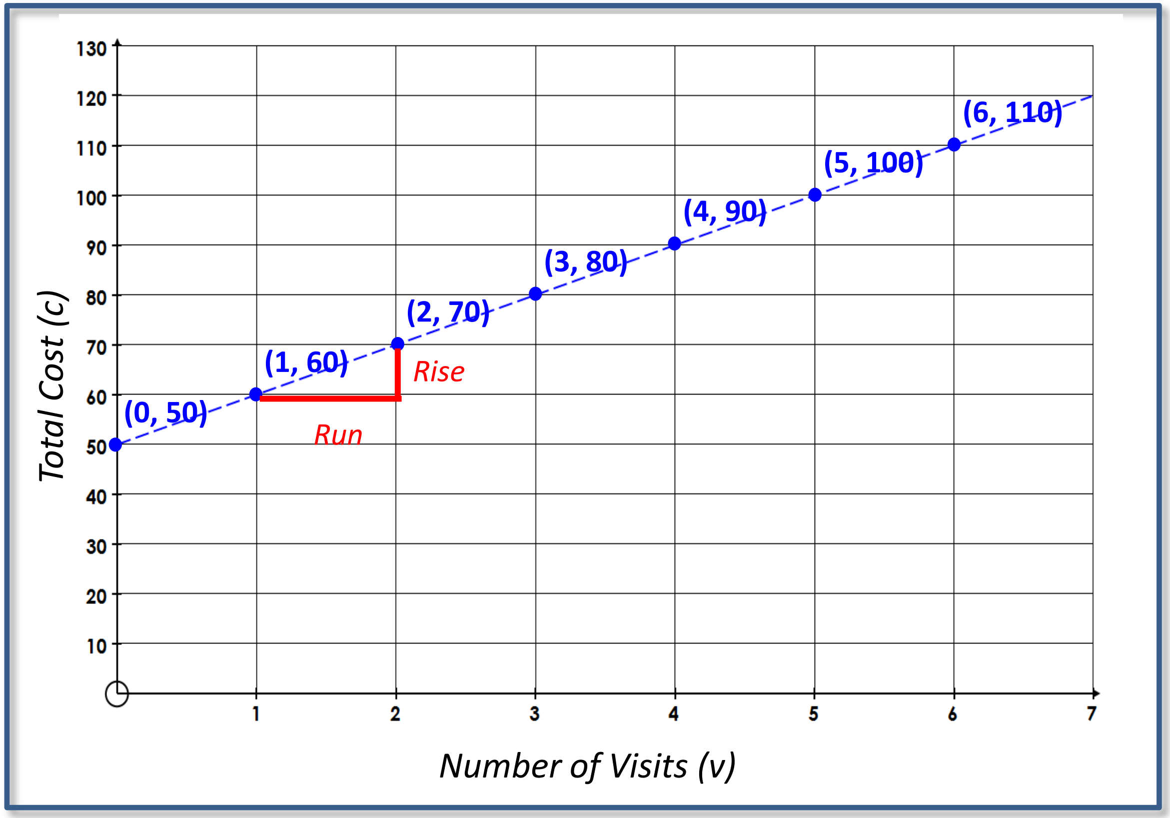


The Mathematics of Change

Slope (m)



Gym Membership



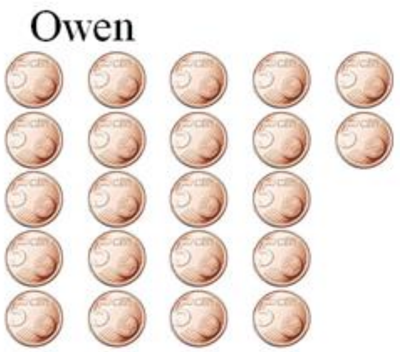
Visits	Total cost c in €	Change
0	50	
1	60	+10
2	70	+10
3	80	+10
4	90	+10
5	100	+10
6	110	+10

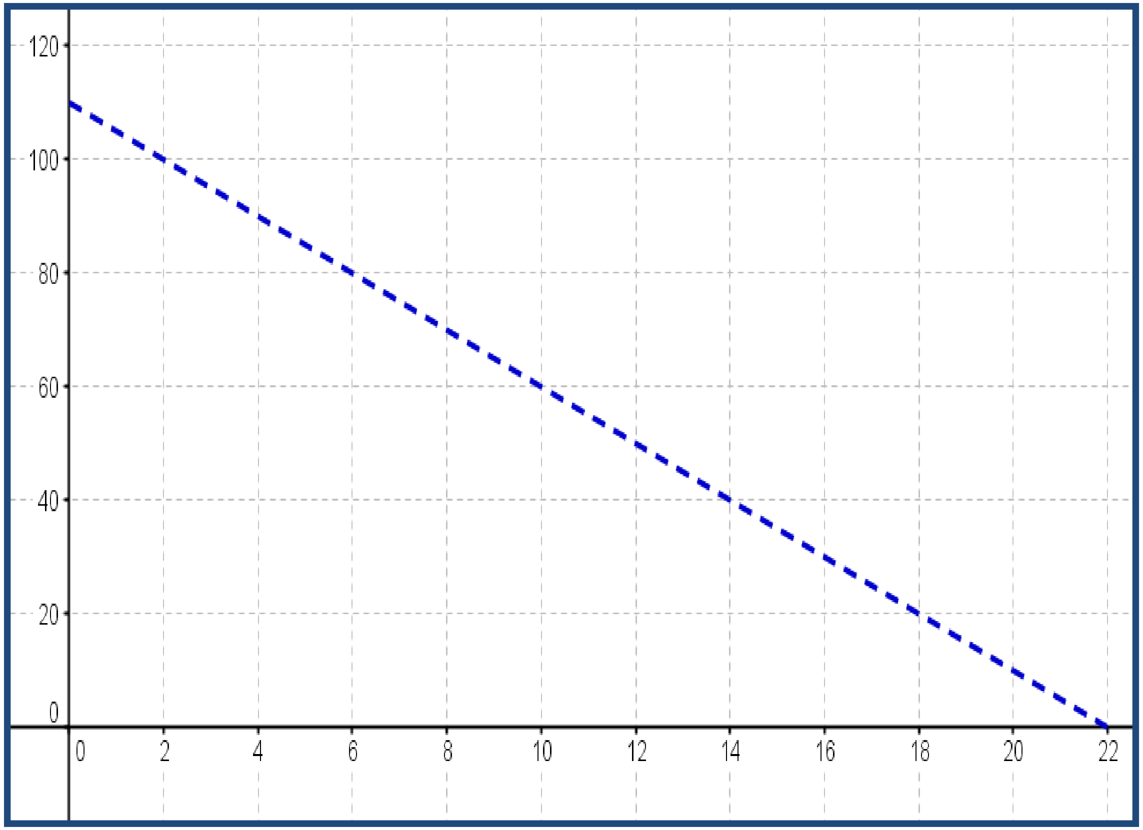
$$\text{Rate of change} = \text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{10}{1} = 10$$

$$\text{Total Cost } c = 50 + 10v$$

Problem Solving

Owen has 22 five-cent coins in his wallet.
Each day, he decides to take one coin from his wallet and put it into a money box.



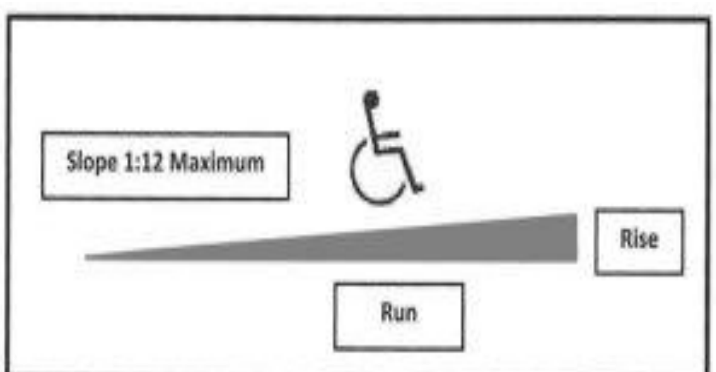


Days Elapsed	*Amount (C)*
0	110
1	105
2	100
3	95
4	90

$$\text{Slope} = \frac{\text{Rise}}{\text{Run}} = \frac{-5}{1} = -5$$

Can you see Rates of Change?

Rates of Change Around Us



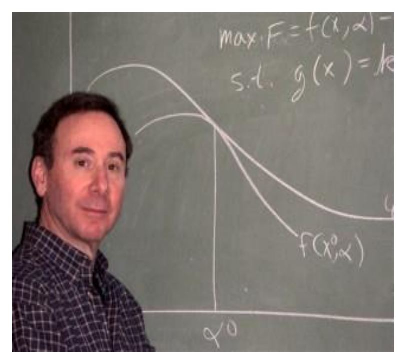
World in Motion

A **microbiologist** might be interested in rate at which number of bacteria in a colony changes with time;

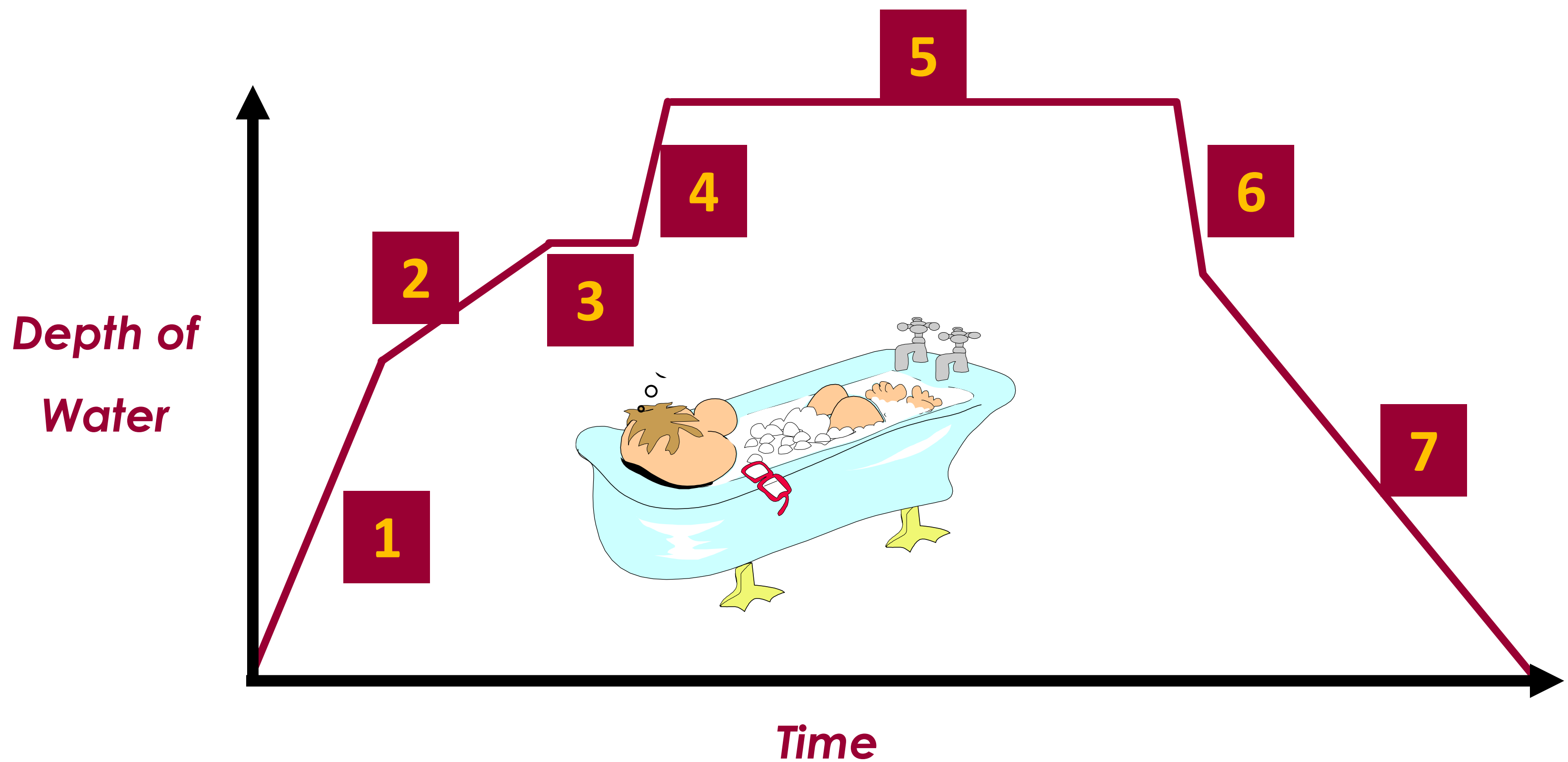
An **engineer** might be interested in rate at which length of a metal rod changes with temperature.

An **economist** might be interested in rate at which production cost changes with quantity of a product manufactured.

A **medical researcher** might be interested in rate at which radius of an artery changes with concentration of alcohol in bloodstream.

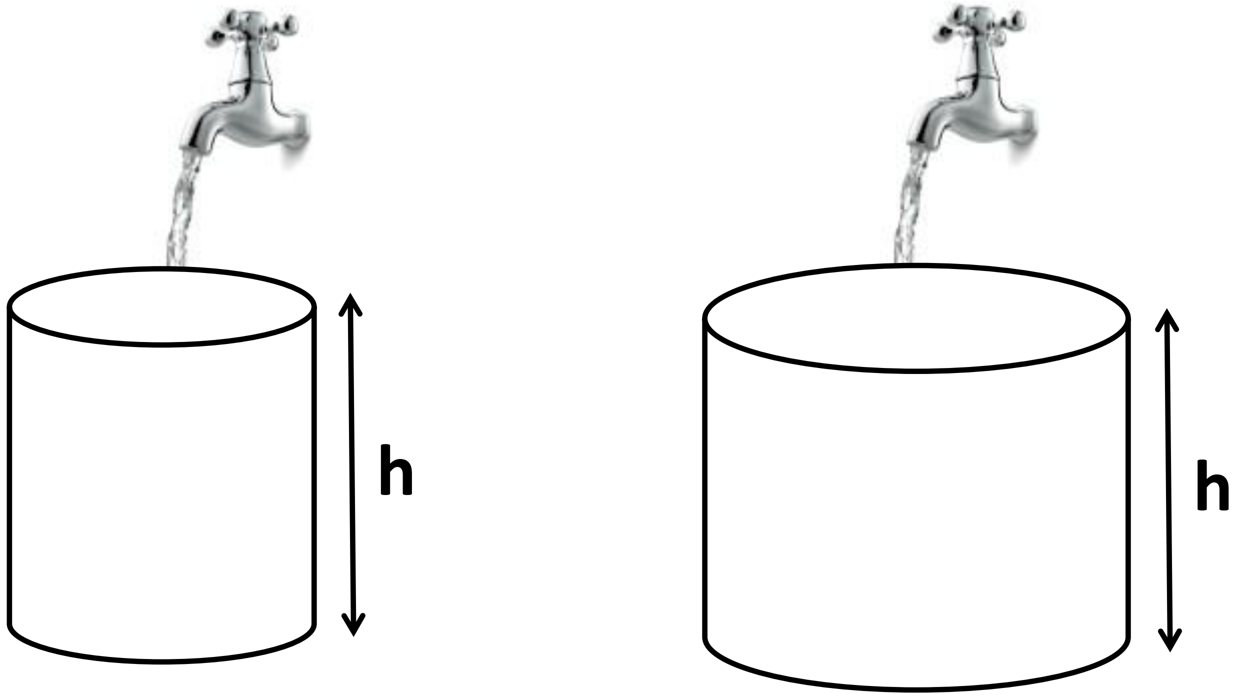


Graphs of Real Life Situations



Discuss how the depth of water changes over time as Archie takes his bath. Create a word bank of the terms which arise during your discussion.

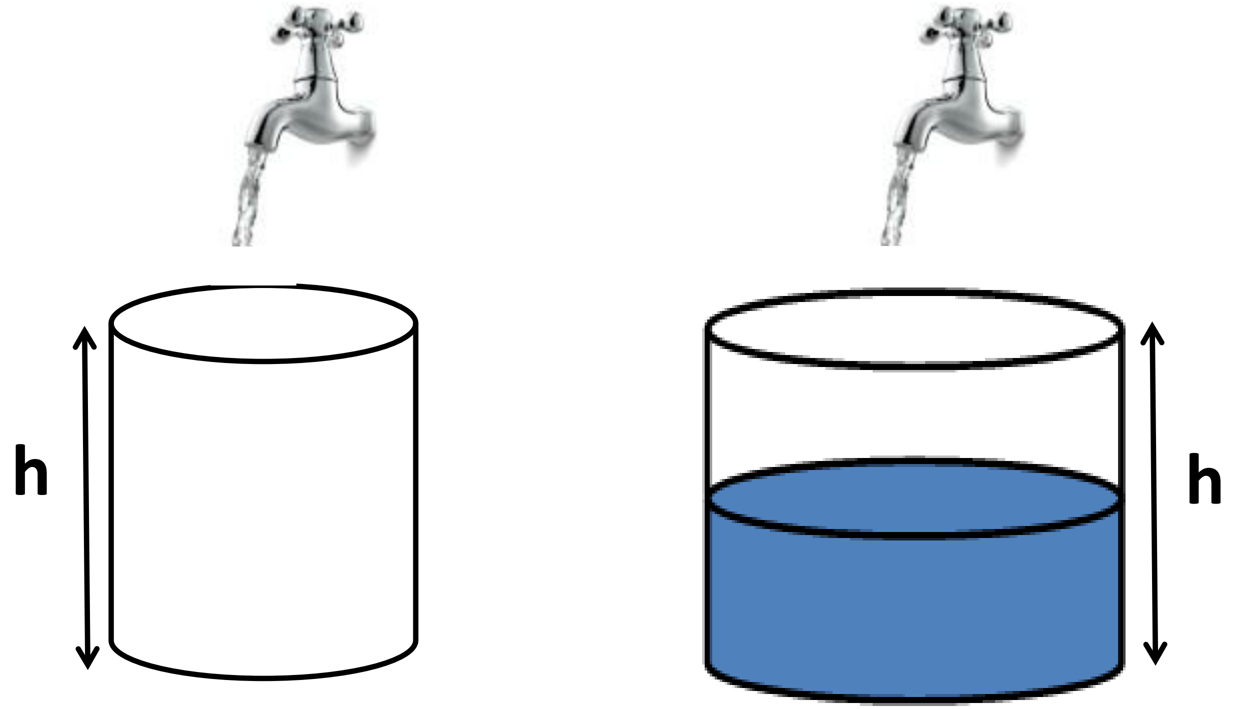
Using the same diagram, draw a rough sketch of how the height of water in these containers changes with respect to time as they are being filled up. They both are of the same height and are being filled at the same rate.



Container A ***Container B***

What about if container B has some water in it already?

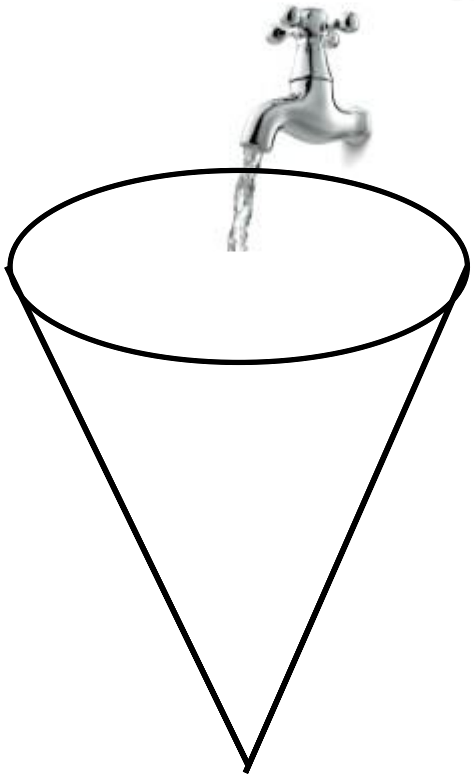
What would your graph look like now?



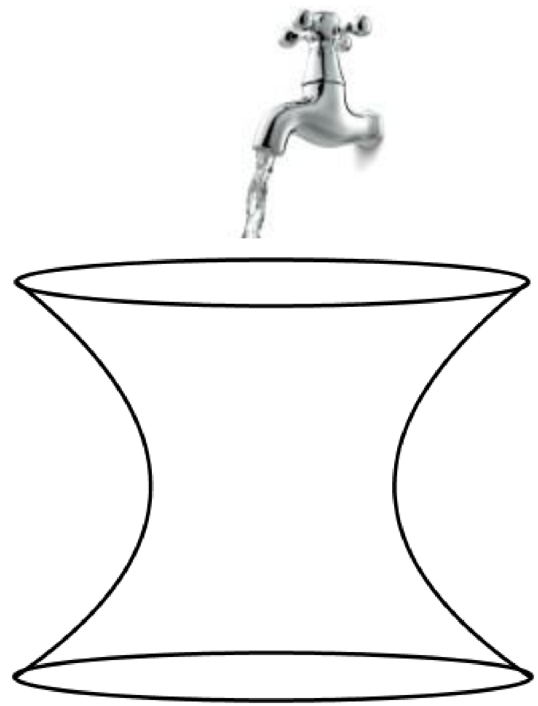
Container A   Container B

Now, sketch <u>separate</u> graphs for these two containers.

How are they different to what you have drawn already?

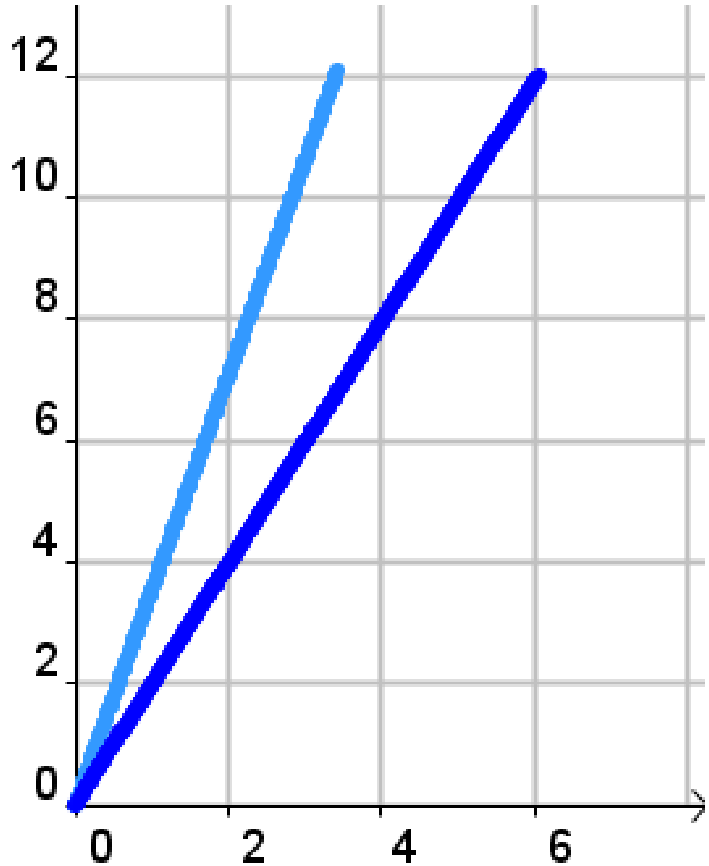


Container C

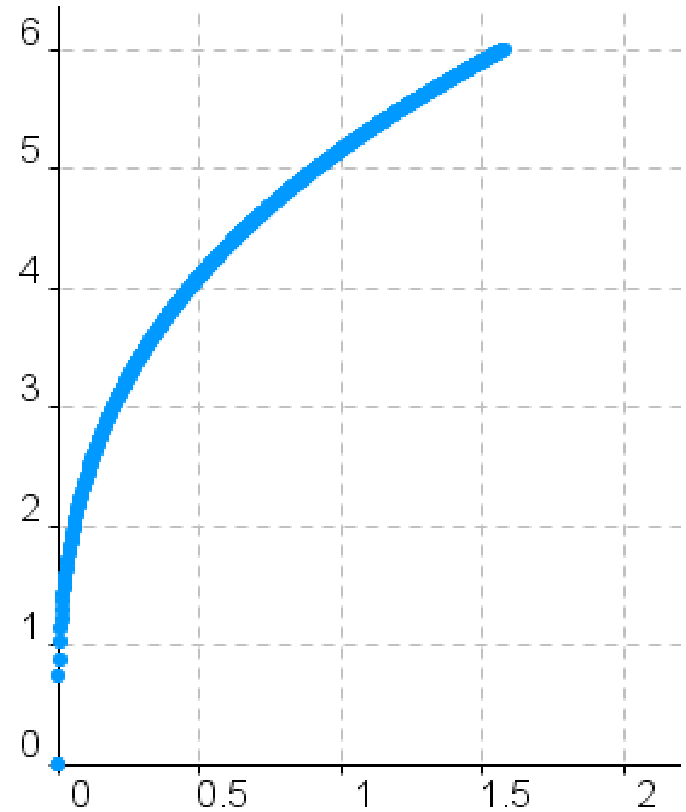


Container D

What's the difference?

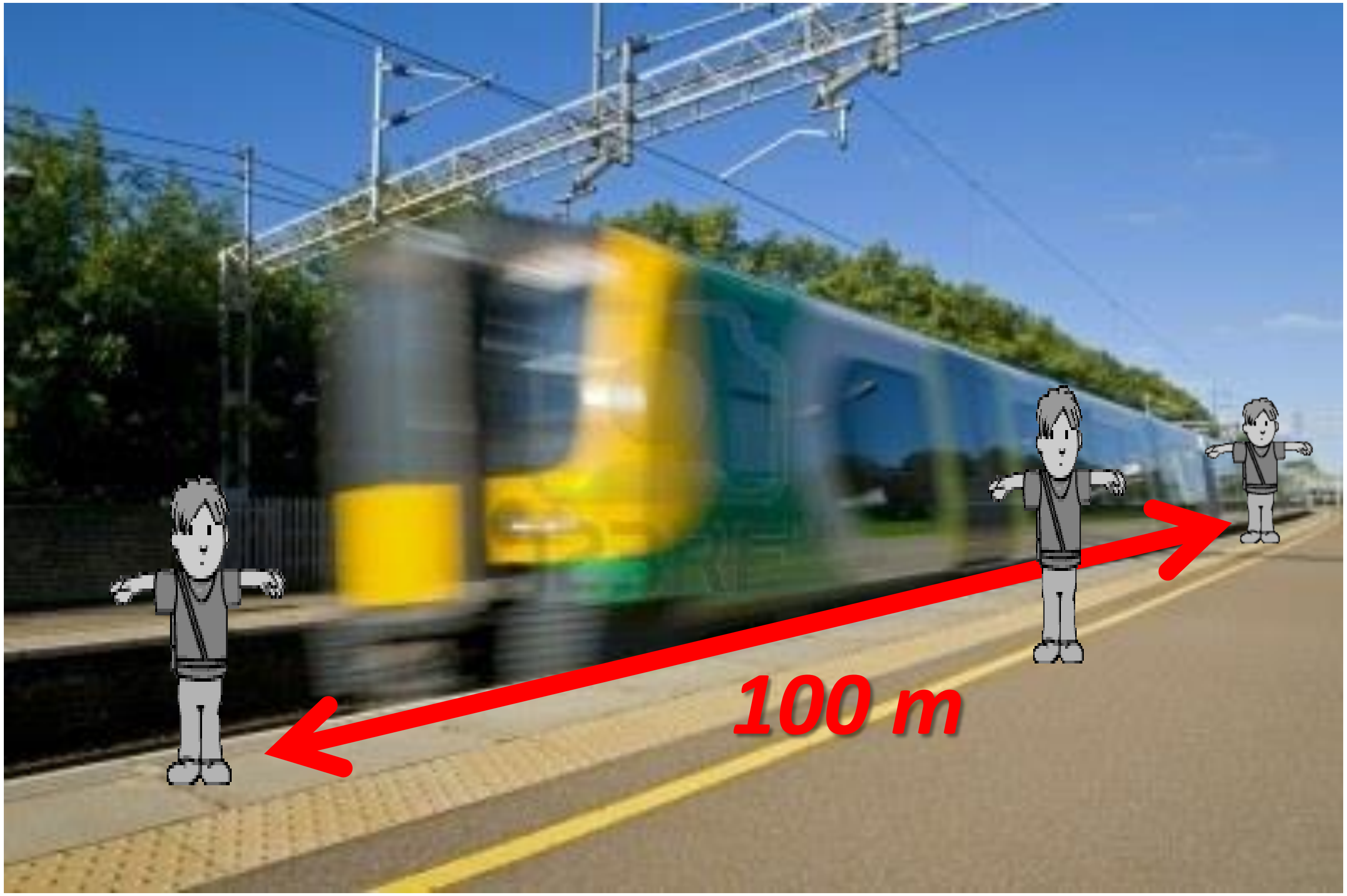


Cylinders



Cone

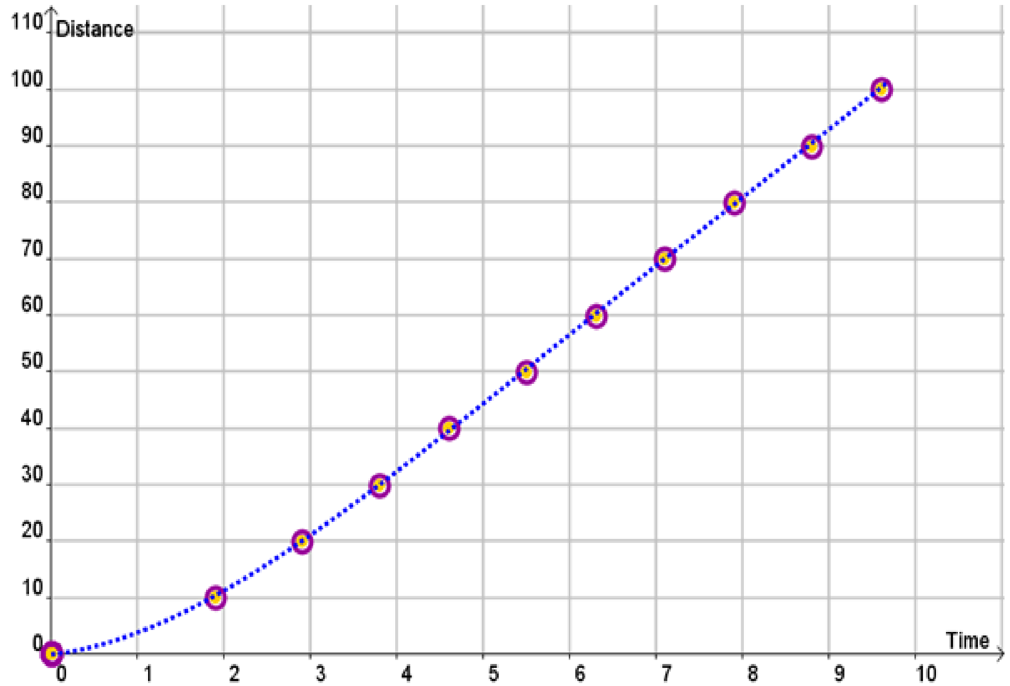
Constant Speed in the Real World



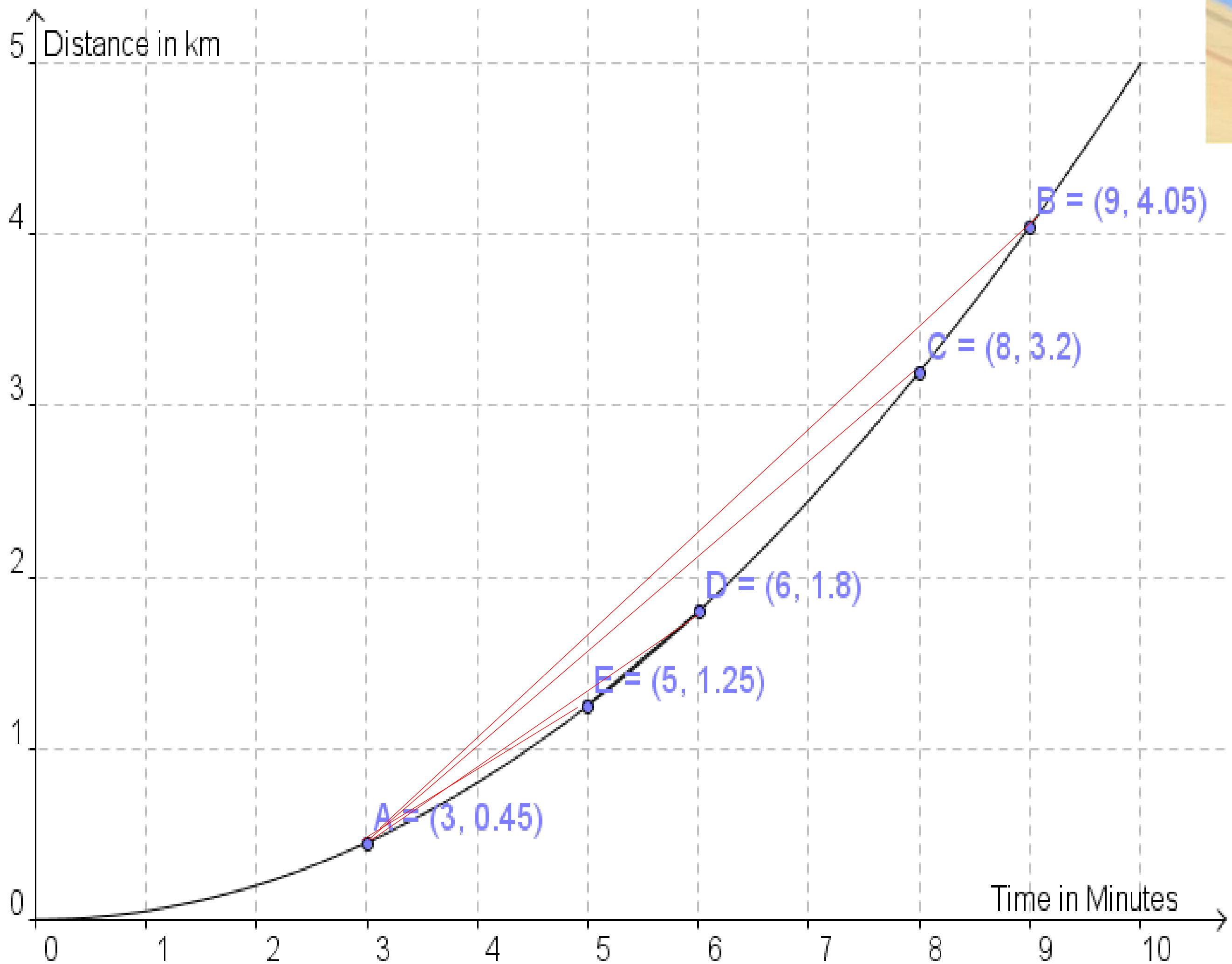
The World's Fastest Man

Student Activity

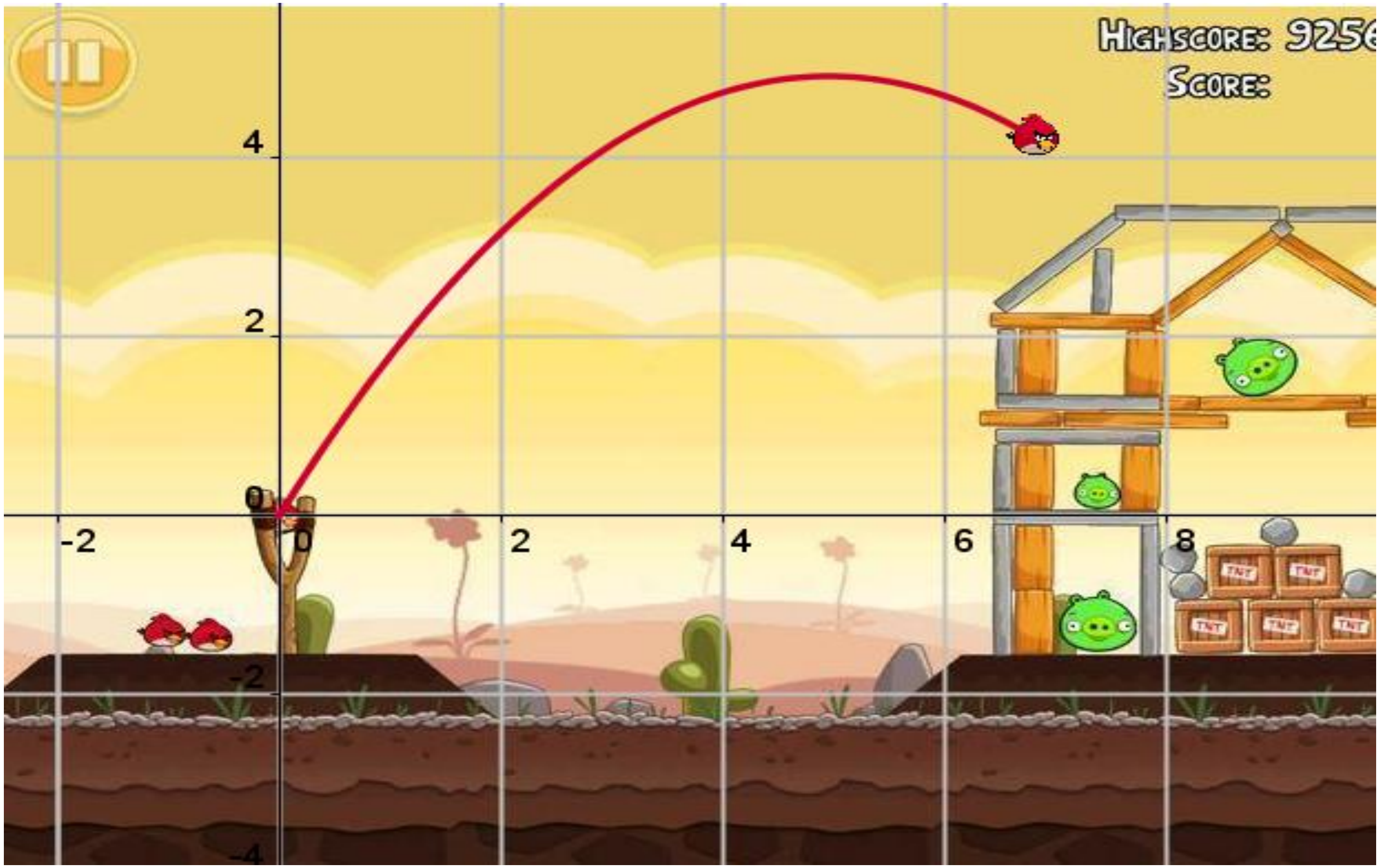
Distance (m)	10	20	30	40	50	60	70	80	90	100
Time (s)	1.89	2.88	3.78	4.64	5.47	6.29	7.10	7.92	8.75	9.58



Student Activity



How are we going to find the instantaneous rate of change?

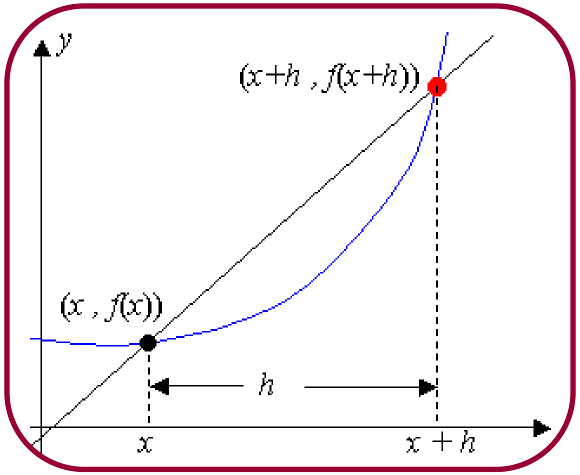


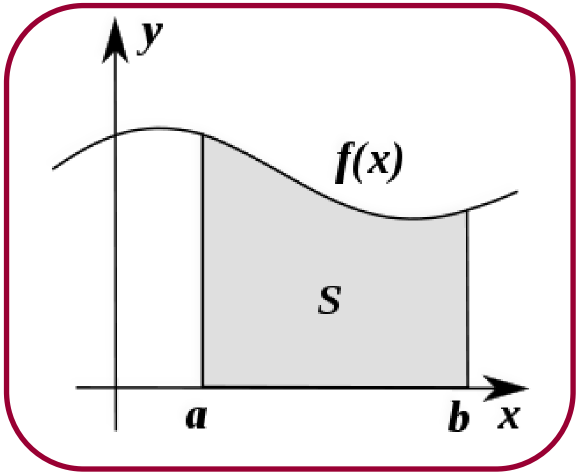
Newton

Leibniz

Calculus

The study of change – how things change and how quickly they change

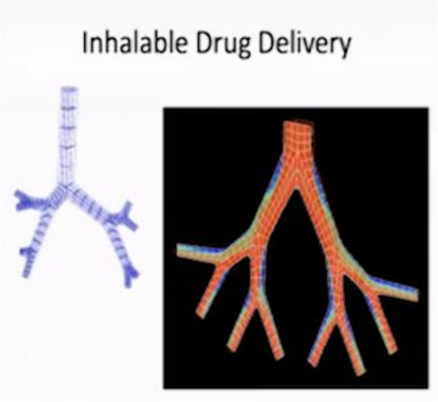


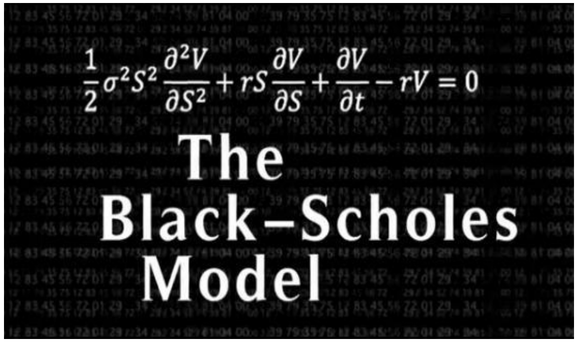



QinetiQ


FUEL
E
F
WALLET


TOMORROW
30
9
8
8
35
9
45
11
11
40

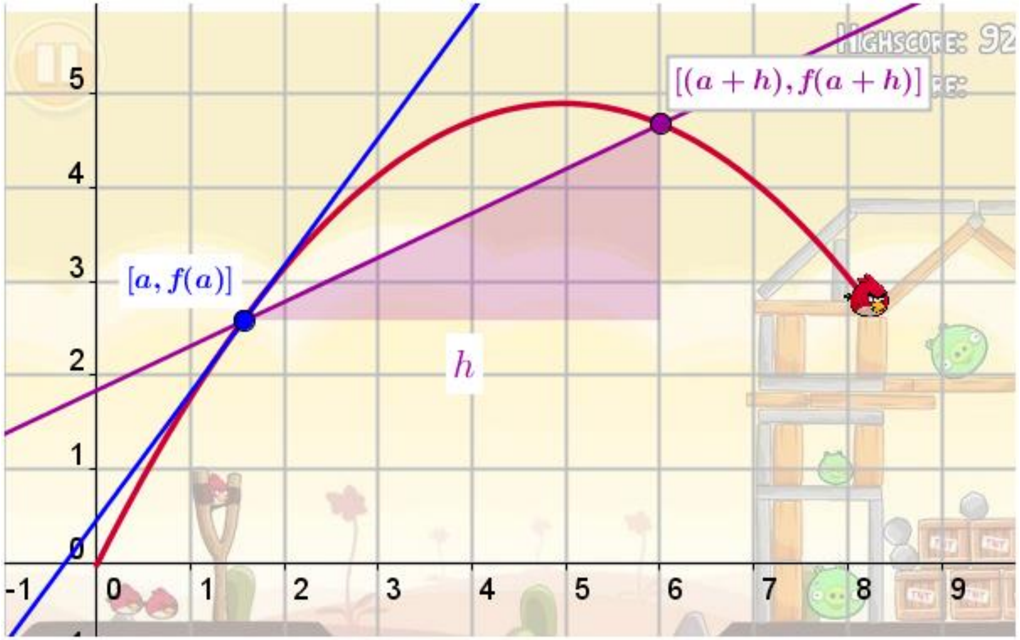

Inhalable Drug Delivery


$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} - rV = 0$
The Black–Scholes Model

Finding the Derivative using First Principles

$$\text{Average Rate of Change} = \frac{f(a+h) - f(a)}{h}$$

$$\text{Instantaneous Rate of Change} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



Deriving the Slope Function Using First Principles

$$f(x) = x^2 - x - 6$$

$$f(x+h) = (x+h)^2 - (x+h) - 6$$

$$f(x+h) = x^2 + 2xh + h^2 - x - h - 6$$

$$f(x+h) - f(x) = x^2 + 2xh + h^2 - x - h - 6 - (x^2 - x - 6)$$

$$f(x+h) - f(x) = 2xh + h^2 - h$$

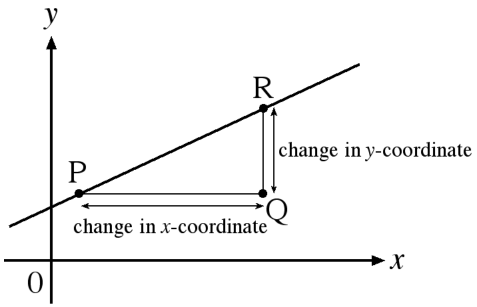
$$\frac{f(x+h) - f(x)}{h} = 2x + h - 1$$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 2x - 1$$

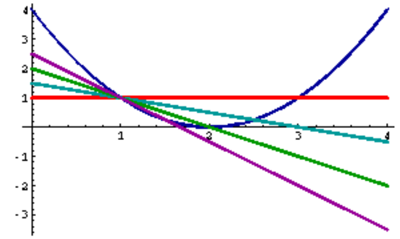
$$f'(x) = 2x - 1$$

The Central Ideas

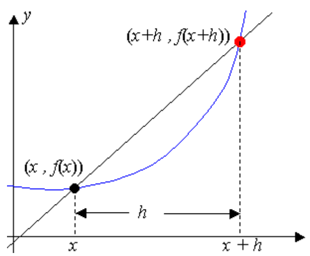
Slope as average rate of change of a function



Successive secants to approximate the Instantaneous rate of change



The Derivative will do this for us most efficiently.



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$