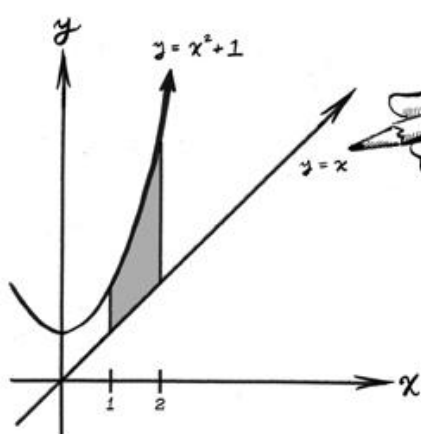
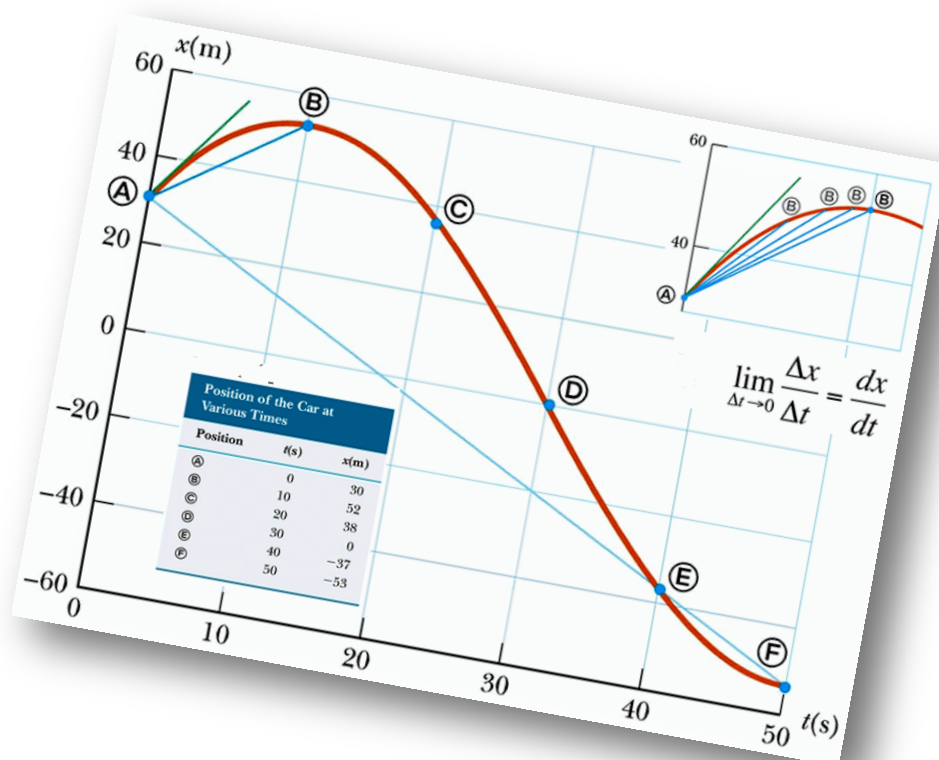


Project Maths Workshop 8



Name: _____

School: _____

Contents

WS08.01	Problem Solving as a “Means” not as an “End”	2
WS08.02	Informal Introduction to Limits and Continuity of Functions	3
	Student Activity: Concept of a Limit	3
	Task 1	4
	Task 2	5
WS08.03	An Introduction to Calculus	7
	Section A: Student Activity 3 – Part 1	7
	Section A: Student Activity 4	8
	Section A: Student Activity 5	9
	Section A: Student Activity 6 – Part 1	11
WS08.04	Exploring the Slope of Tangents and Rates of Change	12
	Describing Curves	24
	Curve Sketching Derivatives	25
WS08.05	GeoGebra Session	28
	Activity A – Functions and Algebra	28
	Activity B – Functions and Calculus	32
	Activity C – Functions and Calculus	35
	Summary of Useful Commands for Teaching Functions and Calculus	40

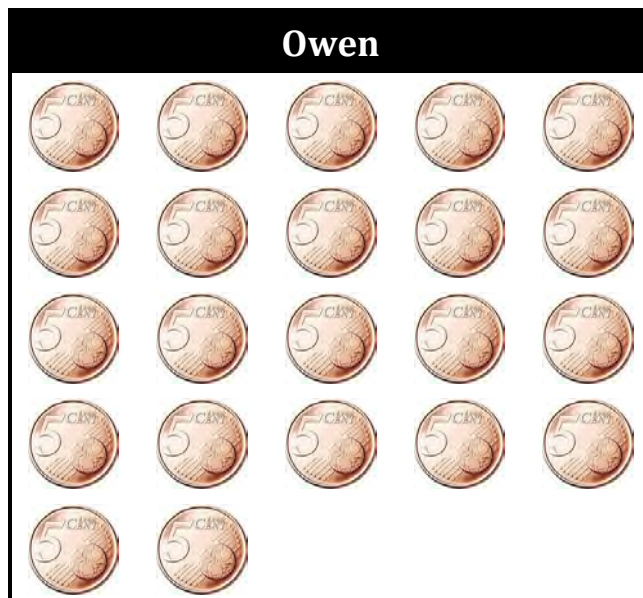
WS08.01 Problem Solving as a “Means” not as an “End”

John has 18 ten-cent coins in his wallet and Owen has 22 five-cent coins in his wallet.

Each day, they decide to take one coin from their wallets and put it into a money box, until one of them has no more coins left in their wallet.



When does Owen have more money than John in his wallet?



WS08.02 Informal Introduction to Limits and Continuity of Functions

Student Activity: Concept of a Limit

Aim: To investigate the trends in the values of different functions as x approaches x_0

Given: $f(x) = x + 3$, $g(x) = \frac{x^2 - 9}{x - 3}$ and $h(x) = \frac{1}{x - 3}$.

1. Fill in the following table:

x	2.9	2.99	2.999	2.9999	3	3.0001	3.001	3.01	3.1
$f(x) = x + 3$									
$g(x) = \frac{x^2 - 9}{x - 3}$									
$h(x) = \frac{1}{x - 3}$									

2. When x approaches 3 (but $x \neq 3$) does the value of each of the following functions approach a fixed value?

If so, find that value.

If not, can you describe what you think is happening?

(i) $f(x) = x + 3$

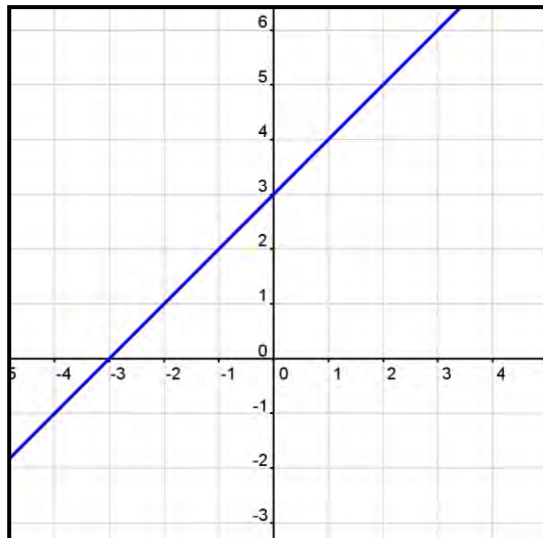
(ii) $g(x) = \frac{x^2 - 9}{x - 3}$

(iii) $h(x) = \frac{1}{x - 3}$

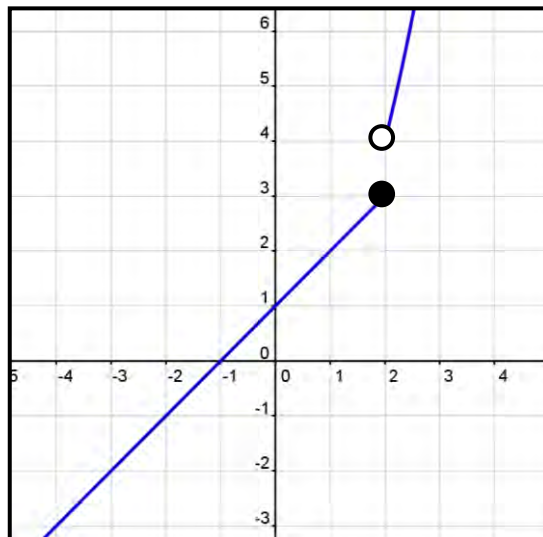
Task 1

Analyse each graph below and determine whether the function it represents is continuous:
If not state why the function is discontinuous.

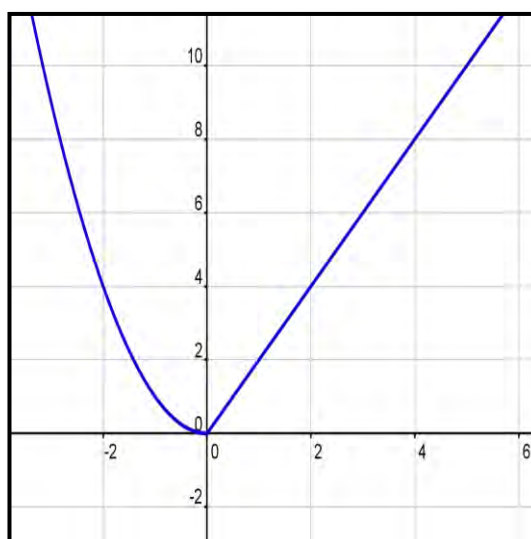
(i) $f: x \mapsto x+3$



(ii) $y = \begin{cases} x+1, & \text{if } x \leq 2 \\ x^2, & \text{if } x > 2 \end{cases}$



(iii) $g(x) = \begin{cases} x^2, & \text{if } x < 0 \\ 2x, & \text{if } x \geq 0 \end{cases}$



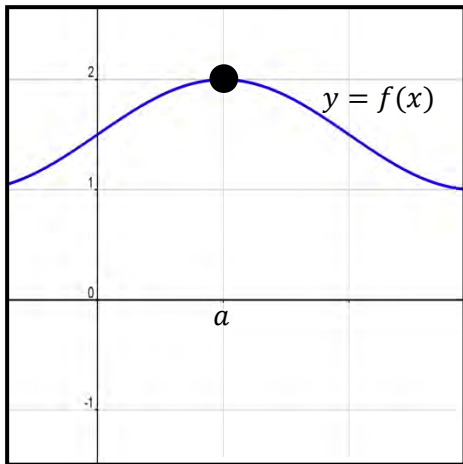
Task 2

According to each of the following graphs of functions $f(x)$:

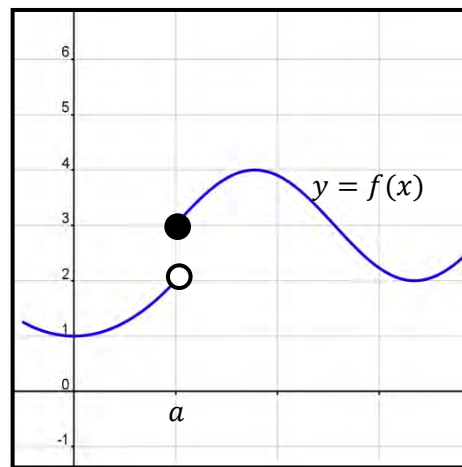
(i) Determine whether $f(x)$ is continuous at $x = a$.

(ii) Find $\lim_{x \rightarrow a} f(x)$.

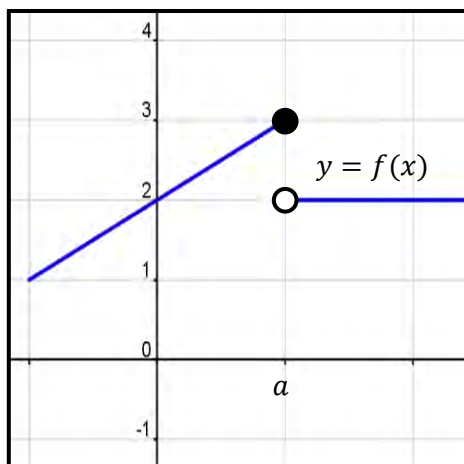
(a)



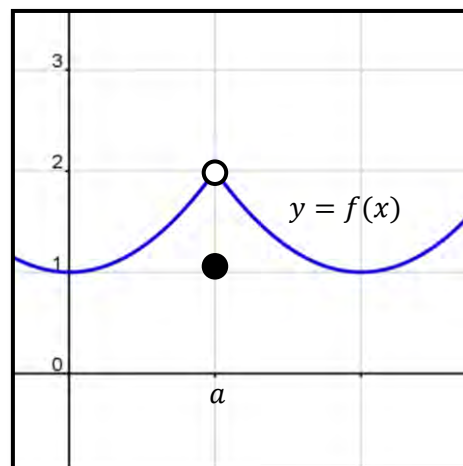
(b)



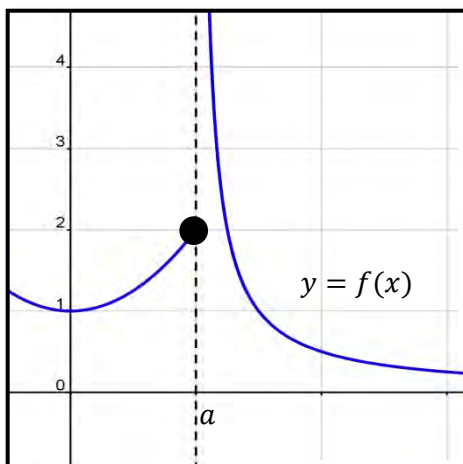
(c)



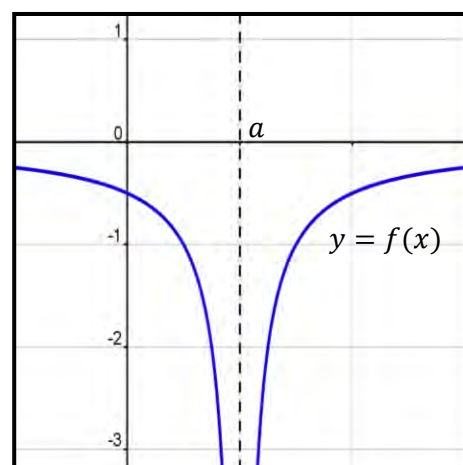
(d)



(e)



(f)



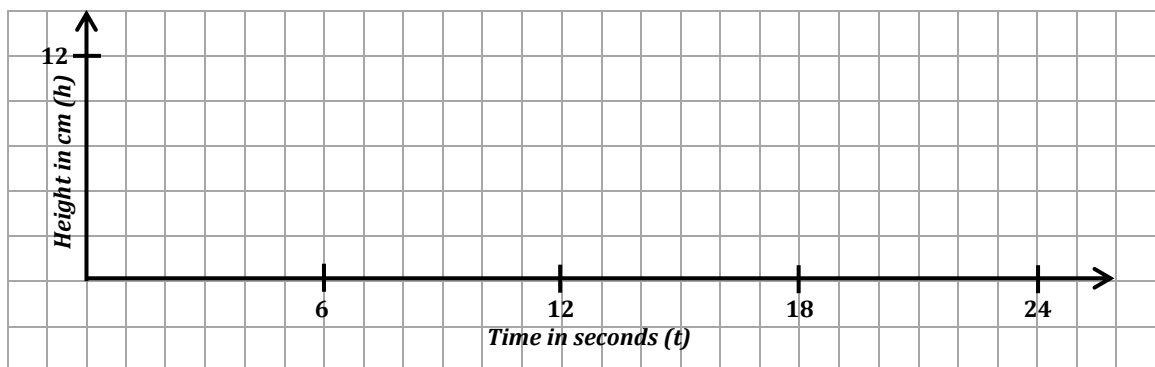
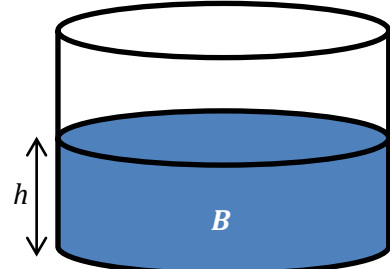
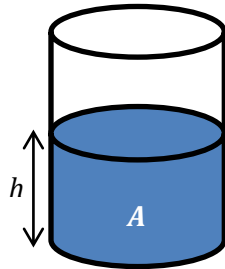
Solutions to **(a)** to **(f)** above:

- | | | | | |
|------------|------------|------------------------|-------------|----------------------|
| (a) | (i) | Yes continuous at a | (ii) | Limit is 2 |
| (b) | (i) | No not continuous at a | (ii) | Limit does not exist |
| (c) | (i) | No not continuous at a | (ii) | Limit does not exist |
| (d) | (i) | No not continuous at a | (ii) | Limit is 2 |
| (e) | (i) | No not continuous at a | (ii) | Limit does not exist |
| (f) | (i) | No not continuous at a | (ii) | Limit does not exist |

WS08.03 An Introduction to Calculus

Section A: Student Activity 3 – Part 1

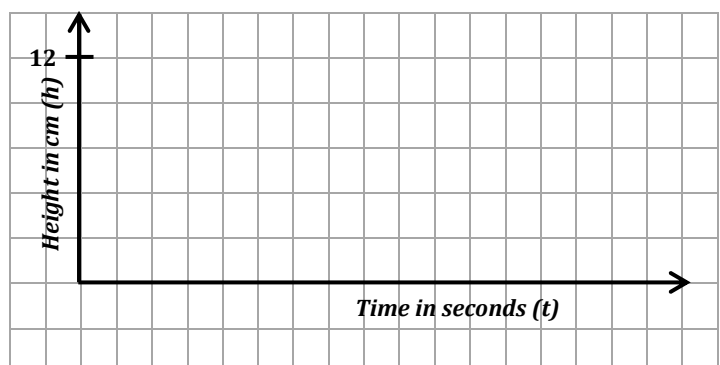
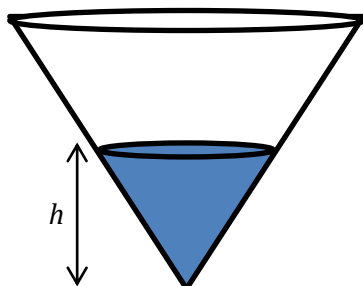
1. (i) Two cylindrical containers, *A* and *B* are being filled with water. The volume of water increases at the same rate in both and the height of both containers is 12 cm. Sketch a graph to show the rate at which the height of the water level changes with time for both containers. Put both containers on one graph. Container *A* is full after 6 seconds and container *B* is full after 24 seconds.



- (ii) Why does it take container *B* longer to fill?

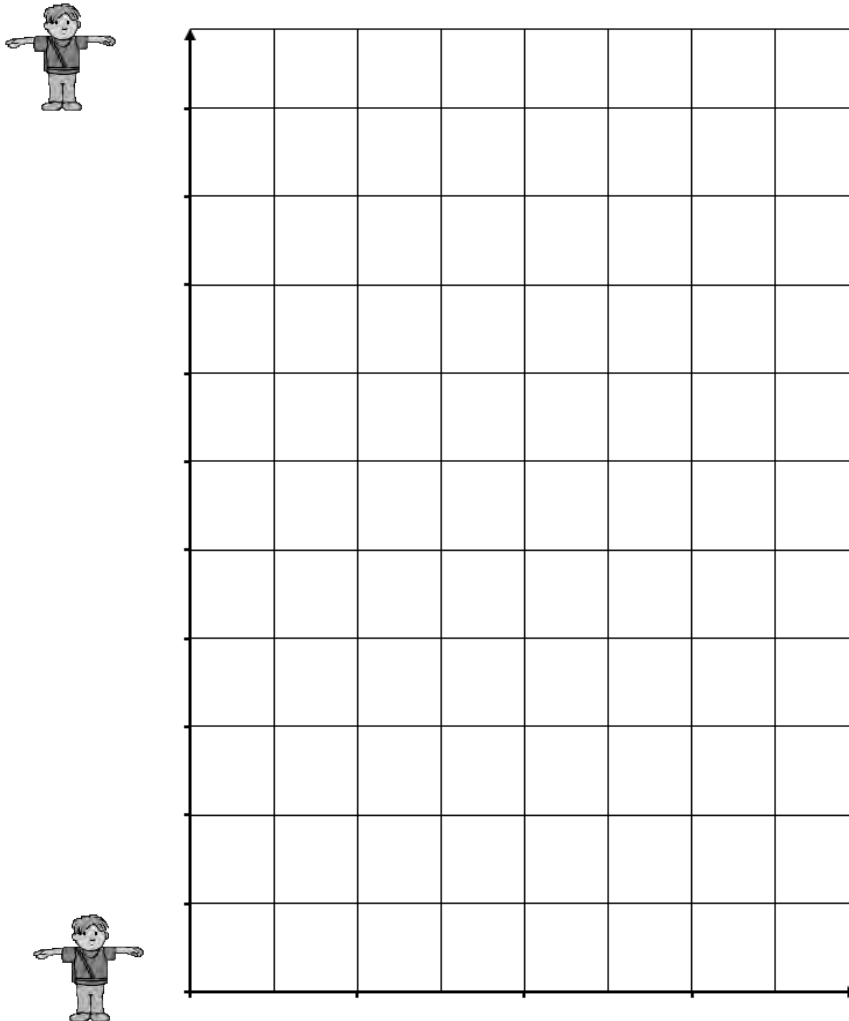
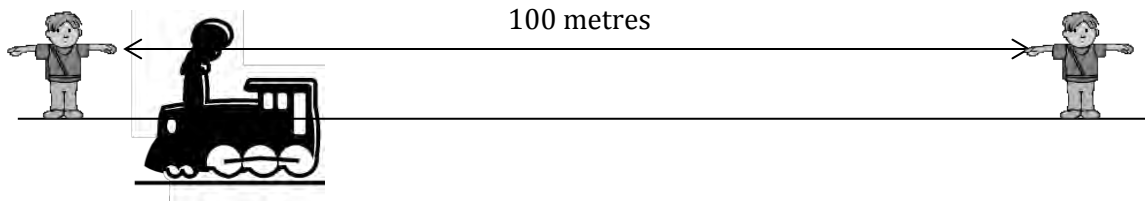
2. (i) Water flows into a vessel in the shape of an inverted cone as shown below. The volume of water increases at the same rate as for the two cylinders above. The vessel has the same height and radius as container *B*. How long will it take to fill the vessel?

- (ii) As water is poured into the vessel, sketch a rough graph to show how the height of the water level changes with time.



Section A: Student Activity 4

Some Transition Year students decide to carry out an experiment on constant speed. They have a class discussion on where they might see a model for constant speed. They decide that if they go to a train station and choose a train that is not scheduled to stop there, that the train will most likely pass them at a constant speed. Two students from the class arrange to stand 100 meters apart at either end of the platform and time the train between these two positions.



2. At what speed does the train pass the two students in km/hour?

3. If the teacher was standing half way between the students during the experiment to supervise, at what speed do you think the train passed the teacher? Give a reason for your answer.

Section A: Student Activity 5

In the 2009 World Championships in Berlin, Usain Bolt set the World Record for the Men's 100 m sprint, running it in 9.58 seconds.

Below is a table of Usain Bolt's split times every 10 metres during the race.

Distance (m)	10	20	30	40	50	60	70	80	90	100
Time (s)	1.89	2.88	3.78	4.64	5.47	6.29	7.10	7.92	8.75	9.58

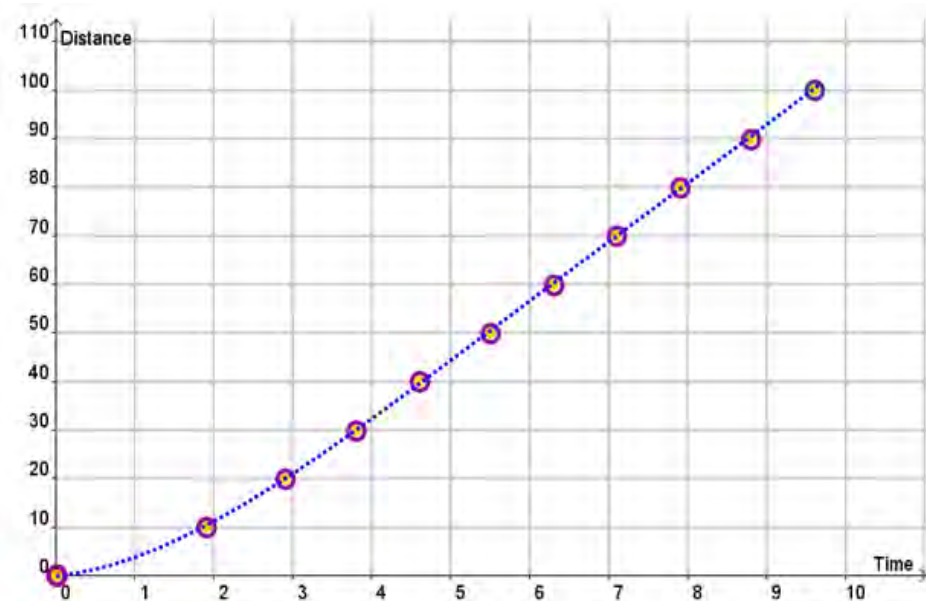


- How fast do you think Usain Bolt ran during the race? Give your answer correct to 2 decimal places in m/sec.

- Do you think he ran at this speed throughout the whole race? Give *two* reasons for your answer.

- What do you think your answer for Question 1 represents?

4. (i) Using a ruler, join the points $(0, 0)$ and $(9.58, 100)$ on the graph below.



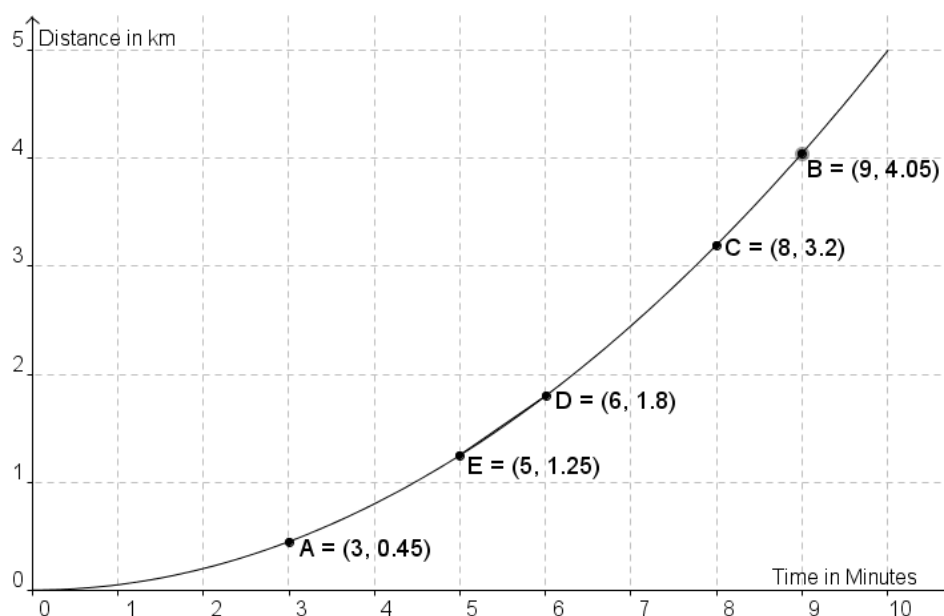
- (ii) Find the slope of this line.

- (iii) The line that joins $(0,0)$ to $(9.58,100)$ has a special name.
It is called a *secant line* to the above curve.
What observation can you make about the slope of this secant line?

5. How do you think we could calculate Usain's speed at *precisely* 1 second into the race?

Section A: Student Activity 6 – Part 1

Below is a distance-time graph of the first ten minutes of a warm up cycle by Olympic Gold medallist Victoria Pendleton.



1. Over these 10 minutes, what is Victoria Pendleton's average speed in km/min?
2. The coach wants to know what her speed is at exactly 3 minutes during this warm up. To help answer this question do the following:
 - (i) Using your ruler, draw in the secants $[AB]$, $[AC]$, $[AD]$, $[AE]$.
 - (ii) Fill in the following table. Answers correct to 2 decimal places.

Slope of Secant $[AB]$ =	Average speed between A and B =
Slope of Secant $[AC]$ =	Average speed between A and C =
Slope of Secant $[AD]$ =	Average speed between A and D =
Slope of Secant $[AE]$ =	Average speed between A and E =

3. The slope of which secant is the nearest estimate to Victoria's speed after exactly 3 minutes?

4. How might you find a better estimate for Victoria's speed after exactly 3 minutes?

WS08.04 Exploring the Slope of Tangents and Rates of Change

1. Slides with the various functions will be shown. Fill in the following table.

Function	Slope at all points on the function	Equation of the slope function
$f(x) = 3$		$f'(x) =$
$g(x) = 1$		$g'(x) =$
$h(x) = -1$		$h'(x) =$
$y = -3$		$\frac{dy}{dx} =$
\vdots	\vdots	\vdots
$f(x) = n$		$f'(x) =$

The derivative of a constant is _____.

2. Slides with the various functions will be shown. Fill in the following table.

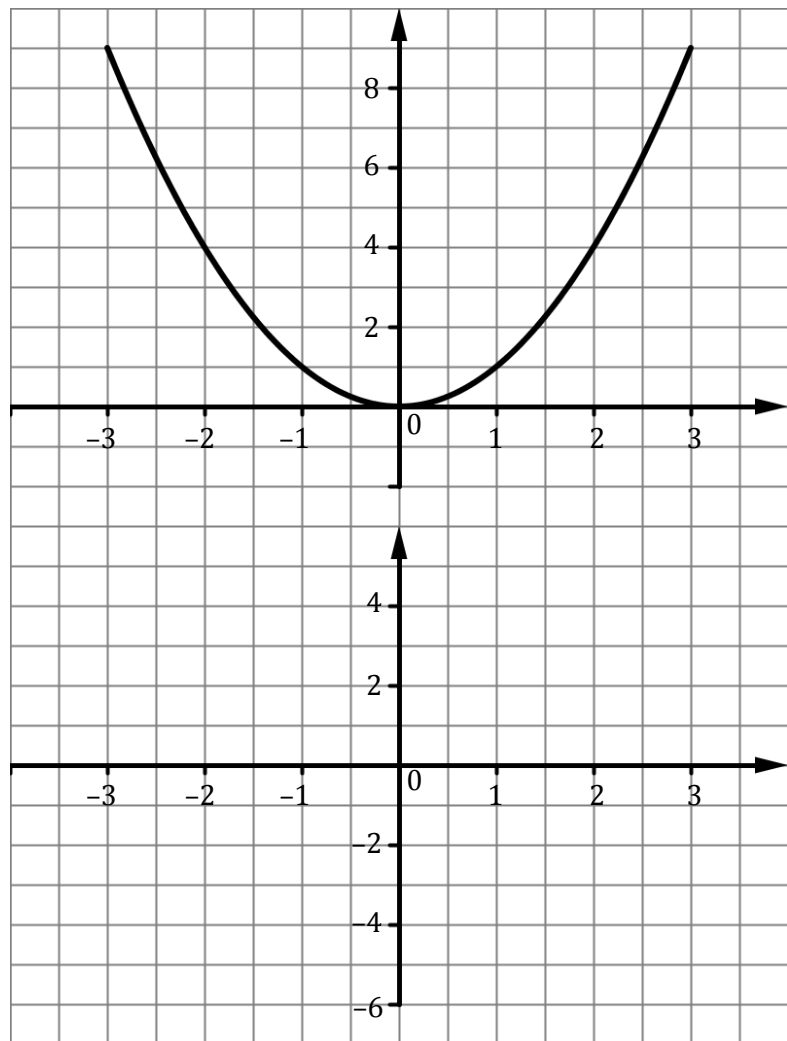
Function	Slope at all points on the function	Equation of the slope function
$f(x) = x$		$f'(x) =$
$g(x) = x + 2$		$g'(x) =$
$h(x) = \frac{3}{2}x$		$h'(x) =$
$y = \frac{3}{2}x + 3$		$\frac{dy}{dx} =$
$p(x) = -x$		
$k(x) = -x + 3$		
\vdots	\vdots	\vdots
$f(x) = nx$ (where n is a constant)		
$f(x) = nx + c$ (where n and c are constants)		

The derivative of $f(x) = nx$ is _____.

The derivative of $f(x) = nx + c$ is _____.

3. Page 14 has a graph of the function $f(x) = x^2$.
- (a) Calculate the slopes of the tangents at the points indicated and enter your answers in the table below.
- Hint:** A ruler will help you read the slopes of the tangents.
- (b) Graph the values from the table (*on the lower graph*) in the space provided.

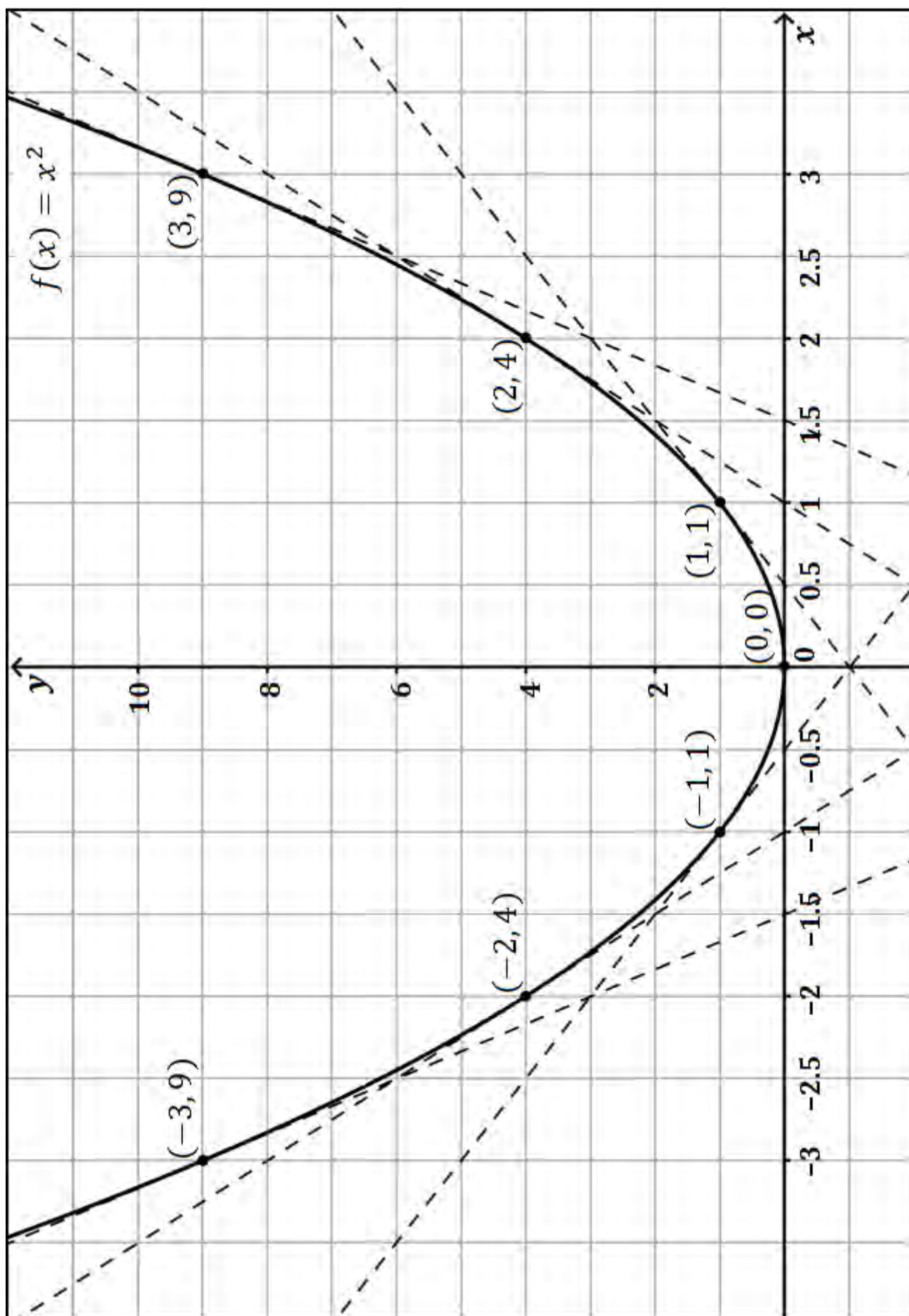
x	Slope of Tangent
-3	
-2	
-1	
0	
1	
2	
3	



- (c) Which of these represents the slope function?
- (i) $f'(x) = x + 2$ (ii) $f'(x) = 2$ (iii) $f'(x) = 2x$

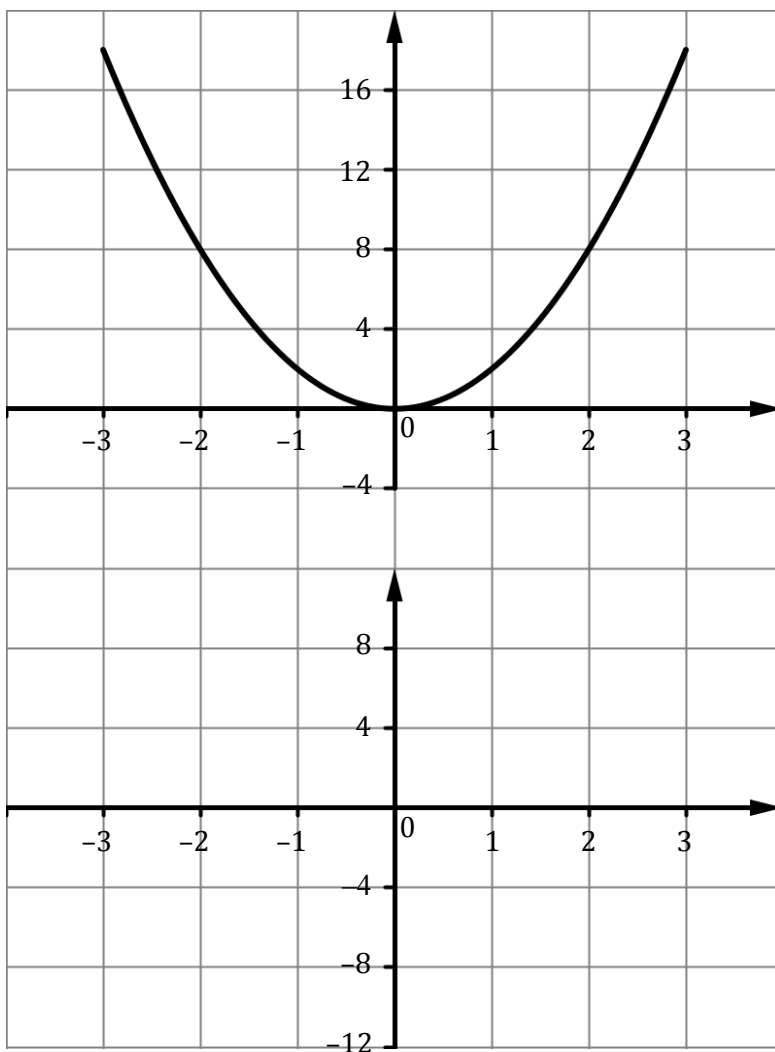
Give a reason for your answer.

Graph of $f(x) = x^2$



4. To complete the exercise below refer to the graph of the function $g(x) = 2x^2$ which is on the board.
- (a) Fill in the slopes of the tangents at the points indicated and enter your answers in the table below.
- (b) Graph the values from the table (*on the lower graph*) in the space provided.

x	Slope of Tangent
-3	
-2	
-1	
0	
1	
2	
3	



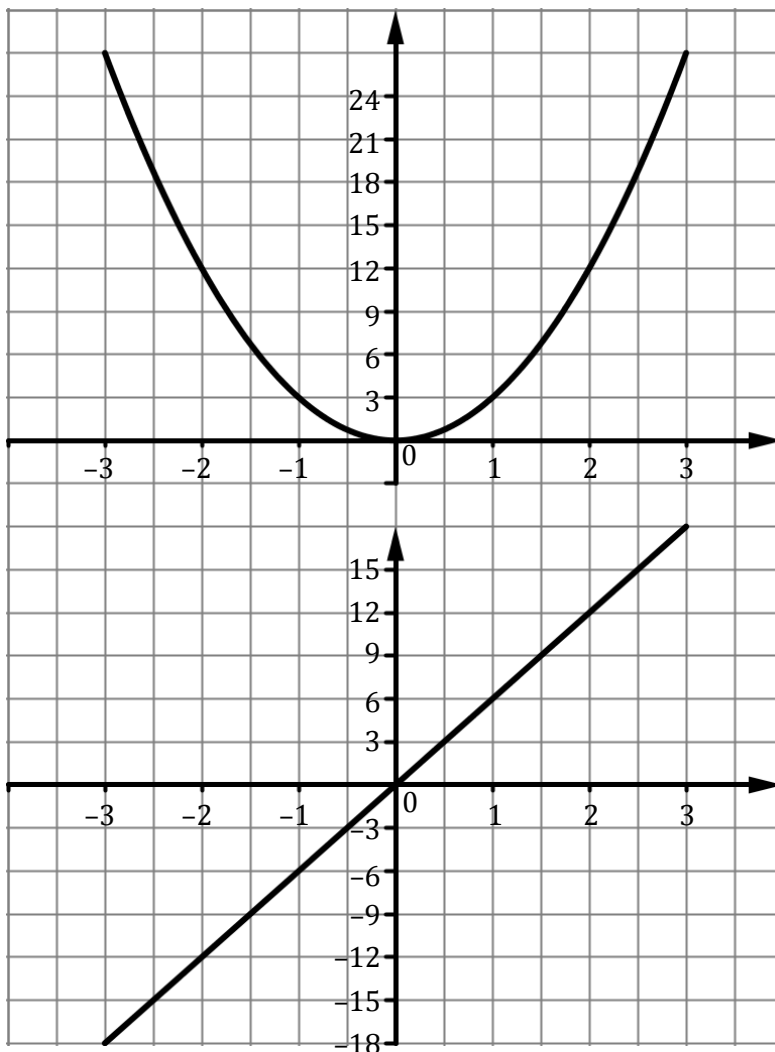
- (c) The equation of the slope function, $g'(x)$, is:

5. To complete the exercise below refer to the graph of the function $h(x) = 3x^2$ which is on the board.

(a) Fill in the slopes of the tangents at the points indicated.

(b) Graph the values from the table (*on the lower graph*) in the space provided.

x	Slope of Tangent
-3	-18
-2	-12
-1	-6
0	0
1	6
2	12
3	18



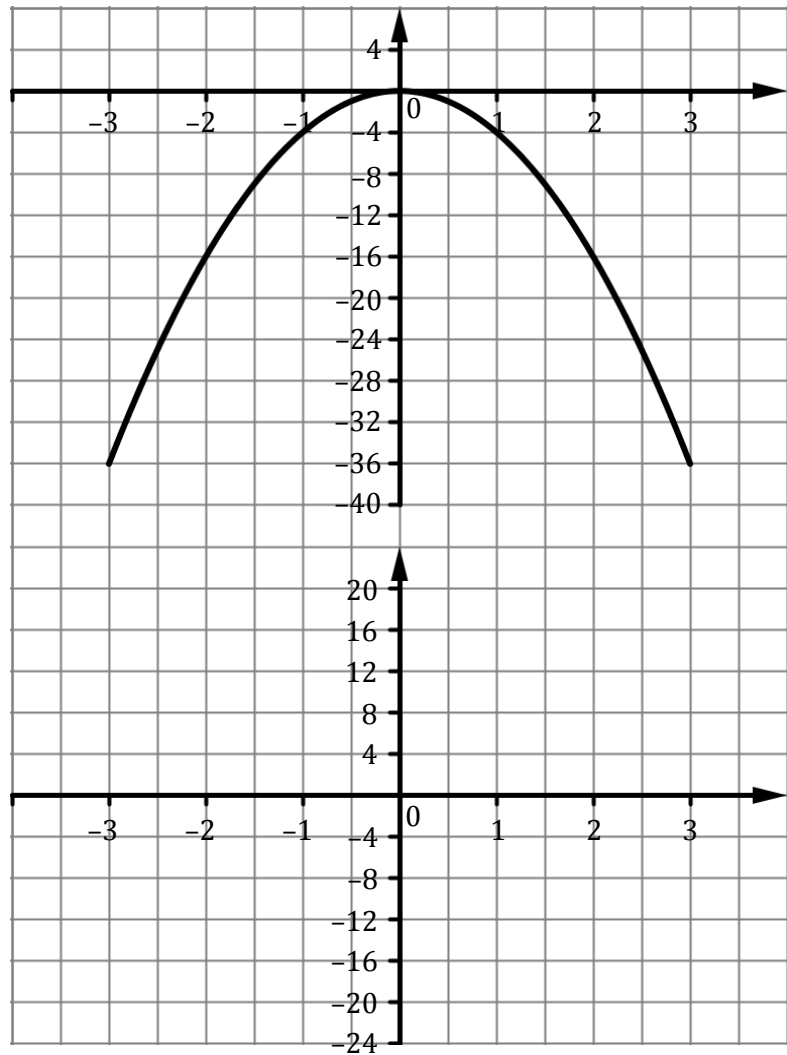
(c) The equation of the slope function, $h'(x)$, is: $h'(x) = 6x$

6. To complete the exercise below refer to the graph of the function $y = -4x^2$ which is on the board.

(a) Fill in the slopes of the tangents at the points indicated.

(b) Graph the values from the table (*on the lower graph*) in the space provided.

x	Slope of Tangent
-3	
-2	
-1	
0	
1	
2	
3	



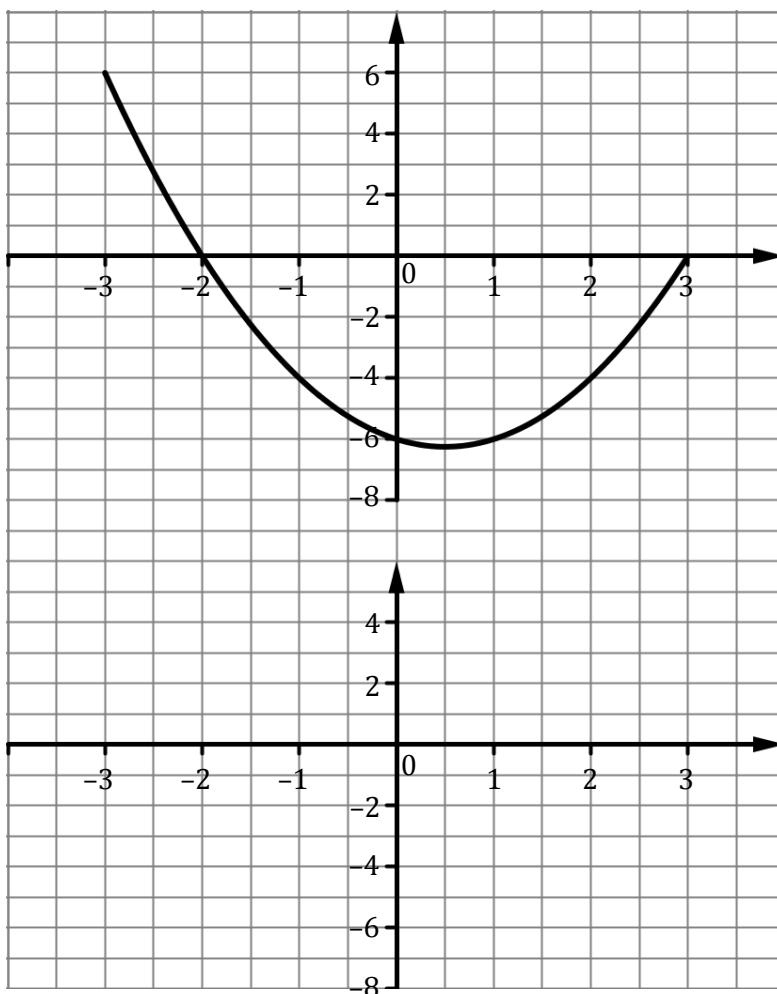
(c) The equation of the slope function, $\frac{dy}{dx}$, is:

7. Refer to questions 3, 4, 5, and 6 in order to fill in the table below.

Function, $f(x)$	Slope of Function or $f'(x)$
$f(x) = x^2$	$f'(x) =$
$g(x) = 2x^2$	$g'(x) =$
$h(x) = 3x^2$	$h'(x) =$
$y = -4x^2$	$\frac{dy}{dx} =$
$k(x) = 10x^2$	$k'(x) =$
\vdots	\vdots
$f(x) = ax^2$	$f'(x) =$

8. To complete the exercise below refer to the graph of the function $f(x) = x^2 - x - 6$ which is on the board.
- (a) What do you think the equation of the slope function, $f'(x)$, is? _____
- (b) Fill in the slopes of the tangents at the points indicated.
- (c) Graph the values from the table (*on the lower graph*) in the space provided.

x	Slope of Tangent
-3	
-2	
-1	
0	
1	
2	
3	



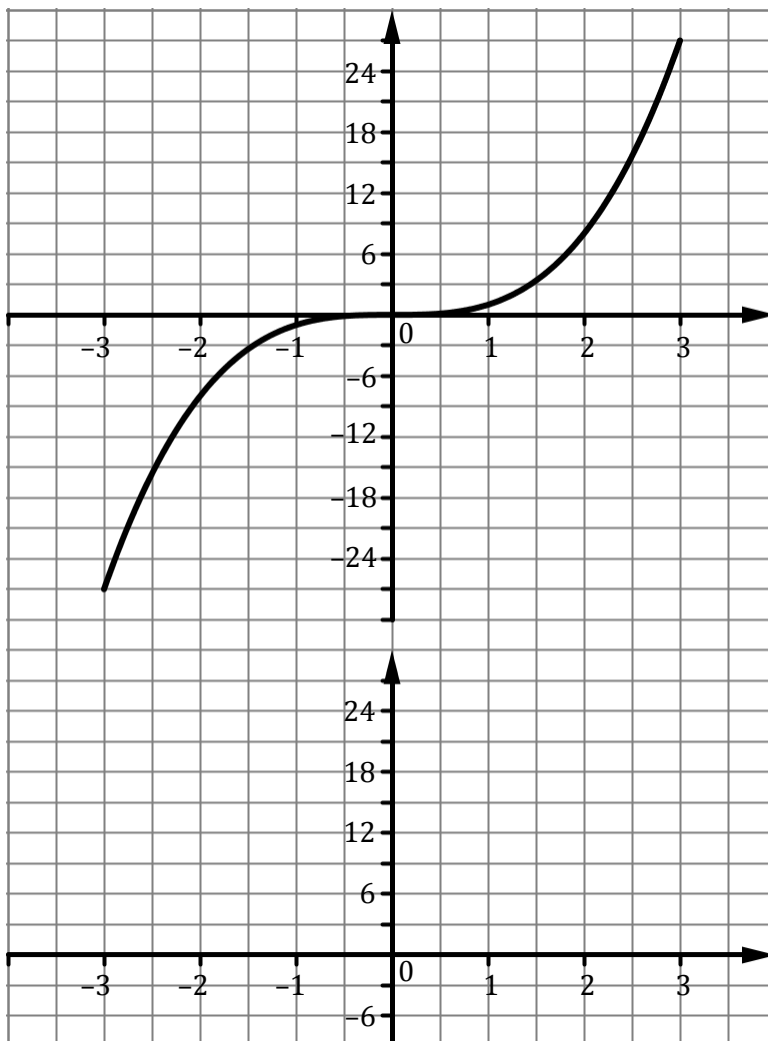
- (d) The equation of the slope function, $f'(x)$, is:
- (e) Compare this to your prediction in part (a).
What conclusion can be drawn from finding the slope function of this last function?
- (f) Patrick wants to know what the slope of $f(x)$ when $x = 1.5$.
There are a few ways of estimating/calculating the slope of $f(x)$ when $x = 1.5$.
- (i) From the graph of $f(x)$ in the GeoGebra file what would be a good estimate of the slope of $f(x)$ when $x = 1.5$?
- (ii) From the list of slopes in the table what would be a good estimate of the slope of $f(x)$ when $x = 1.5$?
- (iii) From the graph of $f'(x)$ what would be a good estimate of the slope of $f(x)$ for $x = 1.5$? (Show your workings on the graph)
- (iv) From the equation $f'(x)$ what is the slope of $f(x)$ when $x = 1.5$?
9. (a) What is the equation of the slope function ($f'(x)$) of $f(x) = x^2 - x$?
- (b) What is the slope of the tangent of the function $f(x) = x^2 - x$ when $x = 3$?

10. To complete the exercise below refer to the graph of the function $f(x) = x^3$ which is on the board.

(a) Fill in the slopes of the tangents at the points indicated.

(b) Graph the values from the table (*on the lower graph*) in the space provided.

x	Slope of Tangent
-3	
-2	
-1	
0	
1	
2	
3	



(c) What is the shape of the slope function? _____

(d) How could you confirm this? _____

The parent function for quadratics is $y = x^2$.

$y = x^2$ contains the points: $(-3, 9)$, $(-2, 4)$, $(-1, 1)$, $(0, 0)$, $(1, 1)$, $(2, 4)$, $(3, 9)$.

Your slope function contains the points: $(-3, \quad)$, $(-2, \quad)$, $(-1, \quad)$, $(0, \quad)$, $(1, \quad)$, $(2, \quad)$, $(3, \quad)$.

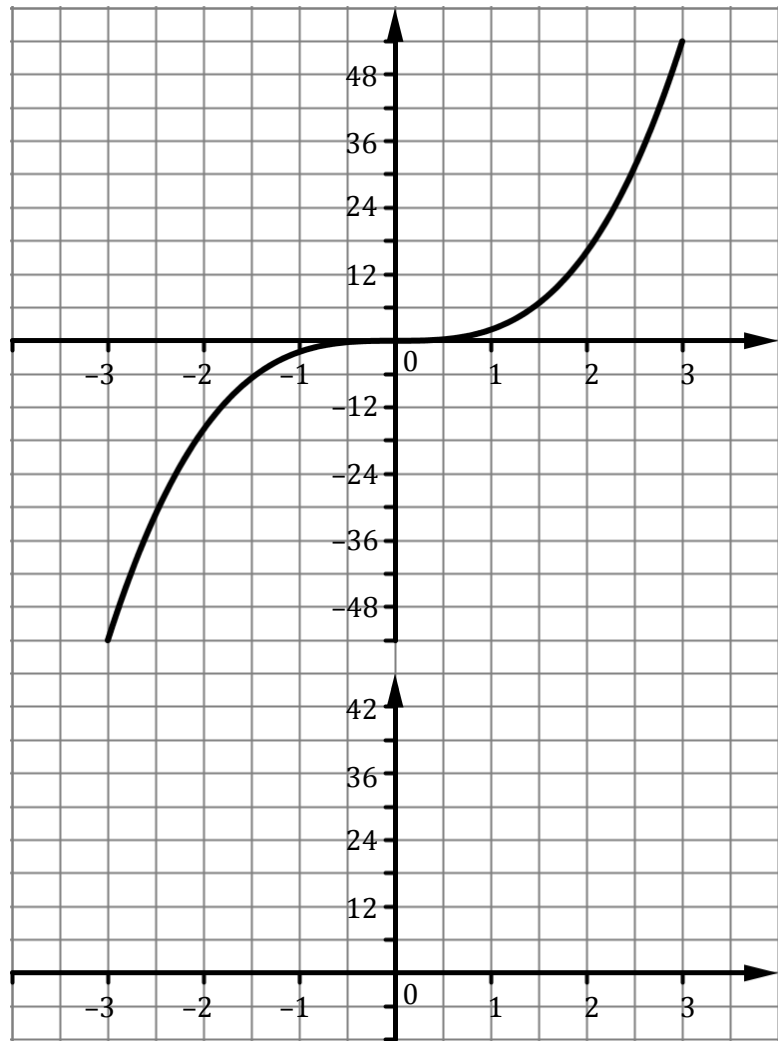
The equation of the slope function, $f'(x)$, is: _____

11. To complete the exercise below refer to the graph of the function $g(x) = 2x^3$ which is on the board.

(a) Fill in the slopes of the tangents at the points indicated.

(b) Graph the values from the table (*on the lower graph*) in the space provided.

x	Slope of Tangent
-3	
-2	
-1	
0	
1	
2	
3	



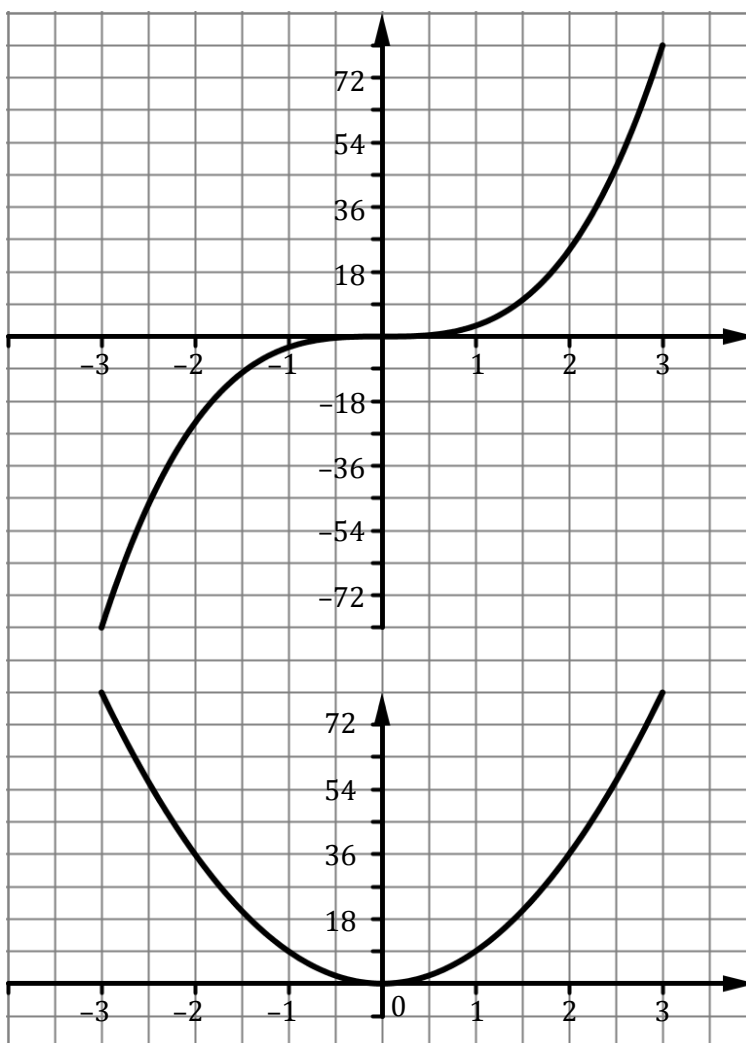
(c) The equation of the slope function, $g'(x)$, is:

12. To complete the exercise below refer to the graph of the function $h(x) = 3x^3$ which is on the board.

(a) Fill in the slopes of the tangents at the points indicated.

(b) Graph the values from the table (*on the lower graph*) in the space provided.

x	Slope of Tangent
-3	81
-2	36
-1	9
0	0
1	9
2	36
3	81



(c) The equation of the slope function, $h'(x)$ is: $9x^2$

- 13.** Refer to questions 10, 11, and 12 in order to fill in the table below.

Function, $f(x)$	Slope Function or $f'(x)$
$f(x) = x^3$	$f'(x) =$
$g(x) = 2x^3$	$g'(x) =$
$h(x) = 3x^3$	$h'(x) =$
\vdots	\vdots
$f(x) = ax^3$	$f'(x) =$

- 14.** What are the slopes of the functions:

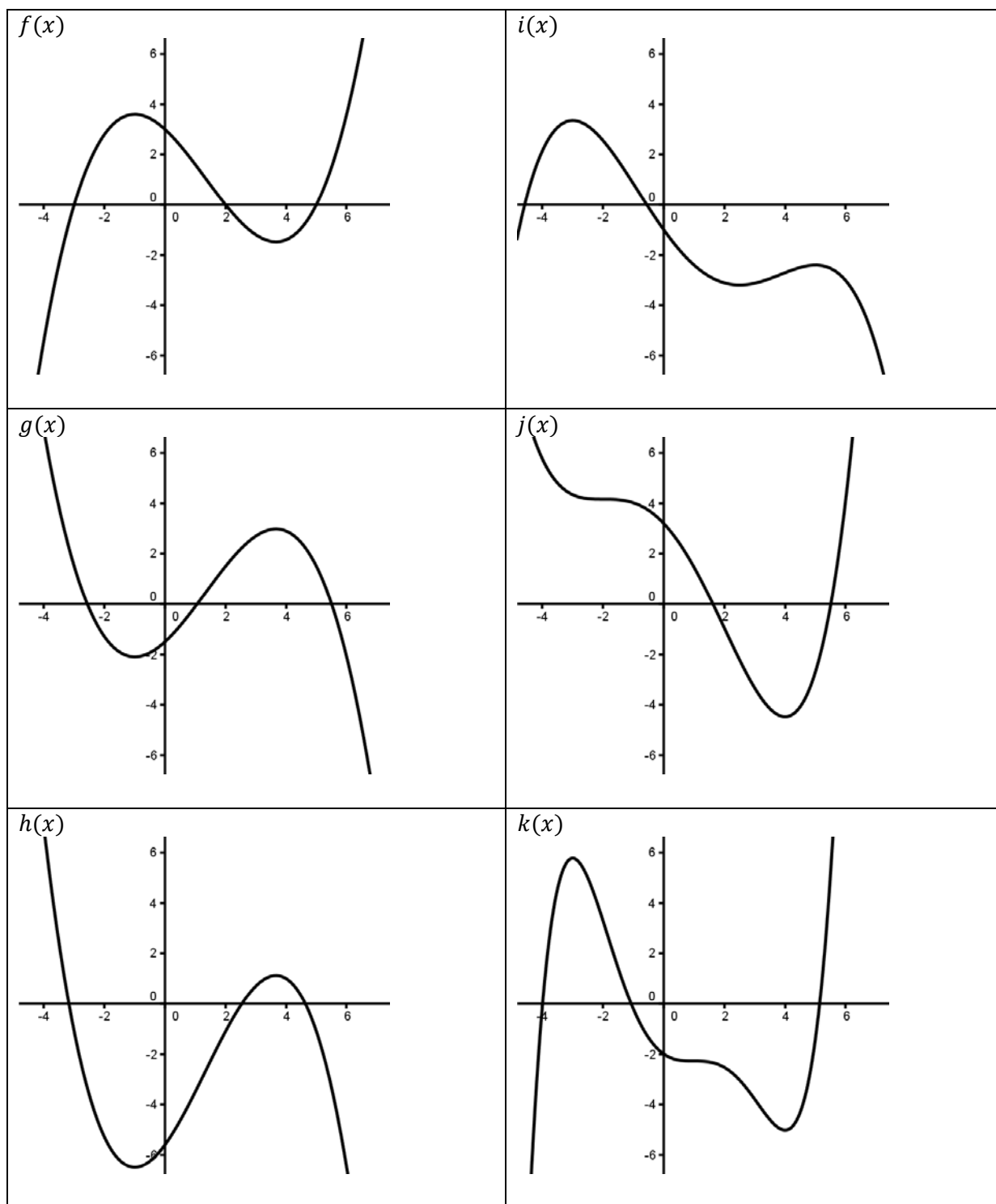
(i) $h(x) = x^8$

(ii) $g(x) = 3x^{10}$

(iii) $f(x) = 5x^2 - 3x - 6?$

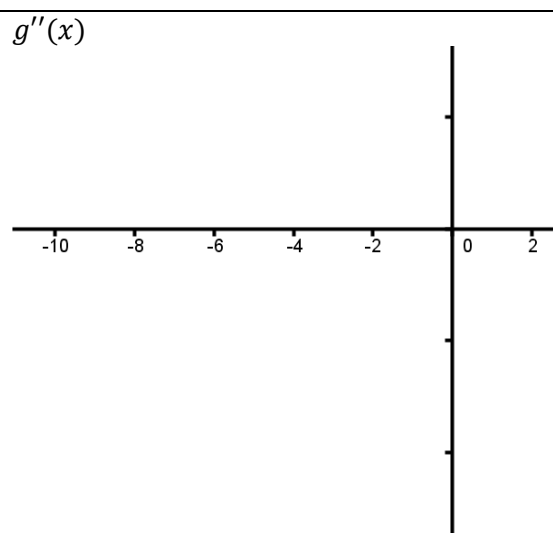
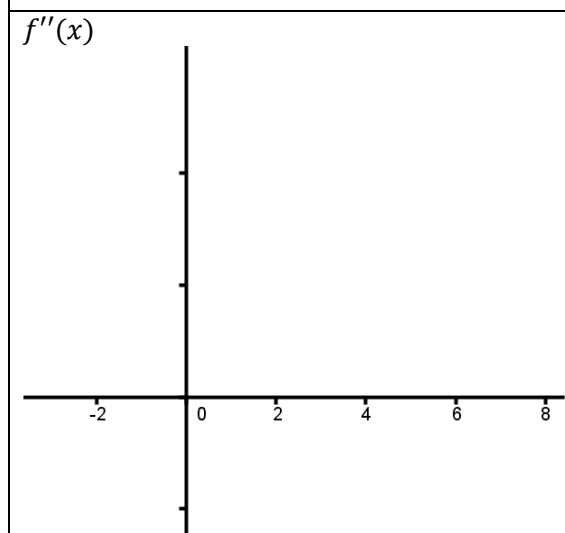
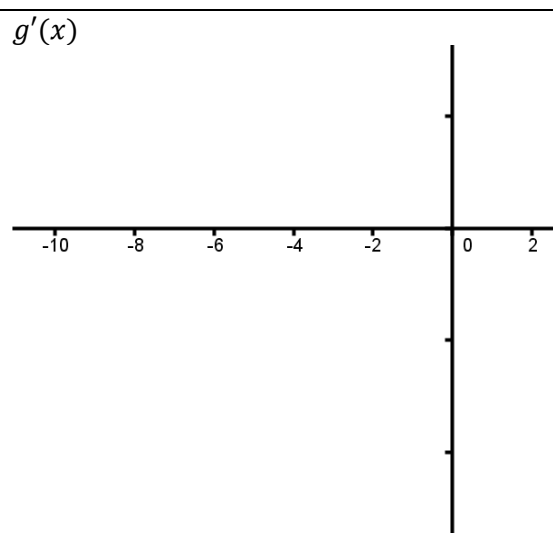
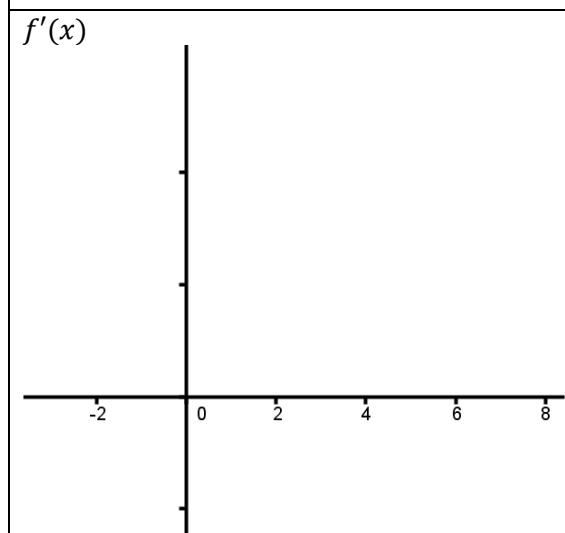
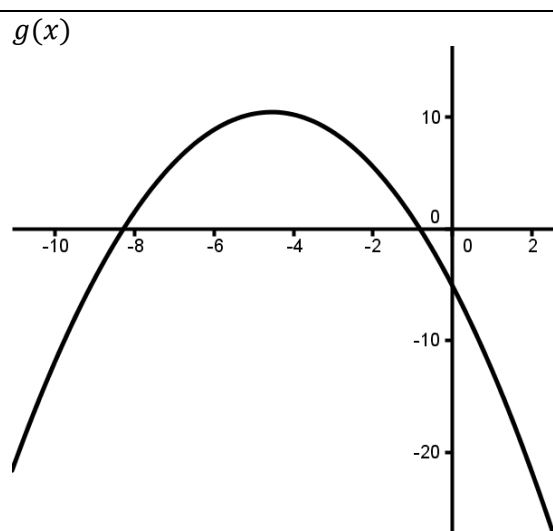
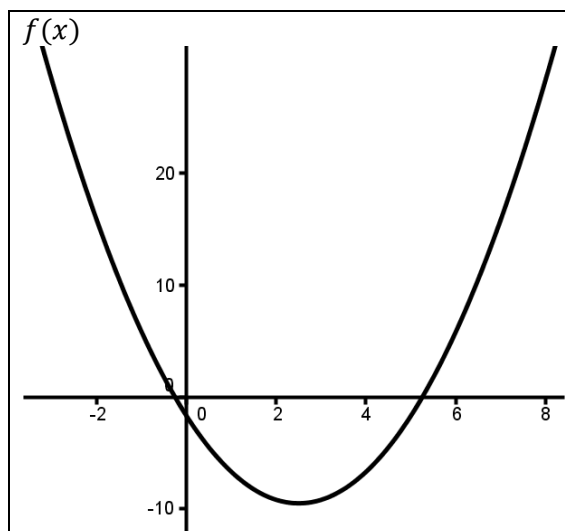
Describing Curves

Pair up. One person describes a function. The other person draws the function on their whiteboard using only the descriptions their partner gives them i.e. they cannot look at this page.

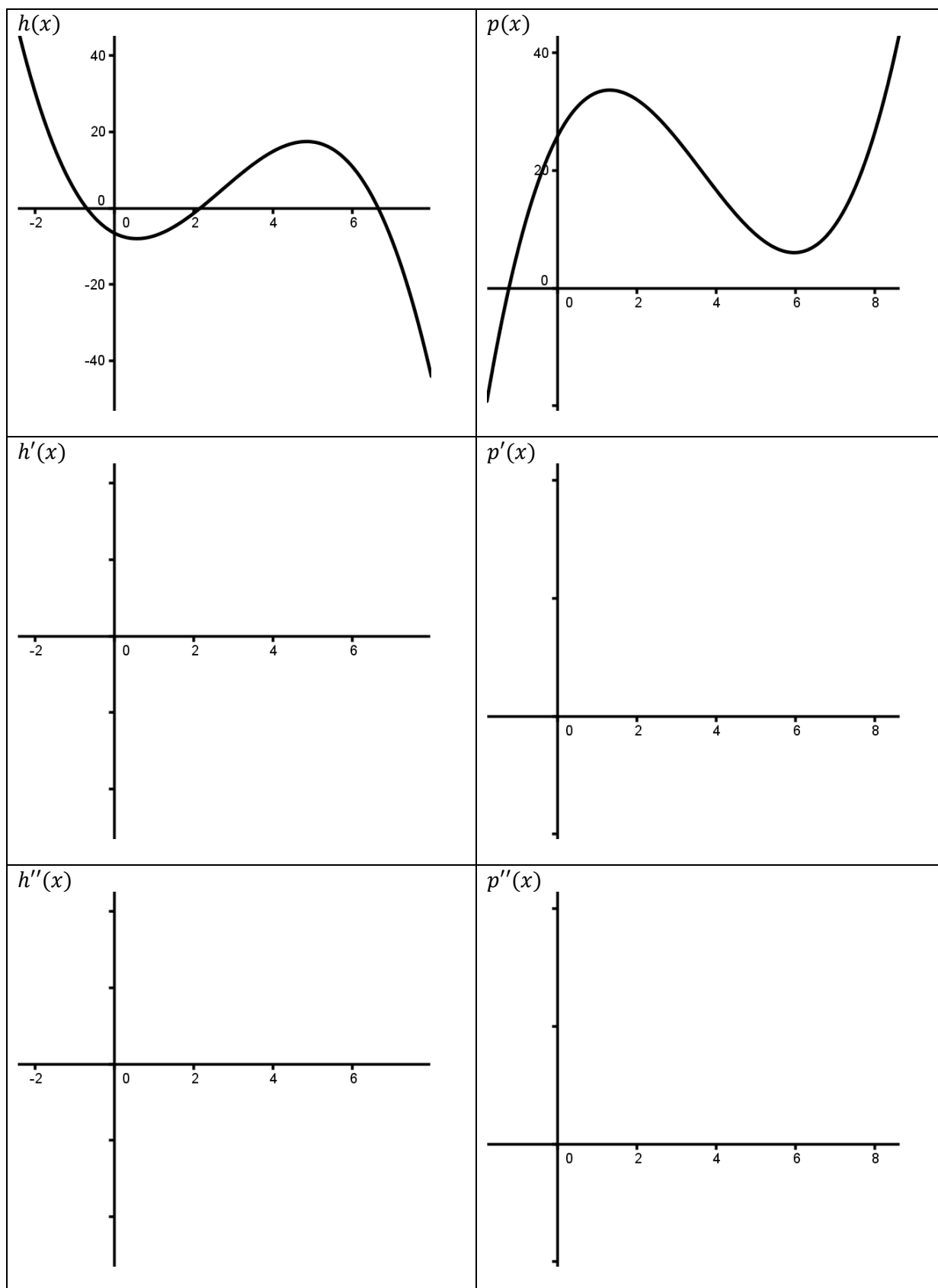


Curve Sketching Derivatives

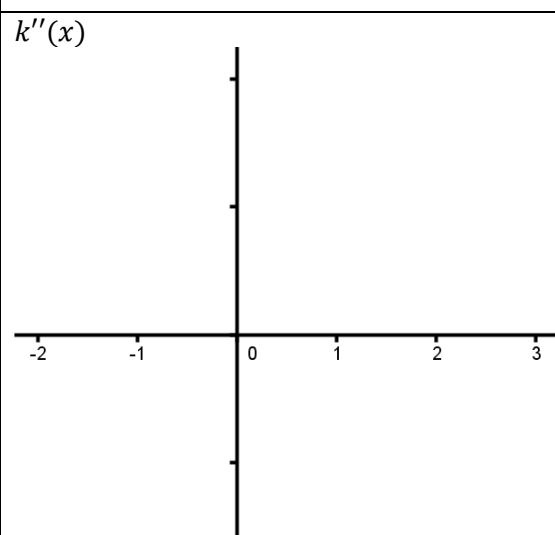
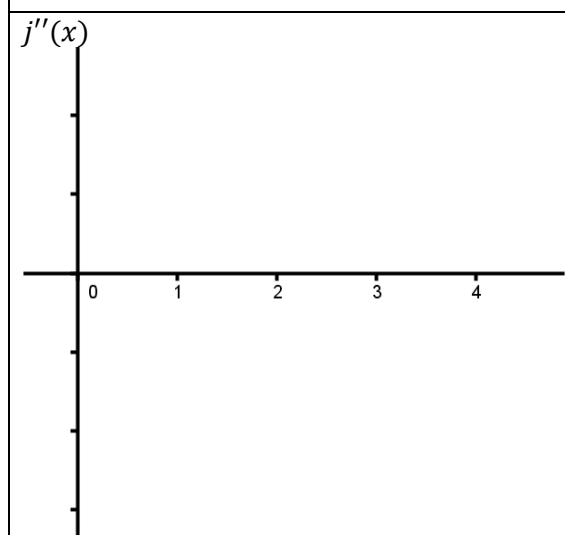
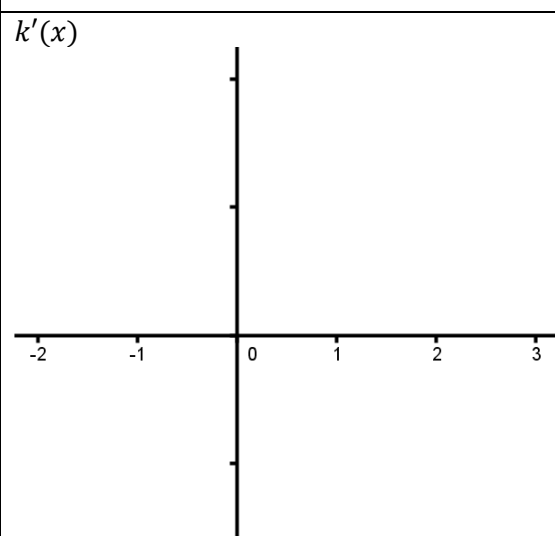
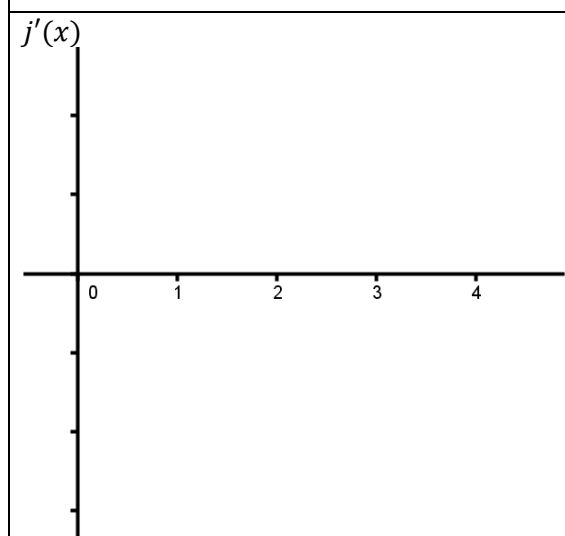
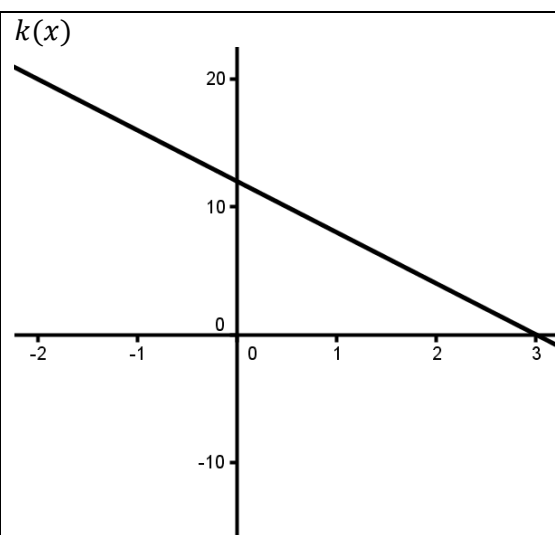
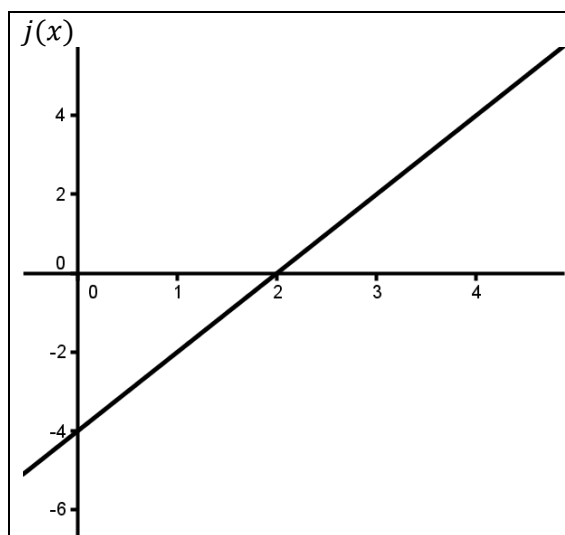
Sketch the graphs of the first and second derivatives of each function below:



Sketch the graphs of the first and second derivatives of each function below:



Sketch the graphs of the first and second derivatives of each function below:



WS08.05 GeoGebra Session

Activity A – Functions and Algebra

Task 1 – Graphing a Function

1. Open a *GeoGebra* Window.
2. Make sure to have the Graphics View, the Algebra View and the Input View displayed.
3. Click on the Input Bar with your mouse and then type $f(x) = x^2$. Click Enter. A quadratic function should appear in the Graphics View.

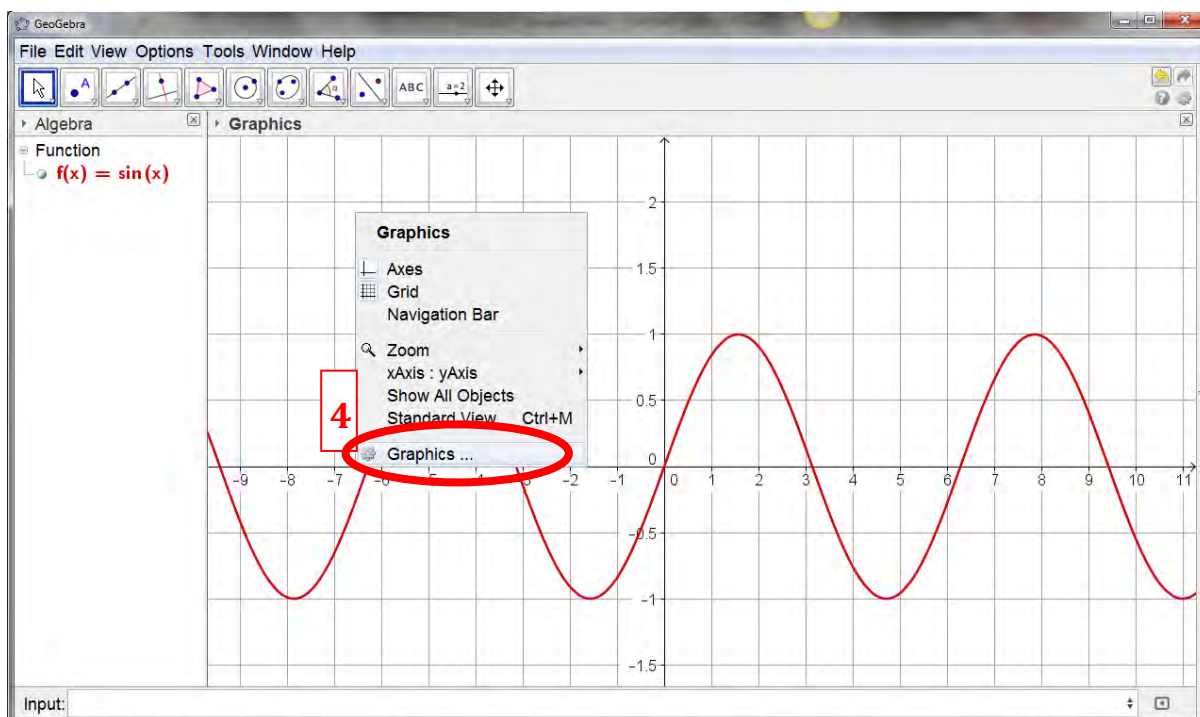
Task 2 – Graphing Multiple Functions


Often it is valuable to be able to observe the relationship between two or more functions at the same time (for example when solving simultaneous equations). It is easy to graph several functions at the same time in *GeoGebra*.

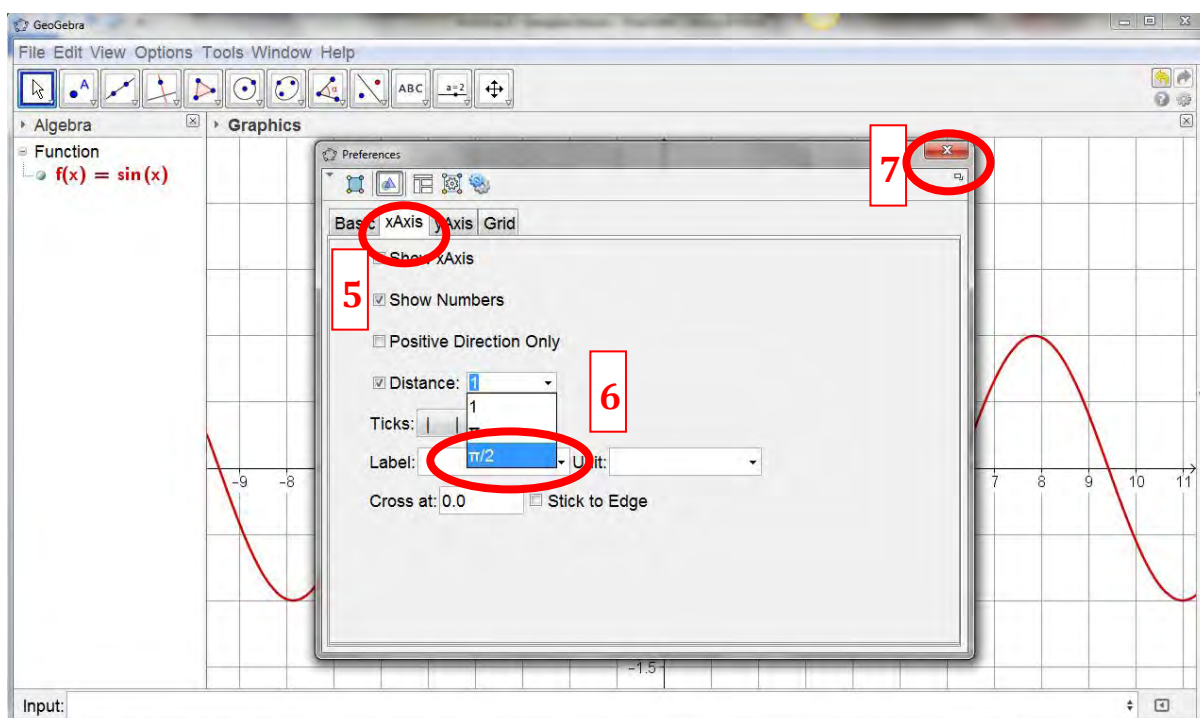
1. In the Input Bar enter a new function e.g. $f(x) = x + 3$.
2. In the Input Bar enter a second function e.g. $g(x) = x^2$. Note that when creating the second function you must give it a different name ($g(x)$ here). If you use $f(x)$ again it will overwrite the previous function. If you want to create a third function you must give it its own unique name and so on.
3. You can graph practically any function using *GeoGebra*. The table below shows you different types of function and the command used to graph them. Try graphing a few of them on the same axes.

Function	Example	<i>GeoGebra</i> Input Bar Command
Linear	$f(x) = 3x + 2$	$f(x) = 3x + 2$
Quadratic	$f(x) = 4x^2$	$f(x) = 4 * x^2$
Cubic	$f(x) = 2x^3$	$f(x) = 2 * x^3$
Exponential	$f(x) = 2^x$	$f(x) = 2^x$
Logarithmic	$f(x) = \log_2(x)$	$f(x) = \log(2, x)$
Trigonometric	$f(x) = \sin\left(x + \frac{\pi}{2}\right)$	$f(x) = \sin(x + \pi/2)$
Modulus	$f(x) = x + 3 $	$f(x) = \text{abs}(x + 3)$

4. If you are graphing trigonometric functions – it is usually a good idea to change the x -axes so that it is written in terms of radians. To do so, right-click on the *Graphics View*. Click on *Graphics* at the bottom of the window which appears (see next page).



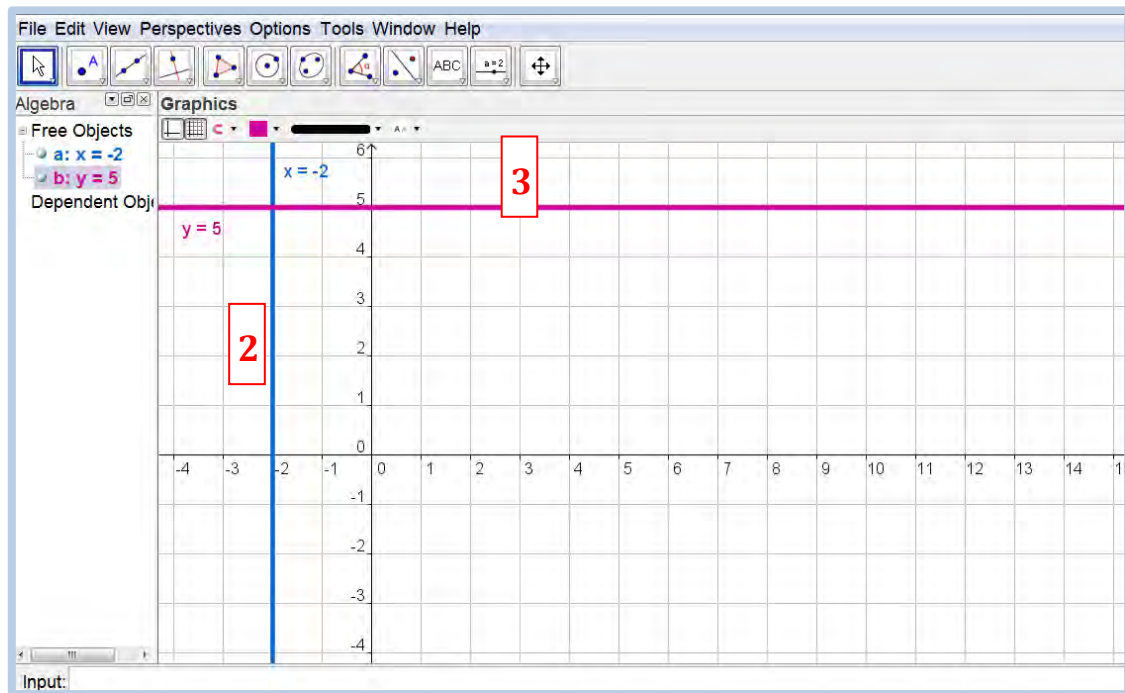
5. Click on the xAxis tab.
6. Click on the drop-down menu beside *Distance*. Select $\pi/2$. This will change the x-axis to radians.
7. Click on  to exit the *Preferences* window.



Task 3 – Graphing Vertical and Horizontal Lines

When solving many problems it is often useful to graph a specific vertical or horizontal line. This is easily done.

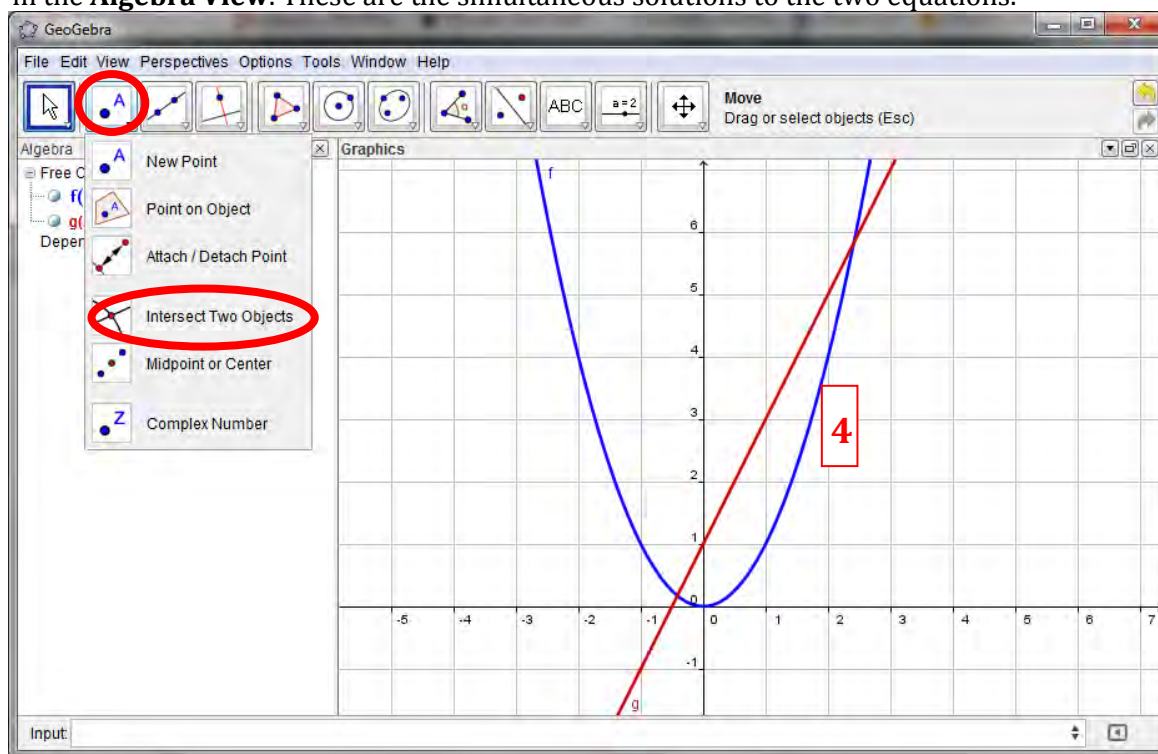
1. Open a new *GeoGebra* window.
2. In the **Input Bar** enter $x = -2$. This will create a vertical line, parallel to the y – axis passing through the point $(-2, 0)$.
3. In the **Input Bar** enter $y = 5$. This will create a horizontal line, parallel to the x – axis passing through the point $(0, 5)$.



Task 4 – Solving Simultaneous Equations Graphically

A graphical approach to solving two equations simultaneously is often useful – particularly if the algebraic solution is difficult. While a graph will often only provide an approximate solution it is useful for students to have access to this. *GeoGebra* can be used to demonstrate solving two equations simultaneously.

1. Re-write the following equations, such that y is a function of x (Note *GeoGebra* will graph these equations when input in the form below, however if students want to graph these functions by hand to solve simultaneous equations, they will have to write the equation in function form).
 $x^2 - y = 0$
 $2x - y + 1 = 0$
2. Open new *GeoGebra* window. Enter both functions (remember to name them differently) in the **Input Bar**. It may help to assign a different colour the each function.
3. The x -coordinate of the points of intersection of the graphs of the functions represents the solutions to the simultaneous equations.. These points may be estimated by eye – however we can get an exact solution using *GeoGebra*. On the **Toolbar** click on the **New Point** button and from the drop-down menu select the **Intersect Two Objects** button.
4. Click anywhere on function 1 and then anywhere on function 2 (in the **Graphics View**) and *GeoGebra* will find the points of intersection. The coordinates of the intersection points will appear in the **Algebra View**. These are the simultaneous solutions to the two equations.



Task 5 – Solving Modulus Inequalities

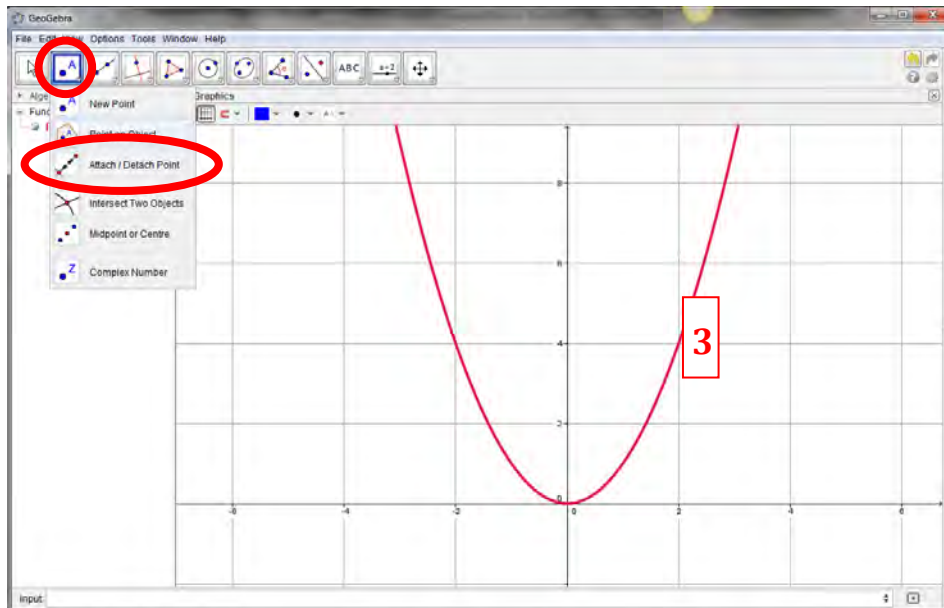
Using *GeoGebra* as an aid, solve the modulus inequality $|x - 2| < 3$. (Hint: Use a graphical approach)

Activity B – Functions and Calculus

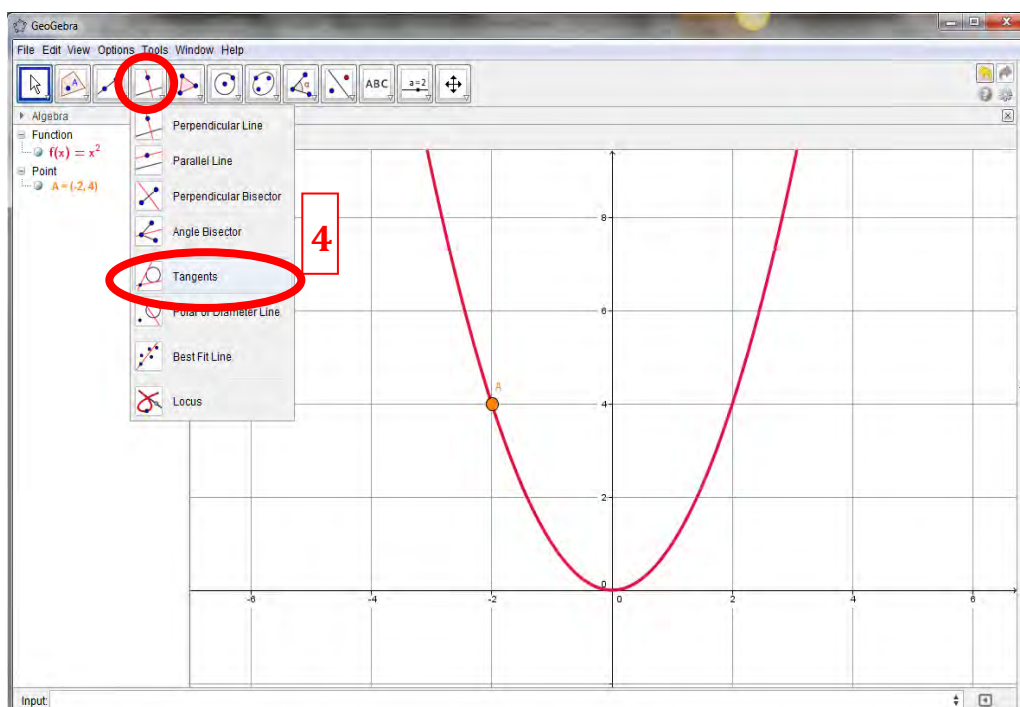
Task 1 – Investigating the Slope of Tangents to the Curve

One of the key concepts in calculus is that the instantaneous rate of change of a function is given by the slope of the tangent at that point and that this slope can change as you move along the function. We can show this with *GeoGebra*.

1. Open a new window in *GeoGebra*. Make sure the **Algebra View**, **Graphics View1** and **Input Bar** are visible.
2. Create a function $f(x) = x^2$ by typing $f(x) = x^2$ in the **Input Bar**.
3. Select the **Point on Object** button from the **Toolbar** and click anywhere on the function to create a point on it (called *A*). You may move point *A* around by dragging it but it will stay on the function at all times.

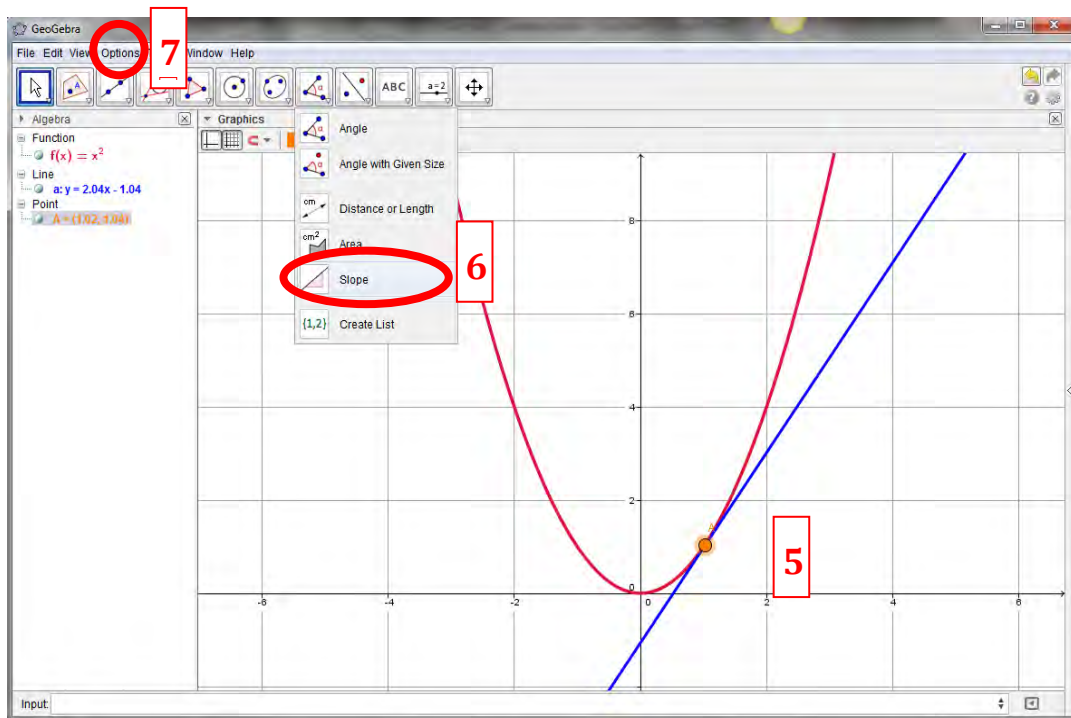


4. Select the **Tangents** button from the **Toolbar** and click on point *A* then click on the function. This will create a tangent to the function through the point *A*.

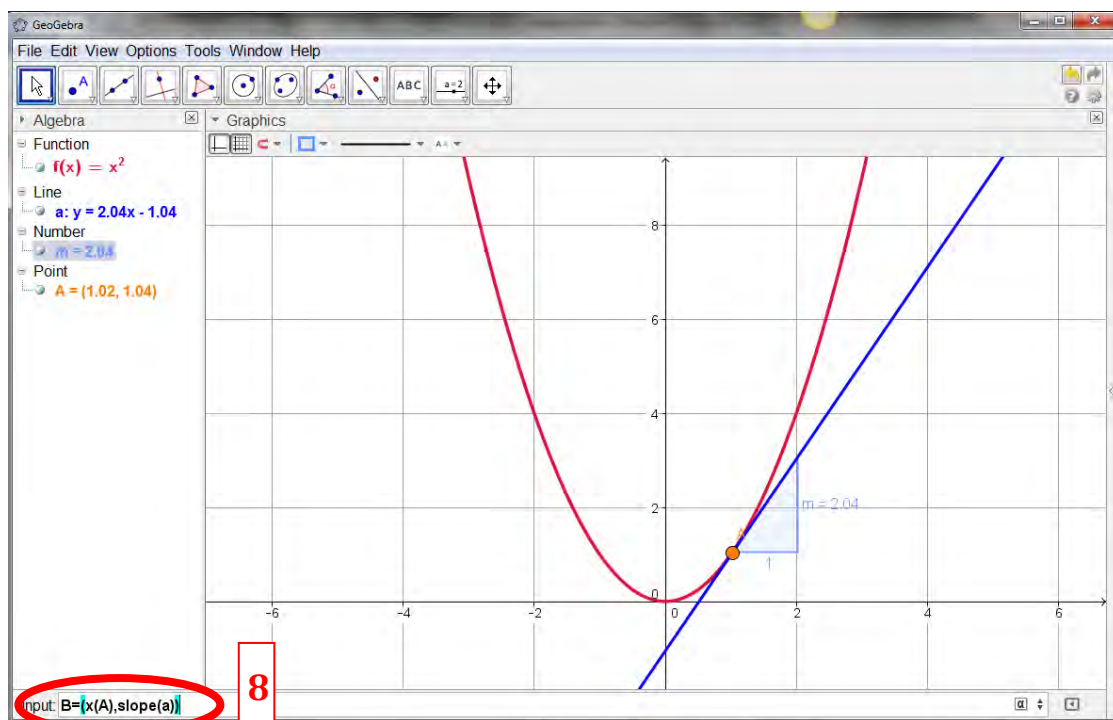


5. You can inspect the tangent to the curve at any point, simply by dragging point *A* around the curve.

6. We would like to investigate how the slope of the curve (derivative) changes from point to point. To do so, select **Slope** from the **Toolbar** and click on the tangent. This will create a small rise/run triangle where the rise is the slope. As you drag the point A around the function you can see how the slope changes.
7. If you only want to look at the Slope at fixed values of x (e.g. $-2, -1, 0, 1, 2$ etc.) click on **Options** in the **Menu Bar**, then **Point Capturing** and **Fixed to Grid**. Now as you drag the point A around the function it will only land on specific values.

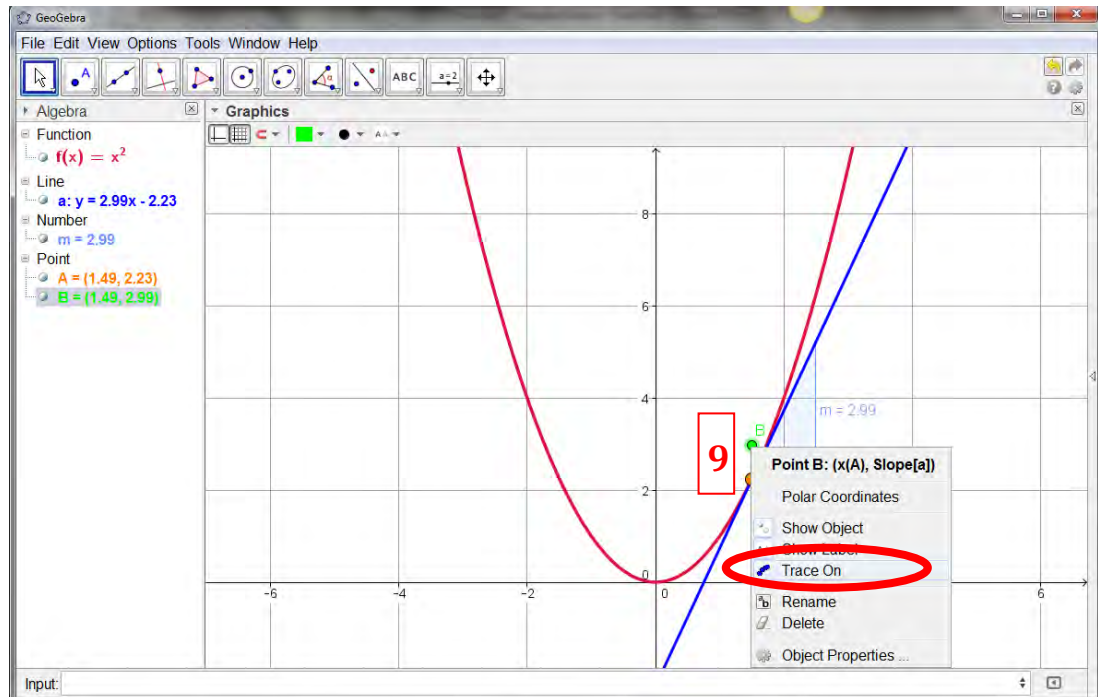


8. We can demonstrate the pattern of slopes or slope function of $f(x)$ in a few short steps. We need to create a new point B which has the same x -coordinate as point A but a y -coordinate equal to the slope of the tangent to the curve at point A . To do so, in the Input bar, type $B = (x(A), \text{slope}(a))$ and click ENTER. A new point B will appear in the Graphics View.



9. As you move point A around the curve, point B will also move. Point B shows the value of the slope at each point along the curve. If you want to see the shape of this pattern, RIGHT-CLICK on point B

and choose *Trace On*. Now as you move point *A* around the curve, point *B* will trace out slope, thereby demonstrating the pattern the slopes make (i.e. the derivative).



Task 2 – What is a Turning Point?

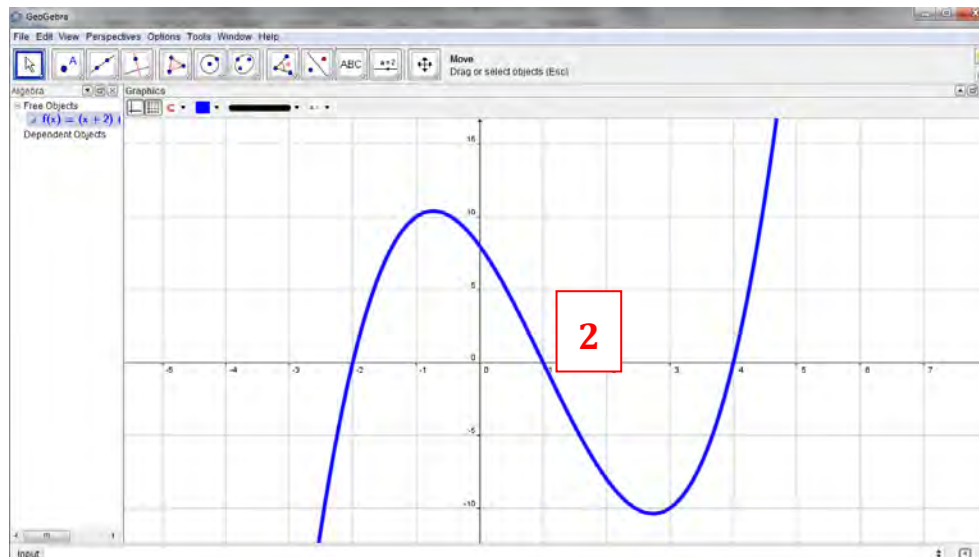
Peter says “A point on a function is a turning point provided the slope of the tangent at that point is zero”. Using *GeoGebra*, investigate this statement with some simple functions e.g. $f(x) = x^2$ and $f(x) = x^3$. Is Peter correct? Explain.

1. Open a new *GeoGebra* window.
2. Graph the function $f(x) = x^2$.
3. Create a point *A* on the curve and attach a tangent to the curve at that point.
4. Calculate the slope of the tangent.
5. Move the point *A* so that it is located exactly on the origin $(0, 0)$. What do you notice about the slope of the tangent? Does this result agree with Peter’s statement? Explain.
6. Double-click on the function in the *Algebra View* and change it so that it is now $f(x) = x^3$.
7. Move the point *A* so that it is located exactly on the origin $(0, 0)$. Is this a turning point? What do you notice about the slope of the tangent? Does this agree with Peter’s statement? Explain.

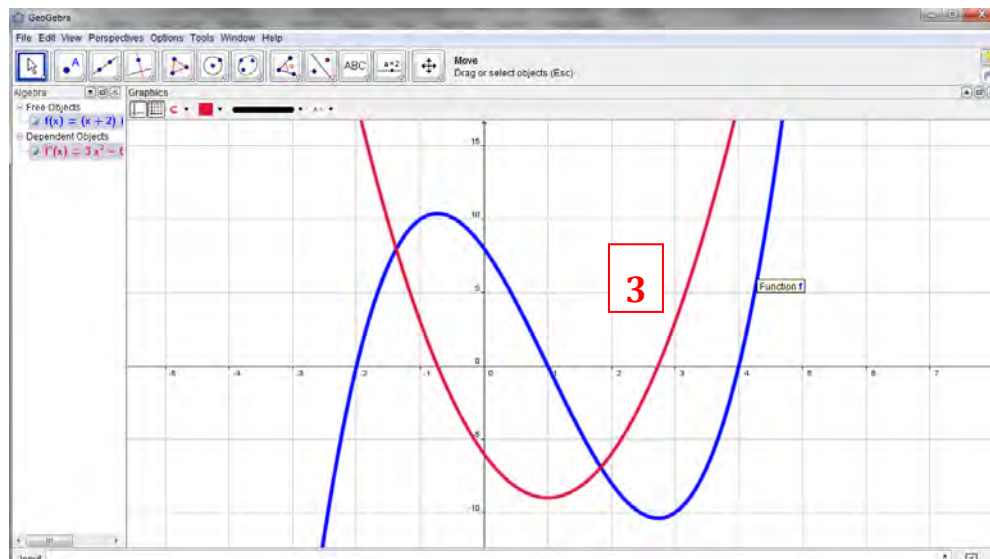
Activity C – Functions and Calculus

Leaving certificate students are expected to be able to sketch the derivatives of various functions – using their understanding of how the slope of the original function changes. They are also expected to be able to identify the derivatives of different functions. The derivatives of a function can be graphed easily using *GeoGebra*.

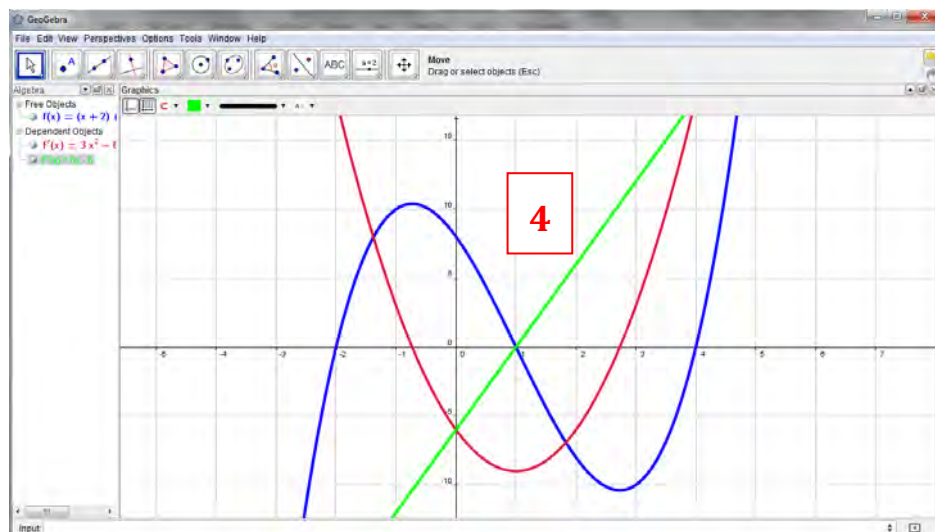
1. Open a new *GeoGebra* window.
2. Graph the cubic function $f(x) = (x + 2)(x - 1)(x - 4)$. Rescale and reposition the axes to show all the important features of $f(x)$.



3. In the **Input Bar** – type `Derivative[f]`. This will graph $f'(x)$ – the derivative of $f(x)$ – on top of the original function. It should be clear that the derivative is quadratic.



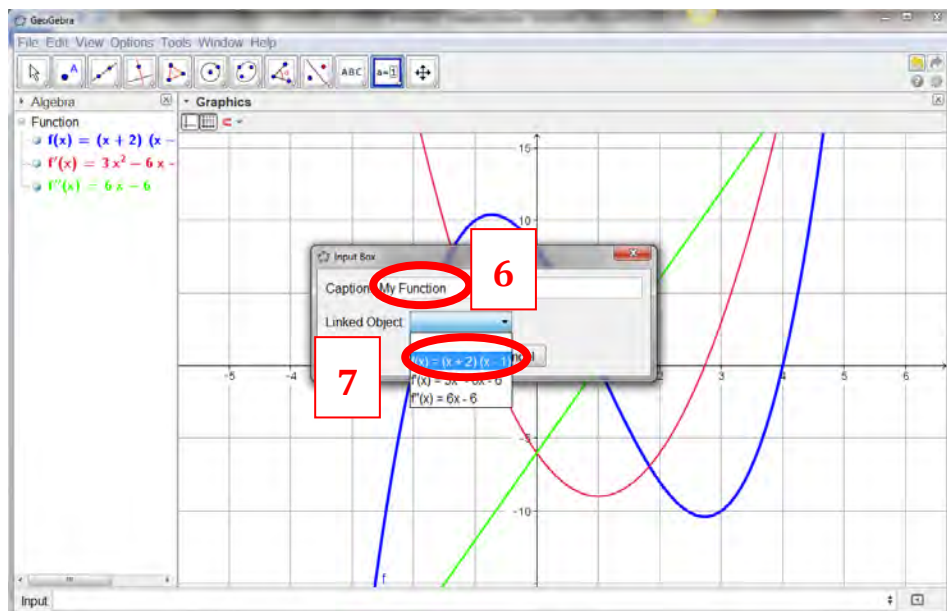
4. In the **Input Bar** – type $\text{Derivative}[f]$. This will graph $f''(x)$ – the second derivative of $f(x)$ – on top of the original function and the first derivative. It should be clear that the second derivative is linear.



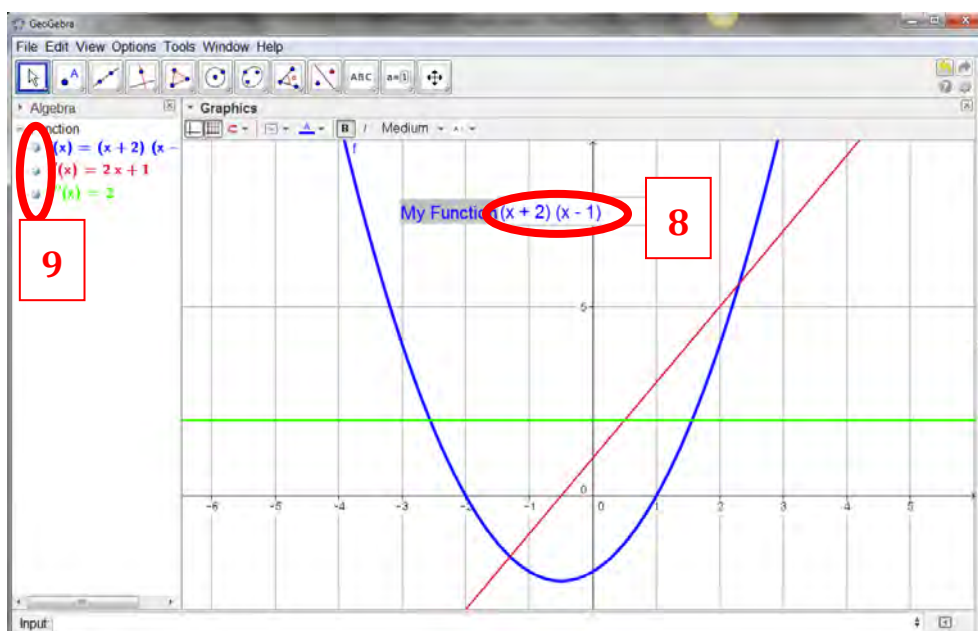
5. You could add an input box to this file which would allow you to change the original function. To create an input box click on the *Insert Input Box* in the toolbar.



6. Click anywhere in the *Graphics View* to create the input box. A new window will appear. In the box entitled *Caption* type in a name for your input box e.g. My Function.
7. In the drop-down menu beside *Linked Object* select the first entry from the list – $f(x) = (x + 2)(x - 1)(x - 4)$. Click on *Apply* to close this window and create the input box.



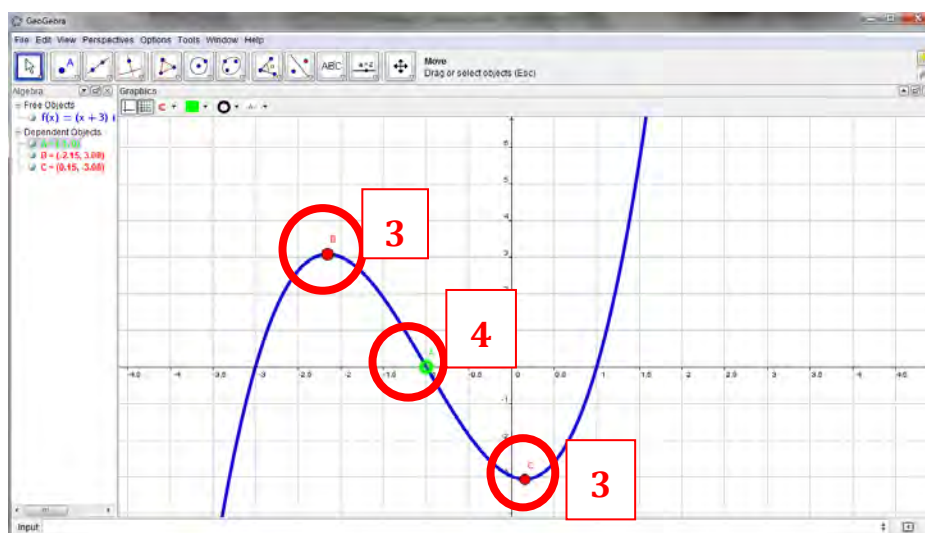
8. Click anywhere in the input box and edit the entry to modify your function. When you do so – *GeoGebra* will automatically re-draw the first and second derivatives.
9. You can use the show/hide button in **Algebra View** to show one of the three graphs or all three graphs at the same time. This could form the basis of a simple revision activity for your students where you show them a function and ask them to sketch the first and second derivatives. You then show the derivative functions to provide solutions



Task 2 – Locating Turning Points and Points of Inflection

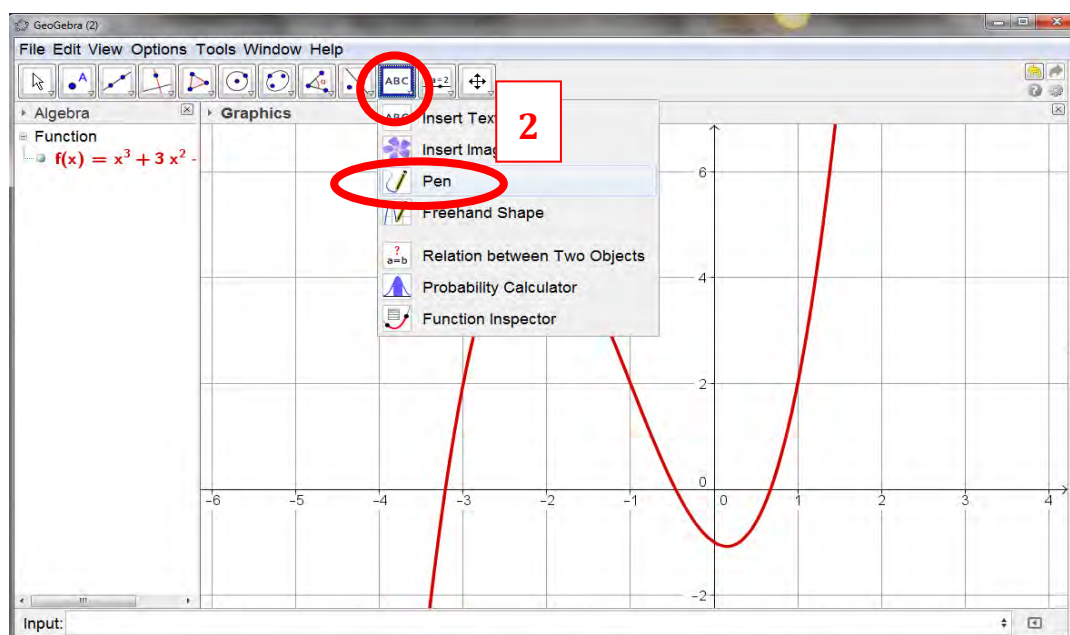
Turning points and points of inflection of a function may be identified by looking at the first and second derivative of a function. If you need to identify these points quickly, *GeoGebra* has built-in commands for doing so.

1. Open a new *GeoGebra* window.
2. Enter the function $f(x) = (x+3)(x+1)(x-1)$ in the **Input Bar**.
3. Into the **Input Bar** type `TurningPoint[f]`. The turning points of the function will be displayed in the **Graphics View** and in the **Algebra View**. Note in some versions of *GeoGebra* the `TurningPoint[f]` command is replaced by the `Extremum[f]` command.
4. To locate points of inflection, type `InflectionPoint[f]`. All points of inflection should now be shown in both the **Algebra View** and the **Graphics View**.

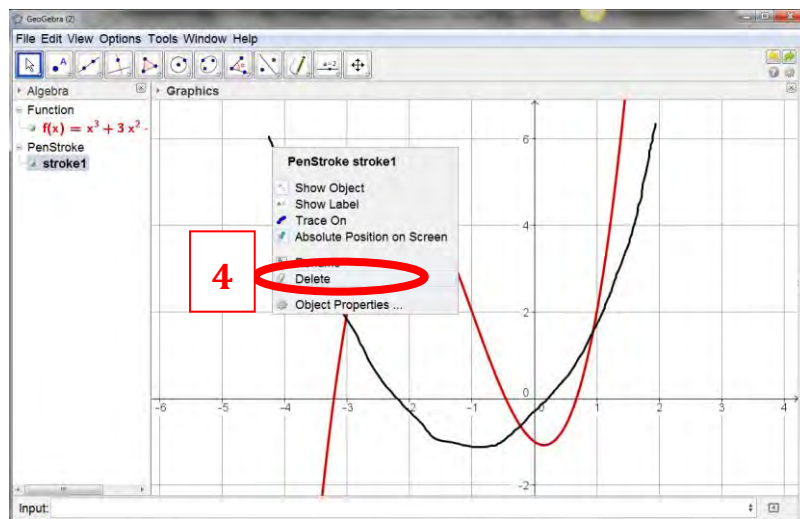


Task 3 – The effect on the Derivative of translating a function parallel to the y-axis (a vertical shift)

1. Create a *GeoGebra* file to draw the cubic function $f(x) = x^3 + 3x^2 - x - 1$.
2. Predict what the derivative of the function will look like and sketch it in *GeoGebra* with the **Pen** tool.



3. Check your prediction by using *GeoGebra* to find the derivative of $f(x)$.
4. Once you have checked your sketch delete the sketch by right-clicking on it in the *Graphics View* (or the *Algebra View*) and choosing *Delete*.



1. Create a **slider** c (to vary between -5 and 5 in steps of 1).
2. Modify your original function $f(x)$ so that it is now $f(x) = x^3 + 3x^2 - x + c$, where c is a **slider**.
3. Predict what will happen to the graph of your function when you change the value of c ? Explain your reasoning. Check your prediction with *GeoGebra* by sliding c .
4. We have already looked at the derivative of $f(x)$. Can you predict what will happen to the derivative of $f(x)$ as the value of c is changed? Explain your reasoning.
5. Check your prediction by graphing the derivative of $f(x)$ in *GeoGebra* and then changing the value of c using the slider. Were you correct in your prediction? Explain the effect that a vertical shift on a function has on its derivative.
6. Write down three different cubic functions which have the same slope function as your original function $f(x)$. Explain your choice.
7. An unknown function $w(x)$ has a slope function $w'(x) = 10x - 1$. Suggest a possible form for $w(x)$. Explain your reasoning. Is this the only possible form for $w(x)$? Could you write down the general form of $w(x)$? Confirm your answers using *GeoGebra*.
8. Use *GeoGebra* to investigate the effect of a horizontal shift on the derivative. To horizontally shift the above function, modify $f(x)$ so that it is $f(x) = (x + c)^3 + 3(x + c)^2 - (x + c) - 3$. Changing c will now produce a horizontal shift. What happens to the derivative if the function is shifted horizontally instead of vertically? Does the shape of the function change? Does the derivative change?
9. The function $A(r) = \cos(r^3 - 2)$ has a slope of -3.64 (rounded to two places) when $r = 2$ (and expressed in radians). Write down any other trigonometric function which has the same slope when $r = 2$. Confirm your answer using *GeoGebra*.

Summary of Useful Commands for Teaching Functions and Calculus

Functions 1

- | | |
|---|--|
| 1. $f(x) = x^2 - 3 * x + 2$ | creates a function $f(x) = x^2 - 3 * x + 2$ |
| 2. <code>Function[f,-1,4]</code> | creates a new function $g(x)$ which is $f(x)$ in the domain $-1 < x < 4$. |
| 3. Un-tick blue dot in the Algebra View beside $f(x)$. | hides $f(x)$ in the Graphics View |
| 4. $y = 4$ | creates a horizontal line a . |
| 5. <code>Intersect[g, a]</code> | displays the point of intersection of g and a . |






Functions 2

- | | |
|-----------------------------|--|
| 1. $f(x) = x^2 - 3 * x + 2$ | creates a function $f(x) = x^2 - 3 * x + 2$ |
| 2. <code>Slowplot[f]</code> | Hides $f(x)$ and creates a new function $g(x)$ and a slider which controls how much of $g(x)$ is shown on the screen. You may pause the animation by right-clicking the slider and unticking Animation On . You may then control the slider manually. |

Calculus

- | | |
|--|--|
| 1. $f(x) = (x - 2)(x - 1)(x + 2)$ | creates a function $f(x)$ |
| 2. <code>root[f]</code> | displays the roots of $f(x)$. |
| 3. <code>inflectionpoint[f]</code> | displays the inflection point of $f(x)$. |
| 4. <code>derivative[f]</code> or $f'(x)$ | displays the derivative of $f(x)$. |
| 6. <code>turningpoint[f]</code> | displays the turning points of $f(x)$. |
| 7. <code>derivative[f']</code> or $f''(x)$ | displays the second derivative of $f(x)$. |

Useful Commands

- | | | |
|---|--------------------|--|
|  | Check Box | used to show/hide objects in the graphics view |
|  | Slider | used to investigate the effect of variables |
|  | Move Graphics View | used to look at different parts of the graph |
|  | Move Graphics View | used to change the scale of the axes |
|  | Function Inspector | used to investigate lots of properties of functions (e.g. change, change of change, first derivative etc.) |

Notes:

Notes: