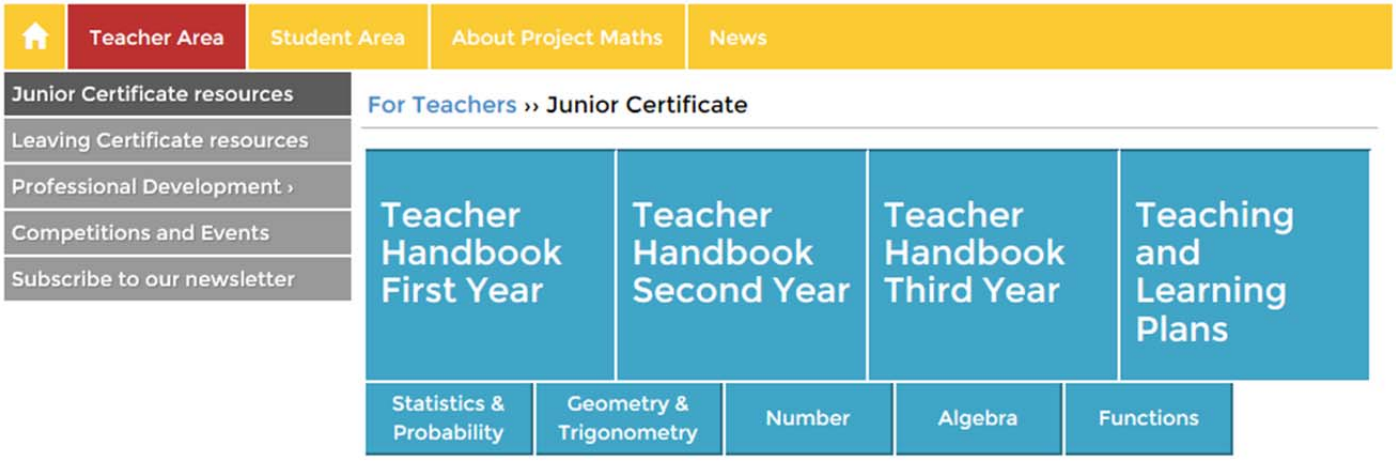

TEACHER HANDBOOK
First Year
DRAFT
Based on the 2016 Syllabus
Project
Maths
Tionscadal Mata
Development Team

Project Maths Tionscadal Mata Development Team

Contents

The strand structure of the syllabus should not be taken to imply that topics are to be studied in isolation. Where appropriate, connections should be made within and across the strands and with other areas of learning. (NCCA JC syllabus page 10 and LC syllabus page 8)

Resources which will allow teachers plan lessons, easily access specific learning outcomes in the syllabus and relevant support material such as "Teaching & Learning Plans" and suggested activities to support learning and teaching are available on the Project Maths website www.projectmaths.ie



Introduction: Student Learning

While this is a handbook for teachers, it must be emphasised that student learning and the process of mathematical thinking and building understanding are the main focus of this document.

Information and Communications Technologies are used whenever and wherever appropriate to help to support student learning. It is also envisaged that, at all levels, learners will engage with a dynamic geometry software package.

Students with mild general learning disabilities

Teachers are reminded that the NCCA Guidelines on mathematics for students with mild general learning disabilities can be accessed at

http://www.ncca.ie/uploadedfiles/PP_Maths.pdf

This document includes

- Exemplars (from page 7)
- Approaches and Methodologies

Note on the Common Introductory Course

This first year handbook contains a suggested sequence for teaching First Year students. It includes the material in the Common Introductory Course (see JC syllabus) which is a minimum course for first year students and also some other sections of the Junior Certificate syllabus which teachers may wish to deal with in first year. The lesson ideas which include the **Common Introductory Course** are marked "**CIC**".

Note: Synthesis and problem solving listed below must be incorporated into all of the Strands.
The list of skills below is taken from Strand 1of the syllabus but, an identical list is given at the end of each Strand in the syllabus.

All Strands **Synthesis and problem-solving skills**	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Useful websites

	http://www.projectmaths.ie/ http://ncca.ie/en/Curriculum_and_Assessment/Post-Primary_Education/Project_Maths/ http://www.examinations.ie/

Bridging Framework

http://action.ncca.ie/resource/Bridging-Documents/47

The adoption of a strands structure in Junior Certificate Mathematics continues the pathways which different topics of mathematics follow as the learner progresses from primary school. To facilitate a smooth transition between mathematics in the primary school and in junior cycle a Bridging Framework has been developed. This contains three elements, a *Common Introductory Course*, a *bridging content document* and a *bridging glossary*.

Literacy and Numeracy Strategy

The National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020

Numeracy encompasses the ability to use mathematical understanding and skills to solve problems and meet the demands of day-to-day living in complex social settings. To have this ability, a young person needs to be able to think and communicate quantitatively, to make sense of data, to have a spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems.

Literacy includes the capacity to read, understand and critically appreciate various forms of communication including spoken language, printed text, broadcast media, and digital media.

Colour coding used in the suggested sequence below:

Strand 1 **Statistics and Probability**	**Strand 2** **Geometry and Trigonometry**	**Strand 3** **Number**	**Strand 4** **Algebra**	**Strand 5** **Functions**

Suggested Sequence of topics-

Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
Section 1	3.5	1.1	Sets	9	
	3.1	1.2	Number system N	9	
	3.1	1.3	Number system Z	10	
	3.1	1.4	Number system Q	11	
	3.1	1.5	Ratio and Proportion	12	
Section 2	1.4	1.6	Introduction to Statistics	13	
	1.5	1.7	Formulating the question, collecting and organising data	13	
	1.6	1.8	Representing data graphically and numerically	14	
	1.1	1.9	Fundamental principal of counting	14	
	1.2	1.10	Introduction to probability	14	
	1.2	1.11	Probability and relative frequency	15	
	1.2	1.11a (**optional for 1st year**)	Fair trials with two dice or outcome of two coin tosses	15	
Section 3	2.1	1.12	Introduction to geometry (planes and points)	18	
	2.1	1.13	Introduction to angles	18	
	2.1	1.14	Measuring angles, introduction to the protractor	19	
	2.1	1.15	Vertically opposite angles(Theorem 1)	19	
	2.1	1.16	Use of the compass	20	
	2.1	1.17	Constructions 8 &	20	

Section number	Strand (Syllabus section)	Corresponding Lesson Number	Title of lesson idea	Page number	
			9		
	2.1	1.18	Constructions 5 & 6	21	
Section 4	2.4	1.19	Introduction to axial symmetry	22	
	2.4	1.20	Introduction to central symmetry and rotations	22	
Section 5	2.2	1.21	Introduction to co-ordinate geometry	23	
Section 6	3.4	1.22	Applied measure	24	
Section 7	4.1,4.2,4.3	1.23	Introduction to patterns	25	
	4.6	1.24	Algebraic Expressions	26	
	4.7	1.25	Simple Linear Equations	26	
Section 8	2.1	1.26	Constructions 10 & 11 Congruent triangles 1	27	
	2.1	1.27	Congruent triangles 2	27	
	2.1	1.28	Theorem 2	28	
	2.1	1.29	Alternate angles Theorems 3 & 4	28	
	2.1	1.30	Corresponding angles and Theorems 5 & 6	29	
	2.1	1.31	Constructions 1 & 2	30	
	2.1	1.32	Constructions 3 & 4	31	

Lesson Ideas

Section 1: Number

Lesson Idea 1.1 (CIC)

Title
Sets

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The concept of a set as being a collection of well-defined objects or elements
- How to list elements of a finite set
- How to describe the rule that defines a set
- The idea that equality of sets is a relationship in which two equal sets have the same elements
- How to use the cardinal number terminology when referring to set membership
- Venn diagrams, universal set, null set, sub-set, set builder notation
- How to perform the operations of union and intersection on two sets
- The commutative property for intersection and union
- How to solve problems involving sets

Lesson Idea 1.2

Title
Number system **N**

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Place value (CIC)

- Use the number line to order numbers in **N**
- Models such as decomposition, skip counting, arranging items in arrays and accumulating groups of equal size to make sense of the operations of addition, subtraction, multiplication and division in **N** where the answer is in **N**, including the inverse operations and the relationships between these operations for **N** (CIC)
- The order of operations including the use of brackets (CIC)
- How to generalise and articulate observations of arithmetic operations (CIC)
- The concept of inverse operations (CIC)
- Commutative, associative and distributive laws (CIC)
- How to consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value(CIC)
- How to analyse solution strategies to problems (CIC)
- Begin to look at the idea of mathematical proof (CIC)
- Indices (exponents ϵ **N**, square roots)
- Rules for multiplication and division of powers of the same number
- Factors, multiples, prime numbers and prime factors (CIC)
- How to check a result by considering whether it is of the right order of magnitude.
- How to check a result by working a problem backwards
- Rounding, approximating, estimating and justifying these approximations and estimates(CIC)
- How to present numerical answers to the degree of accuracy specified (CIC)

Lesson Idea 1.3 (CIC)

Title
Number system **Z**

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Use the number line to order numbers in **Z**

- The operations of addition, subtraction, multiplication and division for **Z** using models such as the number line
- Consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value

Lesson Idea 1.4(CIC)

Title
Number system **Q**

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- Use the number line to order numbers in
- Models to help think about the operations of addition, subtraction, multiplication and division and the relationships between these operations for
- How to generalise and articulate observations of arithmetic operations
- How to consolidate the idea that equality is a relationship in which two mathematical expressions hold the same value
- How to analyse solution strategies to problems
- Begin to look at the idea of mathematical proof
- Decimals
- Percentages
- The equivalence of fractions, decimals and percentages to compare proportions
- How to check a result by considering whether it is of the right order of magnitude.
- How to check a result by working a problem backwards
- How to justify approximations and estimates of calculations
- How to present numerical answers to the degree of accuracy specified

Lesson Idea 1.5

Title
Ratio and proportion

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The relationship between ratio and proportion (a proportion as a statement of equivalent ratios)
- Problems involving rates and finding unit rate (unitary method)

Section 2: Statistics and Probability

Lesson Idea 1.6

Title
Introduction to Statistics

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The purpose of statistics
- The use of statistics to gather information from a sample of the population with the intention of making generalisations about the whole population
- Misconceptions and misuses of statistics

Lesson Idea 1.7

Title
Formulating the question, collecting and categorising data

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to formulate one (or more) questions that can be answered with data
- The different ways of collecting data, e.g. *CensusAtSchool* questionnaire(See Appendix C) (CIC)
- How to plan an investigation involving statistics and conduct the investigation using the data handling cycle(CIC):
    - Pose one (or more) questions that can be answered with data
    - Collect data
    - Analyse the data (summarise the data in diagrammatic form)
    - Interpret the results
    - Refine the question if necessary

- **How to select a sample (Simple Random Sample)**
- **The importance of representativeness so as to avoid biased samples**
- The different types of data: categorical or numerical

Lesson Idea 1.8

Title
Representing data graphically and numerically

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to use bar charts(CIC), line plots(CIC), stem and leaf plots to display data (CIC)
- How to select appropriate graphical methods to describe the sample (univariate data only) (CIC)
- How to select appropriate numerical methods to describe the sample (univariate data only) CIC:
    - Measures of central tendency: mean, mode, median
    - Range as a measure of spread

Lesson Idea 1.9

Title
Fundamental principle of counting

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to list all possible outcomes of an experiment
- How to apply the fundamental principle of counting

Lesson Idea 1.10

Title
Introduction to probability

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- How to distinguish certain from uncertain events
- How to describe events as being more or less likely from experience
- How to order events from least likely to most likely and be able to justify their choice
- How to use the scale from 0 to 1 to informally place everyday chance-related events
- How to represent and interpret probabilities as fractions, decimals and percentages
- How to represent the probability of an event as a fraction or decimal between 0 and 1 or as a percentage
- How to list all possible outcomes for practical experiments such as rolling one die

Lesson Idea 1.11

Title
Probability and relative frequency

Resources
Online resources on the Project Maths website
Dynamic software package

Content
These lessons will involve the students in investigating and understanding:

- The concepts of a fair die and equally likely outcomes
- How to list all the possible outcomes when rolling a fair die
- That the outcomes on successive throws of a die are independent of each other
- How to calculate the relative frequency for each outcome by experiment and note how it approaches the theoretical probability as the number of trials increases i.e. probability as long term relative frequency
- The principle that, in the case of equally likely outcomes, the probability is given by the number of outcomes of interest divided by the total number of outcomes
- The following terminology: trial, outcome, set of all possible outcomes, relative frequency, event, theoretical probability, equally likely outcomes
- How to estimate probabilities from experimental data

Lesson Idea 1.11a (Optional for first year)

Title
Fair trials with two dice or two coins

Resources
Online resources on the Project Maths website
Dynamic software package

Content

These lessons will involve the students in investigating and understanding:

- How to list all the possible outcomes for throwing two dice using a two way table
- How to relate the number of outcomes to the fundamental principle of counting
- What the *concept* of *fairness* means in a game with two dice
- How to come up with rules for a game which make it fair/unfair
- How to construct a probability table
- The relationship between an event and its complement
- How to determine the probability of an event using the results of an experiment and use this to predict the result of a repetition of the experiment, for equally likely outcomes

Section 3: Synthetic Geometry 1

While proofs are not the issue as regards informal introduction, it is important that students are kept aware that the theorems build logically.

Concepts relevant to this section:
Set, plane, point, line, ray, angle, real number, length, degree. Triangle, right-angle, congruent triangles, parallel lines, area, line segment, collinear points, distance, reflex angle, ordinary angle, straight angle, null angle, full angle, supplementary angles, vertically-opposite angles, acute angle, obtuse angle, angle bisector, perpendicular lines, perpendicular bisector of a line segment, isosceles triangle, equilateral triangle, scalene triangle, right-angled triangle, exterior angles of a triangle, interior opposite angles, alternate angles, corresponding angles, transversal line, circle.

Refer to the syllabus for the **"Geometry Course for Post - primary School Mathematics"** which sets out the agreed course in geometry for both Junior Certificate Mathematics and Leaving Certificate Mathematics. Strand 2 of the relevant syllabus document specifies the learning outcomes at the different syllabus levels. Details of instruments which may be used for constructions are outlined in
"Section 7: Constructions to Study"

Refer to Appendix B for the "Guide to Axioms, Theorems and Constructions for all Levels". In Appendix B, * indicates that proof of the relevant theorem is required for JCHL and LCHL and ** indicates that proof of the relevant theorem is required for LCHL only.

Information and Communications Technologies are used whenever and wherever appropriate to help to support student learning. (Dynamic software package (for example GeoGebra))

As outlined at the workshops, the use of manipulative products such as "geostrips", "anglegs", geo-boards etc. can make the learning so much more enjoyable for students of all perceived abilities.

Note on experimentation and experimental results:
With experimentation, when we measure, the results are only approximations and won't agree exactly. It is important for students to report faithfully what they find e.g. for a triangle they could find the sum of the of the angles to be 179^0 or 181.5^0 etc. The conclusion is that the angles appear to add up to 180^0 .....this is a plausible working assumption. There is a distinction between what you can discover and what you can prove.

See Section 8.2 (From Discovery to Proof) of *Geometry for Post - primary School Mathematics.*

Lesson Idea 1.12 (CIC)

Title
Introduction to geometry

"Discussing and verbalising concepts are important aspects of the phases of learning. Students clarify and reorganise their thoughts through talking about them". (Van Hiele theory)

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Plane, points, lines, line segments, rays, collinear points, length of a line segment
- **Axiom 1:** There is exactly one line through any two given points.
- **Axiom 2:** [Ruler Axiom]: The properties of the distance between points

Suggested class activities
For students with mild general learning disabilities see the activities on pages 128 – 134 of the NCCA *Guidelines for Teachers of Students with Mild General Learning Disabilities* at http://www.ncca.ie/uploadedfiles/PP_Maths.pdf

Lesson Idea 1.13 (CIC)

Title
Introduction to angles

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Angle as a rotation, angles in different orientations
- How to estimate angles in degrees, naming angles
- Terms: Perpendicular, parallel, vertical, horizontal

- **Axiom 3:** Protractor Axiom (The properties of the degree measure of an angle).

Suggested class activities
For students with mild general learning disabilities see the activities on pages 128 – 134 of the NCCA *Guidelines for Teachers of Students with Mild General Learning Disabilities* at http://www.ncca.ie/uploadedfiles/PP_Maths.pdf

Lesson Idea 1.14(CIC)

Title
Measuring angles, introduction to the protractor

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Measurement of angles using a protractor
- Possible misconceptions: Students thinking that size of an angle varies with arm or arc-length; failure to recognise equal angles in different orientations
  Common error: Reading from the incorrect scale on the protractor
- The addition of angles
  (**Axiom 3**: see ***Geometry Course for Post-Primary School Mathematics*** – see syllabus p.10)

Lesson Idea 1.15(CIC)

Title
Vertically opposite angles

"Theorems are full of potential for surprise and delight. Every theorem can be taught by considering the unexpected matter which theorems claim to be true. Rather than simply telling students what the theorem claims, it would be helpful if we assumed we didn't know it… it is the mathematics teacher's responsibility to recover the surprise embedded in the theorem and convey it to the pupils. The method is simple: just imagine you do not know the fact. This is where the teacher meets the students".

Resources
Online resources on the Project Maths website
Dynamic software package

A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- A straight angle has 180°
- Supplementary angles
- Vertically opposite angles
- Theorem 1: Vertically opposite angles are equal in measure. (Students convince themselves through investigation that the theorem appears to be true.)

Lesson Idea 1.16(CIC)

Title
Use of the Compass

Resources

Online resources on the Project Maths website

Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- The use of the compass to draw circles, arcs and various shapes

Lesson Idea 1.17

Title
Constructions 8 and 9

Resources

Online resources on the Project Maths website

Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Construction 8: Line segment of a given length on a given ray (CIC)

- Construction 9: Angle of a given number of degrees with a given ray as one arm

Lesson Idea 1.18(CIC)

Title
Constructions 5 and 6

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Construction 5: Line parallel to a given line, through a given point
- **Axiom 5:** Given any line *l* and a point P, there is exactly one line through P that is parallel to *l*.
- Construction 6: Division of a line segment into two or three equal segments without measuring it

Section 4: Transformation Geometry

Lesson Idea 1.19(CIC)

Title
Introduction to axial symmetry, axis of symmetry

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Axial symmetry
- Axis of symmetry

Lesson Idea 1.20(CIC)

Title
Introduction to central symmetry, centre of symmetry, rotation, centre of rotation, angle of rotation, direction of rotation

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Central symmetry
- Centre of symmetry
- Rotation of points: centre of rotation, angle of rotation, direction of rotation

Section 5: Co-ordinate Geometry

Lesson Idea 1.21(CIC)

Title
Introduction to coordinate geometry

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- How to coordinate the plane
- How to locate points on the plane using coordinates

Section 6: Applied measure

Lesson Idea 1.22

Title
Applied measure

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Units of measure, time, mass in context
- Problems involving average speed, distance and time
- Problems involving perimeter and area of the following plane figures: rectangle, square and figures made from combinations of these
- The nets of rectangular solids
- The surface area of rectangular solids

Section 7: Patterns and Algebra

Lesson Idea 1.23(CIC)

Title
Introduction to patterns

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- The use of tables, graphs, diagrams and manipulatives to represent and analyse patterns (e.g. using unifix cubes) and introduce concepts of variables and constants
- A relationship as that which involves a set of inputs, a set of outputs and a correspondence from each input to each output (e.g. "money box" problems as in the teaching and learning plan), identifying variables and constants
- Relations derived from some kind of context – familiar, everyday situations, imaginary contexts or arrangements of tiles or blocks.
- The use of tables, diagrams and graphs as tools for analysing relations
- How to use patterns to make predictions about what comes next
- How to develop and use their own generalising strategies and ideas and consider those of others
- How to present and interpret solutions, explaining and justifying methods, inferences and reasoning
- How to generalise and explain patterns and relationships in words and numbers
- How to write arithmetic expressions for particular terms in a sequence, linear only
- Change and rate of change linked to slope

Lesson Idea 1.24

Title
Algebraic expressions

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
This lesson will involve the students in investigating and understanding:

- The use of letters to represent quantities that are variable
- Indices in algebra (exponents $\in \mathbb{N}$)
- Terms, coefficients and expressions
- How to add terms
- How to generate algebraic expressions from simple contexts
- How to evaluate expressions
- How to multiply terms and expressions including use of brackets and the distributive law using a model such as the array model

Lesson Idea 1.25

Title
Simple linear equations

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
This lesson will involve the students in investigating and understanding:

- The concept of equality and what is meant by an equation
- Simple problems in context involving the solution of first degree equations in one variable

Section 8: Synthetic geometry 2

Lesson Idea 1.26

Title
Construction of triangles and the meaning of congruent triangles

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Triangles: scalene, isosceles, equilateral, right-angled
- Construction 10: Triangle given SSS – Congruent triangles (Axiom 4)
- Construction 11: Triangle given SAS - Congruent triangles (Axiom 4)

Suggested class activities
Geometry Lesson Idea Ideas 1 and 14 in *Junior Certificate Guidelines for Teachers*
Lesson Idea 1 (pp. 58 - 59) aims to help students to recognise various types of triangles and provide them with concrete experience of dealing with triangles.
Lesson Idea 14 (pp. 72 – 73) introduces students to the idea of congruency with concrete materials.

Lesson Idea 1.27

Title
Congruent triangles (continued)

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Construction 12: Triangle given ASA - Congruent triangles (Axiom 4)
- More constructions of triangles with SSS, SAS and ASA
- By construction, show that AAA and AAS are not sufficient conditions for congruence.

Lesson Idea 1.28(CIC)

Title
Theorem 2

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Theorem 2: (i) In an isosceles triangle the angles opposite the equal sides are equal.
  (ii) Conversely, if two angles are equal, then the triangle is isosceles

Suggested class activities
Students draw their own isosceles triangle (more practice with the compass and ruler) and then measure the angles. Each pupil will have a different triangle; compare results….. cut out and fold…

Use of geostrips: put two identical triangles on top of each other, compare the equal angles and then flip the top one over…

Draw an isosceles triangle containing a 90º angle; discover that the 45^0 setsquare is one of these.

Draw equilateral triangles in a variety of orientations and mark in equal parts.
Real-life: pyramids and architecture; Maslow's pyramid of human needs; the food-pyramid

Lesson Idea 1.29(CIC)

Title
Alternate angles, Theorem 3 and Theorem 4

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Alternate angles by examples and measuring
  (New words: transversal, alternate angles)

- Theorem 3: (i) If a transversal makes equal alternate angles on two lines, then the lines are parallel. (ii) Conversely, if two lines are parallel, then any transversal will make equal alternate angles with them.
- *Theorem 4: The angles in any triangle add to 180°.

Suggested class activities
Teacher draws two parallel lines with a transversal and names the angles using numbers (1- 8). The students draw the diagram into their copy, e.g. by drawing lines along both edges of a ruler and a line cutting across these. The teacher draws two "Z" shapes on the board. Students are asked can they spot the two "Z" shapes in the diagram with the parallel lines and the transversal.
Now the teacher asks students to fill in the numbered angles in each of the "Z" shapes. From then on, students can remember alternate angles as "Z" angles. Of course students must remember the correct terminology also.

Students investigate by drawing a number of examples of a transversal on two parallel lines and then on two non - parallel lines. **By measuring** ,they find that the alternate angles are (approximately) equal only when the lines are parallel.

Note: As it is experimentation the word approximately is used. This is not a proof.

The students draw a number of triangles or use the ones from the previous class and measure their angles with the protractor. Include some obtuse-angled triangles.

Tear (don't cut) off the three angles, having first marked them and put them along the edge of a ruler. It will be obvious that they add to 180°.
See Geometry Lesson Idea 4 in "Junior Certificate Guidelines for Teachers" page 62. Available at www.projectmaths.ie

Lesson Idea 1.30(CIC)

Title
Corresponding angles, Theorem 5 and Theorem 6

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Junior Certificate Guidelines for Teachers (DES 2002, Government Publications Sales Office). It is also available to download at http://www.projectmaths.ie/learningResources/reference.asp

Content
These lessons will involve the students in investigating and understanding:

- Corresponding angles explained by examples and measuring
  (New words: corresponding angles)
- Theorem 5: Two lines are parallel if and only if for any transversal, corresponding angles are equal.
- *Theorem 6: Each exterior angle of a triangle is equal to the sum of the interior opposite angles.

Suggested class activities
Corresponding angles: Teacher draws two parallel lines with a transversal and names the angles using numbers (1-8). The students draw the diagram into their copy, e.g. by drawing lines along both edges of a ruler and a line cutting across these. The teacher draws out four "F" shapes (the four pairs of corresponding angles).Students are asked can they spot four "F" shapes in the diagram with the parallel lines and the transversal.

Now the teacher asks students to fill in the numbered angles in each of the "F" shapes. From then on, students can remember corresponding angles as "F" angles. Of course students must remember the correct terminology also.

Remind students again of the alternate angles from the previous lesson.

Students measure all the angles (protractor practice) in the F shapes and see what they notice. Compare results across the class.

Have the students draw (obvious) non-parallel lines with a transversal, and check the same angles.

Use plenty of student activities in this lesson; different orientations can be really challenging for some pupils.

For Theorem 6: Don't tell students the theorem first. Have the students draw several cases and see if they come up with the theorem.

Very important to have triangles in various orientations

See Geometry Lesson Idea 5 in "Junior Certificate Guidelines for Teachers" Page 63.
Available at www.projectmaths.ie.

Lesson Idea 1.31(CIC)

Title
Constructions 1 and 2

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Construction 1: Use of a compass and straight edge only to bisect an angle
- Construction 2: Use of a compass and straight edge only to draw the perpendicular bisector of a line segment

Lesson Idea 1.32(CIC)

Title
Constructions 3 and 4

Resources
Online resources on the Project Maths website
Dynamic software package
A mathematical instruments set

Content
These lessons will involve the students in investigating and understanding:

- Construction 3: Line perpendicular to a given line l, passing through a given point not on l
  (2 methods: ruler and compass or ruler and set-square)
- Construction 4: Line perpendicular to a given line l, passing through a given point on l (CIC)
  (2 methods: ruler and compass or ruler and set-square)

Appendix A

Geometry: Thinking at Different Levels

The Van Hiele Theory

The **Van Hiele model** describes how students learn geometry. Pierre van Hiele and Dina van Hiele-Geldof, mathematics teachers from the Netherlands, observed their geometry students in the 1950's.The following is a brief summary of the Van Hiele theory. According to this theory, students progress through 5 levels of thinking starting from merely recognising a shape to being able to write a formal proof. The levels are as follows:

***Visualisation (Level 0)**

The objects of thought are shapes and what they look like.

Students have an overall impression of a shape. The appearance of a shape is what is important. They may think that a rotated square is a "diamond" and not a square because it is different from their visual image of a square. They will be able to distinguish shapes like triangles, squares, rectangles etc. but will not be able to explain, for example, what makes a rectangle a rectangle. **Vocabulary**: Students use visual words like "pointy", "curvy", "corner" as well as correct language like angle, rectangle and parallelogram.

Analysis (Level 1)

The objects of thought are "classes" of shapes rather than individual shapes.

- Students think about what makes a rectangle a rectangle and can separate the defining characteristics of a rectangle from irrelevant information like size and orientation. They recognize its parts (sides, diagonals and angles) and compare their properties (similar, congruent)
- They understand that if a shape belongs to a class like "rectangle", then it has all the properties of that class (two pairs of equal sides, right angles, two equal diagonals, two axes of symmetry).
- **Vocabulary:** words like parallel, perpendicular and congruent relating to properties within a figure and the words all, always, sometimes, never, alike, different.
- A concise definition of a figure, using a sufficient rather than an exhaustive list of properties is not possible at this level.
- They do not deal with questions like "Is a square a parallelogram?" but just look at the properties of each class of shape, without comparing the classes.

*Some visualisation and analysis is covered in Primary School.

Relational/ Ordering/Informal Deduction (Level 2)

The objects of thought are the properties of shapes.

- Students are ready to understand interrelationships of properties within figures and between figures. Opposite sides of a parallelogram are parallel and opposite angles are equal.
- A rectangle is a parallelogram since it has all the properties of a parallelogram as well as having all 90^0 angles.
- Students can recognise the difference between a statement and its converse. All squares are rectangles (true) is different to the statement all rectangles are squares (not true).
- Capable of **"if –then"** thinking – if a shape is a rectangle then all the angles in it are right angles. If |<A |= |<B |and |<B| = |<C| then |< A| =|<C|
- They can select one or two properties to define a figure rather than an exhaustive list. If a quadrilateral has 4 equal sides and one right angle it must be a square.
- Students can discover new properties by simple deduction. The two acute angles in a right angled triangle add to 90^0 because all the angles in a triangle add up to 180^0. They can explain logically without having to measure everything.
-

Formal deduction (Level 3)

Students learn how to use an axiomatic system to establish geometric theory. This is the level at which proof of Theorems is learned. The sequence of theorems given in the appendix is arranged in such a manner that each theorem builds on the previous theorem(s).

Rigor (Level 4)

Comparing different axiomatic systems – not done at secondary level

Characteristics of these levels: Students cannot function at any particular level unless they are competent at all previous levels. The teacher's role is crucial in structuring activities to bring students from one level to the next.

How does the teacher bring students from any one level to the next?

5 phases of learning:

1. In an informal discussion of the topic, students are asked to give their initial observations.
2. The teacher provides structured activities such as drawing, making and measuring.
3. The students then verbalise and write down what they have learned and report back in groups to the class, which leads to a class discussion.
4. The teacher then provides an activity which will require students to apply what they have discovered
5. In the last stage students are required to summarise all they have learned and should be able to remember it as they have discovered it through guidance.

A PowerPoint presentation of the Van Hiele theory can be got at www.projectmaths.ie (Workshop 2 Show 2) http://www.projectmaths.ie/workshops/WS2_NR/WS2_NR_PPTS.zip

A mind map of the Van Hiele theory can be found at
http://agutie.homestead.com/files/mindmap/van_hiele_geometry_level.html

Appendix B

Guide to Theorems, Axioms and Constructions at all Levels*

This is intended as a quick guide to the various axioms, theorems and constructions as set out in the ***Geometry Course for Post-Primary School Mathematics (see Syllabus).*** You can get this from the project maths website: www.projectmaths.ie

It is not intended as a replacement for this document, merely as an aid to reading at a glance which material is required to be studied at various levels. The sequence of theorems as given must be followed.

As stated in the heading, these theorems and constructions are underpinned by 46 definitions and 20 propositions which are all set out in the ***Geometry Course for Post-Primary School Mathematics (see Syllabus)***, along with many undefined terms and definable terms used without explicit definition.

*An **axiom** is a statement accepted without proof, as a basis for argument

*A **theorem** is a statement deduced from the axioms by logical argument. Theorems can also be deduced from previously established theorems.

* A **proposition** is a useful or interesting statement that could be proved at this point, but whose proof is not stipulated as an essential part of the programme. Teachers are free to deal with them as they see fit, but they should be mentioned, at least (Appendix p. 20, footnote).

*The instruments that may be used for **constructions** are listed and described in Section 7 (Constructions to Study) of the Appendix and are a straight edge, compass, ruler, protractor and set-square.

Terms: Students at **Junior Certificate Higher level and Leaving Certificate Ordinary level** will be expected to understand the meanings of the following terms related to logic and deductive reasoning:
Theorem, proof, axiom, corollary, converse, implies.

Synthetic Geometry

Guide to Axioms, Theorems and Constructions for all Levels

Interactive files are available in the **Student Area** on the Project Maths website.

	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL only ** proof required for LCHL only ♦ These results are required as background knowledge for constructions and/or applications of trigonometry.	CMN Introd. Course	JC ORD	JC HR	LC FDN	LC ORD	LC HR
	Axiom 1: There is exactly one line through any two given points	✓	✓	✓	✓♦	✓	✓
	Axiom 2: [Ruler Axiom]: The properties of the distance between points.	✓	✓	✓	✓♦	✓	✓
	Axiom 3: Protractor Axiom (The properties of the degree measure of an angle).	✓	✓	✓	✓♦	✓	✓
1	Vertically opposite angles are equal in measure.	✓	✓	✓		✓	✓
	Axiom 4: Congruent triangles conditions (SSS, SAS, ASA)	✓	✓	✓		✓	✓
2	In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.	✓	✓	✓	✓♦	✓	✓
	Axiom 5: Given any line l and a point P, there is exactly one line through P that is parallel to l.	✓	✓	✓	✓♦	✓	✓
3	If a transversal makes equal alternate angles on two lines then the lines are parallel. Conversely**,** if two lines are parallel, then any transversal will make equal alternate angles with them.	✓	✓	✓		✓	✓
4*	The angles in any triangle add to 180^0.	✓	✓	✓	✓♦	✓	✓
5	Two lines are parallel if, and only if, for any transversal, the corresponding angles are equal.	✓	✓	✓		✓	✓
6*	Each exterior angle of a triangle is equal to the sum of the interior opposite angles.	✓	✓	✓		✓	✓
7	The angle opposite the greater of two sides is greater than the angles opposite the lesser. Conversely, the side opposite the greater of two angles is greater than the side opposite the lesser angle.					✓	✓
8	Two sides of a triangle are together greater than the third.					✓	✓
9*	In a parallelogram, opposite sides are equal, and opposite angles are equal. Conversely, (1) if the opposite angles of a convex quadrilateral are equal, then it is a parallelogram; (2) if the opposite sides of a convex quadrilateral are equal, then it is a parallelogram.		✓	✓		✓	✓
	Corollary 1. A diagonal divides a parallelogram into two congruent triangles.			✓			✓
10	The diagonals of a parallelogram bisect each other. Conversely, if the diagonals of a quadrilateral bisect one another, then the quadrilateral is a parallelogram.		✓	✓		✓	✓

	Axioms and Theorems (supported by 46 definitions, 20 propositions) *proof required for JCHL only ** proof required for LCHL only ♦ These results are required as background knowledge for constructions and/or applications of trigonometry.	CMN Introd. Course	JC ORD	JC HR	LC FDN	LC ORD	LC HR
11**	If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.			✓		✓	✓
12**	Let ABC be a triangle. If a line l is parallel to BC and cuts [AB] in the ratio m:n, then it also cuts [AC] in the same ratio. Conversely, if the sides of two triangles are in proportion, then the two triangles are similar.			✓		✓	✓
13**	If two triangles are similar, then their sides are proportional, in order (and converse)		✓	✓		✓	✓
14*	[Theorem of Pythagoras]In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.		✓	✓	✓♦	✓	✓
15	[Converse to Pythagoras]. If the square of one side of a triangle is the sum of the squares of the other two, then the angle opposite the first side is a right angle.		✓	✓		✓	✓
	Proposition 9: (RHS). If two right-angled triangles have hypotenuse and another side equal in length respectively, then they are congruent.		✓	✓		✓	✓
16	For a triangle, base x height does not depend on the choice of base.					✓	✓
	Definition 38: The area of a triangle is half the base by the height.					✓	✓
17	A diagonal of a parallelogram bisects the area.					✓	✓
18	The area of a parallelogram is the base x height.					✓	✓
19*	The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc.			✓			✓
	Corollary 2†: All angles at points of a circle, standing on the same arc are equal (and converse).			✓			✓
	Corollary 3: Each angle in a semi-circle is a right angle.		✓	✓		✓	✓
	Corollary 4: If the angle standing on a chord [BC] at some point of the circle is a right-angle, then [BC] is a diameter.		✓	✓		✓	✓
	Corollary 5: If ABCD is a cyclic quadrilateral, then opposite angles sum to 180^0 (and converse).			✓			✓
20	(i) Each tangent is perpendicular to the radius that goes to the point of contact. (ii) If P lies on the circle S, and a line l is perpendicular to the radius to P, then l is a tangent to S.					✓	✓
	Corollary 6: If two circles intersect at one point only, then the two centres and the point of contact are collinear.					✓	✓
21	(i) The perpendicular from the centre to a chord bisects the chord. (ii) The perpendicular bisector of a chord passes through the centre.					✓	✓

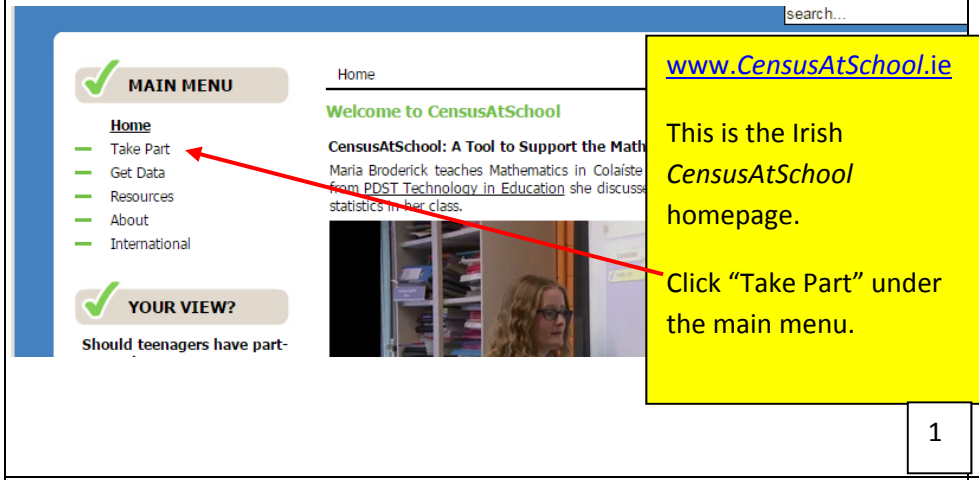
† The corollaries are numbered as in the *Geometry for Post-primary School Mathematics*; corollary 2 is the first one relating to theorem 19

♦ These results are required as background knowledge for constructions and/or applications of trigonometry

	Constructions (Supported by 46 definitions, 20 propositions, 5 axioms and 21 theorems)	CMN Introd. Course	JC ORD	JC HR	LC FN	LC ORD	LC HR
1	Bisector of an angle, using only compass and straight edge.	✓	✓	✓		✓	✓
2	Perpendicular bisector of a segment, using only compass and straight edge.	✓	✓	✓		✓	✓
3	Line perpendicular to a given line l, passing through a given point not on l.			✓			✓
4	Line perpendicular to a given line l, passing through a given point on l.	✓	✓	✓	✓	✓	✓
5	Line parallel to given line, through a given point.	✓	✓	✓	✓	✓	✓
6	Division of a line segment into 2 or 3 equal segments without measuring it.	✓	✓	✓		✓	✓
7	Division of a line segment into any number of equal segments, without measuring it.			✓			✓
8	Line segment of a given length on a given ray.	✓	✓	✓		✓	✓
9	Angle of a given number of degrees with a given ray as one arm.		✓	✓		✓	✓
10	Triangle, given lengths of 3 sides.		✓	✓	✓	✓	✓
11	Triangle, given SAS data.		✓	✓		✓	✓
12	Triangle, given ASA data		✓	✓		✓	✓
13	Right-angled triangle, given length of hypotenuse and one other side		✓	✓	✓	✓	✓
14	Right-angled triangle, given one side and one of the acute angles.		✓	✓		✓	✓
15	Rectangle given side lengths.		✓	✓	✓	✓	✓
16	Circumcentre and circumcircle of a given triangle, using only straight edge and compass.					✓	✓
17	Incentre and incircle of a triangle of a given triangle, using only straight edge and compass.					✓	✓
18	Angle of 60^0 without using a protractor or set square.					✓	✓
19	Tangent to a given circle at a given point on it.					✓	✓
20	Parallelogram, given the length of the sides and the measure of the angles.					✓	✓
21	Centroid of a triangle.					✓	✓
22	Orthocentre of a triangle.						✓

Appendix C

How to register for *CensusAtSchool*, complete the online questionnaire and retrieve class data for analysis and interpretation



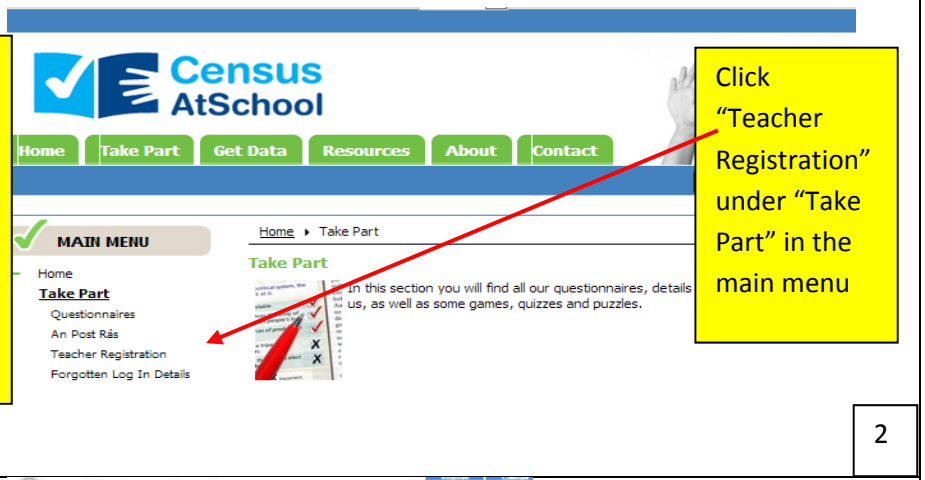
www.*CensusAtSchool*.ie

This is the Irish *CensusAtSchool* homepage.

Click "Take Part" under the main menu.

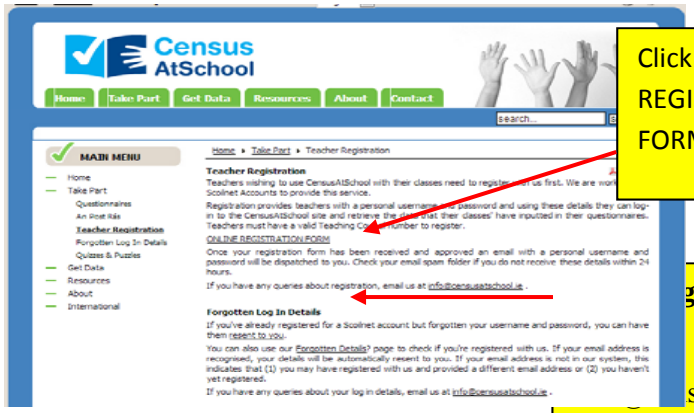
1

Part 1: Registration



Click "Teacher Registration" under "Take Part" in the main menu

2



Click "ONLINE REGISTRATION FORM"

gotten Log In

susatschool.ie

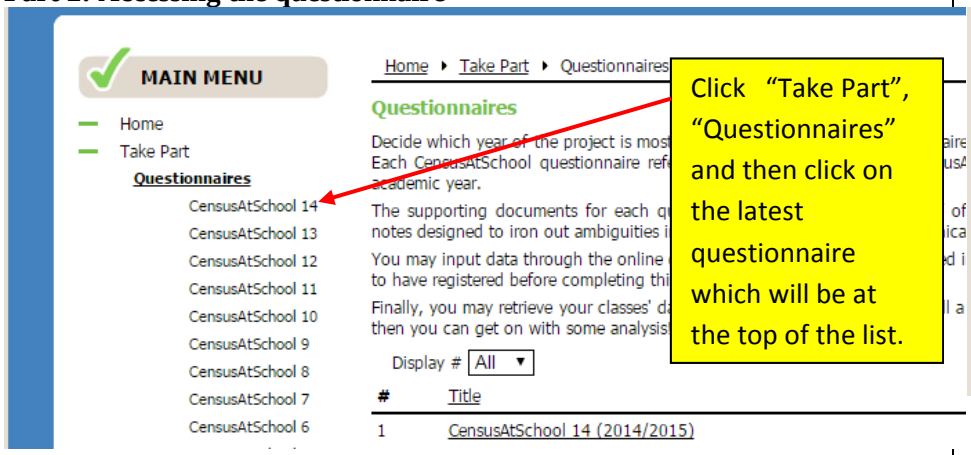
3

Fill out and click on "Submit"

Once your registration form has been received and approved an email with a personal username and password will be dispatched to you. Check your email spam folder if you do not receive these details within 48 hours.

4

Part 2: Accessing the questionnaire



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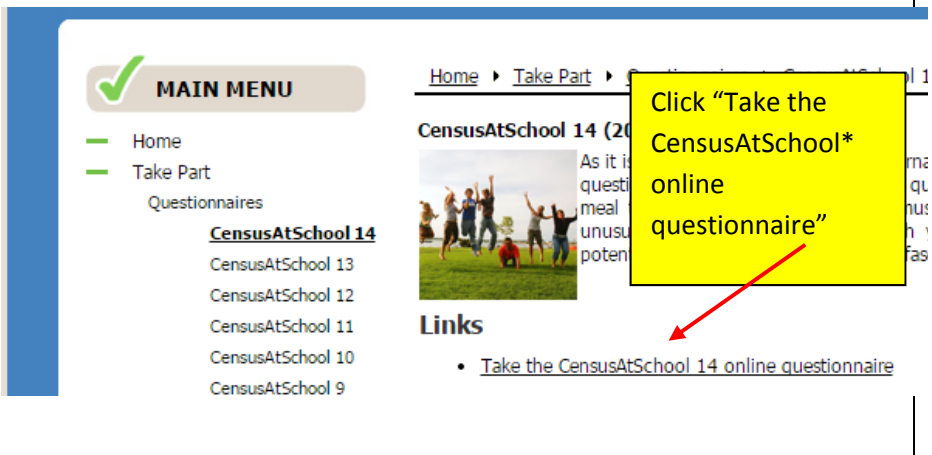
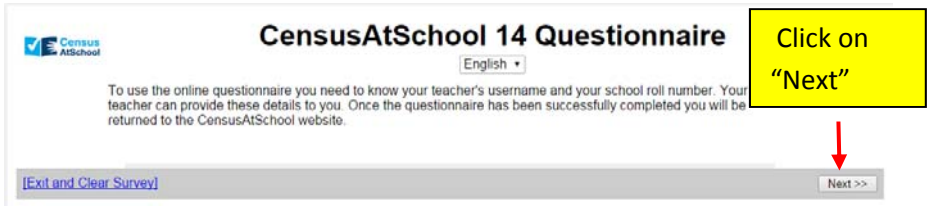


6

Part 3: Filling out the questionnaire



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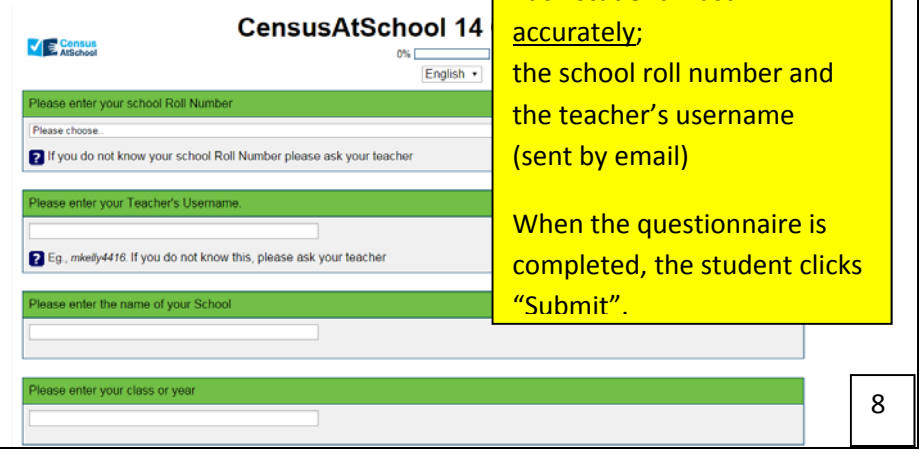
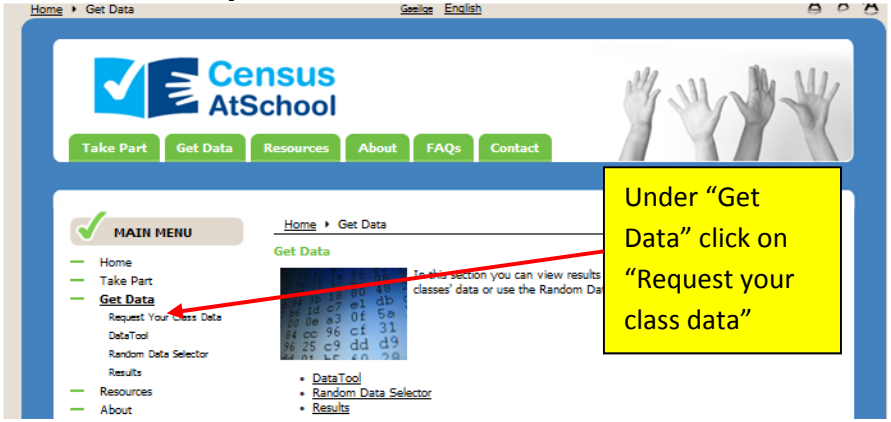


8

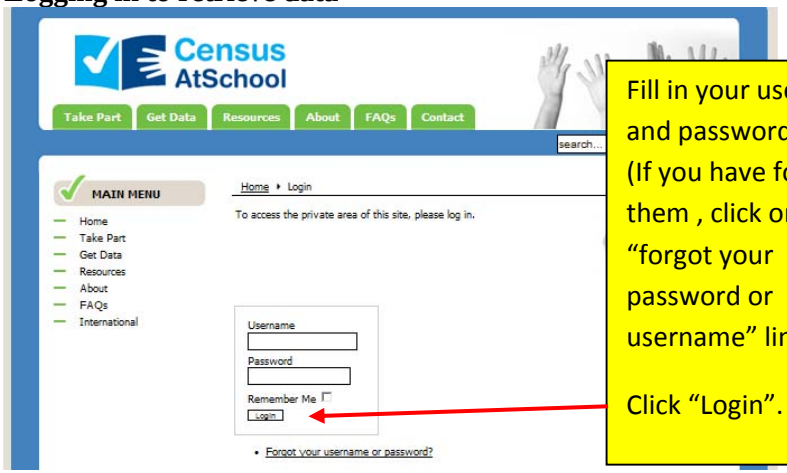
Part 4 Now that my students have filled in the data – what next?



Under "Get Data" click on "Request your class data"

9

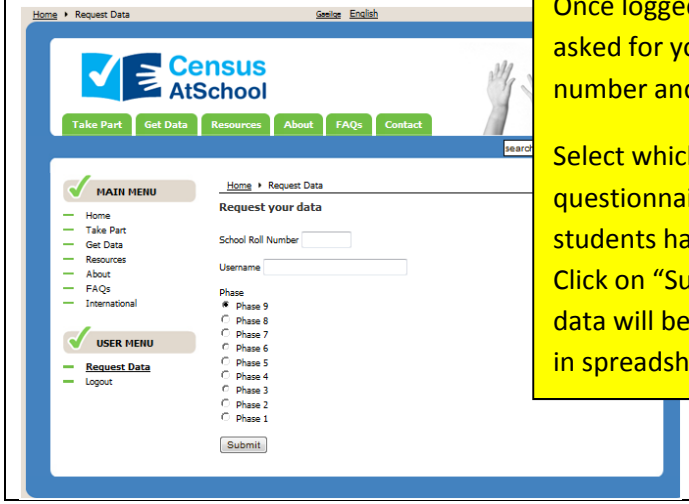
Logging in to retrieve data



Fill in your username and password.
(If you have forgotten them , click on the "forgot your password or username" link.)

Click "Login".

10



Once logged in you will be asked for your school roll number and username.

Select whichever questionnaire (phase) your students have completed. Click on "Submit" and your data will be returned to you in spreadsheet form.

11

Your data will look similar to the spreadsheet below.

Sex	BrthMnth	BrthYear	BrthPlce	BedRms	PeoplHme	InetAxs	NumPts	PtsNone	PtsDog	PtsCat	PtsMouse	PtsRabt
Female	July	1992	New Sout	4	5	Yes - broa	4	No	Yes	Yes	No	No
Female	February	1996	Australian	4	5	Yes - broa	4	No	Yes	Yes	No	No
Male	Septembe	1994	Victoria	4	5	Yes - dial-	5	No	Yes	No	No	No
Female	Septembe	1991	Fiji	4	4	No - Interi	0	Yes	No	No	No	No
Male	August	1993	Queenslai	3	4	No - Interi	0	No	No	No	No	No
Female	Decembei	1993	Tasmania	3	5	Yes - dial-	3	No	No	Yes	No	No
Female	August	1990	Queenslai	4	5	Yes - broa	3	No	Yes	No	No	No
Female	Novembe	1991	New Sout	11	5	Yes - dial-	4	No	Yes	No	No	Yes
Male	June	1995	Victoria	3	5	Yes - othe	6	No	Yes	No	No	No
Female	March	1993	Western A	30	122	No - Interi	2	No	Yes	No	No	No
Female	Decembei	1989	Queenslai	3	3	Yes - broa	2	No	No	No	No	No
Female	April	1993	South Aus	3	5	No - Interi	4	No	Yes	Yes	No	No
Male	March	1996	New Sout	3	2	No - Interi	0	Yes	No	No	No	No
Male	Septembe	1993	Queenslai	4	4	Yes - dial-	18	No	Yes	No	No	No
Male	May	1995	Tasmania	3	4	Yes - broa	2	No	No	No	No	Yes
Female	March	1994	Western A	4	4	Yes - dial-	4	No	Yes	Yes	No	No

12